CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

3.1 INTRODUCTION

In the middle school mathematics, we have learnt about linear equations in one variable and their applications in solving word problems. If a and b are two real numbers such that $x \neq 0$ and x is a variable, then as we have learnt that an equation of the form $ax = b$ or, $ax + b = 0$ is called a linear equation in one variable. Recall that a value of the variable which satisfies a given linear equation in one variable is known as its solution.

In class IX, we have learnt about linear equations in two variables. The general form of a linear equation in two variables is $ax + by + c = 0$ or, $ax + by = c$ where a, b, c are real numbers such that $a \neq 0, b \neq 0$ and x, y are variables (we often denote the condition a and b are not both zero by $a^2 + b^2 \neq 0$). Any pair of values of x and y which satisfies the equation $ax + by + c = 0$ or $ax + by = c$ is called its solution. For example, $x = 2$ and $y = 1$ is a solution of the equation $4x - 3y = 5$. We have also learnt about the graph of a linear equation. The graph of a linear equation in one variable is a straight line parallel to x-axis or y-axis according as the equation is of the form $ay = b$ or $ax = b$, where $a \neq 0$. The graph of a linear equation in two variables is also a straight line. The coordinates of every point on the line representing a linear equation determine a solution of the equation and every solution of linear equation is represented by a point on the line represented by it. Thus, there is one-to-one correspondence between the solutions of a linear equation and points lying on the straight line represented by it.

In this chapter, we shall study about systems of linear equations in two variables, solution of a system of linear equations in two variables and graphical and algebraic methods of solving a system of linear equations in two variables. In the end of the chapter, we shall be discussing some applications of linear equations in two variables in solving simple problems from different areas.

3.2 SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES

In earlier class, we have studies about a linear equation in two variables. In this section, we shall introduce the notion of system of simultaneous linear equations as defined below.

DEFINITION *A pair of linear equations in two variables is said to form a system of simultaneous linear equations.*

Each of the following pairs of linear equations forms a system of two simultaneous linear equations in two variables:

(i) $x + 2y = 3$
$2x - y = 5$

(ii) $2u + 5v + 1 = 0$
$u - 2v + 9 = 0$

(iii) $\frac{3}{x}+\frac{2}{y}=9$

$\frac{1}{x}-\frac{1}{y}=5$

(iv) $2a+b-1=0$

$a+b+5=0.$

The general form of a pair of linear equations in two variables x and y is

$$a_1x+b_1y+c_1=0$$

and

$$a_2x+b_2y+c_2=0,$$

where $a_1, b_1, c_1, a_2, b_2\ c_2$ are all real numbers and $a_1^2+b_1^2 \neq 0,\ a_2^2+b_2^2 \neq 0$. This is known as the algebraic representation of a system of simultaneous linear equations in two variables.

SOLUTION *A pair of values of the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system.*

Clearly, $x=2, y=-1$ is a solution of the system of simultaneous linear equations

$$x+y=1$$
$$2x-3y=7.$$

ILLUSTRATION 1 *Show that $x=2, y=1$ is a solution of the system of simultaneous linear equations*

$$3x-2y=4$$
$$2x+y=5.$$

SOLUTION The given system of equations is

$$3x-2y=4 \quad \text{...(i)}$$
$$2x+y=5 \quad \text{...(ii)}$$

Putting $x=2$ and $y=1$ in equation (i), we have

$$\text{LHS}=3\times 2-2\times 1=4=\text{RHS}$$

Putting $x=2$ and $y=1$ in equation (ii), we have

$$\text{LHS}=2\times 2+1\times 1=5=\text{RHS}$$

Thus, $x=2$ and $y=1$ satisfy both the equations of the given system.

Hence, $x=2, y=1$ is a solution of the given system.

ILLUSTRATION 2 *Show that $x=2, y=1$ is not a solution of the system of simultaneous linear equations*

$$2x+7y=11$$
$$x-3y=5$$

SOLUTION The given system of equations is

$$2x+7y=11 \quad \text{...(i)}$$
$$x-3y=5. \quad \text{...(ii)}$$

Putting $x=2, y=1$ in equation (i), we have

$$\text{LHS}=2\times 2+7\times 1=11=\text{RHS}$$

So, $x=2$ and $y=1$ satisfy equation (i)

Putting $x = 2, y = 1$ in equation (i), we have,

$$\text{LHS} = 2 \times 1 - 3 \times 1 = -1 \neq \text{RHS}$$

So, $x = 2$ and $y = 1$ do not satisfy equation (ii)

Hence, $x = 2, y = 1$ is not a solution of the given system of equations.

ILLUSTRATION 3 *Show that $x = 2, y = 1$ and $x = 4, y = 4$ are solutions of the system of equations*

$$3x - 2y = 4$$
$$6x - 4y = 8.$$

SOLUTION The given system of equations is

$$3x - 2y = 4 \quad \ldots(i)$$
$$6x - 4y = 8 \quad \ldots(ii)$$

Putting $x = 2$ and $y = 1$ in equation (i) and (ii) respectively, we get

$$\text{LHS} = 3 \times 2 - 2 \times 1 = 4 = \text{RHS}$$
$$\text{LHS} = 6 \times 2 - 4 \times 1 = 8 = \text{RHS}$$

So, $x = 2, y = 1$ is a solution of the given system of equations.

Similarly, it can be checked that $x = 4, y = 4$ is also a solution of the given system.

Hence $x = 2, y = 1$ and $x = 4, y = 4$ are solutions of the given system of equations.

In the above discussion, we have seen that a system of linear equations will have either a unique solution or an infinitely many solutions or no solution. If a system of simultaneous linear equations has a solution (either unique or infinitely many), then the system is said to be consistent of other wise it is said to be an in-consistent system as defined below.

CONSISTENT SYSTEM *A system of simultaneous linear equations is said to be consistent, if it has at least one solution.*

IN-CONSISTENT SYSTEM *A system of simultaneous linear equations is said to be in-consistent, if it has no solution.*

Clearly, systems of equations discussed in illustrations 1, 2, and 3 are consistent whereas the system of equations $x - 2y = 1, 2x - 4y = 3$ is in-consistent because there is no pair of values of x and y which satisfies the two equations simultaneously.

3.3 GRAPHICAL REPRESENTATION OF LINEAR EQUATIONS

In the previous section, we have seen what a pair of linear equations in two variables look like algebraically? In class IX, we have learnt that the graphical (i.e. geometric) representation of a linear equation in two variables is a straight line such that every point on the line represents a solution of the equation and every solution of the equation is represented by a point on the line. Let us now see what a pair of linear equations in two variables will look like, graphically? Since a linear equation in two variables represents a straight line. Therefore, a pair of linear equations in two variables will be represented by two straight lines, both to be considered together. We know that given two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines intersect at one point.

(ii) The two lines are parallel i.e. they do not intersect however far they are extended.

(iii) The two lines are coincident lines i.e. one line overlaps the other line.

Thus, the graphical representation of a pair of simultaneous linear equations in two variables will be in one of the following forms.

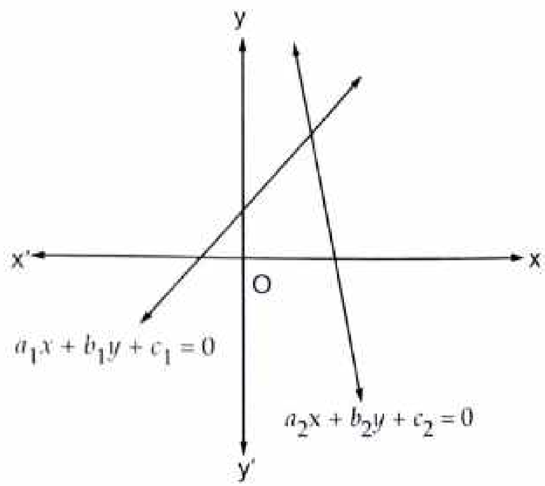

Fig. 3.1 (Intersecting lines)

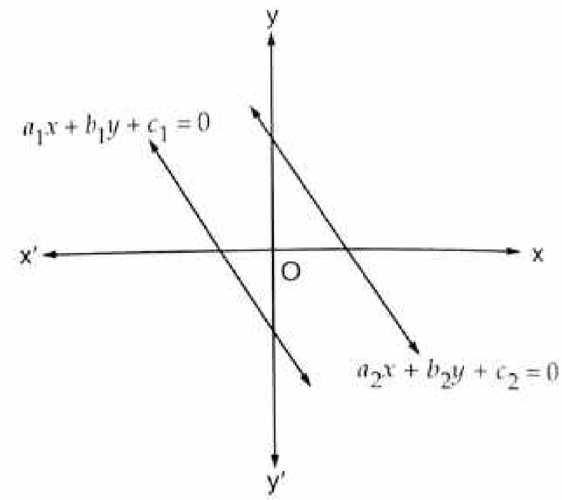

Fig. 3.2 (Parallel lines)

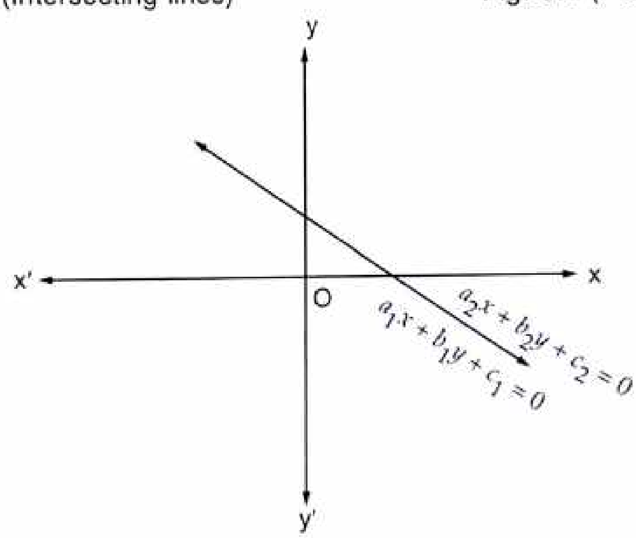

Fig. 3.3 (Coincident lines)

Let us now consider some examples on formulation, algrbraic and graphical representation of a pair of linear equations in two variables.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Ten students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys. Represent this situation algebraically and graphically.*

SOLUTION *Formulation:* Let the number of girls be x and the number of boys be y.

It is given that total ten students took part in the quiz.

$\therefore$ Number of girls + Number of boys = 10

i.e. $x + y = 10$

It is also given that the number of girls is 4 more than the number of boys.

$\therefore$ Number of girls = Number of boys + 4

i.e. $x = y + 4$

or, $x - y = 4$

Algebraic Representation: Thus, the algebraic representation of the given situation is

$$x + y = 10 \qquad \ldots(i)$$

$$x - y = 4 \qquad \ldots(ii)$$

Graphical Representation: In order to represent the above pair of linear equations graphically, we will have to find two points on the line represented by each equation. That is, we will have to find two solutions of each equation. As we have in class IX that there are infinitely many solutions of each linear equation. So, we can choose any two solutions of each equation. We know that it is always convenient to plot points having integral coordinates on the graph paper in comparison to points with fractional coordinates. So, we choose solutions having integral values. For this, we give such an integral value to one of the variables that the value of the other variable is also an integer. The most convenient integer value is zero. So, putting $y = 0$ in $x + y = 10$, we get $x = 10$. Similarly, by putting $x = 0$ in $x + y = 10$, we get $y = 10$.

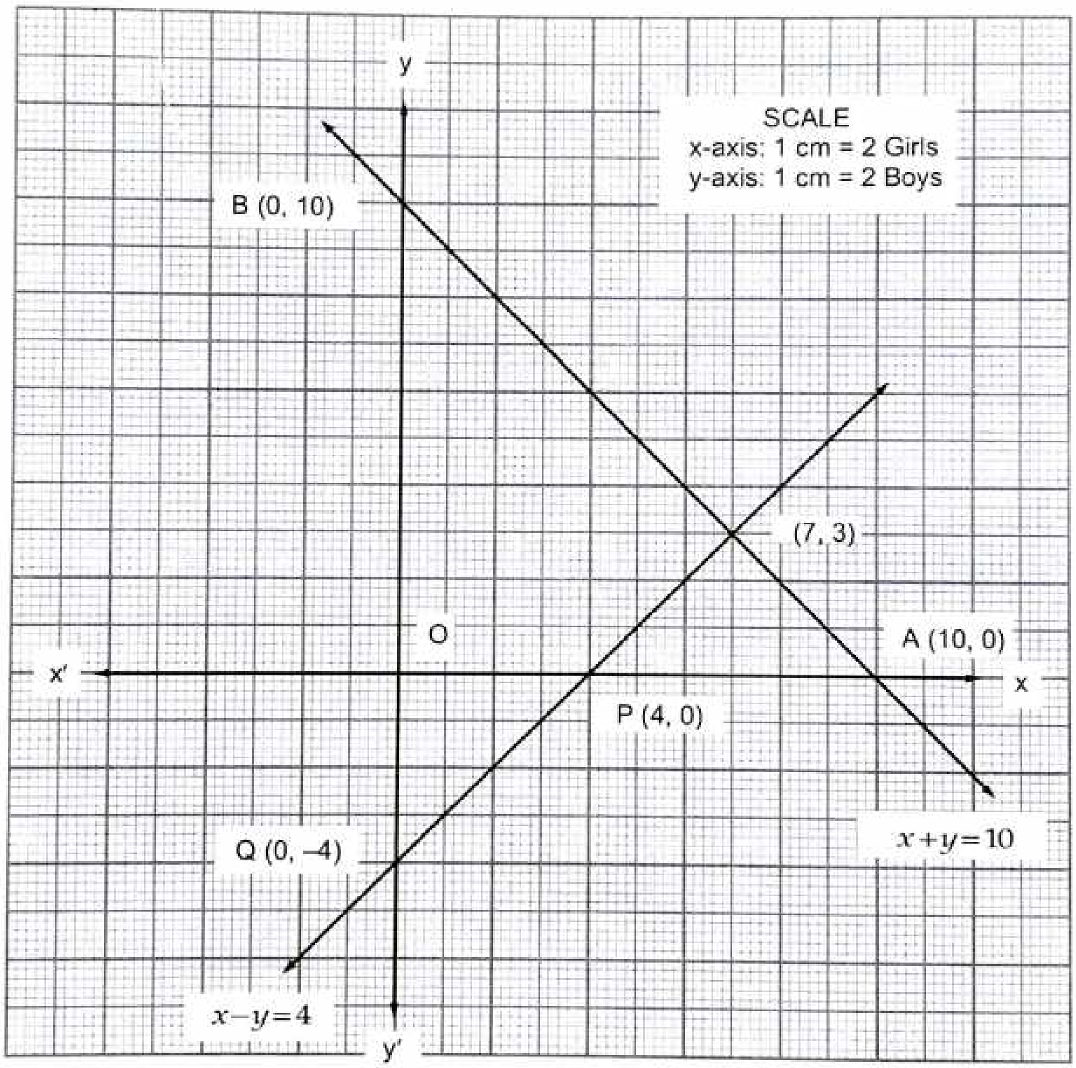

Fig. 3.4

Thus, two solutions of equation (i) are:

x	10	0
y	0	10

Similarly, two solutions of equation (ii) are:

x	4	0
y	0	– 4

Now, we plot the points A (10, 0), B (0, 10), P (4, 0) and Q (0, –4) corresponding to these solutions on the graph paper and draw the lines AB and PQ representing the equations $x + y = 10$ and $x - y = 4$ as shown in Fig. 3.4.

We observe that the two lines representing the two equations are intersecting at the point (7, 3).

EXAMPLE 2 *The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, he buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.* **[NCERT]**

SOLUTION *Formulation:* Let the price of a bat be ₹ x and that of a ball be ₹ y.
It is given that 3 bats and 6 balls are bought for ₹ 3900.

$$\therefore \quad 3x + 6y = 3900$$

It is also given that one bat and 3 balls of the same kind cost ₹ 1300.

$$\therefore \quad x + 3y = 1300$$

Algebraic Representation: The algebraic representation of the given situation is

$$3x + 6y = 3900 \quad \ldots(i)$$

$$x + 3y = 1300 \quad \ldots(ii)$$

Graphical Representation: In order to obtain the equivalent graphical representation, we find two points on the line representing each equation. That is, we find two solutions of each equation.

We have,

$$3x + 6y = 3900$$

When $y = 0$, we have

$$3x + 0 = 3900 \Rightarrow x = \frac{3900}{3} = 1300$$

When $x = 0$, we have

$$0 + 6y = 3900 \Rightarrow y = \frac{3900}{6} = 650$$

Thus, two solutions of equation (i) are:

x	1300	0
y	0	650

We have,

$$x + 3y = 1300$$

When $y = 100$, we have

$$x + 300 = 1300 \Rightarrow x = 1000$$

When $x = 100$, we have

$$100 + 3y = 1300 \Rightarrow 3y = 1200 \Rightarrow y = 400$$

Thus, two solutions of equation (ii) are:

x	1000	100
y	100	400

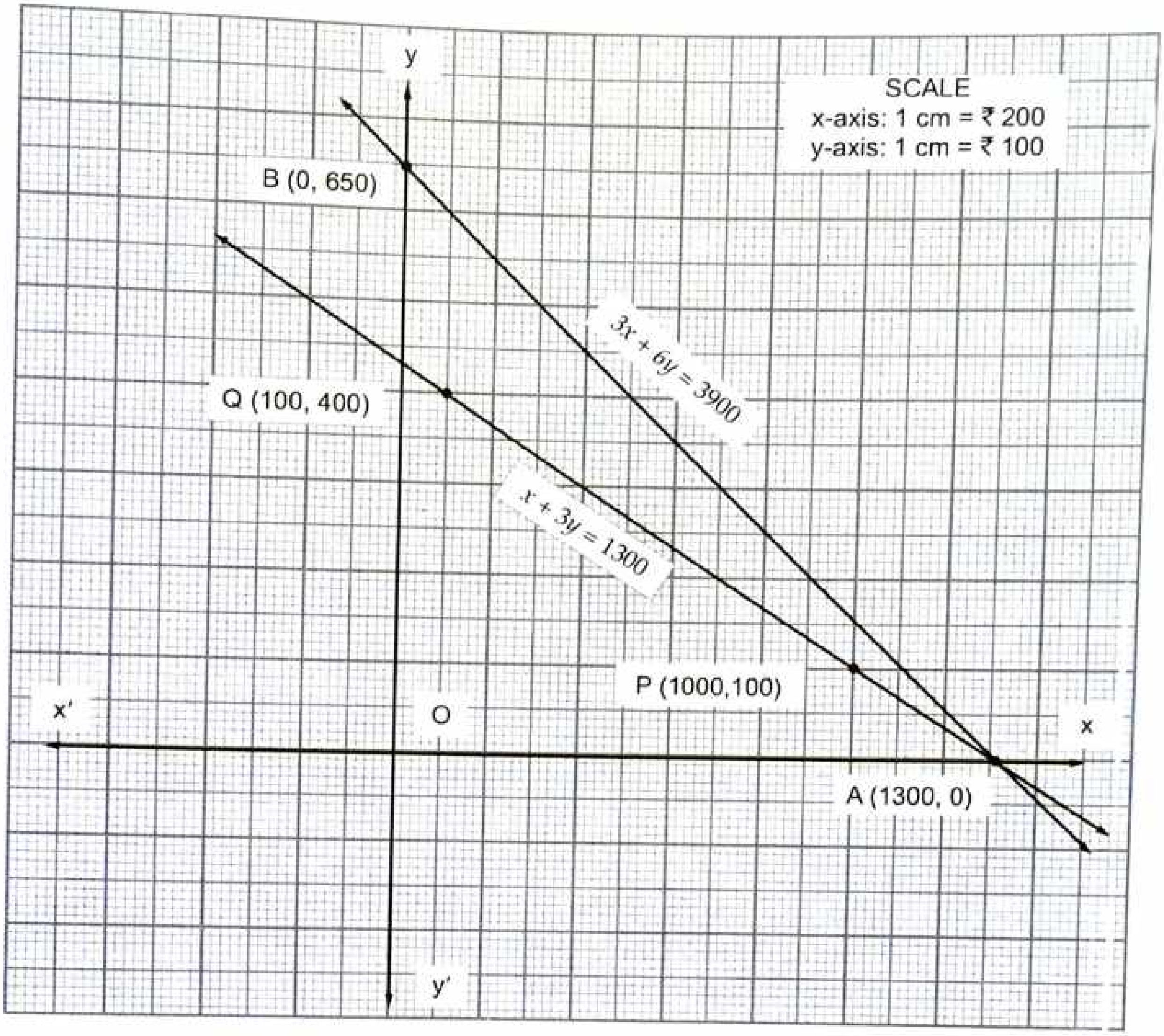

Fig. 3.5

Now, we plot the points A (1300, 0) and B (0, 650) and draw the line AB passing through these two points to represent equation $3x + 6y = 3900$ as shown in Fig. 3.5. To represent the equation $x + 3y = 1300$, we plot the points P (1000, 100) and Q (100, 400) and the line passing through these points is as shown in Fig. 3.5.

We observe that the two lines representing the two equations are intersecting at the point A (1300, 0).

REMARK *If we look at the graphical (geometrical) representation of the pair of linear equations in the above examples, we find that each pair represents intersecting lines. The pair of linear equations in Example 2 is*

$$3x + 6y - 3900 = 0$$

$$x + 3y - 1300 = 0$$

or,

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 3, b_2 = 6, c_1 = -3900, \; a_2 = 1, b_2 = 3, c_2 = -1300$

We have,

$$\frac{a_1}{a_2} = \frac{3}{1} = 3 \text{ and } \frac{b_1}{b_2} = \frac{6}{3} = 2$$

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

will represent intersecting lines, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. *The converse is also true for any pair of linear equations.*

EXAMPLE 3 *Romila went to a stationary stall and purchased 2 pencils and 3 erasers for ₹9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this situation algebraically and graphically.* **[NCERT]**

SOLUTION *Formulation:* Let the cost of 1 pencil be ₹ x and that of one eraser be ₹ y.

It is given that Romila purchased 2 pencils and 3 erasers for ₹9.

$\therefore \quad 2x + 3y = 9$

It is also given that Sonali purchased 4 pencils and 6 erasers for ₹18.

$\therefore \quad 4x + 6y = 18$

Algebraic Representation: The algebraic representation of the given situation is

$$2x + 3y = 9 \quad \ldots(i)$$
$$4x + 6y = 18 \quad \ldots(ii)$$

Graphical Representation: In order to obtain the graphical representation of the above pair of linear equations, we find two points on the line representing each equation. That is, we find two solutions of each equation. Let us find these solutions. We will try to find solutions having integral values.

We have,

$$2x + 3y = 9$$

Putting $x = -3$, we get

$$-6 + 3y = 9 \Rightarrow 3y = 15 \Rightarrow y = 5$$

Putting $x = 0$, we get

$$0 + 3y = 9 \Rightarrow y = 3$$

Thus, two solutions of $2x + 3y = 9$ are:

x	-3	0
y	5	3

We have,

$$4x + 6y = 18$$

Putting $x = 3$, we get

$$12 + 6y = 18 \Rightarrow 6y = 6 \Rightarrow y = 1$$

Putting $x = -6$, we get

$$-24 + 6y = 18 \Rightarrow 6y = 42 \quad y = 7$$

Thus, two solutions of $4x + 6y = 18$ are

x	3	-6
y	1	7

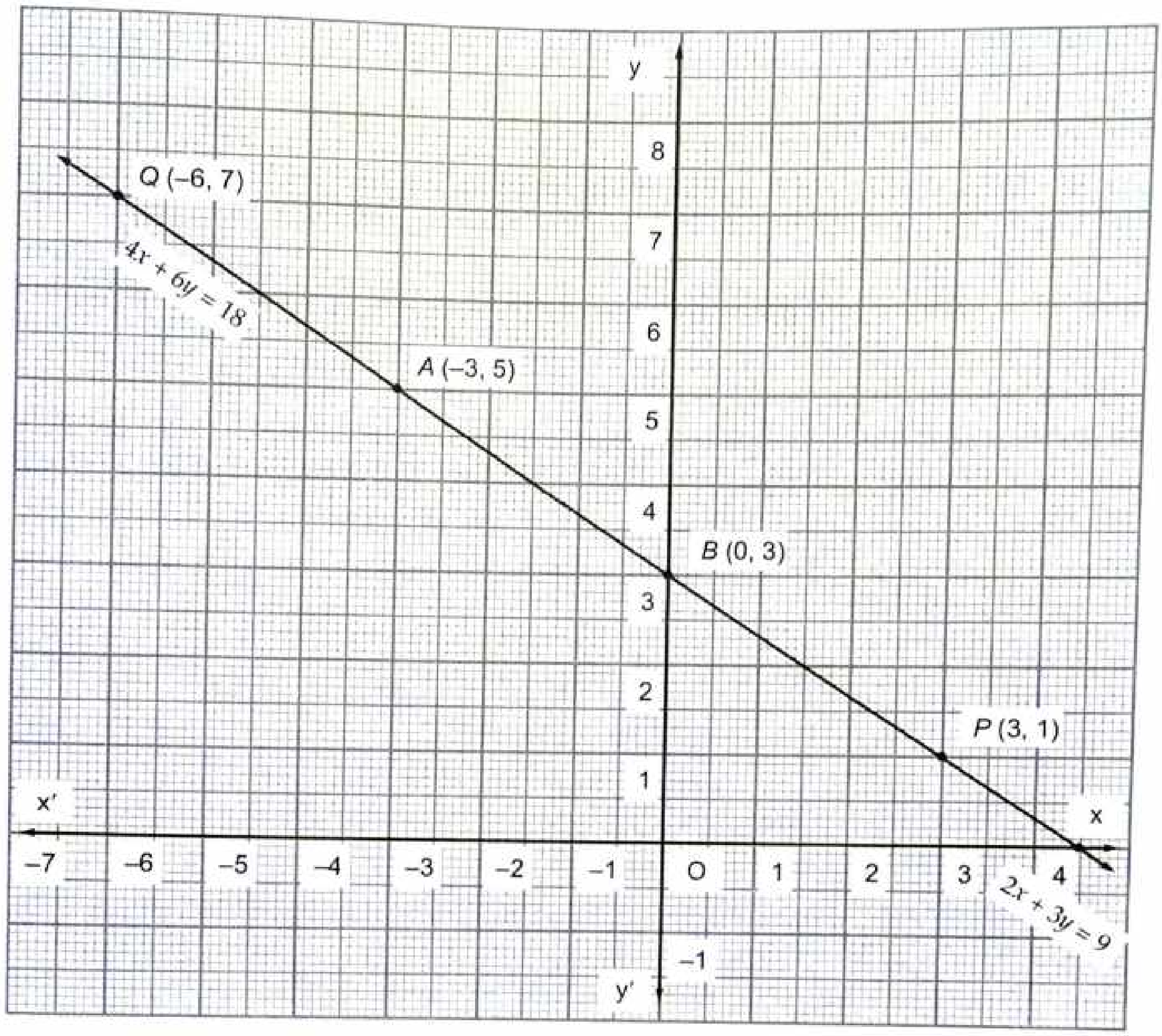

Fig. 3.6

Now, we plot the points $A(-3, 5)$ and $B(0, 3)$ and draw the line passing through these points to obtain the graph of the line $2x + 3y = 9$. Points $P(3, 1)$ and $Q(-6, 7)$ are plotted on the graph paper and we join them to obtain the graph of the line $4x + 6y = 18$. We find that both the lines AB and PQ coincide.

REMARK *Graphical representation of the pair of linear equations in the above example provides us coincident lines. Let us write the above pair of linear equations as*

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where $a_1 = 2, b_1 = 3, c_1 = -9, a_2 = 4, b_2 = 6, c_2 = -18$

We observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Thus, the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

will represent coincident lines, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The converse is also true for any pair of linear equations.

EXAMPLE 4 *The path of a train A is given by the equation $x + 2y - 4 = 0$ and the path of another train B is given by the equation $2x + 4y - 12 = 0$. Represent this situation graphically.* **[NCERT]**

SOLUTION The paths of two trains are given by the following pair of linear equations.

$$x + 2y - 4 = 0 \quad \ldots(i)$$
$$2x + 4y - 12 = 0 \quad \ldots(ii)$$

In order to represent the above pair of linear equations graphically, we need two points on the line representing each equation. That is, we find two solutions of each equation as given below:

We have,

$$x + 2y - 4 = 0$$

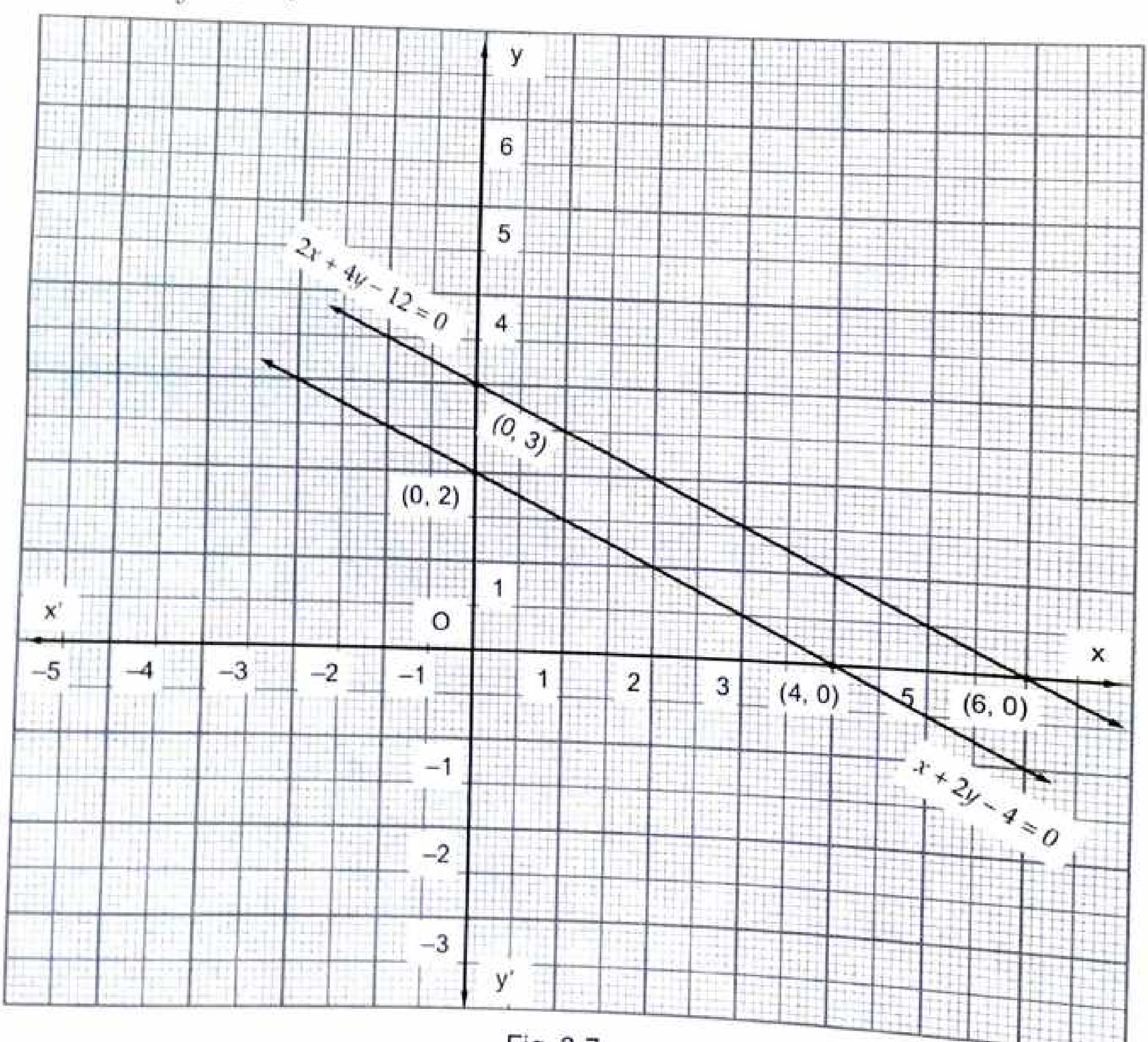

Fig. 3.7

Putting $y = 0$, we get

$$x + 0 - 4 = 0 \Rightarrow x = 4$$

Putting $x = 0$, we get

$$0 + 2y - 4 = 0 \Rightarrow y = 2$$

Thus, two solutions of equation $x + 2y - 4 = 0$ are:

x	4	0
y	0	2

We have,

$$2x + 4y - 12 = 0$$

Putting $x = 0$, we get

$$0 + 4y - 12 = 0 \Rightarrow y = 3$$

Putting $y = 0$, we get

$$2x + 0.12 = 0 \Rightarrow x = 6$$

Thus, two solutions of equation $2x + 4y - 12 = 0$ are:

x	0	6
y	3	0

Now, we plot the points A (4, 0) and B (0, 2) and draw a line passing through these two points to get the graph of the line represented by the equations (i).

We also plot the points P (0, 3) and Q (6, 0) and draw a line passing through these two points to get the graph of the line represented by the equation (ii).

We observe that the lines are parallel and they do not intersect any where.

REMARK *The graphical representation of the above pair of linear equations provides us a pair of parallel lines.*

Let us write the pair of linear equations.

$$x + 2y - 4 = 0$$

$$2x + 4y - 12 = 0$$

as

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where $a_1 = 1, b_1 = 2, c_1 = -4, a_2 = 2, b_2 = 4$ *and* $c_2 = -12$.

We have,

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-4}{-12} = \frac{1}{3}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

will represent parallel lines, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The converse is also true for any pair of linear equations.

It follows from the above examples that the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

will represent:

(i) intersecting lines, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) coincident lines, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) parallel lines, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

EXERCISE 3.1

LEVEL-1

1. Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs ₹ 3, and a game of Hoopla costs ₹ 4. If she spent ₹ in the fair, represent this situation algebraically and graphically. [NCERT]
2. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically. [NCERT]
3. The path of a train A is given by the equation $3x + 4y - 12 = 0$ and the path of another train B is given by the equation $6x + 8y - 48 = 0$. Represent this situation graphically.
4. Gloria is walking along the path joining (– 2, 3) and (2, – 2), while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.
5. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

   (i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

   (ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

   (iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$
6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

   (i) intersecting lines (ii) parallel lines (iii) coincident lines.
7. The cost of 2kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4kg of apples and 2kg of grapes is ₹ 300. Represent the situation algebraically and geometrically. [NCERT]

ANSWERS

1. $x - 2y = 0$, $3x + 4y = 20$
2. $x - 7y + 42 = 0$, $x - 3y - 6 = 0$

5. (i) Intersecting lines (ii) Coincident lines (iii) Parallel lines
6. (i) $x + 2y - 4 = 0$ (ii) $4x + 6y - 12 = 0$ (iii) $4x + 6y - 16 = 0$
7. $2x + y = 160$, $4x + 2y = 300$.

3.4 GRAPHICAL METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATIONS

In this section, we shall use the knowledge of construction of graphs of linear equations in solving systems of simultaneous linear equations in two variables. We have learnt that the coordinates of every point on the line representing a linear equation in two variables determine a solution of the equation and every solution of the equation is represented by a point on the line. Thus, if there is a system of simultaneous linear equations in two variables such that the lines representing the equations intersect at a point $P(\alpha, \beta)$. Clearly, point P lies on both the lines, so its coordinates will satisfy both the equations in the system. Thus, $x = \alpha, y = \beta$ is the solution of the given system of equations. If the lines represented by the two equations are coincident, then they have infinitely many common points. Therefore, every point on the line provides a solution of the given system of equations and hence it has infinitely many solutions. If the lines represented by the two equations are parallel, then they do not have a common point and so the system has no solution i.e. it is in-consistent.

The procedure of solving a system of simultaneous linear equations in two variables by drawing their graphs is known as the graphical method.

We may use the following algorithm to solve a system of simultaneous linear equations in two variables by graphical method:

ALGORITHM

STEP I *Obtain the given system of simultaneous linear equations in x and y.*
Let the system of simultaneous linear equations be

$$a_1x + b_1y = c_1 \quad \ldots(i)$$
$$a_2x + b_2y = c_2 \quad \ldots(ii)$$

STEP II *Draw the graphs of the equations (i) and (ii) in step I.*
Let the lines l_1 and l_2 represent the graphs of (i) and (ii) respectively.

STEP III *If the lines l_1 and l_2 intersect at a point and (α, β) are the coordinates of this point, then the given system has a unique solution given by $x = \alpha, y = \beta$. Otherwise go to step IV.*

STEP IV *If the lines l_1 and l_2 are coincident, then the system is consistent and has infinitely many solutions. In this case, every solution of one of the equations is a solution of the system. Otherwise go step V.*

STEP V *If the lines l_1 and l_2 are parallel, then the given system of equations is in-consistent i.e. it has no solution.*

Following examples illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Solve graphically the system of equations:*

$$x + y = 3$$
$$3x - 2y = 4$$

SOLUTION Graph of the equation $x + y = 3$:

$$x + y = 3 \Rightarrow y = 3 - x$$

When $x = 1$, we have

$$y = 3 - 1 = 2$$

When $x = 2$, we have

$$y = 3 - 2 = 1$$

Thus, we have the following table:

x	1	2
y	2	1

Plotting the points $A(1, 2)$ and $B(2, 1)$ and drawing a line joining them, we get the graph of the equation $x + y = 3$ as shown in Fig. 3.8.

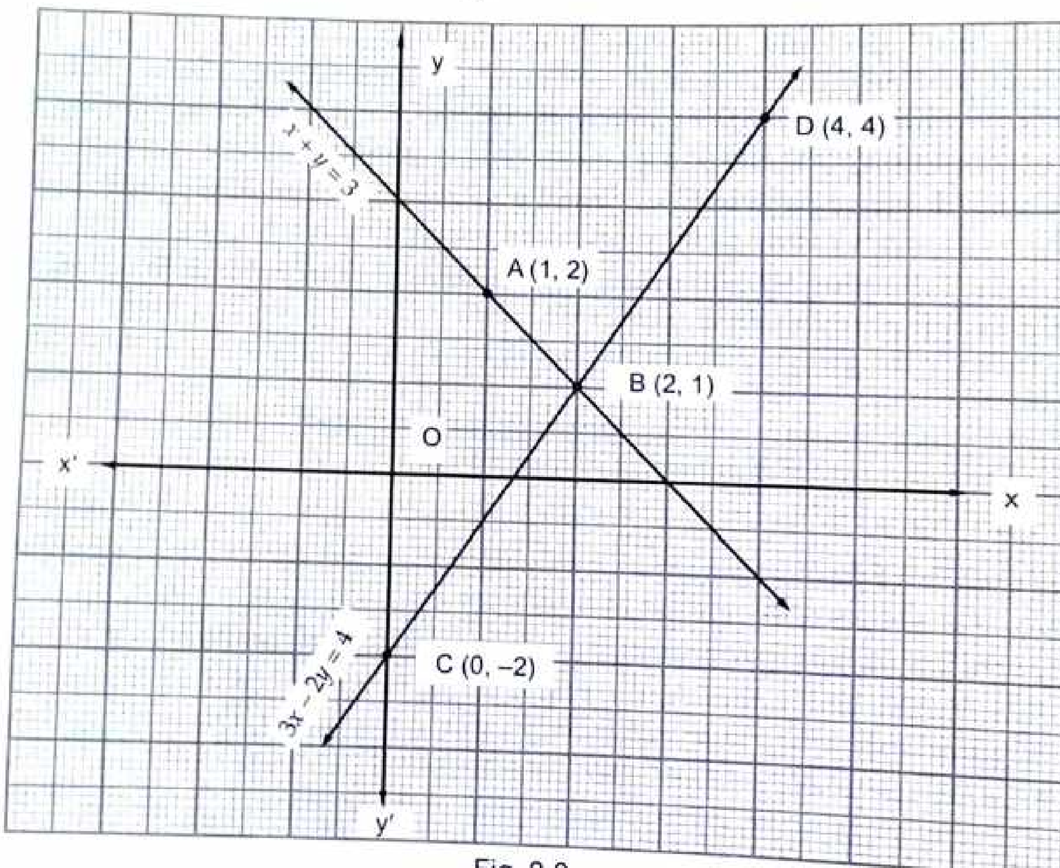

Fig. 3.8

Graph of the equation $3x - 2y = 4$:

We have,

$$3x - 2y = 4 \Rightarrow 2y = 3x - 4 \Rightarrow y = \frac{3x - 4}{2}$$

When $x = 0$, we have

$$y = \frac{3 \times 0 - 4}{2} = -2$$

When $x = 4$, we have

$$y = \frac{3 \times 4 - 4}{2} = 4$$

Thus, we have the following table:

x	0	4
y	-2	4

Plotting the point $C(0, -2)$ and $D(4, 4)$ on the same graph paper and drawing a line joining them, we obtain the graph of the equation $3x - 2y = 4$.

Clearly, the two lines intersect at point $P(2, 1)$.

Hence, $x = 2, y = 1$ is the solution of the given system.

EXAMPLE 2 *Show graphically that the system of equations*

$$2x + 4y = 10$$
$$3x + 6y = 12$$

has no solution.

SOLUTION Graph of $2x + 4y = 10$:

We have,

$$2x + 4y = 10 \Rightarrow 4y = 10 - 2x \Rightarrow y = \frac{5 - x}{2}$$

When $x = 1$, we have

$$y = \frac{5 - 1}{2} = 2$$

When $x = 3$, we have

$$y = \frac{5 - 3}{2} = 1$$

Thus, we have the following table:

x	1	3
y	2	1

Plot the points $A(1, 2)$ and $B(3, 1)$ on a graph paper. Join A and B and extend it on both sides to obtain the graph of $2x + 4y = 10$ as shown in Fig. 3.9.

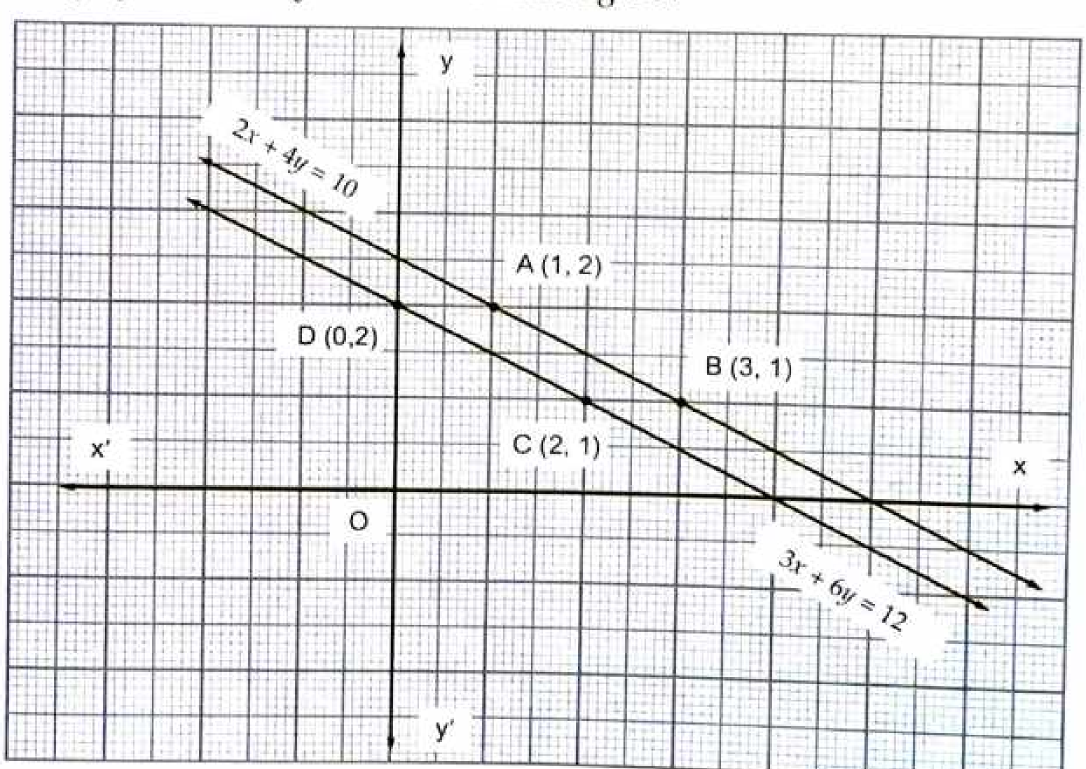

Fig. 3.9

Graph of $3x + 6y = 12$:

We have, $3x + 6y = 12 \Rightarrow 6y = 12 - 3x \Rightarrow y = \dfrac{4 - x}{2}$

When $x = 2$, we have

$$y = \frac{4-2}{2} = 1$$

When $x = 0$, we have

$$y = \frac{4-0}{2} = 2.$$

Thus, we have the following table:

x	2	0
y	1	2

Plot the points C(2, 1) and D (0, 2) on the same graph paper. Join C and D and extend it on both sides to obtain the graph of $3x + 6y = 12$ as shown in Fig. 3.9.

We find the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

EXAMPLE 3 *Show graphically that the system of equations*

$$3x - y = 2$$
$$9x - 3y = 6$$

has infinitely many solutions.

SOLUTION Graph of $3x - y = 2$:

We have, $3x - y = 2 \Rightarrow y = 3x - 2$

When $x = 2$, we have

$$y = 3 \times 2 - 2 = 4$$

When $x = 1$, we have

$$y = 3 \times 1 - 2 = 1$$

Thus, we have the following table:

x	2	1
y	4	1

Plotting the points $A(2, 4)$ and $B(1, 1)$ on the graph paper and drawing a line passing through A and B, we obtain the graph of $3x - y = 2$ as shown in Fig. 3.10.

Graph of $9x - 3y = 6$:

We have, $9x - 3y = 6$

$$\Rightarrow \quad y = 9x - 6$$

$$\Rightarrow \quad y = \frac{9x-6}{3}$$

When $x = 0$, we have

$$y = \frac{9 \times 0 - 6}{3} = -2$$

When $x = -1$, we have

$$y = \frac{9 \times -1 - 6}{3} = -5$$

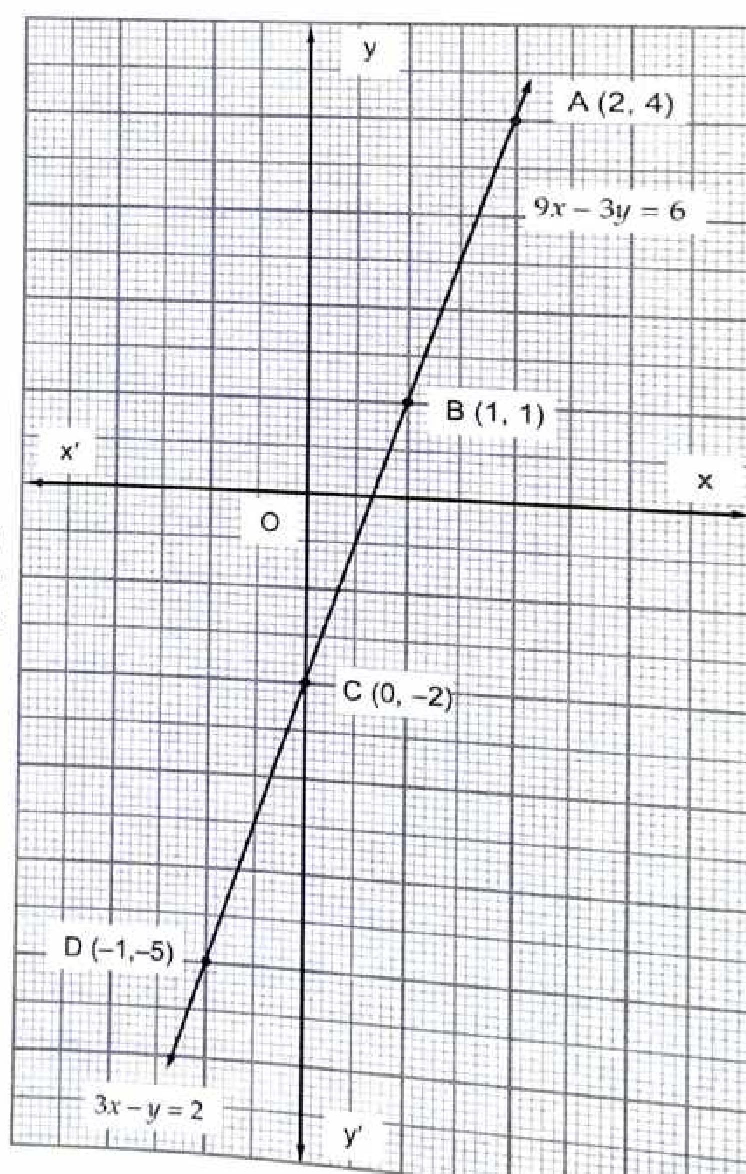

Fig. 3.10

Thus, we have the following table:

x	0	-1
y	-2	-5

Plotting the points $C(0,-2)$ and $D(-1,-5)$ on the graph paper and drawing a line passing through these two points on the same graph paper we obtain the graph of $9x-3y=6$. We find the C and D both lie on the graphy of $3x-y=2$. Thus, the graphs of the two equations are coincident. Consequently, every solution of one equation is a solution of the other. Hence, the system of equations has infinitely many solutions.

EXAMPLE 4 *Use a single graph paper and draw the graph of the following equations:*

$$2y-x=8;\ 5y-x=14,\ y-2x=1.$$

Obtain the vertices of the triangle so obtained.

SOLUTION Graph of $2y-x=8$:

We have, $2y-x=8 \Rightarrow x=2y-8$

When $y=2$, we have

$$x=2\times2-8=-4$$

When $y=3$, we have

$$x=2\times3-8=-2.$$

Thus, we have the following table:

x	-4	-2
y	2	3

Plot the points $A_1(-4,2)$ and $B_1(-2,3)$ on the graph paper. Join A_1 and B_1 and extend it on both sides to obtain the graph of $2y-x=8$ as shown in Fig. 3.11.

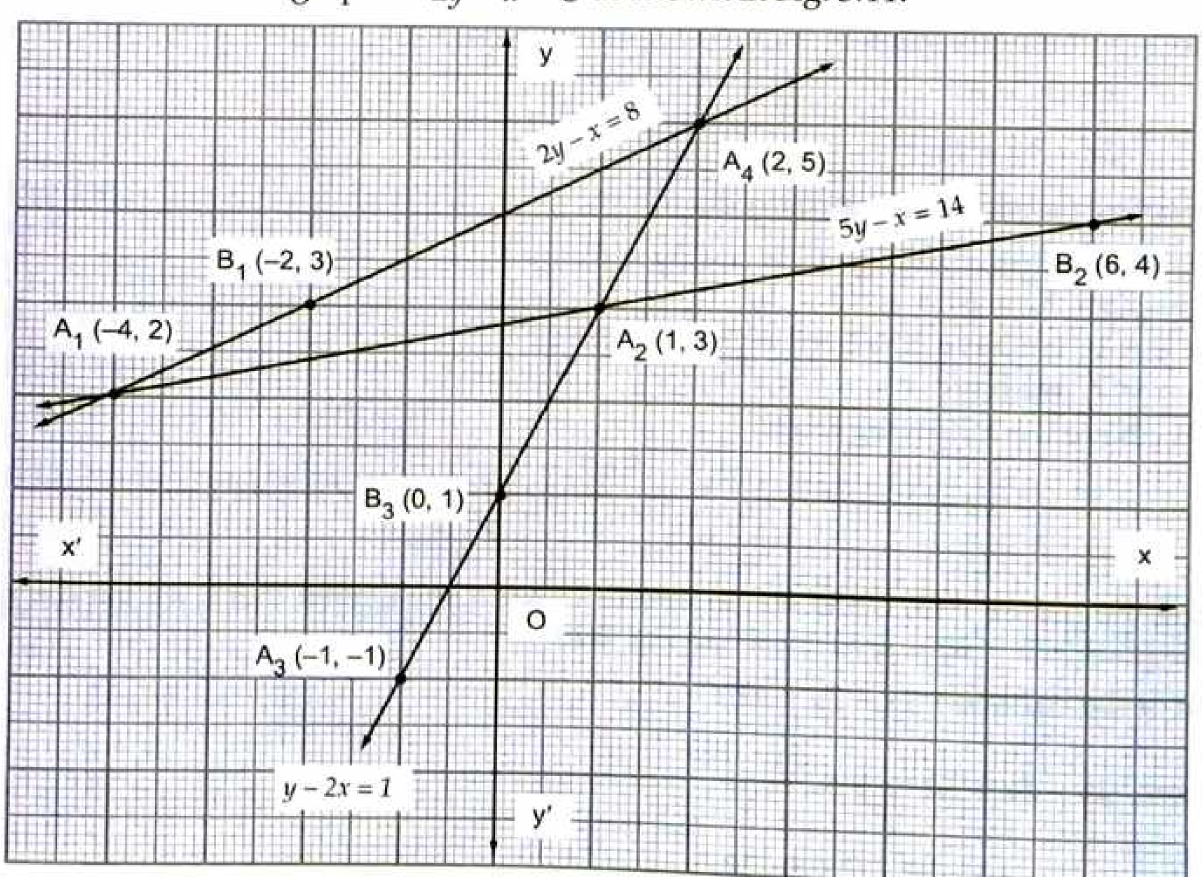

Fig. 3.11

Graph of $5y - x = 14$:

We have, $5y - x = 14 \Rightarrow x = 5y - 14$

When $y = 3$, we have $x = 5 \times 3 - 14 = 1$

When $y = 4$, we have $x = 5 \times 4 - 14 = 6$

Thus, we have the following table:

x	1	6
y	3	4

Plot the points $A_2(1, 3)$ and $B_2(6, 4)$ on a graph paper. Join A_2 and B_2 and extend it on both sides to obtain the graph of $5y - x = 14$ as shown in Fig. 3.11.

Graph of $y - 2x = 1$:

We have, $y - 2x = 1 \Rightarrow y = 2x + 1$

When $x = -1$, we have $y = 2 \times -1 + 1 = -1$

When $x = 0$, we have $y = 2 \times 0 + 1 = 1$

Thus, we have the following table:

x	-1	0
y	-1	1

Plot the points $A_3(-1 - 1)$ and $B_3(0, 1)$ on the same graph paper. Join A_3 and B_3 and extend it on both sides to obtain the graph of $y - 2x = 1$ as shown in Fig. 3.11.

From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A_1(-4, 2)$, $A_2(1, 3)$ and $A_4(2, 5)$.

Hence, the vertices of the required triangle are $(-4, 2)$, $(1, 3)$ and $(2, 5)$.

EXAMPLE 5 *Solve the following system of equations graphically*

$$x + 3y = 6$$

$$2x - 3y = 12$$

and hence find the value of a, if $4x + 3y = a$. **[CBSE 2008]**

SOLUTION Graph of the equation $x + 3y = 6$:

We have, $x + 3y = 6 \Rightarrow x = 6 - 3y$

When $y = 1$, we have $x = 6 - 3 = 3$

When $y = 2$, we have $x = 6 - 6 = 0$

Thus we have the following table:

x	3	0
y	1	2

Plotting the points $A(3, 1)$ and $B(0, 2)$ and drawing a line joining them, we get the graph of the equation $x + 3y = 6$ as shown in Fig. 3.12.

Graph of the equation $2x - 3y = 12$:

We have, $2x - 3y = 12 \Rightarrow y = \dfrac{2x - 12}{3}$

When $x = 3$, we have $y = \dfrac{2 \times 3 - 12}{3} = -2$

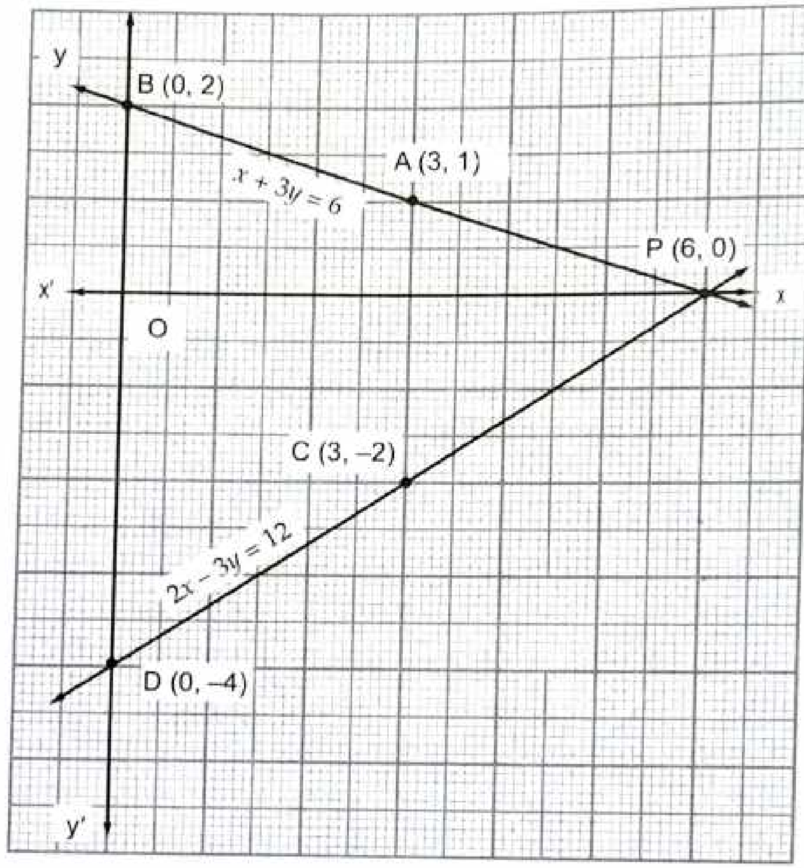

Fig. 3.12

When $x = 0$, we have $y = \frac{0-12}{3} = -4$

Thus, we have the following table:

x	3	0
y	– 2	– 4

Plotting the points $C(3,-2)$ and $D(0,-4)$ on the same graph paper and drawing a l joining them, we obtain the graph of the equation $2x - 3y = 12$ as shown in Fig. 3.12. Clearly, two lines intersect at $P(6, 0)$.

Hence, $x = 6, y = 0$ is the solution of the given system of equations.

Putting $x = 6, y = 0$ in $a = 4x + 3y$, we get

$$a = (4 \times 6) + (3 \times 0) = 24$$

EXAMPLE 6 *Solve the following system of linear equations graphically:*

$$2x - y - 4 = 0$$
$$x + y + 1 = 0$$

Find the points where the lines meet y-axis. [CBSE 200

SOLUTION We have,

$$2x - y - 4 = 0$$
$$x + y + 1 = 0$$

Graph of the equation $2x - y - 4 = 0$:

We have,

$$2x - y - 4 = 0$$

$\Rightarrow \quad y = 2x - 4$

When $x = 0$, we have $y = -4$

When $x = 2$, we have $y = 0$

Thus, we have the following table giving points on the line $2x - y - 4 = 0$.

x	0	2
y	– 4	0

Plotting the points $A(0,-4)$ and $B(2,0)$ on the graph paper on a suitable scale and drawing a line passing through these two points we obtain the graph of the line given by the equation $2x - y - 4 = 0$ as shown in Fig. 3.13.

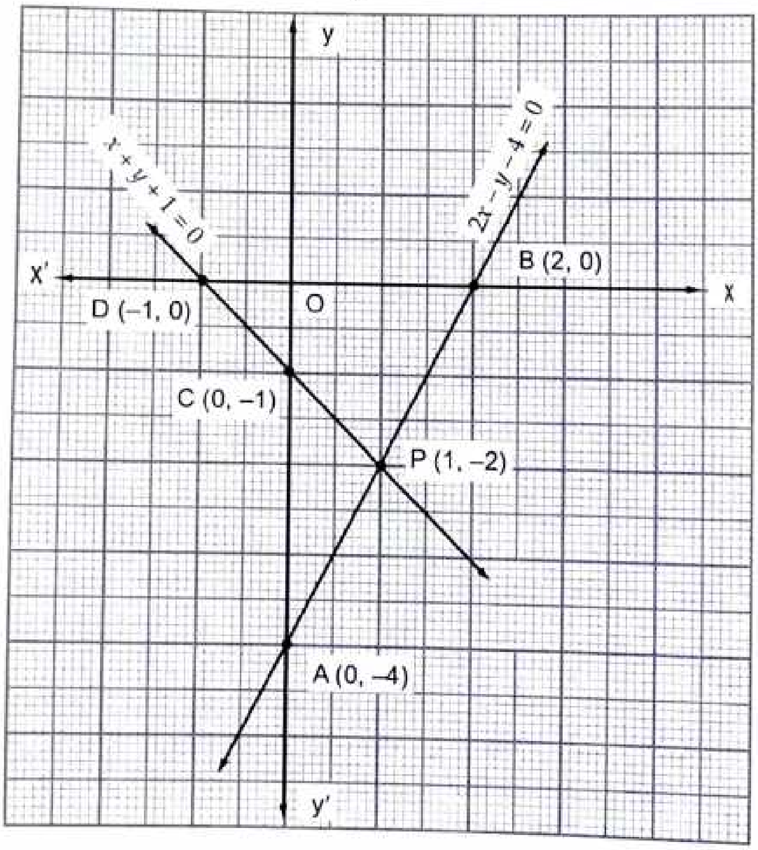

Fig. 3.13

Graph of the equation $x + y + 1 = 0$:

We have,

$$x + y + 1 = 0 \Rightarrow y = -x - 1 \text{ and } x = -y - 1.$$

When $x = 0$, we have $y = -1$

When $x = -1$, we have $y = 0$

Thus we have the following table giving points on the line $x + y + 1 = 0$

x	0	– 1
y	– 1	0

Plotting the points $C(0,-1)$ and $D(-1,0)$ on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $x + y + 1 = 0$ as shown in Fig. 3.13.

Clearly, the two lines intersect at $P(1,-2)$. Hence, $x = 1, y = -2$ is the solution of the given system of equations.

From Fig. 3.13, we observe that the lines represented by the equations $2x - y - 4 = 0$ and $x + y + 1 = 0$ meet y-axis at $A(0,-4)$ and $C(0,-1)$ respectively.

EXAMPLE 7 *Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.* **[CBSE 2002]**

SOLUTION We have,

$$2x + y = 6 \quad \text{...(i)}$$
$$2x - y + 2 = 0 \quad \text{...(ii)}$$

Graph of the equation $2x + y = 6$:

We have,

$$2x + y = 6 \Rightarrow y = 6 - 2x$$

When $x = 0$, we have $y = 6$

When $x = 2$, we have $y = 2$

Thus, we have the following table giving two points on the line represented by the equation $2x + y = 6$

x	0	3
y	6	0

Plotting the points $A(0, 6)$ and $B(3, 0)$ on the graph paper on a suitable scale and drawing a line joining them, we obtain the graph of the line represented by the equation $2x + y = 6$ as shown in Fig. 3.14.

Graph of the equation $2x - y + 2 = 0$:

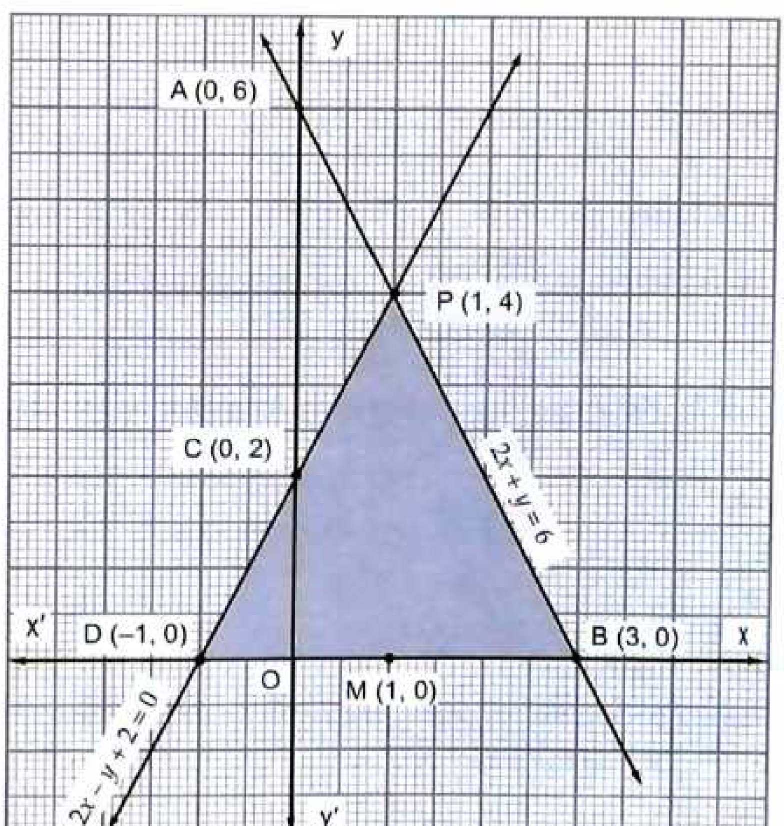

Fig. 3.14

We have,

$$2x - y + 2 = 0 \Rightarrow y = 2x + 2$$

When $x = 0$, we have $y = 2$

When $x = -1$, we have $y = 0$

Thus, we have the following table giving two points on the line representing the given equation

x	0	−1
y	2	0

Plotting the points $C(0, 2)$ and $D(-1, 0)$ on the same graph paper and joining them, we obtain the graph of the line represented by the equation $2x - y + 2 = 0$ as shown in Fig. 3.14.

It is evident from the graph that the two lines intersect at point $P(1, 4)$. The area enclosed the lines and x-axis is shown in Fig. 3.14.

Thus, $x = 1, y = 4$ is the solution of the given system of equations. Draw PM perpendicular from P on x-axis

Clearly, we have

$PM = y$-coordinate of point $P(1, 4)$

$\Rightarrow \quad PM = 4$

and, $\quad DB = 4$

$\therefore \quad$ Area of the shaded region = Area of ΔPBD

$\Rightarrow \quad$ Area of the shaded region $= \frac{1}{2}(\text{Base} \times \text{Height})$

$\Rightarrow \quad$ Area of the shaded region $= \frac{1}{2}(DB \times PM)$

$\Rightarrow \quad$ Area of the shaded region $= \left(\frac{1}{2} \times 4 \times 4\right)$ sq. units = 8 sq. units

EXAMPLE 8 *Solve the following system of linear equations graphically:*

$$x - y = 1$$
$$2x + y = 8.$$

Shade the area bounded by these two lines and y-axis. Also, determine this area.

[CBSE 2001]

SOLUTION We have,

$$x - y = 1$$
$$2x + y = 8$$

Graph of the equation $x - y = 1$:

We have,

$$x - y = 1 \Rightarrow y = x - 1 \text{ and } x = y + 1$$

Putting $x = 0$, we get $y = -1$

Putting $y = 0$, we get $x = 1$

Thus, we have the following table for the points on the line $x - y = 1$:

x	0	1
y	-1	0

Plotting points $A(0, -1)$, $B(1, 0)$ on the graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $x - y = 1$ as shown in Fig. 3.15.

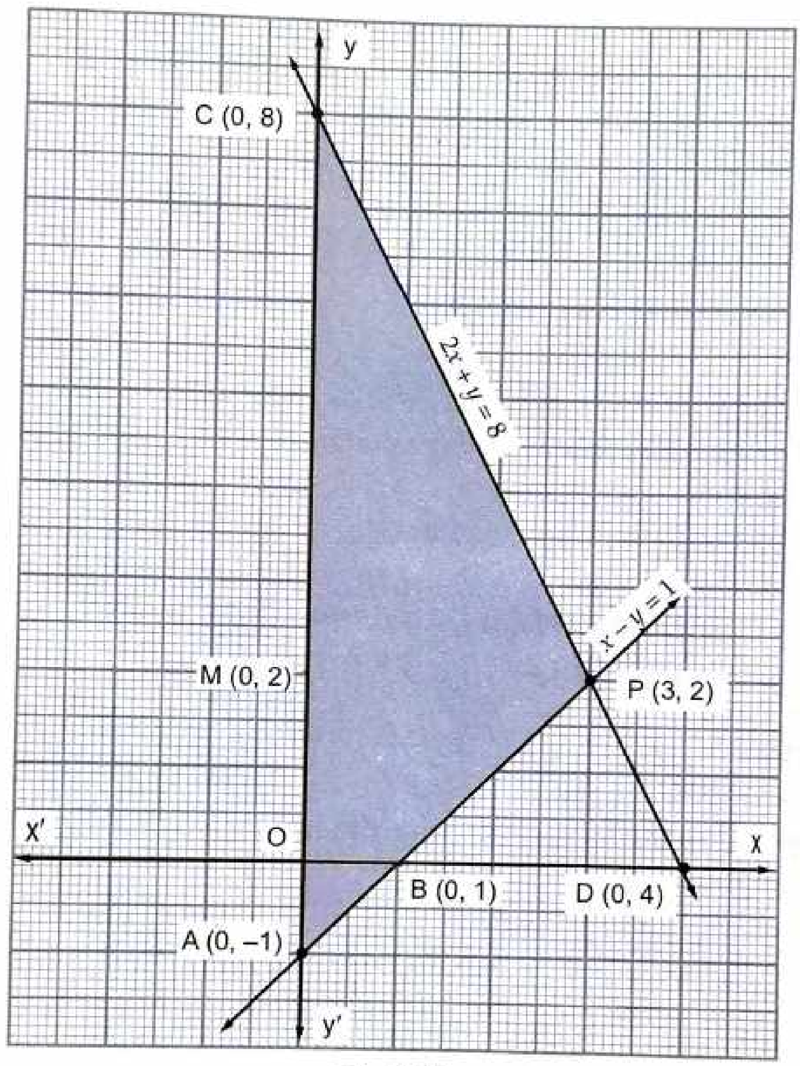

Fig. 3.15

Graph of the equation $2x + y = 8$:
We have,

$$2x + y = 8 \Rightarrow y = 8 - 2x \text{ and } x = \frac{8 - y}{2}$$

Putting $x = 0$, we get $y = 8$

Putting $y = 0$, we get $x = 4$

Thus, we have the following table giving two points on the line represented by the equation $2x + y = 8$.

x	0	4
y	8	0

Plotting points $C(0, 8)$ and $D(4, 0)$ on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 8$ as shown in Fig. 3.15.

Clearly, the two lines intersect at $P(3, 2)$. The area enclosed by the lines represented by the given equations and the y-axis is shaded in Fig. 3.15.

Now, Required area = Area of the shaded region

$\Rightarrow$ Required area = Area of ΔPAC

$\Rightarrow$ Required area $= \frac{1}{2}$(Base × Height)

$\Rightarrow$ Required area $= \frac{1}{2}(AC \times PM)$

$\Rightarrow$ Required area $= \frac{1}{2}(9 \times 3)$ sq. units [$\because PM = x$-coordinate of $P = 3$]

$= 13.5$ sq. units.

LEVEL-2

EXAMPLE 9 *Draw the graphs of the following equations:*

$2x - y - 2 = 0$

$4x + 3y - 24 = 0$

$y + 4 = 0.$

Obtain the vertices of the triangle so obtained. Also, determine its area.

SOLUTION Graph of the equation $2x - y - 2 = 0$:

We have, $2x - y - 2 = 0$

When $y = 0$, we have $x = 1$.

When $x = 0$, we have $y = -2$.

Thus, we obtain the following table giving coordinates of two poitns on the line represented by the equation $2x - y - 2 = 0$.

x	1	0
y	0	−2

Plotting points $A(1, 0)$ and $B(0, -2)$ on the graph paper on a suitable scale and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x - y - 2 = 0$ as shown in Fig. 3.16.

Graph of the equation $4x + 3y - 24 = 0$:

We have, $4x + 3y - 24 = 0 \Rightarrow y = \frac{24 - 4x}{3}$ and $x = \frac{24 - 3y}{4}$

When $y = 0$, we have $x = 6$.

When $x = 0$, we have $y = 8$.

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $4x + 3y - 24 = 0$.

x	6	0
y	0	8

Plotting points $C(6, 0)$ and $D(0, 8)$ Plotting points $C(6, 0)$ and $D(0, 8)$ on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $4x + 3y - 24 = 0$ as shown in Fig. 3.16.

Graph of the equation $y + 4 = 0$:

Clearly, $y = -4$ for every value of x. So, let $E(2, -4)$ and $F(0, -4)$ be two points on the line represented by $y + 4 = 0$. Plotting these points on the same graph and drawing a line passing through them, we obtain the graph of the line represented by the equation $y + 4 = 0$ as shown in Fig. 3.16.

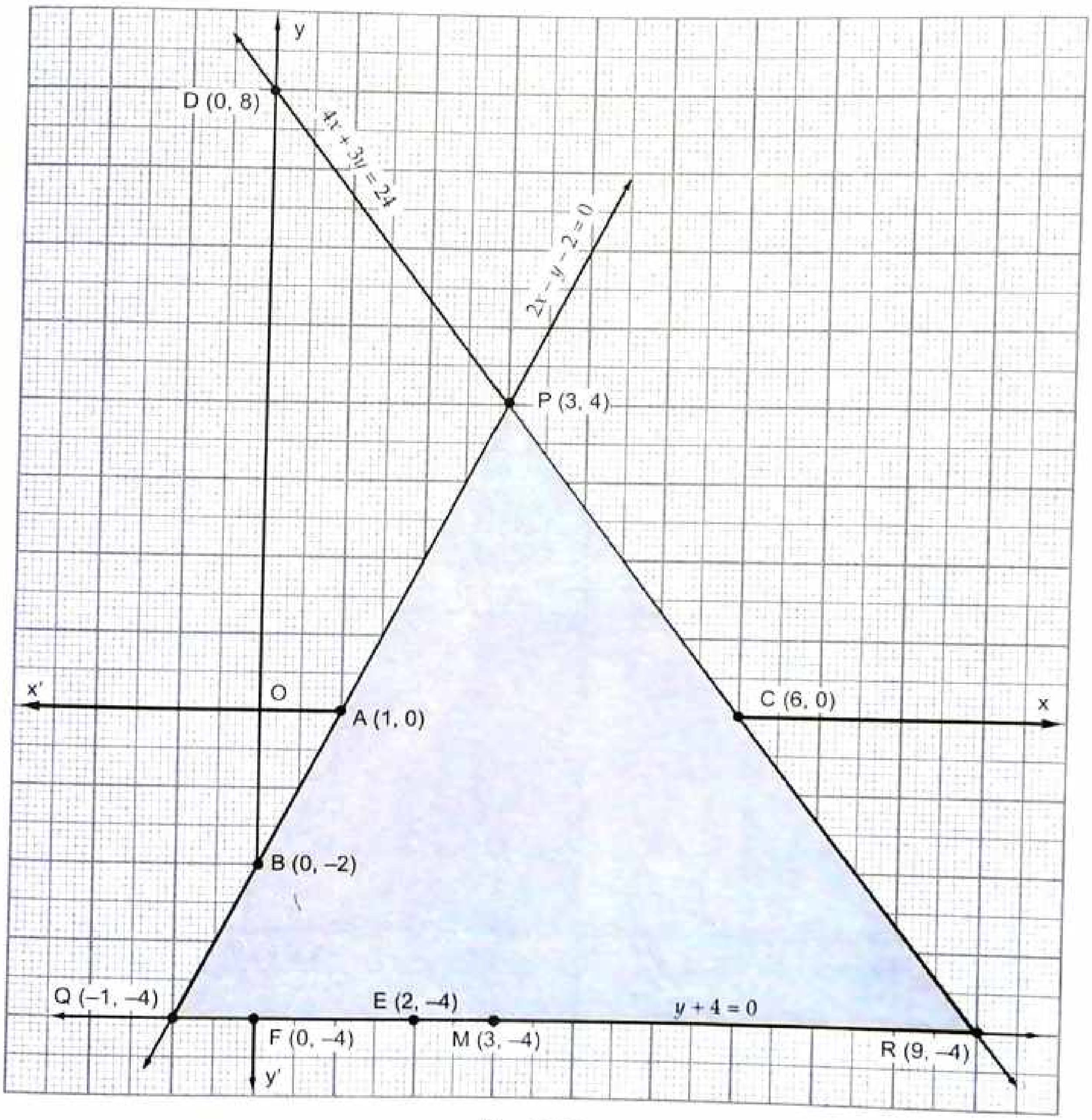

Fig. 3.16

From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $P(3, 4)$, $Q(-1, -4)$ and $R(9, -4)$ as shown in Fig. 3.16.

From Fig. 3.16, we have

$$PM = 8 \text{ and } QR = 10.$$

$$\therefore \quad \text{Area of } \Delta PQR = \frac{1}{2}(\text{Base} \times \text{Height})$$

$\Rightarrow$ Area of $\Delta PQR = \frac{1}{2}(QR \times PM) = \frac{1}{2}(10 \times 8)$ sq. units

$\Rightarrow$ Area of $\Delta PQR = 40$ sq. units.

EXAMPLE 10 *Determine graphically the vertices of a trapezium, the equations of whose sides are: $x = 0$, $y = 0$, $y = 4$ and $2x + y = 6$ Also, determine its area.*

SOLUTION Clearly, $x = 0$ represents y-axis and $y = 0$ represents x-axis.

Graph of the equation $y = 4$:

Clearly, $y = 4$ for every value of x. So, let A (3, 4) and B (0, 4) be two points on the line represented by $y = 4$. Plotting these points on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $y = 4$ as shown in Fig. 3.17. It is a line parallel to x-axis at a distance of 4 units from it.

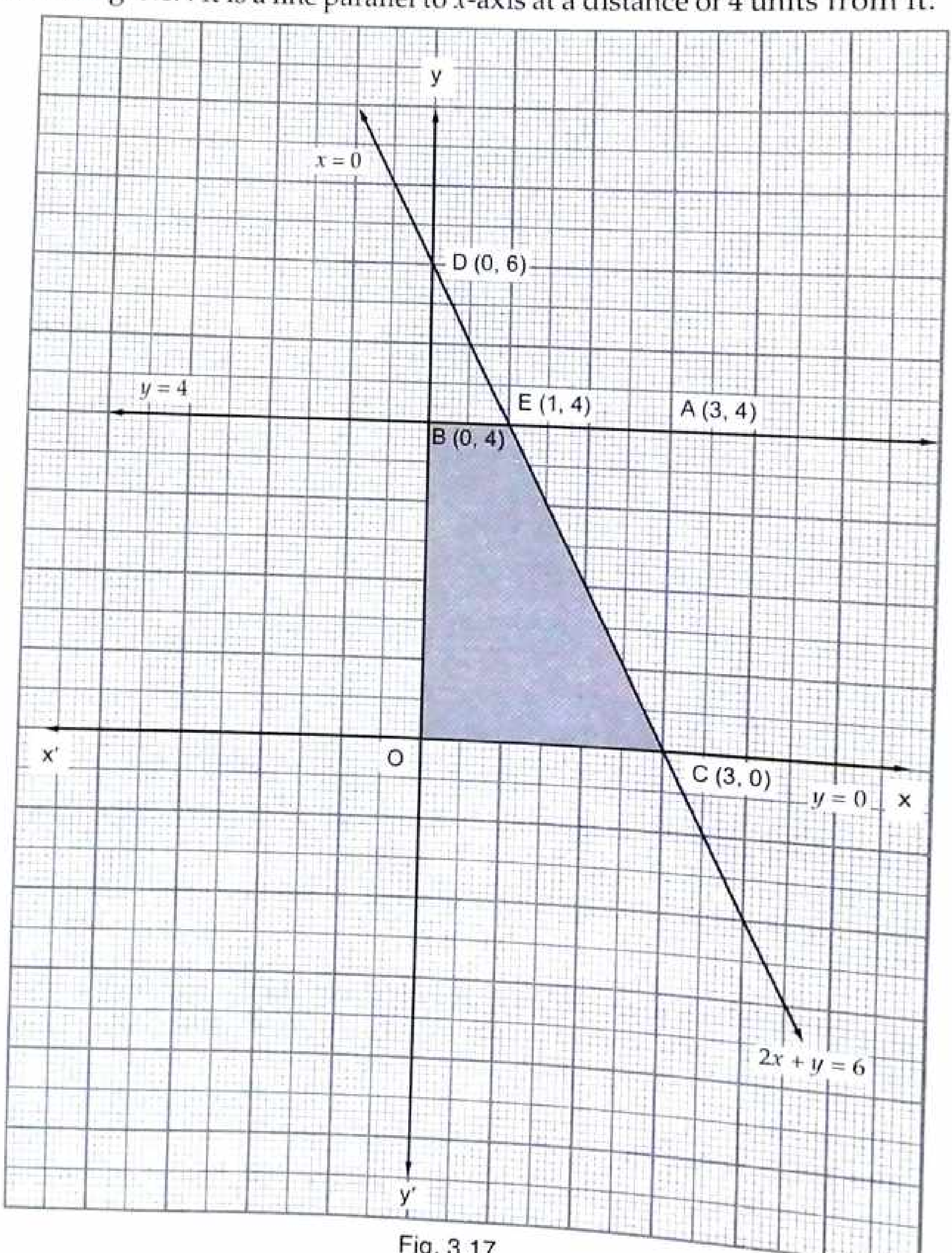

Fig. 3.17

Graph of the equation $2x + y = 6$:

We have $2x + y = 6$

When $y = 0$, we get $x = 3$ and $x = 0$ gives $y = 6$.

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$.

x	3	0
y	0	6

Plotting point C (3, 0) and D (0, 6) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 6$ as shown in Fig. 3.17.

We find that the lines represented by the given equations form the trapezium OCEB as shown in Fig. 3.17. The coordinates of its vertices are $O(0, 0)$, $C(3, 0)$, $E(1, 4)$ and $B(0, 4)$.

Area of trapezium $OCEB = \frac{1}{2}(OC + BE) \times OB = \frac{1}{2}(3 + 1) \times 4 = 8$ sq. units

EXAMPLE 11 *Draw the graphs of the following equations on the same graph paper*

$$2x + y = 2;\ 2x + y = 6$$

Find the coordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium so formed.

SOLUTION *Graph of the equation $2x + y = 2$:*

We have, $2x + y = 2$

When $y = 0$, we have $x = 1$

When $x = 0$, we have $y = 2$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 2$.

x	1	0
y	0	2

Plotting points $A(1, 0)$ and $B(0, 2)$ on the graph paper on a suitable scale and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 2$ as shown in Fig. 3.18.

Graph of the equation $2x + y = 6$:

We have, $2x + y = 6$

When $y = 0$, we get $x = 3$

When $x = 0$, we get $y = 6$

Thus, we obtain the following table giving coordinates of two points on the line represented by the equation $2x + y = 6$.

x	3	0
y	0	6

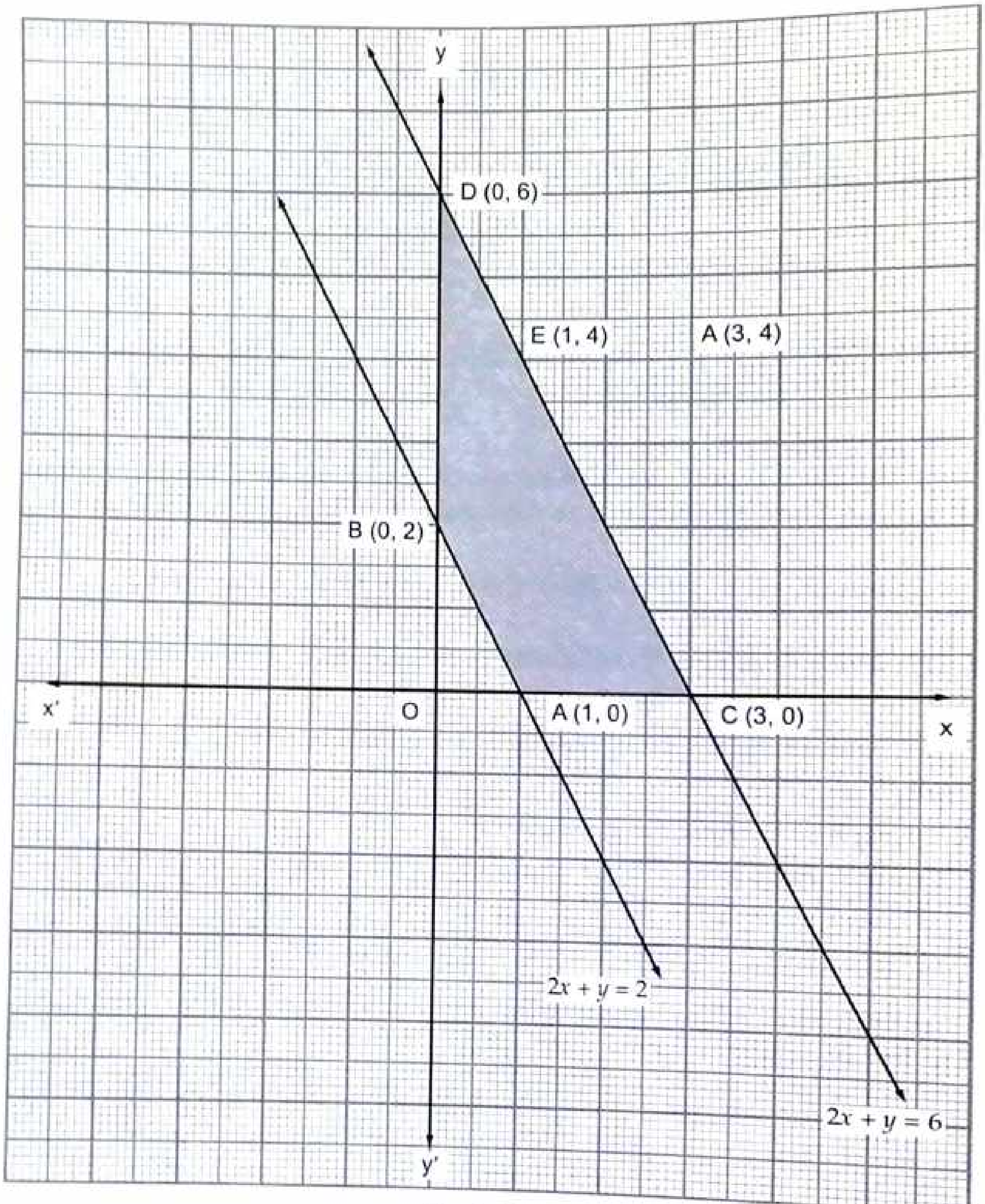

Fig. 3.18

Plotting point C (3, 0) and D (0, 6) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 6$ as shown in Fig. 3.18.

Clearly, lines AB and CD form trapezium $ABCD$.

Also,

Area of trapezium $ACDB$ = Area of ΔOCD – Area of ΔOAB

$$= \frac{1}{2}(OC \times OD) - \frac{1}{2}(OA \times OB)$$

$$= \frac{1}{2}(3 \times 6) - \frac{1}{2}(1 \times 2)$$

$$= 8 \text{ sq. units}$$

EXERCISE 3.2

LEVEL-1

Solve the following systems of equations graphically:

1. $x + y = 3$
   $2x + 5y = 12$
2. $x - 2y = 5$
   $2x + 3y = 10$
3. $3x + y + 1 = 0$
   $2x - 3y + 8 = 0$
4. $2x + y - 3 = 0$
   $2x - 3y - 7 = 0$
5. $x + y = 6$
   $x - y = 2$
6. $x - 2y = 6$
   $3x - 6y = 0$
7. $x + y = 4$
   $2x - 3y = 3$
8. $2x + 3y = 4$
   $x - y + 3 = 0$
9. $2x - 3y + 13 = 0$
   $3x - 2y + 12 = 0$ **[CBSE 2001C]**
10. $2x + 3y + 5 = 0$
    $3x - 2y - 12 = 0$ **[CBSE 2001C]**

Show graphically that each one of the following systems of equations has infinitely many solutions:

11. $2x + 3y = 6$
    $4x + 6y = 12$ **[CBSE 2010]**
12. $x - 2y = 5$
    $3x - 6y = 15$
13. $3x + y = 8$
    $6x + 2y = 16$
14. $x - 2y + 11 = 0$
    $3x - 6y + 33 = 0$

Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):

15. $3x - 5y = 20$
    $6x - 10y = -40$
16. $x - 2y = 6$
    $3x - 6y = 0$
17. $2y - x = 9$
    $6y - 3x = 21$
18. $3x - 4y - 1 = 0$
    $2x - \frac{8}{3}y + 5 = 0$
19. Determine graphically the vertices of the triangle, the equations of whose sides are given below:
    (i) $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$
    (ii) $y = x$, $y = 0$ and $3x + 3y = 10$
20. Determine, graphically whether the system of equations $x - 2y = 2$, $4x - 2y = 5$ is consistent or in-consistent.
21. Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not:
    (i) $2x - 3y = 6$, $x + y = 1$
    (ii) $2y = 4x - 6$, $2x = y + 3$
22. Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.
    (i) $2x - 5y + 4 = 0$,
    $2x + y - 8 = 0$ **[CBSE 2005]**
    (ii) $3x + 2y = 12$
    $5x - 2y = 4$ **[CBSE 2006C]**
    (iii) $2x + y - 11 = 0$,
    $x - y - 1 = 0$ **[CBSE 2000C]**
    (iv) $x + 2y - 7 = 0$,
    $2x - y - 4 = 0$ **[CBSE 2000C]**

(v) $3x + y - 5 = 0,$
$2x - y - 5 = 0$ [CBSE 2002C]

(vi) $2x - y - 5 = 0,$
$x - y - 3 = 0$ [CBSE 2002C]

23. Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

(i) $2x + 3y = 12,$
$x - y = 1$ [CBSE 2001]

(ii) $3x + 2y - 4 = 0,$
$2x - 3y - 7 = 0$ [CBSE 2006C]

(iii) $3x + 2y - 11 = 0$
$2x - 3y + 10 = 0$ [CBSE 2006C]

24. Draw the graphs of the following equations on the same graph paper:
$$2x + 3y = 12$$
$$x - y = 1.$$
Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis. [CBSE 2001]

25. Draw the graphs of $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and x- axis and shade the triangular area. Calculate the area bounded by these lines and x-axis. [CBSE 2002]

26. Solve graphically the system of linear equations:
$$4x - 3y + 4 = 0$$
$$4x + 3y - 20 = 0$$
Find the area bounded by these lines and x-axis. [CBSE 2002]

27. Solve the following system of linear equations graphically:
$3x + y - 11 = 0,\ x - y - 1 = 0.$
Shade the region bounded by these lines and y -axis. Also, find the area of the region bounded by the these lines and y-axis. [CBSE 2002C]

28. Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

(i) $2x + y = 6$
$x - 2y = -2$

(ii) $2x - y = 2$
$4x - y = 8$

(iii) $x + 2y = 5$
$2x - 3y = -4$ [CBSE 2005]

(iv) $2x + 3y = 8$
$x - 2y = -3$ [CBSE 2005]

29. Draw the graphs of the following equations:
$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 = 0$$
$$y - 2 = 0$$
Find the vertices of the triangle so obtained. Also, find the area of the triangle.

30. Solve the following system of equations graphically:
$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 = 0$$
Also, find the area of the region bounded by these two lines and y-axis.

31. Solve the following system of linear equations graphically:
$$4x - 5y - 20 = 0$$
$$3x + 5y - 15 = 0$$
Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis. [CBSE 2004]

32. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and y-axis. Calculate the area of the triangle so formed. [NCERT]

33. Form the pair of linear equations in the following problems, and find their solution graphically:
   (i) 10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz. [NCERT]
   (ii) 5 pencil and 7 pens together cost ` 50, whereas 7 pencils and 5 pens together cost ` 46. Find the cost of one pencil and a pen. [NCERT]
   (iii) Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased." Help her friends to find how many pants and skirts Champa bought. [NCERT]

34. Solve the following system of equations graphically:
   Shade the region between the lines and the y-axis
   (i) $3x - 4y = 7$
   $5x + 2y = 3$ [CBSE 2006C]
   (ii) $4x - y = 4$
   $3x + 2y = 14$ [CBSE 2006C]

35. Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y-axis
   $x + 3y = 6$
   $2x - 3y = 12$ [CBSE 2008]

36. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is
   (i) intersecting lines (ii) Parallel lines (iii) coincident lines [NCERT]

LEVEL-2

37. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are:
   (i) $y = x, y = 2x$ and $y + x = 6$ [CBSE 2000]
   (ii) $y = x,\ 3y = x,\ x + y = 8$ [CBSE 2000]

38. Graphically, solve the following pair of equations:
   $2x + y = 6$
   $2x - y + 2 = 0$
   Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis. [NCERT EXEMPLAR]

39. Determine, graphically, the vertices of the triangle formed by the lines $y = x, 3y = x, x + y = 8$. [NCERT EXEMPLAR]

40. Draw the graph of the equations $x = 3, x = 5$ and $2x - y - 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x-axis. [NCERT EXEMPLAR]

41. Draw the graphs of the lines $x = -2$, and $y = 3$. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure. [NCERT EXEMPLAR]

42. Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis. [NCERT EXEMPLAR]

ANSWERS

1. $x = 1, y = 2$ 2. $x = 5, y = 0$ 3. $x = -1, y = 2$
4. $x = 2, y = -1$ 5. $x = 4, y = 2$ 6. No solution
7. $x = 3, y = 1$ 8. $x = -1, y = 2$ 9. $x = -2, y = 3$
10. $x = 2, y = -3$ 19. (i) $(-4, 2)$ $(1, 3)$, $(2, 5)$ (ii) $(0, 0)$ $(10/3, 0)$, $(5/3, 5/3)$
20. Consistent
21. (i) Unique solution (ii) Infinitely many solutions.
22. (i) $x = 3, y = 2$; $(0, 4/5)$, $(0, 8)$ (ii) $x = 2, y = 3$; $(0, 6)$ and $(0, -2)$
(iii) $x = 4, y = 3$; $(0, 11)$ and $(0, -1)$ (iv) $x = 3, y = 2$, $(0, 3.5)$, $(0, -4)$
(v) $x = 2, y = -1$, $(0, 5)$ $(0, -5)$ (vi) $x = 2, y = -1$, $(0, -5)$, $(0, -3)$
23. (i) $x = 3, y = 2$ (ii) $x = 2, y = -1$ (iii) $x = 1, y = 4$
24. $x = 3, y = 2$, $A(0, 4)$, $B(0, -1)$, $C(3, 2)$
25. 7.5 sq. units 26. $x = 2, y = 4$, 12 sq. units
27. $x = 3, y = 2$, 18 sq. units
28. (i) $x = 2, y = 2$, $(3, 0)$ $(-2, 0)$ (ii) $x = 3, y = 4$, $(1, 0)$, $(2, 0)$
(iii) $x = 1, y = 2$, $(5, 0)$, $(-2, 0)$ (iv) $x = 1, y = 2$, $(4, 0)$, $(-3, 0)$
29. $(3, 4)$, $(0, 2)$, $(6, 2)$, Area = 6 sq. units.
30. $x = 3, y = 4$, Area = 6 sq. units
31. $x = 5, y = 0$; $(5, 0)$ $(0, 3)$ $(0, -4)$.
32. $(1, 0)$, $(0, -3)$, $(0, -5)$, and 1 sq. unit.
33. (i) Girls = 7, Boys = 3 (ii) Pencil: ₹ 3, Pen: ₹ 5 (iii) Pant = 1, Skirt = 0.
34. (i) $x = 1, y = -1$. (ii) $x = 2, y = 4$ 35. $(0, 2)$, $(0, -4)$
36. (i) $3x + 2y - 6 = 0$ (ii) $4x + 6y = 15$ (iii) $4x + 6y = 16$
37. (i) $A(0, 0)$, $B(2, 4)$, $C(3, 3)$ (ii) $A(0, 0)$, $B(4, 4)$ $C(6, 2)$
38. 4 : 1 39. $(0, 0)$, $(4, 4)$, $(6, 2)$ 40. 8 sq. units
41. $(0, 0)$, $A(0, 3)$, $B(-2, 3)$, $C(-2, 0)$; 6 sq. units 42. 6 sq. units

3.5 ALGEBRAIC METHODS OF SOLVING SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES

In the previous sections, we have discussed graphical method of solving two simultaneous linear equations in two variables. Most of the times the graphical method is not convenient, particularly when the point of intersection of the lines represented by two given equations has coordinates as rational numbers. In such situations, graphical method does not give an accurate answer. For example, if the solution to a system of two linear equations is $x = -11/13, y = 5/7$, then by the graphical method, the point of intersection would be $(-11/13, 5/7)$. This point is so close to $(-0.8, 0.7)$ that on the graph paper it is not convenient to distinguish these two points and while reading the coordinates of the point of intersection we are likely to make an error. We may read $x = -0.8, y = 0.7$ as the solution of the given system of equations whereas the correct solution is $x = -11/13, y = 5/7$. Hence, it is necessary to use some precise method to obtain an accurate answer. The algebraic methods described below determine the accurate answer.

The most commonly used algebraic methods of solving simultaneous linear equations in two variables are:

(i) Method of elimination by substitution.
(ii) Method of elimination by equating the coefficients.
(iii) Method of cross-multiplication.

3.5.1 METHOD OF ELIMINATION BY SUBSTITUTION

In this method, we express one of the variables in terms of the other variable from either of the two equations and then this expression is put in the other equation to obtain an equation in one variable as explained in the following algorithm.

ALGORITHM

STEP I *Obtain the two equations. Let the equations be*

$$a_1x + b_1y + c_1 = 0 \quad \ldots(i)$$

and, $$a_2x + b_2y + c_2 = 0 \quad \ldots(ii)$$

STEP II *Choose either of the two equations, say (i), and find the value of one variable, say y, in terms of the other, i.e. x.*

STEP III *Substitute the value of y, obtained in step II, in the other equation i.e. (ii) to get an equation in x.*

STEP IV *Solve the equation obtained in step III to get the value of x.*

STEP V *Substitute the value of x obtained in step IV in the expression for y in terms of x obtained in step II to get the value of y.*

STEP VI *The values of x and y obtained in steps IV and V respectively constitute the solution of the given system of two linear equations.*

Following solved examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Solve the following systems of equations by using the method of substitution:*

(i) $3x - 5y = -1$, $x - y = -1$ (ii) $x + 2y = -1$, $2x - 3y = 12$

SOLUTION (i) The given system of equations is

$$3x - 5y = -1 \quad \ldots(i)$$

$$x - y = -1 \quad \ldots(ii)$$

From (ii), we get

$$y = x + 1$$

Substituting $y = x + 1$ in (i), we get

$$3x - 5(x + 1) = -1$$

$\Rightarrow$ $$-2x - 5 = -1$$

$\Rightarrow$ $$-2x = 4 \Rightarrow x = -2$$

Putting $x = -2$ in $y = x + 1$ we get $y = -1$.

Hence, the solution of the given system of equations is $x = -2, y = -1$.

(ii) The given system of equations is

$$x + 2y = -1 \quad \ldots(i)$$

$$2x - 3y = 12 \quad \ldots(ii)$$

From equation (i), we get

$$x = -1 - 2y$$

Substituting $x = -1 - 2y$ in equation (ii) we get

$\Rightarrow \quad 2(-1 - 2y) - 3y = 12$

$\Rightarrow \quad -2 - 4y - 3y = 12$

$\Rightarrow \quad -7y = 14$

$\Rightarrow \quad y = -2$

Putting $y = -2$ in $x = -1 - 2y$, we get

$$x = -1 - 2 \times (-2) = 3$$

Hence, the solution of the given system of equations is $x = 3, y = -2$.

EXAMPLE 2 *Solve the following systems of equations by using the method of substitution:*

(i) $2x + 3y = 9$
$3x + 4y = 5$

(ii) $\dfrac{2x}{a} + \dfrac{y}{b} = 2$
$\dfrac{x}{a} - \dfrac{y}{b} = 4$

SOLUTION (i) The given system of equations is

$$2x + 3y = 9 \quad \ldots(i)$$

$$3x + 4y = 5 \quad \ldots(ii)$$

From equation (i), we get

$$3y = 9 - 2x \Rightarrow y = \frac{9 - 2x}{3}$$

Substituting $y = \dfrac{9 - 2x}{3}$ in equation (ii), we get

$$3x + 4\left(\frac{9 - 2x}{3}\right) = 5$$

$\Rightarrow \quad \dfrac{9x + 36 - 8x}{3} = 5$

$\Rightarrow \quad x + 36 = 15$

$\Rightarrow \quad x = -21$

Putting $x = -21$ in $y = \dfrac{9 - 2x}{3}$, we get

$$y = \frac{9 + 42}{3} = 17$$

Hence, the solution of the given system of equations is $x = -21, y = 17$

(ii) The given system of equation is

$$\frac{2x}{a} + \frac{y}{b} = 2 \quad \ldots(i)$$

$$\frac{x}{a} - \frac{y}{b} = 4 \quad \ldots(ii)$$

From equation (i), we get

$$\frac{y}{b} = 2 - \frac{2x}{a} \Rightarrow y = b\left(2 - \frac{2x}{a}\right)$$

Substituting $y = b\left(2 - \frac{2x}{a}\right)$ in equation (ii), we get

$$\frac{x}{a} - \frac{b}{b}\left(2 - \frac{2x}{a}\right) = 4$$

$$\Rightarrow \quad \frac{x}{a} - 2 + \frac{2x}{a} = 4$$

$$\Rightarrow \quad \frac{3x}{a} = 6$$

$$\Rightarrow \quad 3x = 6a$$

$$\Rightarrow \quad x = 2a$$

Putting $x = 2a$ in equation (i), we get

$$4 + \frac{y}{b} = 2 \Rightarrow \frac{y}{b} = -2 \Rightarrow y = -2b$$

Hence, the solution of the given system of equations is $x = 2a$, $y = -2b$.

3.5.2 METHOD OF ELIMINATION BY EQUATING THE COEFFICIENTS

In this method, we eliminate one of the two variables to obtain an equation in one variable which can easily be solved. Putting the value of this variable in any one of the given equations, the value of the other variable can be obtained.

Following algorithm explains the procedure.

ALGORITHM

STEP I *Obtain the two equations.*

STEP II *Multiply the equations so as to make the coefficients of the variable to be eliminated equal.*

STEP III *Add or subtract the equations obtained in step II according as the terms having the same coefficients are of opposite or of the same sign.*

STEP IV *Solve equation in one variable obtained in step III.*

STEP V *Substitute the value found in step IV in any one of the given equations and find the value of the other variable.*

The values of the variables in steps IV and V constitute the solution of the given system of equations.

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I **SOLVING SIMULTANEOUS LINEAR EQUATION IN TWO VARIABLES**

EXAMPLE 1 *Solve the following systems of linear equations by using the method of elimination by equating the coefficients:*

(i) $3x + 2y = 11$
$2x + 3y = 4$

(ii) $8x + 5y = 9$
$3x + 2y = 4$

SOLUTION (i) The given systems of equations is

$$3x + 2y = 11 \quad \ldots(i)$$

$$2x + 3y = 4 \quad \ldots(ii)$$

Let us eliminate y from the given equations. The coefficients of y in the given equations are 2 and 3 respectively. The l.c.m. of 2 and 3 is 6. So, we make the coefficients of y equal to 6 in the two equations.

Multiplying (i) by 3 and (ii) by 2, we get

$$9x + 6y = 33 \quad \ldots(iii)$$

$$4x + 6y = 8 \quad \ldots(iv)$$

Subtracting (iv) from (iii), we get

$$5x = 25 \Rightarrow x = 5$$

Substituting $x = 5$ equation in (i), we get

$$15 + 2y = 11 \Rightarrow 2y = -4 \Rightarrow y = -2$$

Hence, the solution of the given system of equations is $x = 5, y = -2$

(ii) The given system of equation is

$$8x + 5y = 9 \quad \ldots(i)$$

$$3x + 2y = 4 \quad \ldots(ii)$$

Let us eliminate x from the given equations. The coefficients of x in the given equations are 8 and 3 respectively. The l.c.m. of 8 and 3 is 24. So, we make both the coefficients equal to 24.

Multiplying both sides of equation (i) by 3 and equation (ii) by 8, we get

$$24x + 15y = 27$$

$$24 + 16y = 32$$

Subtracting (iv) from (iii), we get

$$-y = -5 \Rightarrow y = 5$$

Putting $y = 5$ in (i), we get

$$8x + 25 = 9 \Rightarrow 8x = -16 \Rightarrow x = -2$$

Hence, the solution of the given system of equations is $x = -2, y = 5$.

EXAMPLE 2 *Solve the following system of equations by using the method of elimination by equating the coefficients:*

$$\frac{x}{10} + \frac{y}{5} + 1 = 15$$

$$\frac{x}{8} + \frac{y}{6} = 15$$

SOLUTION (i) The given system of equations is

$$\frac{x}{10} + \frac{y}{5} = 14$$

$$\frac{x}{8} + \frac{y}{6} = 15$$

This system of equations can be re-written as

$$x + 2y = 140$$

$$3x + 4y = 360 \quad \ldots(i)$$

$$\ldots(ii)$$

Let us eliminate y from the equations (i) and (ii). The coefficients of y in the given equations are 2 and 4 respectively. The l.c.m. of 2 and 4 is 4.

Multiplying (i) by 2, we get

$$2x + 4y = 280 \quad \ldots\text{(iii)}$$

$$3x + 4y = 360 \quad \ldots\text{(iv)}$$

Subtracting (iv) from (iii), we get

$$-x = -80 \Rightarrow x = 80$$

Putting $x = 80$ in equation (i), we get

$$80 + 2y = 140 \Rightarrow 2y = 60 \Rightarrow y = 30$$

Hence, the solution of the given system of equations is $x = 80, y = 30$.

Type II SOLVING A SYSTEM OF EQUATIONS WHICH IS REDUCIBLE TO A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

EXAMPLE 3 *Solve the following system of equations:*

$$\frac{1}{2x} - \frac{1}{y} = -1$$

$$\frac{1}{x} + \frac{1}{2y} = 8, \text{ where } x \neq 0, y \neq 0.$$

SOLUTION Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become

$$\frac{u}{2} - v = -1 \Rightarrow u - 2v = -2 \quad \ldots\text{(i)}$$

and, $$u + \frac{v}{2} = 8 \Rightarrow 2u + v = 16 \quad \ldots\text{(ii)}$$

Let us eliminate u from equations (i) and (ii). Multiplying equation (i) by 2, we get

$$2u - 4v = -4 \quad \ldots\text{(iii)}$$

$$2u + v = 16 \quad \ldots\text{(iv)}$$

Subtracting (iv) from (iii), we get

$$-5v = -20 \Rightarrow v = 4$$

Putting $v = 4$ in equation (i), we get

$$u - 8 = -2 \Rightarrow u = 6$$

Hence, $x = \frac{1}{u} = \frac{1}{6}$ and $y = \frac{1}{v} = \frac{1}{4}$

So, the solution of the given system of equation is $x = \frac{1}{6}, y = \frac{1}{4}$.

EXAMPLE 4 *Solve:* $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$

$$\frac{3}{x} + \frac{2}{y} = 0$$

and hence find 'a' for which $y = ax - 4$.

SOLUTION Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$. The given system of equations become

$$2u + \frac{2}{3}v = \frac{1}{6}$$

$\Rightarrow \quad 12u + 4v = 1$ ...(i)

and, $\quad 3u + 2v = 0$ ...(ii)

Multiplying equation (ii) by 2 and subtracting from equation (i), we get

$$6u = 1 \Rightarrow u = \frac{1}{6}$$

Putting $u = \frac{1}{6}$ in (i), we get

$$2 + 4v = 1 \Rightarrow v = -\frac{1}{4}$$

Hence, $x = \frac{1}{u} = 6$ and $y = \frac{1}{v} = -4$

So, the solution of the given system of equations is $x = 6, y = -4$

Putting $x = 6,\ y = -4$ in $y = ax - 4$, we get

$$-4 = 6a - 4 \Rightarrow a = 0$$

EXAMPLE 5 *Solve:* $4x + \frac{6}{y} = 15$

$$6x - \frac{8}{y} = 14$$

and hence find 'p' if $y = px - 2$

SOLUTION The given system of equation is

$$4x + \frac{6}{y} = 15 \quad ...(i)$$

$$6x - \frac{8}{y} = 14 \quad ...(ii)$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$16x + \frac{24}{y} = 60 \quad ...(iii)$$

$$18x - \frac{24}{y} = 42 \quad ...(iv)$$

Adding (iv) and (iii), we get

$$34x = 102 \Rightarrow x = 3$$

Putting $x = 3$ in equation (i), we get

$$12 + \frac{6}{y} = 15 \Rightarrow y = 2$$

Hence, the solution of the given system of equations is $x = 3, y = 2$.

Putting $x = 3, y = 2$ in $y = px - 2$, we get

$$2 = 3p - 2 \Rightarrow p = 4/3$$

EXAMPLE 6 *Solve:* $\dfrac{1}{2(2x+3y)} + \dfrac{12}{7(3x-2y)} = \dfrac{1}{2}$

$$\frac{7}{2x+3y} + \frac{4}{3x-2y} = 2,$$

where $2x + 3y \neq 0$ *and* $3x - 2y \neq 0$.

SOLUTION Let $\dfrac{1}{2x+3y} = u$ and $\dfrac{1}{3x-2y} = v$. Then, the given system of equations becomes

$$\frac{1}{2}u + \frac{12}{7}v = \frac{1}{2} \Rightarrow 7u + 24v = 7 \quad \ldots(i)$$

and, $7u + 4v = 2$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$$20v = 5 \Rightarrow v = \frac{1}{4}$$

Putting $v = \dfrac{1}{4}$ in equation (i), we get

$$7u + 6 = 7 \Rightarrow u = \frac{1}{7}$$

Now, $u = \dfrac{1}{7} \Rightarrow \dfrac{1}{2x+3y} = \dfrac{1}{7} \Rightarrow 2x + 3y = 7$ ...(iii)

and, $u = \dfrac{1}{4} \Rightarrow \dfrac{1}{3x-2y} = \dfrac{1}{4} \Rightarrow 3x - 2y = 4$ ...(iv)

Multiplying equation (iii) by 2 and equation (iv) by 3, we get

$4x + 6y = 14$ ...(v)

$9x - 6y = 12$ ...(vi)

Adding equations (v) and (vi), we get

$$13x = 26 \Rightarrow x = 2$$

Putting $x = 2$ in equation (v), we get

$$8 + 6y = 14 \Rightarrow y = 1$$

Hence, $x = 2, y = 1$ is the solution of the given system of equations.

EXAMPLE 7 *Solve:* $\dfrac{5}{x+y} - \dfrac{2}{x-y} = -1$

$$\frac{15}{x+y} + \frac{7}{x-y} = 10,$$

where $x + y \neq 0$ *and* $x - y \neq 0$.

SOLUTION Let $\dfrac{1}{x+y} = u$ and $\dfrac{1}{x-y} = v$. Then, the given system of equations becomes

$5u - 2v = -1$ ...(i)

$15u + 7v = 10$ ...(ii)

Multiplying equation (i) by 3, this system of equations becomes

$$15u - 6v = -3 \quad \ldots\text{(iii)}$$

$$15u + 7v = 10 \quad \ldots\text{(iv)}$$

Subtracting equation (iv) from equation (iii), we get

$$-13v = -13 \Rightarrow v = 1$$

Putting $v = 1$ in equation (i), we get

$$5u - 2 = -1 \Rightarrow u = \frac{1}{5}$$

Now, $u = \frac{1}{5} \Rightarrow \frac{1}{x+y} = \frac{1}{5} \Rightarrow x + y = 5$ ...(v)

and, $v = 1 \Rightarrow \frac{1}{x-y} = 1 \Rightarrow x - y = 1$ ...(vi)

Adding equations (vi) and (v), we get $2x = 6 \Rightarrow x = 3$.
Putting $x = 3$ in equation (v), we get $y = 2$.
Hence, $x = 3, y = 2$ is the solution of the given system of equations.

LEVEL-2

Type III **ON SOLVING SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES**

EXAMPLE 8 *Solve the following system of equations:*

$$8v - 3u = 5uv$$

$$6v - 5u = -2uv$$

SOLUTION Clearly the given equations are not linear in the variables u and v but can be reduced into linear equations by the an appropriate substitution.
If we put $u = 0$ in either of the two equations, we get $v = 0$.
Thus, $u = 0, v = 0$ form one solution of the given system of equations.
To find the other solutions, we assume that $u \neq 0, v \neq 0$.
Since $u \neq 0, v \neq 0$. Therefore, $uv \neq 0$.
On dividing each of the given equations by uv, we get

$$\frac{8}{u} - \frac{3}{v} = 5 \quad \ldots\text{(i)}$$

$$\frac{6}{u} - \frac{5}{v} = -2 \quad \ldots\text{(ii)}$$

Taking $\frac{1}{u} = x$ and $\frac{1}{v} = y$, the above equations become

$$8x - 3y = 5 \quad \ldots\text{(iii)}$$

$$6x - 5y = -2 \quad \ldots\text{(iv)}$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$$24x - 9y = 15 \quad \ldots\text{(v)}$$

$$24x - 20y = -8 \quad \ldots\text{(vi)}$$

Subtracting equation (vi) from equation (v), we get

$$11y = 23 \Rightarrow y = \frac{23}{11}$$

Putting $y = \frac{23}{11}$ in equation (iii), we get

$$8x - \frac{69}{11} = 5 \Rightarrow 8x = \frac{69}{11} + 5 \Rightarrow 8x = \frac{124}{11} \Rightarrow x = \frac{31}{22}$$

Now, $x = \frac{31}{22} \Rightarrow \frac{1}{u} = \frac{31}{22} \Rightarrow u = \frac{22}{31}$

and, $y = \frac{23}{11} \Rightarrow \frac{1}{v} = \frac{23}{11} \Rightarrow v = \frac{11}{23}$

Hence, the given system of equations has two solutions given by

(i) $u = 0, v = 0$ (ii) $u = \frac{22}{31}, v = \frac{11}{23}$

EXAMPLE 9 *Solve:* $3(2u + v) = 7uv$
$3(u + 3v) = 11uv$

SOLUTION Clearly, the given equations are not linear equations in the variables u and v but can be reduced to linear equations by an appropriate substitution.

If we put $u = 0$ in either of the two equations, we get $v = 0$.

So, $u = 0, v = 0$ form a solution of the given system of equations.

To find the other solutions, we assume that $u \neq 0, v \neq 0$.

Now, $u \neq 0, v \neq 0 \Rightarrow uv \neq 0$.

On dividing each one of the given equations by uv, we get

$$\frac{6}{v} + \frac{3}{u} = 7 \quad \text{...(i)}$$

$$\frac{3}{v} + \frac{9}{u} = 11 \quad \text{...(ii)}$$

Taking $\frac{1}{u} = x$ and $\frac{1}{v} = y$, the above equations become

$$3x + 6y = 7 \quad \text{...(iii)}$$

$$9x + 3y = 11 \quad \text{...(iv)}$$

Multiplying equation (iv) by 2, the above system of equations becomes

$$3x + 6y = 7 \quad \text{...(v)}$$

$$18x + 6y = 22 \quad \text{...(vi)}$$

Substracting equation (vi) from equation (v), we get

$$-15x = -15 \Rightarrow x = 1$$

Putting $x = 1$ in equation (iii), we get

$$3 + 6y = 7 \Rightarrow y = \frac{4}{6} = \frac{2}{3}$$

Now, $x = 1 \Rightarrow \frac{1}{u} = 1 \Rightarrow u = 1$

and, $y = \frac{2}{3} \Rightarrow \frac{1}{v} = \frac{2}{3} \Rightarrow v = \frac{3}{2}$

Hence, the given system of equations has two solutions given by

(i) $u = 0, v = 0$ (ii) $u = 1, v = 3/2$

Type IV **EQUATIONS OF THE FORM** $ax + by = c$ **AND** $bx + ay = d$ **WHERE** $a \neq b$

To solve the above type of equations, following algorithm may be used.

ALGORITHM

STEP I *Obtain the two equations.*

Let the equation be $ax + by = c$ *and* $bx + ay = d$

STEP II *Adding and subtracting the two equations, we obtain*

$$(a+b)x + (a+b)y = c + d \Rightarrow x + y = \frac{c+d}{a+b} \quad \text{...(i)}$$

$$(a-b)\,x - (a-b)\,y = c - d \Rightarrow x - y = \frac{c-d}{a-b} \quad \text{...(ii)}$$

STEP III *Add and subtract equations (i) and (ii) to get the values of x and y.*

EXAMPLE 10 *Solve:* $217x + 131y = 913$

$131x + 217y = 827$

SOLUTION We have,

$$217x + 131y = 913 \quad \text{...(i)}$$

$$131x + 217y = 827 \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$348x + 348y = 1740 \Rightarrow x + y = 5 \quad \text{...(iii)}$$

Subtracting equation (ii) from equation (i), we get

$$86x - 86y = 86 \Rightarrow x - y = 1 \quad \text{...(iv)}$$

Adding equation (iii) and (iv), we get

$$2x = 6 \Rightarrow x = 3$$

Putting $x = 3$ in equation (iii), we get $y = 2$.

Hence, $x = 3$ and $y = 2$ is the solution of the given system of equations.

EXAMPLE 11 *Solve:* $37x + 41y = 70$

$41x + 37y = 86$

SOLUTION We have,

$$37x + 41y = 70 \quad \text{...(i)}$$

$$41x + 37y = 86 \quad \text{...(ii)}$$

Adding equation (i) and (ii), we get

$$78x + 78y = 156 \Rightarrow x + y = 2 \quad \text{...(iii)}$$

Subtracting equation (i) from equation (ii), we get

$$4x - 4y = 16 \Rightarrow x - y = 4 \quad \text{...(iv)}$$

Adding equation (iii) and (iv), we get

$$2x = 6 \Rightarrow x = 3$$

Putting $x = 3$ in equation (iii), we get $y = -1$.

Hence, $x = 3$ and $y = -1$ is the solution of the given system of equations.

Type IV EQUATIONS OF THE FORM

$$a_1x + b_1y + c_1 = d_1$$
$$a_2x + b_2y + c_2 = d_2$$
$$a_3x + b_3y + c_3 = d_3$$

To solve the above type of equations, following algorithm may be used.

ALGORITHM

STEP I *Take any one of the three equations.*

STEP II *Obtain the value of one of the variable, say z from it.*

STEP III *Substitute the value of z obtained in Step II in the remaining two equations to obtain two linear equations in x, y.*

STEP IV *Solve the equations in x, y obtained in Step III by elimination method.*

STEP V *Substitute the values of x, y obtained in Step IV and step II to get the value of z.*

Following examples illustrate the above procedure.

EXAMPLE 12 *Solve:* $2x - y = 4$

$$y - z = 6$$
$$x - z = 10$$

SOLUTION We have,

$$2x - y = 4 \quad \ldots(i)$$
$$y - z = 6 \quad \ldots(ii)$$
$$x - z = 10 \quad \ldots(iii)$$

From equation (iii), we get $z = x - 10$

Substituting the value of z in equation (ii), we get

$$y - (x - 10) = 6$$
$$\Rightarrow \quad -x + y = -4 \quad \ldots(iv)$$

Adding equations (i) and (iv), we get

$$x = 0$$

Putting $x = 0$ equation in (i) and (iii) we get

$$y = -4 \text{ and } z = -10$$

Hence, $x = 0, y = -4, z = -10$ is the solution of the given system of equations.

EXAMPLE 13 *Solve:* $x + 2y + z = 7$

$$x + 3z = 11$$
$$2x - 3y = 1$$

SOLUTION We have,

$$x + 2y + z = 7 \quad \ldots(i)$$
$$x + 3z = 11 \quad \ldots(ii)$$
$$2x - 3y = 1 \quad \ldots(iii)$$

From equation (i), we get

$$z = 7 - x - 2y$$

Substituting $z = 7 - x - 2y$ in equation (ii), we get

$$x + 3(7 - x - 2y) = 11$$

$\Rightarrow \quad x + 21 - 3x - 6y = 11$

$\Rightarrow \quad -2x - 6y = -10$ ...(iv)

Adding equations (iii) and (iv), we get

$$-9y = -9 \Rightarrow y = 1$$

Putting $y = 1$ in equation (iii), we get $x = 2$.

Putting $x = 2, y = 1$ in equation (i), we get

$$2 + 2 + z = 7 \Rightarrow z = 3$$

Hence, $x = 2, y = 1, z = 3$

EXERCISE 3.3

LEVEL-1

Solve the following systems of equations:

1. $11x + 15y + 23 = 0$
   $7x - 2y - 20 = 0$

2. $3x - 7y + 10 = 0$
   $y - 2x - 3 = 0$

3. $0.4x + 0.3y = 1.7$
   $0.7x - 0.2y = 0.8$

4. $\frac{x}{2} + y = 0.8$
   $\frac{7}{x + \frac{y}{2}} = 10$

5. $7(y + 3) - 2(x + 2) = 14$
   $4(y - 2) + 3(x - 3) = 2$

6. $\frac{x}{7} + \frac{y}{3} = 5$
   $\frac{x}{2} - \frac{y}{9} = 6$

7. $\frac{x}{3} + \frac{y}{4} = 11$
   $\frac{5x}{6} - \frac{y}{3} = -7$

8. $\frac{4}{x} + 3y = 8$
   $\frac{6}{x} - 4y = -5$ [CBSE 2010]

9. $x + \frac{y}{2} = 4$
   $\frac{x}{3} + 2y = 5$

10. $x + 2y = \frac{3}{2}$
    $2x + y = \frac{3}{2}$

11. $\sqrt{2}x - \sqrt{3}y = 0$
    $\sqrt{3}x - \sqrt{8}y = 0$ [NCERT]

12. $3x - \frac{y + 7}{11} + 2 = 10$
    $2y + \frac{x + 11}{7} = 10$

13. $2x - \frac{3}{y} = 9$
    $3x + \frac{7}{y} = 2, y \neq 0$

14. $0.5x + 0.7y = 0.74$
    $0.3x + 0.5y = 0.5$

15. $\frac{1}{7x} + \frac{1}{6y} = 3$
$\frac{1}{2x} - \frac{1}{3y} = 5$

16. $\frac{1}{2x} + \frac{1}{3y} = 2$
$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

17. $\frac{15}{u} + \frac{2}{v} = 17$
$\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$

18. $\frac{3}{x} - \frac{1}{y} = -9$
$\frac{2}{x} + \frac{3}{y} = 5$

19. $\frac{2}{x} + \frac{5}{y} = 1$
$\frac{60}{x} + \frac{40}{y} = 19$

20. $\frac{1}{5x} + \frac{1}{6y} = 12$
$\frac{1}{3x} - \frac{3}{7y} = 8$

21. $\frac{4}{x} + 3y = 14$
$\frac{3}{x} - 4y = 23$ [NCERT]

22. $\frac{4}{x} + 5y = 7$ [CBSE 2003]
$\frac{3}{x} + 4y = 5$

23. $\frac{2}{x} + \frac{3}{y} = 13$
$\frac{5}{x} - \frac{4}{y} = -2$ [NCERT]

24. $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$
$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ [NCERT]

LEVEL-2

25. $\frac{x+y}{xy} = 2$
$\frac{x-y}{xy} = 6$

26. $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$
$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$

27. $\frac{6}{x+y} = \frac{7}{x-y} + 3$
$\frac{1}{2(x+y)} = \frac{1}{3(x-y)}$

28. $\frac{xy}{x+y} = \frac{6}{5}$
$\frac{xy}{y-x} = 6$

29. $\frac{22}{x+y} + \frac{15}{x-y} = 5$
$\frac{55}{x+y} + \frac{45}{x-y} = 14$

30. $\frac{5}{x+y} - \frac{2}{x-y} = -1$
$\frac{15}{x+y} + \frac{7}{x-y} = 10$

31. $\frac{3}{x+y} + \frac{2}{x-y} = 2$
$\frac{9}{x+y} - \frac{4}{x-y} = 1$

32. $\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$
$\frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$

33. $\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2}$

$\frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2}$, where $x \neq -1$ and $y \neq 1$

34. $x+y=5xy$

$3x+2y=13xy,$

35. $x+y=2xy$

$\frac{x-y}{xy}=6$

36. $2(3u-v)=5uv$

$2(u+3v)=5uv$

37. $\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$

$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$

38. $\frac{44}{x+y}+\frac{30}{x-y}=10$

$\frac{55}{x+y}+\frac{40}{x-y}=13$ [CBSE 2002C]

39. $\frac{5}{x-1}+\frac{1}{y-2}=2$ [NCERT, CBSE 09]

$\frac{6}{x-1}-\frac{3}{y-2}=1$

40. $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{9}{x-y}=-2$ [NCERT]

41. $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$ [NCERT]

42. $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$ [NCERT]

43. $152x-378y=-74$

$-378x+152y=-604$ [NCERT]

44. $99x+101y=499$

$101x+99y=501$

45. $23x-29y=98$
$29x-23y=110$

46. $x-y+z=4$
$x-2y-2z=9$
$2x+y+3z=1$

47. $x-y+z=4$
$x+y+z=2$
$2x+y-3z=0$

48. $21x+47y=110$
$47x+21y=162$ [NCERT EXEMPLAR]

49. If $x+1$ is a factor of $2x^3+ax^2+2bx+1$, then find the values of a and b given that $2a-3b=4$.

[NCERT EXEMPLAR]

50. Find the solution of the pair of equations $\frac{x}{10}+\frac{y}{5}-1=0$ and $\frac{x}{8}+\frac{y}{6}=15$. Hence, find λ, if $y=\lambda x+5$.

[NCERT EXEMPLAR]

51. Find the values of x and y in the following rectangle. [NCERT EXEMPLAR]

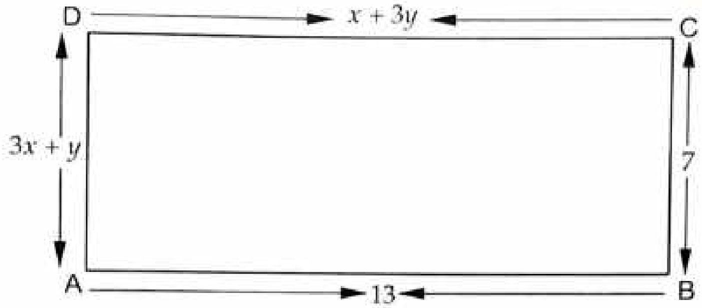

Fig. 3.19

52. Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?. [NCERT EXEMPLAR]

53. Write a pair of linear equations which has the unique solution $x = -1, y = 3$. How many such pairs can you write? [NCERT EXEMPLAR]

ANSWERS

1. C $x = 2, y = -3$	2. $x = -1, y = 1$	3. $x = 2, y = 3$
4. $x = 0.4, y = 0.6$	5. $x = 5, y = 1$	6. $x = 14, y = 9$
7. $x = 6, y = 36$	8. $x = 2, y = 2$	9. $x = 3, y = 2$
10. $x = \frac{1}{2}, y = \frac{1}{2}$	11. $x = 0, y = 0$	12. $x = 3, y = 4$
13. $x = 3, y = -1$	14. $x = 0.5, y = 0.7$	15. $x = \frac{1}{14}, y = \frac{1}{6}$
16. $x = \frac{1}{2}, y = \frac{1}{3}$	17. $u = 5, v = \frac{1}{7}$	18. $x = -\frac{1}{2}, y = \frac{1}{3}$
19. $x = 4, y = 10$	20. $x = \frac{89}{4080}, y = \frac{89}{1512}$	21. $x = \frac{1}{5}, y = -2$
22. $x = \frac{1}{3}, y = -1.$	23. $x = \frac{1}{2}, y = \frac{1}{3}$	24. $x = 4, y = 9$
25. $x = -\frac{1}{2}, y = \frac{1}{4}$	26. $x = 1, y = 3$	27. $x = -\frac{5}{4}, y = -\frac{1}{4}$
28. $x = 2, y = 3$	29. $x = 8, y = 3$	30. $x = 3, y = 2$
31. $x = \frac{5}{2}, y = \frac{1}{2}$	32. $x = \frac{1}{2}, y = \frac{5}{4}$	33. $x = 4, y = 5$
34. $x = \frac{1}{2}, y = \frac{1}{3}$	35. $x = \frac{-1}{2}, y = \frac{1}{4}$	36. $u = 2, v = 1$
37. $x = 1, y = 1$	38. $x = 8, y = 3$	39. $x = 4, y = 5$
40. $x = \frac{21}{8}, y = \frac{9}{8}$	41. $x = 1, y = 1$	42. $x = 1, y = 1$

43. $x = 2, y = 1$
44. $x = 3, y = 2$
45. $x = 3, y = -1$
46. $x = 3, y = -2, z = -1$
47. $x = 2, y = -1, z = 1$
48. $x = 3, y = 1$
49. $a = 5, b = 2$
50. $x = 340, y = -165, \lambda = -\frac{1}{2}$
51. $x = 1, y = 4$
52. $3x + 2y = 5$, Infinitely many
53. $12x + 5y = 3$, Infinitely many

3.5.3 METHOD OF CROSS-MULTIPLICATION

THEOREM *Let* $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

be a system of simultaneous linear equations in two variables x and y such that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ *i.e.* $a_1b_2 - a_2b_1 \neq 0$. *Then the system has a unique solution given by*

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \text{ and } y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

PROOF The given system of equations is

$$a_1x + b_1y + c_1 = 0 \qquad \ldots(i)$$

$$a_2x + b_2y + c_2 = 0 \qquad \ldots(ii)$$

Multiplying equation (i) by b_2, (ii) by b_1 and subtracting, we get

$$b_2(a_1x + b_1y + c_1) - b_1(a_2x + b_2y + c_2) = 0$$

$$\Rightarrow \quad x(a_1b_2 - a_2b_1) = (b_1c_2 - b_2c_1)$$

$$\Rightarrow \quad x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \qquad [\because (a_1b_2 - a_2b_1) \neq 0]$$

Multiplying equation (i) by a_2, (ii) by a_1, and subtracting, we get

$$a_2(a_1x + b_1y + c_1) - a_1(a_2x + b_2y + c_2) = 0$$

$$\Rightarrow \quad y(a_2b_1 - a_1b_2) + (c_1a_2 - c_2a_1) = 0$$

$$\Rightarrow \quad y(a_1b_2 - a_2b_2) = (c_1a_2 - c_2a_1)$$

$$\Rightarrow \quad y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)} \qquad [\because (a_1b_2 - c_2b_1) \neq 0]$$

Hence, $x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)}$ and $y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$

REMARK 1 *The above solution is generally written as*

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

or,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

REMARK 2 *The following procedure is very helpful in determining the solution without remembering the above formula:*

STEP 1 *Obtain the two equations.*

STEP II *Shift all terms on LHS in the two equations to introduce zeros on RHS i.e., write the two equations in the following form:*

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

STEP III *In the above system of equations there are three columns viz. column containing x i.e.* $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, *column containing y i.e.* $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ *and column containing constant terms i.e.* $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

To obtain the solution, write x, –y and 1 separated by equality signs as shown below:

$$\frac{x}{\begin{matrix} b_1 & & c_1 \\ & \times & \\ b_2 & & c_2 \end{matrix}} = \frac{-y}{\begin{matrix} a_1 & & c_1 \\ & \times & \\ a_2 & & c_2 \end{matrix}} = \frac{1}{\begin{matrix} a_1 & & b_1 \\ & \times & \\ a_2 & & b_2 \end{matrix}}$$

In the denominator of x leave column containing x and write remaining two columns in the same order, in the denominator of – y leave column containing y and write the remaining two columns. Similarly, in the denominator of one write columns containing x and y. Mark crossed-arrows pointing downward from top to bottom and pointing upward from bottom to top as shown above.

The arrows between two numbers indicate that the numbers are to be multiplied.

STEP IV *To obtain the denominators of x, – y and 1, multiply the numbers with downward arrow and from their product subtract the product of the numbers with upward arrow. Applying this, we get*

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

STEP V *Obtain the value of x by equating first and third expression in step IV. The value of y is obtained by equating second and third expressions in step IV.*

Following examples illustrate the above procedure

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Solve the each of the following systems of equations by using the method of cross-multiplication:*

(i) $x + y = 7$
$5x + 12y = 7$

(ii) $2x + 3y = 17$
$3x - 2y = 6$

(iii) $2x - y - 3 = 0$
$4x + y - 3 = 0$

(iv) $2x + y - 35 = 0$
$3x + 4y - 65 = 0$

SOLUTION (i) The given system of equations is

$$x + y - 7 = 0$$
$$5x + 12y - 7 = 0$$

By cross-multiplication, we get

$$\frac{x}{\begin{matrix} 1 & & -7 \\ & \times & \\ 12 & & -7 \end{matrix}} = \frac{-y}{\begin{matrix} 1 & & -7 \\ & \times & \\ 1 & & -7 \end{matrix}} = \frac{1}{\begin{matrix} 1 & & 1 \\ & \times & \\ 5 & & 12 \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{1\times -7 - 12\times -7} = \frac{-y}{1\times -7 - 5\times -7} = \frac{1}{1\times 12 - 5\times 1}$$

$$\Rightarrow \quad \frac{x}{-7+84} = \frac{-y}{-7+35} = \frac{1}{12-5}$$

$$\Rightarrow \quad \frac{x}{77} = \frac{-y}{28} = \frac{1}{7}$$

$$\Rightarrow \quad x = \frac{77}{7} \text{ and } y = -\frac{28}{7} \Rightarrow x = 11 \text{ and } y = -4$$

Hence, the solution of the given system of equations is $x = 11, y = -4$.

(ii) The given system of equations is

$$2x + 3y - 17 = 0$$
$$3x - 2y - 6 = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{array}{cc} 3 & -17 \\ -2 & -6 \end{array}} = \frac{-y}{\begin{array}{cc} 2 & -17 \\ 3 & -6 \end{array}} = \frac{1}{\begin{array}{cc} 2 & 3 \\ 2 & -2 \end{array}}$$

$$\Rightarrow \quad \frac{x}{3\times -6 - (-2)\times -17} = \frac{-y}{2\times -6 - 3\times -17} = \frac{1}{2\times -2 - 3\times 3}$$

$$\Rightarrow \quad \frac{x}{-18-34} = \frac{-y}{-12+51} = \frac{1}{-4-9}$$

$$\Rightarrow \quad \frac{x}{-52} = \frac{-y}{39} = \frac{1}{-13}$$

$$\Rightarrow \quad x = \frac{-52}{-13} \text{ and } y = \frac{-39}{-13} \Rightarrow x = 4 \text{ and } y = 3$$

Hence, $x = 4, y = 3$ is the solution of the given system of equations.

(iii) The given system of equations is

$$2x - y - 3 = 0$$
$$4x + y - 3 = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{array}{cc} -1 & -3 \\ 1 & -3 \end{array}} = \frac{-y}{\begin{array}{cc} 2 & -3 \\ 4 & -3 \end{array}} = \frac{1}{\begin{array}{cc} 2 & -1 \\ 4 & 1 \end{array}}$$

$$\Rightarrow \quad \frac{x}{-1\times -3 - 1\times -3} = \frac{-y}{2\times -3 - 4\times -3} = \frac{1}{2\times 1 - 4\times -1}$$

$$\Rightarrow \quad \frac{x}{3+3} = \frac{-y}{-6+12} = \frac{1}{2+4}$$

$$\Rightarrow \quad \frac{x}{6} = \frac{y}{-6} = \frac{1}{6}$$

$\Rightarrow \qquad x = \dfrac{6}{6} = 1$ and $y = -\dfrac{6}{6} = -1$

Hence, the solution of the given system of equations is $x = 1, y = -1$

(iv) The given system of equations is

$$2x + y - 35 = 0$$

$$3x + 4y - 65 = 0$$

$$\frac{x}{\begin{array}{cc} 1 & -35 \\ 4 & -65 \end{array}} = \frac{-y}{\begin{array}{cc} 2 & -35 \\ 3 & -65 \end{array}} = \frac{1}{\begin{array}{cc} 2 & 1 \\ 3 & 4 \end{array}}$$

$$\Rightarrow \qquad \frac{x}{1 \times -65 - 4 \times -35} = \frac{-y}{2 \times -65 - 3 \times -35} = \frac{1}{2 \times 4 - 3 \times 1}$$

$$\Rightarrow \qquad \frac{x}{-65 + 140} = \frac{-y}{-130 + 105} = \frac{1}{8 - 3}$$

$$\Rightarrow \qquad \frac{x}{75} = \frac{y}{25} = \frac{1}{5} \Rightarrow x = \frac{75}{5} = 15 \text{ and } y = \frac{25}{5} = 5$$

Hence, the solution of the given system of equations is $x = 15, y = 5$.

EXAMPLE 2 *Solve* : $\dfrac{x}{a} + \dfrac{y}{b} = a + b$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

SOLUTION The given system of equations may be written as

$$\frac{1}{a} \cdot x + \frac{1}{b} \cdot y - (a + b) = 0$$

$$\frac{1}{a^2} \cdot x + \frac{1}{b^2} \cdot y - 2 = 0$$

By cross-multiplication, we have

$$\frac{x}{\begin{array}{cc} 1/b & -(a+b) \\ 1/b^2 & -2 \end{array}} = \frac{-y}{\begin{array}{cc} 1/a & -(a+b) \\ 1/a^2 & -2 \end{array}} = \frac{1}{\begin{array}{cc} 1/a & 1/b \\ 1/a^2 & 1/b^2 \end{array}}$$

$$\Rightarrow \qquad \frac{x}{\frac{1}{b} \times (-2) - \frac{1}{b^2} \times -(a + b)} = \frac{-y}{\frac{1}{a} \times -2 - \frac{1}{a^2} \times -(a + b)} = \frac{1}{\frac{1}{a} \times \frac{1}{b^2} - \frac{1}{a^2} \times \frac{1}{b}}$$

$$\Rightarrow \qquad \frac{x}{-\frac{2}{b} + \frac{a}{b^2} + \frac{1}{b}} = \frac{-y}{-\frac{2}{a} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{a^2 b}}$$

$$\Rightarrow \qquad \frac{x}{\frac{a}{b^2} - \frac{1}{b}} = \frac{-y}{-\frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{a^2 b}}$$

$$\Rightarrow \qquad \frac{x}{\frac{a-b}{b^2}} = \frac{y}{\frac{a-b}{a^2}} = \frac{1}{\frac{a-b}{a^2b^2}}$$

$$\Rightarrow \qquad x = \frac{a-b}{b^2} \times \frac{1}{\frac{a-b}{a^2b^2}} = a^2 \text{ and } y = \frac{a-b}{a^2} \times \frac{1}{\frac{a-b}{a^2b^2}} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of equations.

EXAMPLE 3 *Solve:* $ax + by = a - b$ **[NCERT, CBSE 2000,**

$bx - ay = a + b$

SOLUTION The given system of equations may be written as

$$ax + by - (a - b) = 0$$

$$bx - ay - (a + b) = 0$$

By cross-multiplication, we get

$$\frac{x}{\begin{array}{ccc} b & \times & -(a-b) \\ -a & & -(a+b) \end{array}} = \frac{-y}{\begin{array}{ccc} a & \times & -(a-b) \\ b & & -(a+b) \end{array}} = \frac{1}{\begin{array}{ccc} a & \times & b \\ b & & -a \end{array}}$$

$$\Rightarrow \qquad \frac{x}{b \times -(a+b) - (-a) \times -(a-b)} = \frac{-y}{a \times -(a+b) - b \times -(a-b)} = \frac{1}{-a^2 - b^2}$$

$$\Rightarrow \qquad \frac{x}{-b(a+b) - a(a-b)} = \frac{-y}{-a(a+b) + b(a-b)} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \qquad \frac{x}{-b^2 - a^2} = \frac{-y}{-a^2 - b^2} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \qquad \frac{x}{-(a^2 + b^2)} = \frac{y}{a^2 + b^2} = \frac{1}{-(a^2 + b^2)}$$

$$\Rightarrow \qquad x = -\frac{(a^2 + b^2)}{-(a^2 + b^2)} = 1 \text{ and } y = \frac{(a^2 + b^2)}{-(a^2 + b^2)} = -1$$

Hence, the solution of the given system of equations is $x = 1, y = -1$.

EXAMPLE 4 *Solve:* $x + y = a + b$

$ax - by = a^2 - b^2$

SOLUTION The given system of equations may be written as

$$x + y - (a + b) = 0$$

$$ax - by - (a^2 - b^2) = 0$$

By cross-multiplication, we get

$$\frac{x}{-(a^2 - b^2) - (-b) \times -(a + b)} = \frac{-y}{-(a^2 - b^2) - a \times -(a + b)} = \frac{1}{1 \times -b - a \times 1}$$

$$\Rightarrow \qquad \frac{x}{-a^2 + b^2 - ab - b^2} = \frac{-y}{-a^2 + b^2 + a^2 + ab} = \frac{1}{-b - a}$$

$$\Rightarrow \quad \frac{x}{-a(a+b)} = \frac{-y}{b(a+b)} = \frac{1}{-(a+b)}$$

$$\Rightarrow \quad \frac{x}{-a(a+b)} = \frac{y}{-b(a+b)} = \frac{1}{-(a+b)}$$

$$\Rightarrow \quad x = \frac{-a(a+b)}{-(a+b)} = a \text{ and } y = \frac{-b(a+b)}{-(a+b)} = b$$

Hence, the solution of the given system of equations is $x = a, y = b$

EXAMPLE 5 *Solve:* $\frac{x}{a} + \frac{y}{b} = 2$

$ax - by = a^2 - b^2$ **[CBSE 2005]**

SOLUTION The given system of equations may be writen as

$bx + ay - 2ab = 0$

$ax - by - (a^2 - b^2) = 0$

By cross-multiplication, we have

$$\frac{x}{-a(a^2-b^2)-(-b)(-2ab)} = \frac{-y}{-b(a^2-b^2)-a(-2ab)} = \frac{1}{b \times -b - a \times a}$$

$$\Rightarrow \quad \frac{x}{-a(a^2-b^2)-2ab^2} = \frac{-y}{-b(a^2-b^2)+2a^2b} = \frac{1}{-b^2-a^2}$$

$$\Rightarrow \quad \frac{x}{-a(a^2-b^2+2b^2)} = \frac{-y}{-(a^2-b^2-2a^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-(a^2+b^2)} = \frac{-y}{-b(-a^2-b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad x = \frac{-a(a^2+b^2)}{-(a^2+b^2)} = a \text{ and } y = \frac{-b(a^2+b^2)}{-(a^2+b^2)} = b$$

Hence, the solution of the given system of equations is $x = a, y = b$.

LEVEL-2

EXAMPLE 6 *Solve the following system of equations in x and y*

$(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$ **[NCERT]**

SOLUTION The given system of equations may be written as

$(a-b)x + (a+b)y - (a^2 - 2ab - b^2) = 0$

$(a+b)x + (a+b)y - (a^2 + b^2) = 0$

By cross-multiplication, we have

$$\frac{x}{(a+b)\times-(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)} = \frac{-y}{(a-b)\times-(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)}$$

$$= \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)} = \frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)} = \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{(a+b)\{-(a^2+b^2)+(a^2-2ab-b^2)\}} = \frac{-y}{(a+b)(a^2-2ab-b^2)-(a-b)(a^2+b^2)} = \frac{1}{(a+b)(a-b-a-b)}$$

$$\Rightarrow \quad \frac{x}{(a+b)(-2ab-2b^2)} = \frac{-y}{a^3-a^2b-3ab^2-b^3-a^3-ab^2+a^2b+b^3} = \frac{1}{-(a+b)\,2b}$$

$$\Rightarrow \quad \frac{x}{-2b(a+b)^2} = \frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad x = \frac{-2b(a+b)^2}{-2b(a+b)} = a+b \text{ and } y = \frac{4ab^2}{-2b(a+b)} = \frac{-2ab}{a+b}$$

Hence, the solution of the given system of equations is $x = a+b,\ y = -\frac{2ab}{a+b}$.

EXAMPLE 7 *Solve the following system of equations in x and y:*

$$\frac{a}{x} - \frac{b}{y} = 0$$

$$\frac{ab^2}{x} + \frac{a^2b}{y} = a^2+b^2, \text{ where } x, y \neq 0.$$

SOLUTION Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system of equations becomes

$$au - bv + 0 = 0$$

$$ab^2u + a^2bv - (a^2+b^2) = 0$$

By cross-multiplication, we have

$$\frac{u}{-b\times-(a^2+b^2)-a^2b\times 0} = \frac{-v}{a\times-(a^2+b^2)-ab^2\times 0} = \frac{1}{a\times a^2b-ab^2\times -b}$$

$$\Rightarrow \quad \frac{u}{b(a^2+b^2)} = \frac{-v}{-a(a^2+b^2)} = \frac{1}{a^3b+ab^3}$$

$$\Rightarrow \quad \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$$

$$\Rightarrow \quad u = \frac{b(a^2+b^2)}{ab(a^2+b^2)} = \frac{1}{a} \text{ and } v = \frac{a(a^2+b^2)}{ab(a^2+b^2)} = \frac{1}{b}$$

Now, $u = \frac{1}{a} \Rightarrow \frac{1}{x} = \frac{1}{a} \Rightarrow x = a$ and $v = \frac{1}{b} \Rightarrow \frac{1}{y} = \frac{1}{b} \Rightarrow y = b$

Hence, the solution of the given system of equation is $x = a,\ y = b$

EXAMPLE 8 *Solve the following system of equations in x and y*

$$ax + by = 1$$

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1 \text{ or, } bx + ay = \frac{2ab}{a^2+b^2}$$

SOLUTION The given system of equations may be written as

$$ax + by - 1 = 0$$

$$bx + ay - \frac{2ab}{a^2+b^2} = 0$$

By cross-multiplication, we have

$$\frac{x}{b \times -\frac{2ab}{a^2+b^2} - a \times -1} = \frac{-y}{a \times -\frac{2ab}{a^2+b^2} - b \times -1} = \frac{1}{a \times a - b \times b}$$

$$\Rightarrow \quad \frac{x}{\frac{-2ab^2}{a^2+b^2} + a} = \frac{-y}{\frac{-2a^2b}{a^2+b^2} + b} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \quad \frac{x}{\frac{-2ab^2 + a^3 + ab^2}{a^2+b^2}} = \frac{-y}{\frac{-2a^2b + a^2b + b^3}{a^2+b^2}} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \quad \frac{x}{\frac{a^3 - ab^2}{a^2+b^2}} = \frac{-y}{\frac{-a^2b + b^3}{a^2+b^2}} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \quad \frac{x}{\frac{a(a^2 - b^2)}{a^2+b^2}} = \frac{-y}{\frac{b(a^2 - b^2)}{a^2+b^2}} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \quad x = \frac{a(a^2-b^2)}{a^2+b^2} \times \frac{1}{a^2-b^2} \text{ and } y = \frac{b(a^2-b^2)}{a^2+b^2} \times \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad x = \frac{a}{a^2+b^2} \text{ and } y = \frac{b}{a^2+b^2}$$

Hence, the solution of the given system of equations is $x = \frac{a}{a^2+b^2}, y = \frac{b}{a^2+b^2}$

EXAMPLE 9 *Solve:* $a(x+y) + b(x-y) = a^2 - ab + b^2$

$$a(x+y) - b(x-y) = a^2 + ab + b^2$$

SOLUTION Taking $x + y = u$ and $x - y = v$ the given system of equations becomes

$$au + bu - (a^2 - ab + b^2) = 0$$

$$au - bv - (a^2 + ab + b^2) = 0$$

By cross-multiplication, we have

$$\Rightarrow \quad \frac{u}{b \times -(a^2+ab+b^2) - (-b) \times -(a^2-ab+b^2)} = \frac{-v}{a \times -(a^2+ab+b^2) + a(a^2-ab+b^2)}$$

$$= \frac{1}{a \times -b - a \times b}$$

$$\Rightarrow \quad \frac{u}{-b(a^2+ab+b^2)-b(a^2-ab+b^2)} = \frac{-v}{-a(a^2+ab+b^2)+a(a^2-ab+b^2)} = \frac{1}{-ab-ab}$$

$$\Rightarrow \quad \frac{u}{-b(a^2+ab+b^2+a^2-ab+b^2)} = \frac{-v}{-a(a^2+ab+b^2-a^2+ab-b^2)} = \frac{1}{-2ab}$$

$$\Rightarrow \quad \frac{u}{-2b(a^2+b^2)} = \frac{-v}{-a(2ab)} = \frac{1}{-2ab}$$

$$\Rightarrow \quad u = \frac{-2b(a^2+b^2)}{-2ab}, v = \frac{2a^2b}{-2ab} \Rightarrow u = \frac{a^2+b^2}{a}, v = -a$$

Now, $u = \dfrac{a^2+b^2}{a} \Rightarrow x+y = \dfrac{a^2+b^2}{a}$ ...(i)

and, $v = -a \Rightarrow x - y = -a$ ...(ii)

Adding equations (i) and (ii), we get

$$2x = \frac{a^2+b^2}{a} - a \Rightarrow 2x = \frac{a^2+b^2-a^2}{a} \Rightarrow 2x = \frac{b^2}{a} \Rightarrow x = \frac{b^2}{2a}$$

Subtracting equation (ii) from equation (i), we get

$$2y = \frac{a^2+b^2}{a} + a \Rightarrow 2y = \frac{a^2+b^2+a^2}{a} \Rightarrow y = \frac{2a^2+b^2}{2a}$$

Hence, the solution of the given system of equations is $x = \dfrac{b^2}{2a}, y = \dfrac{2a^2+b^2}{2a}$

EXAMPLE 10 *Solve:* $ax + by = c$

$bx + ay = 1 + c$ **[NCERT]**

SOLUTION The given system of equations may be written as

$$ax + by - c = 0$$
$$bx + ay - (1+c) = 0$$

By cross-multiplication, we have

$$\frac{x}{b \times -(1+c) - a \times -c} = \frac{-y}{a \times -(1+c) - b \times -c} = \frac{1}{a \times a - b \times b}$$

$$\Rightarrow \quad \frac{x}{-b(1+c)+ac} = \frac{-y}{-a(1+c)+bc} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{ac-bc-b} = \frac{y}{ac-bc+a} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{c(a-b)-b} = \frac{y}{c(a-b)+a} = \frac{1}{(a-b)(a+b)}$$

$$\Rightarrow \quad x = \frac{c(a-b)-b}{(a-b)(a+b)} \text{ and } y = \frac{c(a-b)+a}{(a-b)(a+b)}$$

$$\Rightarrow \quad x = \frac{c}{a+b} - \frac{b}{(a-b)(a+b)} \text{ and } y = \frac{c}{a+b} + \frac{a}{(a-b)(a+b)}$$

Hence, the solution of the given system of equations is

$$x = \frac{c}{a+b} - \frac{b}{a^2-b^2}, \; y = \frac{c}{a+b} + \frac{a}{a^2-b^2}$$

EXAMPLE 11 *Solve the following system of equations:*

$$x + y = a - b$$
$$ax - by = a^2 + b^2$$

SOLUTION The given system of equations may be written as

$$x + y - (a - b) = 0$$
$$ax - by - (a^2 + b^2) = 0$$

By cross-multiplication, we have

$$\frac{x}{-(a^2 + b^2) - b(a - b)} = \frac{y}{-a(a - b) + (a^2 + b^2)} = \frac{1}{-b - a}$$

$$\Rightarrow \quad \frac{x}{-a^2 - ab} = \frac{y}{ab + b^2} = \frac{1}{-b - a}$$

$$\Rightarrow \quad \frac{x}{-a(a + b)} = \frac{y}{b(a + b)} = \frac{1}{-(a + b)}$$

$$\Rightarrow \quad x = \frac{-a(a + b)}{-(a + b)} = a \text{ and } y = \frac{b(a + b)}{-(a + b)} = -b$$

Hence, $x = a, y = -b$ is the solution of the given system of equations.

EXERCISE 3.4

LEVEL-1

Solve each of the following systems of equations by the method of cross-multiplication:

1. $x + 2y + 1 = 0$
   $2x - 3y - 12 = 0$

2. $3x + 2y + 25 = 0$
   $2x + y + 10 = 0$

3. $2x + y = 35$
   $3x + 4y = 65$

4. $2x - y = 6$
   $x - y = 2$

5. $\frac{x + y}{xy} = 2, \quad \frac{x - y}{xy} = 6$

6. $ax + by = a - b$
   $bx - ay = a + b$

7. $x + ay = b$
   $ax - by = c$

8. $ax + by = a^2$
   $bx + ay = b^2$

9. $\frac{5}{x + y} - \frac{2}{x - y} = -1$
   $\frac{15}{x + y} + \frac{7}{x - y} = 10$

10. $\frac{2}{x} + \frac{3}{y} = 13$
    $\frac{5}{x} - \frac{4}{y} = -2$

11. $\frac{57}{x + y} + \frac{6}{x - y} = 5$
    $\frac{38}{x + y} + \frac{21}{x - y} = 9$ [CBSE 2002C]

LEVEL-2

12. $\frac{x}{a} + \frac{y}{b} = 2$
    $ax - by = a^2 - b^2$

13. $\frac{x}{a} + \frac{y}{b} = a + b$ [NCERT EXEMPLAR]
    $\frac{x}{a^2} + \frac{y}{b^2} = 2$

14. $\frac{x}{a} = \frac{y}{b}$

$ax + by = a^2 + b^2$ [NCERT]

15. $2ax + 3by = a + 2b$

$3ax + 2by = 2a + b$

16. $5ax + 6by = 28$

$3ax + 4by = 18$

17. $(a + 2b)x + (2a - b)y = 2$

$(a - 2b)x + (2a + b)y = 3$

18. $x\left(a - b + \frac{ab}{a-b}\right) = y\left(a + b - \frac{ab}{a+b}\right)$

$x + y = 2a^2$

19. $bx + cy = a + b$

$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b}$

20. $(a - b)x + (a + b)y = 2a^2 - 2b^2$

$(a + b)(x + y) = 4ab$

21. $a^2x + b^2y = c^2$

$b^2x + a^2y = d^2$

22. $ax + by = \frac{a+b}{2}$

$3x + 5y = 4$

23. $2(ax - by) + a + 4b = 0$

$2(bx + ay) + b - 4a = 0$ [CBSE 2004]

24. $6(ax + by) = 3a + 2b$

$6(bx - ay) = 3b - 2a$

[CBSE 2004]

25. $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b, x, y \neq 0$

[CBSE 2006C]

26. $mx - ny = m^2 + n^2$

$x + y = 2m$ [CBSE 2006C]

27. $\frac{ax}{b} - \frac{by}{a} = a + b$

$ax - by = 2ab$ [CBSE 2009]

28. $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$

$x + y = 2ab$ [CBSE 2010]

ANSWERS

1. $x = 3, y = -2$
2. $x = 5, y = -20$
3. $x = 15, y = 5$
4. $x = 4, y = 2$
5. $x = \frac{-1}{2}, y = \frac{1}{4}$
6. $x = 1, y = -1$
7. $x = \frac{ac + b^2}{a^2 + b}, y = \frac{ab - c}{a^2 + b}$
8. $x = \frac{a^2 + ab + b^2}{a + b}, y = \frac{-ab}{a + b}$
9. $x = 3, y = 2$
10. $x = \frac{1}{2}, y = \frac{1}{3}$
11. $x = 11, y = 8$
12. $x = a, y = b$
13. $x = a^2, y = b^2$
14. $x = a, y = b$
15. $x = \frac{4a - b}{5a}, y = \frac{-a + 4b}{5b}$
16. $x = \frac{2}{a}, y = \frac{3}{b}$
17. $x = \frac{5b - 2a}{10ab}, y = \frac{a + 10b}{10ab}$
18. $x = \frac{a^3 - b^3}{a}, y = \frac{a^3 + b^3}{a}$

19. $x = \frac{a}{b}, y = \frac{b}{c}$
20. $x = \frac{2ab - a^2 + b^2}{b}, y = \frac{(a-b)(a^2+b^2)}{b(a+b)}$
21. $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}, y = \frac{a^2d^2 - b^2c^2}{a^4 - b^4}$
22. $x = \frac{1}{2}, y = \frac{1}{2}$
23. $x = \frac{-1}{2}, y = 2$
24. $x = \frac{1}{2}, y = \frac{1}{3}$.
25. $x = a^2, y = b^2$
26. $x = m + n, y = m - n$
27. $x = b, y = -a$
28. $x = y = ab$

3.6 CONDITIONS FOR SOLVABILITY (OR CONSISTENCY)

Uptill now we have been discussing various methods of solving a system of simultaneous linear equations in two variables with the assumption that the system has a unique solution. In this section, we shall be discussing the conditions for solvability of a system of simultaneous linear equations in two variables.

Consider the following three systems of simultaneous linear equations:

(i) $2x + y = 5$, $3x - 2y = 4$ (ii) $3x + 4y = 2$, $6x + 8y = 4$ (iii) $2x - 3y = 5$, $4x - 6y = 9$

Clearly, $x = 2, y = 1$ satisfies the first system of equations. So, it is a solution of this system. Also, no other set of values of x and y satisfy this system of equations. So, we say that the system is consistent (solvable) with a unique solution. Graphically, it is due to the reason that the lines represented by the two equations intersect at only one point.

Now, consider the second system of equations. It can easily be checked that $x = 2, y = -1$ is a solution of this system of equations. So, we say that the system is consistent i.e. it possesses a solution. Also, $x = 1, y = -1/4; x = -2, y = 2; x = 6, y = -4$ etc. are solutions of this system. If follows from this that the second system of equations is consistent with infinitely many solutions. Graphically, it is due to the reason that the lines represented by the two equations are coincident.

For the third system of equations there is no set of values of x and y which satisfy the two equations simultaneously. This is because the lines represented by the two equations are parallel. So, we say that the system is inconsistent or not solvable.

It follows from the above discussion that a given system of equations is either inconsistent (does not have a solution) or it is consistent with infinitely many solutions or it is consistent with a unique solution.

In the following discussion, we shall find the conditions for consistency of a system of simultaneous linear equations.

Consider the system of equations

$$a_1x + b_1y + c_1 = 0 \quad \ldots(i)$$
$$a_2x + b_2y + c_2 = 0 \quad \ldots(ii)$$

Multiplying equation (i) by b_2, equation (ii) by b_1 and subtracting, we get

$$x(a_1b_2 - a_2b_1) = (b_1c_2 - b_2c_1) \quad \ldots(iii)$$

Multiplying equation (ii) by a_1, equation (i) by a_2 and subtracting, we get

$$y(a_1b_2 - a_2b_1) = (c_1a_2 - c_2a_1) \quad \ldots(iv)$$

Now, the following cases arise:

CASE I *When* $a_1b_2 - a_2b_1 \neq 0$, *i.e.* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

If $a_1b_2 - a_2b_1 \neq 0$, *then from equations (iii) and (iv), we have*

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \text{ and } y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

It follows from this that the system of equations $a_1x + b_1y + c_1 = 0$ *and,* $a_2x + b_2y + c_2 = 0$ *is consistent with unique solution, if*

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

REMARK 1 *If* $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

is a system of simultaneous linear equations with $a_1b_2 - a_2b_1 \neq 0$, *then the lines represented by the equations* $a_1x + b_1y + c_1 = 0$ *and* $a_2x + b_2y + c_2 = 0$ *intersect at exactly one point having coordinates* $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$ *as shown in Fig. 3.20. Hence, the given system of equations has unique solution.*

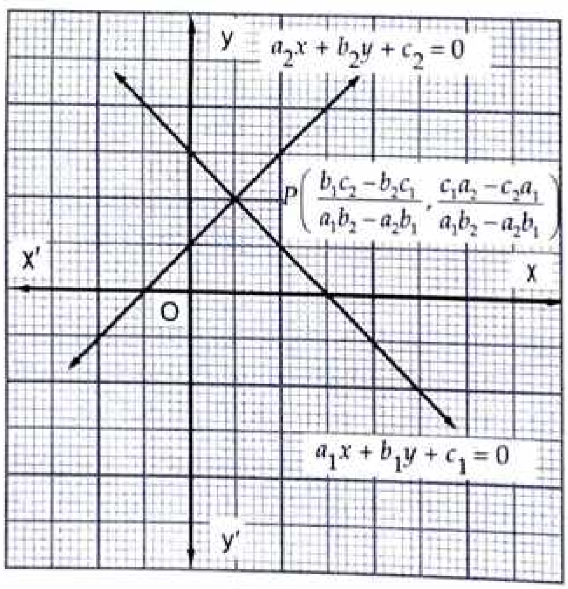

Fig. 3.20

CASE II *When* $a_1b_2 - a_2b_1 = 0$ *i.e.,* $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Let $\frac{a_1}{a_2} = \frac{b_1}{b_2} = k$. *Then,* $a_1 = ka_2$ and $b_1 = kb_2$

Putting $a_1 = ka_2$ and $b_1 = kb_2$ *in equation (i), we get*

$$k(a_2x + b_2y) + c_1 = 0 \quad \ldots(v)$$

From equation (ii), we have

$$a_2x + b_2y = -c_2$$

Putting $a_2x + b_2y = -c_2$ *in equation (v), we get*

$$-kc_2 + c_1 = 0 \Rightarrow c_1 = kc_2$$

Thus, we have

$$a_1 = ka_2, b_1 = kb_2 \text{ and } c_1 = kc_2$$

Substituting the values of a_1, b_1, c_1 in equation (i), we obtain

$$k(a_2x + b_2y + c_2) = 0$$

Thus, the system of equations reduces to

$$k(a_2x + b_2y + c_2) = 0$$

$$a_2x + b_2y + c_2 = 0$$

Clearly, every solution of first of these two equations is a solution of the other and vice-versa.

Thus, in this case, the system has infintely many solutions.

It follows from this that the system of equations $a_1x + b_1y + c_1 = 0$ and, $a_2x + b_2y + c_2 = 0$ is consistent with infinitely many solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

REMARK 2 *If* $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

is a system of simultaneous linear equations with $a_1b_2 - a_2b_1 = 0$ and $a_1c_2 - a_2c_1 = 0$ i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the lines represented by the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident i.e. the two equations represent the same line. Consequently, every point on the line determines a solution of the system. Hence, the system of equations has infinitely many solutions.

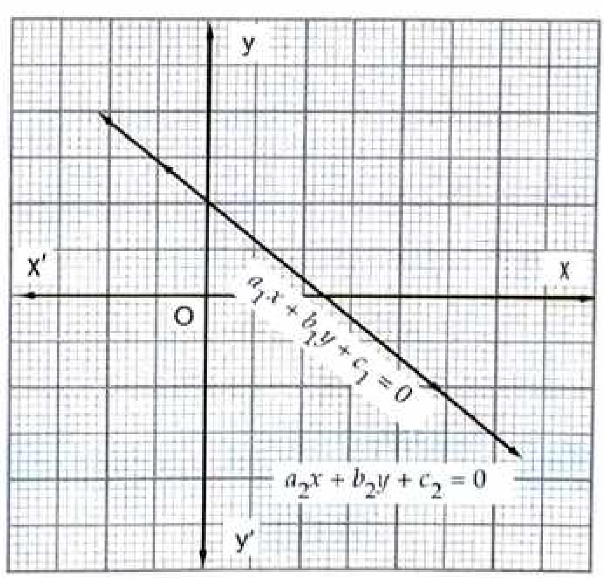

Fig. 3.21

CASE III *When* $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

In this case, it is clear that the given system of equations is inconsistent i.e. it has no solution.

REMARK 3 *If* $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

is a system of simultaneous linear equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, *then the lines represented by the equations* $a_1x + b_1y + c_1 = 0$ *and* $a_2x + b_2y + c_2 = 0$ *are parallel and non-coincident. Consequently, the system has no solution or it is inconsistent.*

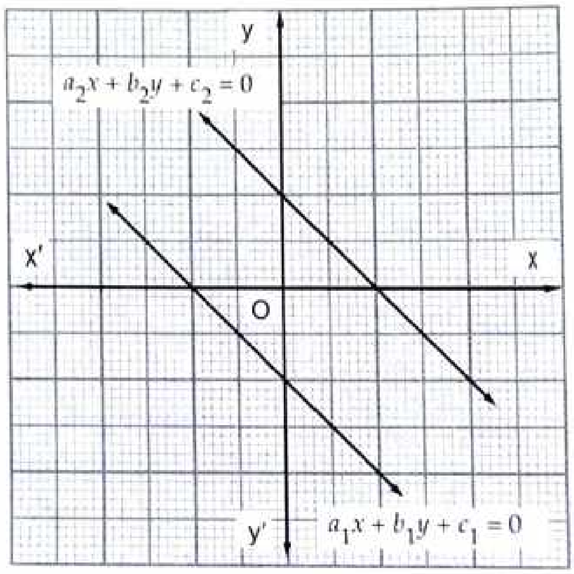

Fig. 3.22

SUMMARY *The above results can be summarised as under:*

The system of equations

$$a_1x + b_1y + c_1 = 0 \quad \ldots\text{(i)}$$

$$a_2x + b_2y + c_2 = 0 \quad \ldots\text{(ii)}$$

(i) *is consistent with unique solution, if* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., lines represented by equations (i) and (ii) are not parallel

(ii) *is consistent with infinitely many solutions, if* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., line represented by equation (i) and (ii) are coincident.

(iii) *is inconsistent, if* $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., lines represented by equations (i) and (ii) are parallel and non-coincident.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it.*

(i) $2x + 3y = 7$
$6x + 5y = 11$

(ii) $6x + 5y = 11$
$9x + \frac{15}{2}y = 21$

(iii) $-3x + 4y = 5$
$\frac{9}{2}x - 6y + \frac{15}{2} = 0$

SOLUTION (i) The given system of equations may be written as

$$2x + 3y - 7 = 0$$

$$6x + 5y - 11 = 0$$

The given system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2, b_1 = 3, c_1 = -7$ and $a_2 = 6, b_2 = 5, c_2 = -11$

We have, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{3}{5}$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given system of equations has unique solution.

To find the solution, we use the cross-multiplication method.
By cross-multiplication, we have

$$\frac{x}{3 \times -11 - 5 \times -7} = \frac{-y}{2 \times -11 - 6 \times -7} = \frac{1}{2 \times 5 - 6 \times 3}$$

$$\Rightarrow \quad \frac{x}{-33 + 35} = \frac{-y}{-22 + 42} = \frac{1}{10 - 18}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{y}{-20} = \frac{1}{-8}$$

$$\Rightarrow \quad x = \frac{-2}{8} = -\frac{1}{4} \text{ and } y = \frac{-20}{-8} = \frac{5}{2}$$

Hence, the given system of equations has unique solution given by $x = -\frac{1}{4}, y = \frac{5}{2}$.

(ii) The given system of equations may be written as

$$6x + 5y - 11 = 0$$
$$9x + \frac{15}{2}y - 21 = 0$$

The given system of equations is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 6, b_1 = 5, c_1 = -11, a_2 = 9, b_2 = \frac{15}{2}, c_2 = -21$

We have, $\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{5}{15/2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations has no solution i.e. it is in-consistent.

(iii) The given system of equations may be written as

$$-3x + 4y - 5 = 0$$
$$\frac{9}{2}x - 6y + \frac{15}{2} = 0$$

The given system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = -3, b_1 = 4, c_1 = -5$ and $a_2 = \frac{9}{2}, b_2 = -6, c_2 = \frac{15}{2}$

We have, $\frac{a_1}{a_2} = \frac{-3}{9/2} = \frac{-2}{3}, \frac{b_1}{b_2} = \frac{4}{-6} = \frac{-2}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{15/2} = \frac{-2}{3}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given system of equations has infinitely many solutions.

EXAMPLE 2 *For each of the following systems of equations determine the value of k for which the given system of equations has a unique solution:*

(i) $x - ky = 2$
$3x + 2y = -5$

(ii) $2x - 3y = 1$
$kx + 5y = 7$

(iii) $2x + 3y - 5 = 0$
$kx - 6y - 8 = 0$

(iv) $2x + ky = 1$
$5x - 7y = 5$

SOLUTION (i) The given system of equations is

$$x - ky - 2 = 0$$
$$3x + 2y + 5 = 0$$

This system of equation is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 1, b_1 = -k, c_1 = -2$ and $a_2 = 3, b_2 = 2, c_2 = 5$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{1}{3} \neq \frac{-k}{2} \Rightarrow k \neq \frac{-2}{3}$$

So, the given system of equations will have unique solution for all real values of k other than $-2/3$.

(ii) The given system of equations is

$$2x - 3y - 1 = 0$$
$$kx + 5y - 7 = 0$$

It is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2, b_1 = -3, c_1 = -1$ and $a_2 = k, b_2 = 5, c_2 = -7$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{2}{k} \neq \frac{-3}{5} \Rightarrow k \neq \frac{-10}{3}$$

So, the given system of equations is consistent with unique solution for all values of k other than $-10/3$.

(iii) The given system of equations is

$$2x + 3y - 5 = 0$$
$$kx - 6y - 8 = 0$$

It is of the form $a_1x + b_1y + c_1 = 0$

and, $a_2x + b_2y + c_2 = 0$

where, $a_1 = 2, b_1 = 3, c_1 = -5$ and $a_2 = k,\ b_2 = -6$ and $c_2 = -8$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{2}{k} \neq \frac{3}{-6} \Rightarrow k \neq -4$$

So, the given system of equations will have unique solution for all real values of k other than –4.

(iv) The given system of equations is

$$2x + ky - 1 = 0$$
$$5x - 7y - 5 = 0$$

It is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2, b_1 = k, c_1 = -1$ and $a_2 = 5, b_2 = -7$ and $c_2 = -5$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{2}{5} \neq \frac{k}{-7} \Rightarrow k \neq \frac{-14}{5}$$

So, the given system of equations will have unique solution for all real values of k other than $\frac{-14}{5}$.

EXAMPLE 3 *For each of the following systems of equations determine the value of k for which the given system of equations has infinitely many solutions.*

(i) $5x + 2y = k$
$10x + 4y = 3$

(ii) $(k - 3)x + 3y = k$
$kx + ky = 12$

(iii) $kx + 3y = k - 3$
$12x + ky = k$

[NCERT]

SOLUTION (i) The given system of equations is

$$5x + 2y - k = 0$$
$$10x + 4y - 3 = 0$$

This system of equation is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 5, b_1 = 2, c_1 = -k$ and $a_2 = 10, b_2 = 4$ and $c_2 = -3$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \text{ i.e., } \frac{5}{10} = \frac{2}{4} = \frac{-k}{-3} \Rightarrow \frac{1}{2} = \frac{k}{3} \Rightarrow k = \frac{3}{2}$$

Hence, the given system of equations will have infinitely many solutions, if $k = \frac{3}{2}$.

(ii) The given system of equations is

$$(k - 3)x + 3y - k = 0$$
$$kx + ky - 12 = 0$$

For infinitely many solutions, we must have

$$\frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$$

$$\Rightarrow \quad \frac{k-3}{k} = \frac{3}{k} \text{ and } \frac{3}{k} = \frac{k}{12}$$

$$\Rightarrow \quad k^2 - 3k = 3k \text{ and } k^2 = 36$$

$$\Rightarrow \quad k^2 - 6k = 0 \text{ and } k^2 = 36$$

$$\Rightarrow \quad (k = 0 \text{ or, } k = 6) \text{ and } (k = \pm 6)$$

$$\Rightarrow \quad k = 6$$

Hence, the given system has infinitely many solutions, if $k = 6$.

(iii) The given system of equations is

$$kx + 3y - (k-3) = 0$$

$$12x + ky - k = 0$$

For infinitely many solutions, we must have

$$\frac{k}{12} = \frac{3}{k} = \frac{-(k-3)}{-k}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} \text{ and } \frac{3}{k} = \frac{k-3}{k}$$

$$\Rightarrow \quad k^2 = 36 \text{ and } k^2 - 3k = 3k$$

$$\Rightarrow \quad k^2 = 36 \text{ and } k^2 - 6k = 0$$

$$\Rightarrow \quad (k = \pm 6) \text{ and } (k = 0 \text{ or, } k = 6)$$

$$\Rightarrow \quad k = 6$$

Hence, the given system of equations has infinitely many solutions, if $k = 6$.

EXAMPLE 4 *For each of the following system of equations determine the values of k for which the given system has no solution:*

(i) $3x - 4y + 7 = 0$
$kx + 3y - 5 = 0$

(ii) $2x - ky + 3 = 0$
$3x + 2y - 1 = 0$

SOLUTION (i) The given system of equations is

$$3x - 4y + 7 = 0$$

$$kx + 3y - 5 = 0$$

This is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0,$$

where, $a_1 = 3, b_1 = -4, c_1 = 7$ and $a_2 = k, b_2 = 3, c_2 = -5$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have, $\frac{b_1}{b_2} = \frac{-4}{3}$ and $\frac{c_1}{c_2} = \frac{-7}{5}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system will have no solution.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = \frac{-4}{3} \Rightarrow k = \frac{-9}{4}$

Clearly, for this value of k, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system of equations has no solution, when $k = \frac{-9}{4}$

(ii) The given system of equations is

$$2x - ky + 3 = 0$$
$$3x + 2y - 1 = 0$$

This is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2, b_1 = -k, c_1 = 3$ and $a_2 = 3, b_2 = 2, c_2 = -1$

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have, $\frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{3}{-1}$

Clearly, $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \text{ i.e., } \frac{2}{3} = \frac{-k}{2} \Rightarrow k = \frac{-4}{3}$$

Hence, the given system of equations will have no solution, if $k = \frac{-4}{3}$.

EXAMPLE 5 *Find the value(s) of k for which the system of equations*

$$kx - y = 2$$
$$6x - 2y = 3$$

has (i) a unique solution (ii) no solution.

Is there a value of k for which the system has infinitely many solutions?

SOLUTION The given system of equations is

$$kx - y - 2 = 0$$
$$6x - 2y - 3 = 0$$

It is of the form $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0,$$

where $a_1 = k, b_1 = -1, c_1 = -2$ and $a_2 = 6, b_2 = -2, c_2 = -3$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., if } \frac{k}{6} \neq \frac{-1}{-2} \text{ i.e., } k \neq 3.$$

So, the given of equations system will have a unique solution, if $k \neq 3$.

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have, $\frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{k}{6} = \frac{-1}{-2} \Rightarrow k = 3$$

Hence, the given system will have no solution, if $k = 3$.

For the given system to have infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We have, $\frac{a_1}{a_2} = \frac{k}{6}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, whatever be the value of k, we cannot have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, there is no value of k, for which the given system of equations has infinitely many solutions.

EXAMPLE 6 *For what value of k will the equations* $x + 2y + 7 = 0, 2x + ky + 14 = 0$ *represent coincident lines?*

SOLUTION The given equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = 1, b_1 = 2, c_1 = 7$ and $a_2 = 2, b_2 = k, c_2 = 14$

The given equations will represent coincident lines if they have infinitely many solutions. The condition for which is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{2} = \frac{2}{k} = \frac{7}{14} \Rightarrow k = 4$$

Hence, the given system of equations will represent coincident lines, if $k = 4$.

EXAMPLE 7 *For what value of k, will the following system of equations have infinitely many solutions?*

$$2x + 3y = 4$$
$$(k + 2)x + 6y = 3k + 2$$

SOLUTION We know that the system of equations

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

has infinitely many solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given system of equations will have infinitely many solutions, if

$$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

$$\Rightarrow \quad \frac{2}{k+2} = \frac{3}{6} \text{ and } \frac{3}{6} = \frac{4}{3k+2}$$

$$\Rightarrow \quad \frac{2}{k+2} = \frac{1}{2} \text{ and } \frac{1}{2} = \frac{4}{3k+2}$$

$$\Rightarrow \quad k + 2 = 4 \text{ and } 3k + 2 = 8$$

$$\Rightarrow \quad k = 2 \text{ and } k = 2$$

$$\Rightarrow \quad k = 2$$

Hence, the given system of equations will have infinitely many solutions, if $k = 2$.

EXAMPLE 8 *Determine the values of a and b for which the following system of linear equations has infinite solutions:*

$$2x - (a-4)\,y = 2b + 1$$
$$4x - (a-1)\,y = 5b - 1$$

SOLUTION We know that the system of equations

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

has infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given system of equations will have infinite number of solutions, if

$$\frac{2}{4} = \frac{-(a-4)}{-(a-1)} = \frac{2b+1}{5b-1}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{a-4}{a-1} = \frac{2b+1}{5b-1}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{a-4}{a-1} \text{ and } \frac{1}{2} = \frac{2b+1}{5b-1}$$

$$\Rightarrow \quad a - 1 = 2a - 8 \text{ and } 5b - 1 = 4b + 2$$

$$\Rightarrow \quad a = 7 \text{ and } b = 3$$

Hence, the given system of equations will have infinitely many solutions, if $a = 7$ and $b = 3$.

EXAMPLE 9 *For what value of k will the following system of linear equations has no solution?*

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

[NCERT, CBSE 2000]

SOLUTION We know that the system of equations

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

has no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system of equations will have no solution, if

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \text{ and } \frac{1}{k-1} \neq \frac{1}{2k+1}$$

Now,

$$\frac{3}{2k-1} = \frac{1}{k-1} \Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 2$$

Clearly, for $k = 2$ we have $\frac{1}{k-1} \neq \frac{1}{2k+1}$

Hence, the given system of equations will have no solution, if $k = 2$.

EXAMPLE 10 *Find the value of k for which the following system of linear equations has infinite solutions:*

$$x + (k + 1)y = 5$$

$$(k + 1)x + 9y = 8k - 1$$

[CBSE 2002C]

SOLUTION The given system of equations will have infinite solutions, if

$$\frac{1}{k+1} = \frac{k+1}{9} = \frac{5}{8k-1}$$

$$\Rightarrow \quad \frac{1}{k+1} = \frac{k+1}{9} \text{ and } \frac{k+1}{9} = \frac{5}{8k-1}$$

$$\Rightarrow \quad (k+1)^2 = 9 \text{ and } (k+1)(8k-1) = 45$$

Now, $(k+1)^2 = 9$

$$\Rightarrow \quad k + 1 = \pm 3 \Rightarrow k = 2, -4$$

We observe that $k = 2$ satisfies the equation $(k+1)(8k-1) = 45$ but, $k = -4$ does not satisfy it.

Hence, the given system of equations will have infinitely many solutions, if $k = 2$.

EXAMPLE 11 *Find the values of p and q for which the following system of equations has infinite number of solutions:*

$$2x + 3y = 7$$

$$(p + q)x + (2p - q)y = 21$$

[CBSE 2001]

SOLUTION The given system of equations will have infinite number of solutions, if

$$\frac{2}{p+q} = \frac{3}{2p-q} = \frac{7}{21}$$

$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{2p-q} = \frac{1}{3}$

$\Rightarrow \quad \frac{2}{p+q} = \frac{1}{3}$ and $\frac{3}{2p-q} = \frac{1}{3}$

$\Rightarrow \quad p + q = 6$ and $2p - q = 9$

$\Rightarrow \quad (p+q) + (2p-q) = 6 + 9$ [On adding]

$\Rightarrow \quad 3p = 15$

$\Rightarrow \quad p = 15$

Putting $p = 5$ in $p + q = 6$ or, $2p - q = 9$, we get $q = 1$.

Hence, the given system of equations will have infinitely many solutions, if $p = 5$ and $q = 1$.

EXAMPLE 12 *For what value of k, will the system of equations*

$x + 2y = 5$

$3x + ky - 15 = 0.$

has (i) a unique solution? (ii) no solution? [CBSE 2001]

SOLUTION The given system of equations can be written as

$x + 2y = 5$

$3x + ky = 15$

(i) The above system of equations will have a unique solution, if

$\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

(ii) The above system of equations will have no solution, if

$\frac{1}{3} = \frac{2}{k} \neq \frac{5}{15}$ $\left[\text{Using: } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right]$

$\Rightarrow \quad \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \left(\frac{1}{3}\right)$

$\Rightarrow \quad k = 6$ and $k \neq 6$, which is not possible.

Hence, there is no value of k for which the given system of equations has no solution.

EXAMPLE 13 *Find the values of α and β for which the following system of linear equations has infinite number of solutions:*

$2x + 3y = 7$

$2\alpha x + (\alpha + \beta) y = 28$

has (i) a unique solution? (ii) no solution? [CBSE 2001]

SOLUTION The given system of equations will have infinite number of solutions, if

$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$

$\Rightarrow \quad \frac{1}{\alpha} = \frac{3}{\alpha+\beta} = \frac{1}{4}$

$\Rightarrow \quad \frac{1}{\alpha} = \frac{1}{4}$ and $\frac{3}{\alpha+\beta} = \frac{1}{4}$

$\Rightarrow \quad \alpha = 4$ and $\alpha + \beta = 12$

$\Rightarrow \quad \alpha = 4$ and $\beta = 8$

Hence, the given system of equations will have infinitely many solutions, if $\alpha = 4$ and $\beta = 8$.

EXAMPLE 14 *Determine the values of m and n so that the following system of linear equations have infinite number of solutions:*

$$(2m - 1)x + 3y - 5 = 0$$

$$3x + (n - 1)y - 2 = 0$$

SOLUTION The given system of equations will have infinite number of solutions, if

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{-5}{-2}$$

$$\Rightarrow \quad \frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{2m-1}{3} = \frac{5}{2} \text{ and } \frac{3}{n-1} = \frac{5}{2}$$

$$\Rightarrow \quad 4m - 2 = 15 \text{ and } 6 = 5n - 5$$

$$\Rightarrow \quad 4m = 17 \text{ and } 5n = 11$$

$$\Rightarrow \quad m = \frac{17}{4} \text{ and } n = \frac{11}{5}$$

Hence, the given system of equations will have infinite number of solutions, if $m = \frac{17}{4}$ and $n = \frac{11}{5}$.

EXAMPLE 15 *Determine the value of k so that the following linear equations have no solution:*

$$(3k + 1)x + 3y - 2 = 0$$

$$(k^2 + 1)x + (k - 2)y - 5 = 0$$

[CBSE 2001C]

SOLUTION The given system of equations will have no solution, if

$$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{-2}{-5}$$

$$\Rightarrow \quad \frac{3k+1}{k^2+1} = \frac{3}{k-2} \text{ and } \frac{3}{k-2} \neq \frac{2}{5}$$

Now, $\quad \frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$$\Rightarrow \quad (3k + 1)(k - 2) = 3(k^2 + 1)$$

$$\Rightarrow \quad 3k^2 - 5k - 2 = 3k^2 + 3$$

$$\Rightarrow \quad -5k - 2 = 3$$

$$\Rightarrow \quad -5k = 5$$

$$\Rightarrow \quad k = -1$$

Clearly, $\quad \frac{3}{k-2} \neq \frac{2}{5}$ for $k = -1$.

Hence, the given system of equations will have no solution for $k = -1$.

EXERCISE 3.5

LEVEL-1

In each of the following systems of equations determine whether the system has a unique solution, no solution or ininitely many solutions. In case there is a unique solution, find it: (1-4)

1. $x - 3y = 3$
   $3x - 9y = 2$
2. $2x + y = 5$
   $4x + 2y = 10$
3. $3x - 5y = 20$
   $6x - 10y = 40$
4. $x - 2y = 8$
   $5x - 10y = 10$

Find the value of k for which the following system of equations has a unique solution: (5-8)

5. $kx + 2y = 5$
   $3x + y = 1$
6. $4x + ky + 8 = 0$
   $2x + 2y + 2 = 0$ [NCERT]
7. $4x - 5y = k$
   $2x - 3y = 12$
8. $x + 2y = 3$
   $5x + ky + 7 = 0$

Find the value of k for which each of the following systems of equations have infinitely many solution: (9-19)

9. $2x + 3y - 5 = 0$
   $6x + ky - 15 = 0$
10. $4x + 5y = 3$
    $kx + 15y = 9$
11. $kx - 2y + 6 = 0$
    $4x - 3y + 9 = 0$
12. $8x + 5y = 9$
    $kx + 10y = 18$
13. $2x - 3y = 7$
    $(k + 2)x - (2k + 1)y = 3(2k - 1)$
14. $2x + 3y = 2$
    $(k + 2)x + (2k + 1)y = 2(k - 1)$ [CBSE 2000, 2003]
15. $x + (k + 1)y = 4$
    $(k + 1)x + 9y = 5k + 2$ [CBSE 2000C]
16. $kx + 3y = 2k + 1$
    $2(k + 1)x + 9y = 7k + 1$ [CBSE 2000C]
17. $2x + (k - 2)y = k$
    $6x + (2k - 1)y = 2k + 5$ [CBSE 2000C]
18. $2x + 3y = 7$
    $(k + 1)x + (2k - 1)y = 4k + 1$ [CBSE 2001]
19. $2x + 3y = k$
    $(k - 1)x + (k + 2)y = 3k$ [CBSE 2001]

Find the value of k for which the following system of equations has no solution: (20-25):

20. $kx - 5y = 2$
    $6x + 2y = 7$
21. $x + 2y = 0$
    $2x + ky = 5$

22. $3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

23. $2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

24. $2x + ky = 11$

$5x - 7y = 5$

25. $kx + 3y = k - 3$

$12x + ky = 6$ **[NCERT EXEMPLAR]**

26. For what value of k the following system of equations will be inconsistent?

$$4x + 6y = 11$$
$$2x + ky = 7$$

27. For what value of α, the system of equations

$$\alpha x + 3y = \alpha - 3$$
$$12x + \alpha y = \alpha$$

[CBSE 2003, 2009]

will have no solution?

28. Find the value of k for which the system

$$kx + 2y = 5$$
$$3x + y = 1$$

has (i) a unique solution, and (ii) no solution.

29. Prove that there is a value of $c\ (\neq 0)$ for which the system

$$6x + 3y = c - 3$$
$$12x + cy = c$$

has infinitely many solutions. Find this value.

30. Find the values of k for which the system

$$2x + ky = 1$$
$$3x - 5y = 7$$

will have (i) a unique solution, and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

31. For what value of k, the following system of equations will represent the coincident lines?

$$x + 2y + 7 = 0$$
$$2x + ky + 14 = 0$$

32. Obtain the condition for the following system of linear equations to have a unique solution

$$ax + by = c$$
$$lx + my = n$$

33. Determine the values of a and b so that the following system of linear equations have infinitely many solutions:

$$(2a - 1)x + 3y - 5 = 0$$
$$3x + (b - 1)y - 2 = 0$$

[CBSE 2001C]

34. Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x - 3y = 7$

$(a + b)x - (a + b - 3)y = 4a + b$ [CBSE 2002]

35. Find the values of p and q for which the following system of linear equations has infinite number of solutions:

$2x + 3y = 9$

$(p + q)x + (2p - q)y = 3(p + q + 1)$ [CBSE 2002]

36. Find the values of a and b for which the following system of equations has infinitely many solutions:

(i) $(2a - 1)x - 3y = 5$
$3x + (b - 2)y = 3$
[CBSE 2002C]

(ii) $2x - (2a + 5)y = 5$
$(2b + 1)x - 9y = 15$
[CBSE 2002C]

(iii) $(a - 1)x + 3y = 2$
$6x + (1 - 2b)y = 6$
[CBSE 2002C]

(iv) $3x + 4y = 12$
$(a + b)x + 2(a - b)y = 5a - 1$
[CBSE 2002C]

(v) $2x + 3y = 7$
$(a - b)x + (a + b)y = 3a + b - 2$
[NCERT]

(vi) $2x + 3y - 7 = 0$
$(a - 1)x + (a + 1)y = (3a - 1)$
[CBSE 2010]

(vii) $2x + 3y = 7$
$(a - 1)x + (a + 2)y = 3a$
[CBSE 2010]

(viii) $x + 2y = 1$
$(a - b)x + (a + b)y = a + b - 2$
[NCERT EXEMPLAR]

(ix) $2x + 3y = 7$
$2ax + ay = 28 - by$
[NCERT EXEMPLAR]

37. For which value(s) of λ, do the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?
(ii) infinitely many solutions?
(iii) a unique solution? [NCERT EXEMPLAR]

ANSWERS

1. No solution
2. Infinitely many solutions
3. Infinitely many solutions
4. No solution
5. $k \neq 6$
6. $k \neq 4$
7. k is any real number
8. $k \neq 10$
9. $k = 9$
10. $k = 12$
11. $k = 8/3$
12. $k = 16$
13. $k = 4$
14. $k = 4$
15. $k = 2$
16. $k = 2$
17. $k = 5$
18. $k = 5$
19. $k = 7$
20. $k = -15$
21. $k = 4$
22. $k = -\frac{9}{4}$
23. $k = -\frac{4}{3}$
24. $k = -\frac{14}{5}$
25. $k = -6$
26. $k = 3$

27. $\alpha = -6$ 28. (i) $k \neq 6$ (ii) $k = 6$ 29. $c = 6$

30. (i) $k \neq \frac{-10}{3}$ (ii) $k = \frac{-10}{3}$, No 31. $k = 4$ 32. $am \neq bl$

33. $a = \frac{17}{4}, b = \frac{11}{5}$ 34. $a = -5, b = -1$ 35. $p = 5/3, q = 1/3$

36. (i) $a = 3, b = \frac{1}{5}$ (ii) $a = -1, b = \frac{5}{2}$ (iii) $a = 3, b = -4$ (iv) $a = 5, b = 1$

(v) $a = 5, b = 1$ (vi) $a = 5$ (vii) $a = 7$ (viii) $a = 3, b = 1$ (ix) $a = 4, b = 8$

37. (i) $\lambda = -1$ (ii) $\lambda = 1$ (iii) All real values of λ except ± 1.

3.7 APPLICATIONS TO WORD PROBLEMS

In this section, we shall learn about some applications of simultaneous linear equations in solving problems related to our day-to-day life. There is a wide variety of such problems which are generally called 'word problems'. In solving such problems, we may use the following algorithm.

ALGORITHM

STEP I *Read the problem carefully and identify the unknown quantities. Give these quantities a variable name like x, y, u, v, w etc.*

STEP II *Identify the variables to be determined.*

STEP III *Read the problem carefully and formulate the equations in terms of the variables to be determined.*

STEP IV *Solve the equations obtained in step III using any one of the methods learnt earlier.*

3.7.1 APPLICATIONS TO PROBLEMS BASED ON ARTICLES AND THEIR COSTS

Following examples will illustrate the applications of linear equations in solving word problems based on articles and their costs.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *4 chairs and 3 tables cost ₹2100 and 5 chairs and 2 tables cost ₹1750. Find the cost of a chair and a table separately.*

SOLUTION Let the cost of a chair be ₹ x and that of a table be ₹ y. Then,

$$4x + 3y = 2100$$

and, $$5x + 2y = 1750$$

This system of equations can be written as

$$4x + 3y - 2100 = 0$$

$$5x + 2y - 1750 = 0$$

By using cross-multiplication, we have

$$\frac{x}{3 \times -1750 - 2 \times -2100} = \frac{-y}{4 \times -1750 - 5 \times -2100} = \frac{1}{4 \times 2 - 3 \times 5}$$

$$\Rightarrow \quad \frac{x}{-5250+4200} = \frac{-y}{-7000+10500} = \frac{1}{8-15}$$

$$\Rightarrow \quad \frac{x}{-1050} = \frac{y}{-3500} = \frac{1}{-7}$$

$$\Rightarrow \quad x = \frac{-1050}{-7} = 150 \text{ and } y = \frac{-3500}{-7} = 500.$$

EXAMPLE 2 *37 pens and 53 pencils together cost ₹320, while 53 pens and 37 pencils together cost ₹400. Find the cost of a pen and that of a pencil.*

SOLUTION Let the cost of a pen be ₹ x and that of a pencil be ₹ y. Then,

$$37x + 53y = 320 \quad \ldots(i)$$

and, $$53x + 37y = 400 \quad \ldots(ii)$$

Adding equations (i) and (ii), we get

$$90x + 90y = 720 \Rightarrow x + y = 8 \quad \ldots(iii)$$

Subtracting equation (i) and (ii), we get

$$16x - 16y = 80 \Rightarrow x - y = 5 \quad \ldots(iv)$$

Adding equations (iii) and (iv), we get

$$2x = 13 \Rightarrow x = 6.5$$

Substituting $x = 6.5$ in equation (iii), we get

$$y = (8 - 6.5) = 1.5$$

Hence, cost of one pen = ₹ 6.50 and cost of one pencil = ₹ 1.50.

EXAMPLE 3 *2 tables and 3 chairs together cost ₹2000 whereas 3 tables and 2 chairs together cost ₹2500. Find the total cost of 1 table and 5 chairs*

SOLUTION Let the cost of a table be ₹ x and that of a chair be ₹ y. Then,

$$2x + 3y = 2000$$

and, $$3x + 2y = 2500$$

This system of equations may be written as

$$2x + 3y - 2000 = 0$$

$$3x + 2y - 2500 = 0$$

By using cross-multiplication, we get

$$\frac{x}{3 \times -2500 - 2 \times -2000} = \frac{-y}{2 \times -2500 - 3 \times -2000} = \frac{1}{2 \times 2 - 3 \times 3}$$

$$\Rightarrow \quad \frac{x}{-7500+4000} = \frac{-y}{-5000+6000} = \frac{1}{4-9}$$

$$\Rightarrow \quad \frac{x}{-3500} = \frac{y}{-1000} = \frac{1}{-5}$$

$$\Rightarrow \quad x = \frac{-3500}{-5} = 700 \text{ and } y = \frac{-1000}{-5} = 200$$

$\therefore$ Cost of a table = ₹ 700 and, cost of a chair = ₹ 200

Hence, cost of one table and 5 chairs = ₹ $(x + 5y)$ = ₹ 1700.

LEVEL-2

EXAMPLE 4 *A and B each have certain number of oranges. A says to B, "if you give me 10 of your oranges, I will have twice the number of oranges left with you." B replies, "if you give me 10 of your oranges, I will have the same number of oranges as left with you." Find the number of oranges with A and B separately.*

SOLUTION Suppose A has x oranges and B has y oranges.

According to the given conditions, we have

$$x + 10 = 2(y - 10) \Rightarrow x - 2y + 30 = 0 \quad \ldots(i)$$

and, $$y + 10 = x - 10 \Rightarrow x - y - 20 = 0 \quad \ldots(ii)$$

Subtracting equation (ii) from equation (i), we get

$$-y + 50 = 0 \Rightarrow y = 50$$

Putting $y = 50$ in equation (i), we get $x = 70$

Hence, A has 70 oranges and B has 50 oranges.

EXAMPLE 5 *A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totalling ₹11.25, how many coins of each kind does he have?*

SOLUTION Let the number of 20 paisa coins be x and that of 25 paisa coins be y. Then,

$$x + y = 50 \quad \ldots(i)$$

Total value of 20 paisa coins = $20x$ paisa

Total value of 25 paisa coins = $25y$ paisa

$\therefore$ $$20x + 25y = 1125$$

$\Rightarrow$ $$4x + 5y = 225 \quad \ldots(ii)$$

Thus, we get the following system of linear equations

$$x + y - 50 = 0$$

$$4x + 5y - 225 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-225 + 250} = \frac{-y}{-225 + 200} = \frac{1}{5 - 4}$$

$\Rightarrow$ $$\frac{x}{25} = \frac{y}{25} = \frac{1}{1} \Rightarrow x = 25 \text{ and } y = 25$$

Hence, there are 25 coins of each kind.

EXERCISE 3.6

LEVEL-1

1. 5 pens and 6 pencils together cost ₹ 9 and 3 pens and 2 pencils cost ₹ 5. Find the cost of 1 pen and 1 pencil.
2. 7 audio cassettes and 3 video cassettes cost ₹ 1110, while 5 audio cassettes and 4 video cassettes cost ₹ 1350. Find the cost of an audio cassette and a video cassette.
3. Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then nuimber of pencils would become 4 times the number of pens. Find the original number of pens and pencils.
4. 4 tables and 3 chairs, together, cost ₹ 2,250 and 3 tables and 4 chairs cost ₹ 1950. Find the cost of 2 chairs and 1 table.

5. 3 bags and 4 pens together cost ₹ 257 whereas 4 bags and 3 pens together cost ₹ 324. Find the total cost of 1 bag and 10 pens.

6. 5 books and 7 pens together cost ₹ 79 whereas 7 books and 5 pens together cost ₹ 77. Find the total cost of 1 book and 2 pens.

7. Jamila sold a table and a chair for ₹ 1050, thereby making a profit of 10% on a table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹ 1065. Find the cost price of each. [NCERT EXEMPLAR]

8. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investment in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme? [NCERT EXEMPLAR]

9. The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, he buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball. [NCERT]

10. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day. [NCERT]

11. The cost of 4 pens and 4 pencil boxes is ₹ 100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box. [NCERT EXEMPLAR]

LEVEL-2

12. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital? [NCERT]

13. A and B each have a certain number of mangoes. A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have?

14. Vijay had some bananas, and he divided them into two lots A and B. He sold first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 per five bananas, his total collection would have been ₹ 460. Find the total number of bananas he had. [NCERT EXEMPLAR]

15. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains ₹ 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains ₹ 1500 on the transaction. Find the actual prices of T.V. and fridge.

ANSWERS

1. Cost of one pen = ₹ 3/2, Cost of one pencil = ₹ 1/4
2. ₹ 30, ₹ 300
3. No. of pens = 13, No. of pencils = 27
4. ₹ 750
5. ₹ 155
6. ₹ 20
7. Table ₹ 500, chair ₹ 400
8. ₹ 10000 in scheme A, ₹ 12000 in scheme B
9. Bat ₹ 500, ball ₹ 50

10. ₹ 15, ₹ 3 11. ₹ 10, ₹ 15 12. ₹ 40, ₹ 170

13. $A:34, B:62$ 14. 500 15. ₹ 20,000, ₹ 10,000

HINTS TO SELECTED PROBLEMS

1. Let the cost of each pen be x and that of each pencil be ₹ y. Then, we have $5x + 6y = 9$ and $3x + 2y = 5$.
2. Let the cost of an audio cassette be ₹ x and that of a video cassette be ₹ y. Then we have $7x + 3y = 1110$ and $5x + 4y = 1350$.
7. Let the cost price of a table and that of a chair be x and y respectively. Then,

CASE I S.P. *of a table* $= ₹\left(x + \frac{10}{100}x\right) = ₹\frac{110}{100}x$

S.P. *of a chair* $= ₹\left(y + \frac{25}{100}y\right) = ₹\frac{125}{100}y$

$$\therefore \quad \frac{110}{100}x + \frac{125}{100}y = 1050 \Rightarrow 110x + 125y = 105000 \quad \text{...(i)}$$

CASE II S.P. *of a table* $= ₹\left(x + \frac{25}{100}x\right) = ₹\frac{125}{100}x$

S.P. *of a chair* $= ₹\left(y + \frac{10}{100}y\right) = ₹\frac{110}{100}y$

$$\therefore \quad \frac{125}{100}x + \frac{110}{100}y = 1065 \Rightarrow 125x + 110y = 106500 \quad \text{...(ii)}$$

Adding (i) and (ii), we obtain

$$235(x + y) = 211500 \Rightarrow x + y = 900 \quad \text{...(iii)}$$

Subtracting (i) from (ii), we obtain

$$15(x - y) = 1500 \Rightarrow x - y = 100 \quad \text{...(iv)}$$

Solving these two equations, we get

$$x = 500, y = 400$$

8. Suppose she invested ₹ x in Scheme A and ₹ y in Scheme B. Then,

$$\frac{8x}{100} + \frac{9y}{100} = 1860 \text{ and } \frac{8y}{100} + \frac{9x}{100} = 1880 \Rightarrow 8x + 9y = 186000 \text{ and } 9x + 8y = 188000$$

Adding and subtracting these two equations, we obtain

$$17(x + y) = 374000 \text{ and } -x + y = 2000$$

$$\Rightarrow x + y = 22000 \text{ and } -x + y = 2000$$

$$\Rightarrow x = 10000, y = 12000$$

10. Let the fixed charge be of ₹ x and the extra charge for each day be ₹ y. Then,

$x + 4y = 27$ and $x + 2y = 21$

14. Let there be x bananas in lot A and y bananas in lot B. Then,

$$\frac{2}{3}x + y = 400 \text{ and } x + \frac{4}{5}y = 460 \Rightarrow 2x + 3y - 1200 = 0 \text{ and } 5x + 4y = 2300$$

Solving these two equations, we get $x = 300$ and $y = 200$

15. Let the price of a T.V. be ₹ x and that of a fridge be ₹ y. Then, we have

$$\frac{5x}{100} + \frac{10y}{100} = 2000 \text{ and, } \frac{10x}{100} - \frac{5y}{100} = 1500$$

3.7.2 APPLICATIONS TO PROBLEMS BASED ON NUMBERS

Following examples, will illustrate the applications of linear equations in solving word problems based on numbers. Recall that the two digit number having a and b as units and ten's digits respectively is equal to $10b + a$ and the number obtained by reversing the order of digits is $10a + b$.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Sum of two numbers is 35 and their difference is 13. Find the numbers.*

SOLUTION Let the two numbers be x and y. Then,

$x + y = 35$ ...(i)

and, $x - y = 13$ ...(ii)

Adding equations (i) and (ii), we get

$2x = 48 \Rightarrow x = 24$

Subtracting equation (ii) from equation (i), we get

$2y = 22 \Rightarrow y = 11$

Hence, the two numbers are 24 and 11.

EXAMPLE 2 *In a two digit number, the unit's digit is twice the ten's digit. If 27 is added to the number, the digits interchange their places. Find the number*

SOLUTION (i) Let the digit in the unit's place be x and digit in the ten's place be y. Then,

$x = 2y$ [Given] ...(i)

and, Number $= 10y + x$

Number obained by reversing the digits $= 10x + y$

It is given that the digits interchange their places if 27 is added to the number.

i.e., Number + 27 = Number obtained by interchanging the digits

$\therefore$ $10y + x + 27 = 10x + y$

$\Rightarrow$ $9x - 9y = 27$

$\Rightarrow$ $x - y = 3$ ...(ii)

Putting $x = 2y$ in equation (ii), we get

$2y - y = 3 \Rightarrow y = 3$

Putting $y = 3$ in equation (ii), we get

$$x = 6$$

Hence, the number is $10y + x = 10 \times 3 + 6 = 36$

EXAMPLE 3 *In a two digit number, the ten's digit is three times the unit's digit. When the number is decreased by 54, the digits are reversed. Find the number.*

SOLUTION Let the digit in the unit's place be x and the digit in the ten's place be y. Then,

$$\text{Number} = 10y + x$$

According to the given condition, we have

$$y = 3x \qquad \text{...(i)}$$

Number obtained by reversing the digits $= 10x + y$

If the number is decreased by 54, the digits are reversed.

$\therefore$ Number − 54 = Number obtained by reversing the digits

$\Rightarrow \quad 10y + x - 54 = 10x + y$

$\Rightarrow \quad 9x - 9y = -54 \Rightarrow x - y = -6 \qquad \text{...(ii)}$

Putting $y = 3x$ in equation (ii), we get

$$x - 3x = -6 \Rightarrow x = 3$$

Putting $x = 3$ in $y = 3x$, we get $y = 9$

Hence, number $= 10y + x = 10 \times 9 + 3 = 93$

EXAMPLE 4 *The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.*

SOLUTION Let the digit at unit's place be x and the digit at ten's place be y. Then,

Number $= 10y + x$

Number formed by reversing the digits $= 10y + x$

According to the given conditions, we have

$$x + y = 8 \qquad \text{...(i)}$$

and, $\quad (10y + x) - (10x + y) = 18$

$\Rightarrow \quad 9(y - x) = 18$

$\Rightarrow \quad y - x = 2 \qquad \text{...(ii)}$

On solving equations (i) and (ii), we get $x = 3, y = 5$

Hence, number $= 10y + x = 10 \times 5 + 3 = 53$

EXAMPLE 5 *The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.*

SOLUTION Let the digit in the unit's place be x and the digit at the ten's place be y. Then,

$$\text{Number} = 10y + x$$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have

$$(10y + x) + (10x + y) = 121$$

$\Rightarrow \quad 11(x + y) = 121$

$\Rightarrow \quad x + y = 11$

and, $\quad x - y = \pm 3 \qquad [\because \text{Difference of digits is } 3]$

Thus, we have the following sets of simultaneous equations

$$\left.\begin{aligned} x + y &= 11 \quad \ldots\text{(i)} \\ \text{and,} \quad x - y &= 3 \quad \ldots\text{(ii)} \end{aligned}\right\} \quad \text{or,} \quad \left\{\begin{aligned} x + y &= 11 \quad \ldots\text{(iii)} \\ x - y &= -3 \quad \ldots\text{(iv)} \end{aligned}\right.$$

On solving equation (i) and (ii), we get $x = 7, y = 4$

On solving equations (iii) and (iv), we get $x = 4, y = 7$

When $x = 7, y = 4$, we have

$$\text{Number} = 10y + x = 10 \times 4 + 7 = 47$$

When $x = 4, y = 7$, we have

$$\text{Number} = 10y + x = 10 \times 7 + 4 = 74$$

Hence, the required number is either 47 or, 74.

EXAMPLE 6 *The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.* **[CBSE 2002C]**

SOLUTION Let the digits at units and tens place in the given number be x and y respectively. Then,

$$\text{Number} = 10y + x \quad \ldots\text{(i)}$$

Number formed by interchanging the digits $= 10x + y$

According to the given conditions, we have

$$(10y + x) + (10x + y) = 110$$

and, $(10y + x) - 10 = 5(x + y) + 4$

$\Rightarrow \quad 11x + 11y = 110$

and, $4x - 5y + 14 = 0$

$\Rightarrow \quad x + y - 10 = 0$

and, $4x - 5y + 14 = 0$

By using cross-multiplication, we have

$$\frac{x}{14 - 50} = \frac{y}{-40 - 14} = \frac{1}{-5 - 4}$$

$$\Rightarrow \quad \frac{x}{-36} = \frac{y}{-54} = \frac{1}{-9}$$

$$\Rightarrow \quad x = \frac{-36}{-9} \text{ and } y = \frac{-54}{-9}$$

$$\Rightarrow \quad x = 4 \text{ and } y = 6.$$

Putting the values of x and y in equation (i), we get

$$\text{Number} = 10 \times 6 + 4 = 64$$

EXAMPLE 7 *The sum of a two digit number and the number formed by interchanging the digit is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.* **[CBSE 2002C]**

SOLUTION Let the digits at units and tens place in the given number be x and y respectively. Then,

$$\text{Number} = 10y + x \quad \ldots\text{(i)}$$

Number formed by interchanging the digits $= 10x + y$

According to the given conditions, we have

$$(10y + x) + (10x + y) = 132$$

and, $(10y + x) + 12 = 5(x + y)$

$\Rightarrow$ $11x + 11y = 132$

and, $4x - 5y = 12$

$\Rightarrow$ $x + y - 12 = 0$

and, $4x - 5y - 12 = 0$

Solving these two equations by cross-multiplication, we have

$$\frac{x}{-12 - 60} = \frac{y}{-48 + 12} = \frac{1}{-5 - 4}$$

$\Rightarrow$ $\dfrac{x}{-72} = \dfrac{y}{-36} = \dfrac{1}{-9}$

$\Rightarrow$ $x = \dfrac{-72}{-9}$ and $y = \dfrac{-36}{-9}$

$\Rightarrow$ $x = 8$ and $y = 4$

Substituting the values of x and y in equation (i), we have

$$\text{Number} = 10 \times 4 + 8 = 48.$$

EXAMPLE 8 *The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.* **[CBSE 2002]**

SOLUTION Let the digits at units and tens place of the given number be x and y respectively. Then,

$$\text{Number} = 10y + x \quad \text{...(i)}$$

Number obtained by reversing the order of the digits $= 10x + y$

According to the given conditions, we have

$$(10y + x) + (10x + y) = 165$$

and, $x - y = 3$ or, $y - x = 3$

$\Rightarrow$ $11x + 11y = 165$

and, $x - y = 3$ or, $y - x = 3$

$\Rightarrow$ $x + y = 15$

and, $x - y = 3$ or, $y - x = 3$

Thus, we obtain the following systems of linear equations.

(i) $x + y = 15$

$x - y = 3$

(ii) $x + y = 15$

$y - x = 3$

Solving first system of equations, we get

$$x = 9, y = 6$$

Solving second system of equations, we get

$$x = 6, y = 9$$

Substituting the values of x and y in equation (i), we have

Number = 69 or, 96.

LEVEL-2

EXAMPLE 9 *A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number.*

SOLUTION Let the digit at units place be x and the digit at ten's place be y. Then,

$$\text{Number} = 10y + x$$

According to the given conditions, we have

$$10y + x = 8(x + y) + 1 \Rightarrow 7x - 2y + 1 = 0$$

and, $$10y + x = 13(y - x) + 2 \Rightarrow 14x - 3y - 2 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-2 \times -2 - (-3) \times 1} = \frac{-y}{7 \times -2 - 14 \times 1} = \frac{1}{7 \times -3 - 14 \times -2}$$

$$\Rightarrow \quad \frac{x}{4 + 3} = \frac{-y}{-14 - 14} = \frac{1}{-21 + 28}$$

$$\Rightarrow \quad \frac{x}{7} = \frac{y}{28} = \frac{1}{7}$$

$$\Rightarrow \quad x = \frac{7}{7} = 1 \text{ and } y = \frac{28}{7} = 4$$

Hence, the number $= 10y + x = 10 \times 4 + 1 = 41$.

REMARK *In the above example, if we take the difference of the digits as $x - y$, then we get fractional values of x and y which are not admissible.*

EXAMPLE 10 *If three times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers.*

SOLUTION Let the larger number be x and smaller one be y. We know that

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder} \quad \ldots(i)$$

When $3x$ is divided by y, we get 4 as quotient and 3 as remainder. Therefore, by using (i), we get

$$3x = 4y + 3 \Rightarrow 3x - 4y - 3 = 0 \quad \ldots(ii)$$

When $7y$ is divided by x, we get 5 as quotient and 1 as remainder. Therefore, by using (i), we get

$$7y = 5x + 1 \Rightarrow 5x - 7y + 1 = 0 \quad \ldots(iii)$$

Solving equations (ii) and (iii), by cross-multiplication, we get

$$\frac{x}{-4 - 21} = \frac{-y}{3 + 15} = \frac{1}{-21 + 20} \Rightarrow x = 25 \text{ and } y = 18$$

Hence, the required numbers are 25 and 18.

EXERCISE 3.7

LEVEL-1

1. The sum of two numbers is 8. If their sum is four times their difference, find the numbers.
2. The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

3. A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.
4. The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number. [CBSE 2004]
5. The sum of a two-digit number and the number formed by reversing the order of digits is 66. If the two digits differ by 2, find the number. How many such numbers are there? [NCERT]
6. The sum of two numbers is 1000 and the difference between their squares is 256000. Find the numbers.
7. The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number. [CBSE 2002]
8. A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number. [CBSE 2001C]
9. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number. [CBSE 2001C]
10. A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number. [CBSE 2001C]
11. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number. [CBSE 2005]
12. A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number. [CBSE 2005]
13. The difference between two numbers is 26 and one number is three times the other. Find them.
14. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number. [NCERT]

LEVEL-2

15. Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.
16. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers. [NCERT EXEMPLAR]
17. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number. [NCERT EXEMPLAR]

ANSWERS

1. 5, 3	2. 49	3. 23	4. 78	5. 42 or, 24	6. 628, 372
7. 63 or, 36	8. 24	9. 35	10. 64	11. 36	12. 45
13. 39, 13	14. 18	15. 36	16. 40, 48	17. 83	

HINTS TO SELECTED PROBLEMS

1. Let the numbers be x and y. Then, we have

$x + y = 8$ and $x + y = 4(x - y)$.

6. Let, the large number be x, and, the smaller number be y. Then,

$x + y = 1000$ and $x^2 - y^2 = 256000$.

Now, $x^2 - y^2 = 256000$

$$\Rightarrow (x+y)(x-y) = 256000 \Rightarrow x - y = \frac{256000}{x+y} \Rightarrow x - y = \frac{256000}{1000} = 256.$$

3.7.3 APPLICATION TO PROBLEMS BASED ON FRACTIONS

Folloging examples will illustrate applications of simultaneous linear equations in solving word problems on fractions.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *A fraction becomes 4/5, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fraction becomes 1/2. What is the fraction?*

SOLUTION Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\frac{x+1}{y+1} = \frac{4}{5} \text{ and } \frac{x-5}{y-5} = \frac{1}{2}$$

$$\Rightarrow \quad 5x + 5 = 4y + 4 \text{ and } 2x - 10 = y - 5$$

$$\Rightarrow \quad 5x - 4y + 1 = 0 \text{ and } 2x - y - 5 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-4 \times -5 - (-1) \times 1} = \frac{-y}{5 \times -5 - 2 \times 1} = \frac{1}{5 \times -1 - 2 \times -4}$$

$$\Rightarrow \quad \frac{x}{20+1} = \frac{y}{25+2} = \frac{1}{-5+8}$$

$$\Rightarrow \quad \frac{x}{21} = \frac{y}{27} = \frac{1}{3} \Rightarrow x = \frac{21}{3} = 7 \text{ and } y = \frac{27}{3} = 9$$

Hence, the given fraction is 7/9.

LEVEL-2

EXAMPLE 2 *A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get 18/11, but if the numerator is increased by 8 and the denominator is doubled, we get 2/5. Find the fraction.*

SOLUTION Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\frac{3x}{y-3} = \frac{18}{11} \text{ and } \frac{x+8}{2y} = \frac{2}{5}$$

$$\Leftrightarrow \quad 11x = 6y - 18 \text{ and } 5x + 40 = 4y$$

$$\Leftrightarrow \quad 11x - 6y + 18 = 0 \text{ and } 5x - 4y + 40 = 0$$

By cross-multiplication, we have

$$\frac{x}{(-6)\times 40-(-4)\times 18}=\frac{-y}{11\times 40-5\times 18}=\frac{1}{11\times(-4)-5\times(-6)}$$

$$\Rightarrow \quad \frac{x}{-240+72}=\frac{-y}{440-90}=\frac{1}{-44+30}$$

$$\Rightarrow \quad \frac{x}{-168}=\frac{y}{-350}=\frac{1}{-14}$$

$$\Rightarrow \quad x=\frac{-168}{-14} \text{ and } y=\frac{-350}{-14}$$

$$\Rightarrow \quad x=12 \text{ and } y=25$$

Hence, the fraction is $\frac{12}{25}$.

EXAMPLE 3 *The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.* [CBSE 2001C]

SOLUTION Let the numerator and denominator of the fraction be x and y respectively. Then,

$$\text{Fraction}=\frac{x}{y}$$

It is given that

$$\text{Denominator}=2\,(\text{Numerator})+4$$

$$\Rightarrow \quad y=2x+4$$

$$\Rightarrow \quad 2x-y+4=0$$

According to the given condition, we have

$$y-6=12(x-6)$$

$$\Rightarrow \quad y-6=12x-72$$

$$\Rightarrow \quad 12x-y-66=0$$

Thus, we have the following system of equations

$$2x-y+4=0 \quad \ldots(i)$$

$$12x-y-66=0 \quad \ldots(ii)$$

Subtracting equation (i) from equation (ii), we get

$$10x-70=0 \Rightarrow x=7$$

Putting $x=7$ in equation (i), we get

$$14-y+4=0 \Rightarrow y=18$$

Hence, required fraction $=\frac{7}{18}$.

EXERCISE 3.8

LEVEL-1

1. The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

2. A fraction becomes 9/11 if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction. [NCERT]
3. A fraction becomes 1/3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/2. Find the fraction.
4. If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction? [NCERT]
5. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 1/2. Find the fraction. [CBSE 2006C]
6. When 3 is added to the denominator and 2 is subtracted from the numerator a fraction becomes 1/4. And, when 6 is added to numerator and the denominator is multiplied by 3, it becomes 2/3. Find the fraction.
7. The sum of a numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to 1/3. Find the fraction.
8. If 2 is added to the numerator of a fraction, it reduces to 1/2 and if 1 is subtracted from the denominator, it reduces to 1/3. Find the fraction.

LEVEL-2

9. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. [CBSE 2001C, 2010]
10. If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes 6/5. And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes 2/5. Find the fraction.
11. The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction. [CBSE 2001C, 2010]

ANSWERS

1. $\frac{3}{7}$ 2. $\frac{7}{9}$ 3. $\frac{3}{7}$ 4. $\frac{3}{5}$ 5. $\frac{5}{7}$ 6. $\frac{4}{5}$
7. $\frac{5}{13}$ 8. $\frac{3}{10}$ 9. $\frac{5}{9}$ 10. $\frac{12}{25}$ 11. $\frac{4}{7}$

3.7.4 APPLICATIONS TO PROBLEMS ON AGES

Following examples will illustrate the use of solutions of simultaneous linear equations in solving word problems on ages.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.*

SOLUTION Suppose father's age (in years) be x and that of son's be y. Then,

$$x + 2y = 70$$

and, $$2x + y = 95$$

This system of equations may be written as

$$x + 2y - 70 = 0$$

$$2x + y - 95 = 0$$

By cross-multiplication, we get

$$\frac{x}{2 \times -95 - (-70)} = \frac{-y}{1 \times -95 - 2 \times -70} = \frac{1}{1 \times 1 - 2 \times 2}$$

$$\Rightarrow \quad \frac{x}{-190 + 70} = \frac{-y}{-95 + 140} = \frac{1}{-3}$$

$$\Rightarrow \quad \frac{x}{-120} = \frac{y}{-45} = \frac{1}{-3} \Rightarrow x = \frac{-120}{-3} = 40 \text{ and } y = \frac{-45}{-3} = 15$$

Hence, father's age is 40 years and the son's age is 15 years.

EXAMPLE 2 *I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son?*

SOLUTION Suppose my age is x years and my son's age is y years. Then,

$$x = 3y \quad \text{...(i)}$$

Five years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$$\therefore \quad x + 5 = \frac{5}{2}(y + 5) \quad \text{[Given]}$$

$$\Rightarrow \quad 2x - 5y - 15 = 0 \quad \text{...(ii)}$$

Putting $x = 3y$ in equation (ii), we get

$$6y - 5y - 15 = 0 \Rightarrow y = 15$$

Putting $y = 15$ in equation (i), we get

$$x = 45$$

Hence, my present age is 45 years and my son's present age is 15 years.

EXAMPLE 3 *Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.*

SOLUTION Let the present ages of father and son be x years and y years respectively.

Ten years ago, Father's age $= (x - 10)$ years

Son's age $= (y - 10)$ years

$$\therefore \quad x - 10 = 12(y - 10) \Rightarrow x - 12y + 110 = 0 \quad \text{...(i)}$$

Ten years later, Father's age $= (x + 10)$ years.

Son's age $= (y + 10)$

$$\therefore \quad x + 10 = 2(y + 10) \Rightarrow x - 2y - 10 = 0 \quad \text{...(ii)}$$

Subtracting (ii) from (i), we get

$$-10y + 120 = 0 \Rightarrow 10y = 120 \Rightarrow y = 12$$

Putting $y = 12$ in (i), we get

$$x - 144 + 110 = 0 \Rightarrow x = 34$$

Thus, present age of father is 34 years and the present age of son is 12 years.

EXAMPLE 4 *Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.* [NCERT]

SOLUTION Let the present age of father be x years and the present age of son be y years.

Five years hence, Father's age = $(x + 5)$ years

Son's age = $(y + 5)$ years

Using the given information, we have

$$x + 5 = 3(y + 5) \Rightarrow x - 3y - 10 = 0 \quad \ldots(i)$$

Five years ago, Father's age = $(x - 5)$ years

Son's age = $(y - 5)$ years

Using the given information, we get

$$(x - 5) = 7(y - 5) \Rightarrow x - 7y + 30 = 0 \quad \ldots(ii)$$

Subtracting equation (ii) from equation (i), we get

$$4y - 40 = 0 \Rightarrow y = 10$$

Putting $y = 10$ in equation (i), we get

$$x - 30 - 10 = 0 \Rightarrow x = 40$$

Hence, present age of father is 40 years and present age of son is 10 years.

LEVEL-2

EXAMPLE 5 *A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.*

SOLUTION Let the ages of A and B be x and y years respectively. Then,

$$x - y = \pm 2 \quad \text{[Given]}$$

D's age = $2x$ years. and, C's age = $\frac{y}{2}$ years.

Clearly, D is older than C

$$\therefore \quad 2x - \frac{y}{2} = 40 \Rightarrow 4x - y = 80$$

Thus, we have the following two systems of linear equations

$$x - y = 2 \quad \ldots(i)$$

and, $$4x - y = 80 \quad \ldots(ii)$$

$$x - y = -2 \quad \ldots(iii)$$

and, $$4x - y = 80 \quad \ldots(iv)$$

Subtracting equation (i) from equation (ii), we get

$$3x = 78 \Rightarrow x = 26$$

Putting $x = 26$ in equation (i), we get $y = 24$

Subtracting equation (iv) from equation (iii), we get

$$-3x = -82 \Rightarrow x = \frac{82}{3} = 27\frac{1}{3}$$

Putting $x = \frac{82}{3}$ in equation (iii), we get

$$y = \frac{82}{3} + 2 = \frac{88}{3} = 29\frac{1}{3}$$

Hence, A's age = 26 years and B's age = 24 years

or,

A's age $= 27\frac{1}{3}$ years and B's age $= 29\frac{1}{3}$ years.

EXERCISE 3.9

LEVEL-1

1. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.
2. Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B ?
3. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu? [NCERT]
4. Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.
5. Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.

LEVEL-2

6. The present age of a father is three years more than three times the age of the son. Three years hence father's age will be 10 years more than twice the age of the son. Determine their present ages.
7. A father is three times as old as his son. In 12 years time, he will be twice as old as his son. Find the present ages of father and the son.
8. Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father. [CBSE 2003]
9. Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son. [CBSE 2004]
10. A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old as his sister S. If the ages of the father and sister differ by 40 years, find the age of A.
11. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju as twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju. [NCERT]
12. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now? [NCERT EXEMPLAR]
13. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father. [NCERT EXEMPLAR]

ANSWERS

1. Father's age = 36 years, Son's age = 12 years
2. A's present age = 50 years, B's present age = 20 years.
3. Nuri's age = 50 years, Sonu's age = 20 years

4. Man's age = 30 years, Son's age = 6 years
5. Father's age = 34 years, Son's age = 12 years
6. Father's age = 33 years, Son's age = 10 years
7. Father's age = 36 years, Son's age = 12 years
8. 45 years
9. Father's age = 42 years, Son's age = 10 years
10. 26 years
11. Ani's age = 19 years, Biju's age = 16 years
12. Salim's age = 38 years, Daughter's age = 14 years
13. 40 years

3.7.5 APPLICATION TO PROBLEMS BASED ON TIME DISTANCE AND SPEED

In solving problems based on time, distance and speed, we use the following formulae:

$$\text{Distance} = \text{Speed} \times \text{Time}; \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}} \text{ and, } \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Also, if

Speed of a boat in still water = u km/hr and, Speed of the current = v km/hr

then,

Speed upstream = $(u - v)$ km/hr

Speed downstream = $(u + v)$ km/hr

Following examples will illustrate the use of these formulae.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction they meet in 9 hours and if they go in opposite directions they meet in 9/7 hours. Find their speeds.*

SOLUTION Let X and Y be two cars starting from points A and B respectively. Let the speed of car X be x km/hr and that of car Y be y km/hr.

CASE I *When two cars move in the same directions:*

Suppose two cars meet at point Q. Then,

Distance travelled by car $X = AQ$,

Distance travelled by car $Y = BQ$.

It is given that two cars meet in 9 hours.

$\therefore$ Distance travelled by car X in 9 hours $= 9x$ km.

$\Rightarrow \quad AQ = 9x$

Distance travelled by car y in 9 hours $= 9y$ km.

$\Rightarrow \quad BQ = 9y$

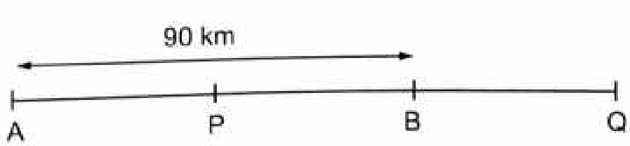

Fig. 3.23

Clearly, $\quad AQ - BQ = AB$

$\Rightarrow \quad 9x - 9y = 90$ [$\because AB = 90$ km]

$\Rightarrow \quad x - y = 10$ ...(i)

CASE II *When two cars move in opposite directions:*

Suppose two cars meet at point P. Then,

Distance travelled by car $X = AP$,

Distance travelled by car $Y = BP$.

In this case, two cars meet in 9/7 hours.

$\therefore$ Distance travelled by car X in $\frac{9}{7}$ hours $= \frac{9}{7}x$ km

$\Rightarrow \quad BP = \frac{9}{7}y$

Clearly, $AP + BP = AB$

$\Rightarrow \quad \frac{9}{7}x + \frac{9}{7}y = 90$

$\Rightarrow \quad \frac{9}{7}(x + y) = 90$

$\Rightarrow \quad x + y = 70$ ...(ii)

Solving equations (i) and (ii), we get

$x = 40$ and $y = 30$.

Hence, speed of car X is 40 km/hr and speed of car Y is 30 km/hr.

EXAMPLE 2 *Ved travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and the rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car.* [NCERT]

SOLUTION Let the speed of the train be x km/hr and the speed of the car be y km/hr.

CASE I *When he travels 120 km by train and the rest by car.*

If Ved travels 120 km by train, then

Distance covered by car is (600 – 120) km = 480 km.

Now, Time taken to cover 120 km by train $= \frac{120}{x}$ hrs $\left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

Time taken to cover 480 km by car $= \frac{480}{y}$ hrs

It is given that the total time of the journey is 8 hours.

$$\therefore \quad \frac{120}{x} + \frac{480}{y} = 8$$

$$\Rightarrow \quad 8\left(\frac{15}{x} + \frac{60}{y}\right) = 8$$

$$\Rightarrow \quad \frac{15}{x} + \frac{60}{y} = 1$$

$$\Rightarrow \quad \frac{15}{x} + \frac{60}{y} - 1 = 0 \quad \text{...(i)}$$

CASE II *When he travels 200 km by train and the rest by car*

If Ved travels 200 km by train, then

Distance travelled by car is (600 – 200) km = 400 km

Now, Time taken to cover 200 km by train $= \frac{200}{x}$ hrs

Time taken to cover 400 km by train $= \frac{400}{y}$ hrs

In this case the total time of journey is 8 hour 20 minutes

$$\therefore \quad \frac{200}{x} + \frac{400}{y} = 8 \text{ hrs } 20 \text{ minutes}$$

$$\Rightarrow \quad \frac{200}{x} + \frac{400}{y} = 8\frac{1}{3} \qquad \left[\because 8 \text{ hrs } 20 \text{ minutes} = 8\frac{20}{60} \text{ hrs} = 8\frac{1}{3} \text{ hrs}\right]$$

$$\Rightarrow \quad \frac{200}{x} + \frac{400}{y} = \frac{25}{3}$$

$$\Rightarrow \quad 25\left(\frac{8}{x} + \frac{16}{y}\right) = \frac{25}{3}$$

$$\Rightarrow \quad \frac{8}{x} + \frac{16}{y} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1$$

$$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} - 1 = 0 \qquad \ldots(ii)$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (i) and (ii), we get

$$15u + 60v - 1 = 0 \qquad \ldots(iii)$$

$$24u + 48v - 1 = 0 \qquad \ldots(iv)$$

By using cross-multiplication, we have

$$\frac{u}{60 \times -1 - 48 \times -1} = \frac{-v}{15 \times -1 - 24 \times -1} = \frac{1}{15 \times 48 - 24 \times 60}$$

$$\Rightarrow \quad \frac{u}{-60 + 48} = \frac{-v}{-15 + 24} = \frac{1}{720 - 1440}$$

$$\Rightarrow \quad \frac{u}{-12} = \frac{v}{-9} = \frac{1}{-720}$$

$$\Rightarrow \quad u = \frac{-12}{-720} = \frac{1}{60} \text{ and } v = \frac{-9}{-720} = \frac{1}{80}$$

Now, $u = \frac{1}{x} \Rightarrow \frac{1}{60} = \frac{1}{x} \Rightarrow x = 60$

and, $v = \frac{1}{y} \Rightarrow \frac{1}{80} = \frac{1}{y} \Rightarrow y = 80$

Hence, speed of train = 60 km/hr and speed of car = 80 km/hr.

EXAMPLE 3 *A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.* **[CBSE 2001]**

SOLUTION Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:

CASE I *When he travels 250 km by train and the rest by car:*

In this case, we have

Time taken by the man to travel 250 km by train $= \frac{250}{x}$ hrs

Time taken by the man to travel $(370-250) = 120$ km by car $= \frac{120}{y}$ hrs

$\therefore$ Total time taken by the man to cover 370 km $= \frac{250}{x} + \frac{120}{y}$

It is given that the total time taken is 4 hours

$$\therefore \quad \frac{250}{x} + \frac{120}{y} = 4$$

$$\Rightarrow \quad \frac{125}{x} + \frac{60}{y} = 2 \qquad \ldots(i)$$

CASE II *When he travels 130 km by train and the rest by car:*

In this case, we have

Time taken by the man to travel 130 km by train $= \frac{130}{x}$ hrs

Time taken by the man to travel $(370-130) = 240$ km by car $= \frac{240}{y}$ hrs.

In this case, total time of the journey is 4 hrs 18 minutes.

$$\therefore \quad \frac{130}{x} + \frac{240}{y} = 4 \text{ hrs } 18 \text{ minutes}$$

$$\Rightarrow \quad \frac{130}{x} + \frac{240}{y} = 4\frac{18}{60}$$

$$\Rightarrow \quad \frac{130}{x} + \frac{240}{y} = \frac{43}{10} \qquad \ldots(ii)$$

Thus, we obtain the following system of equations:

$$\frac{125}{x} + \frac{60}{y} = 2$$

$$\frac{130}{x} + \frac{240}{y} = \frac{43}{10}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system reduces to

$$125u + 60v = 2 \qquad \ldots(iii)$$

$$130u + 240v = \frac{43}{10} \qquad \ldots(iv)$$

Multiplying equation (iii) by 4 the above system of equations becomes

$$500u + 240v = 8 \quad \text{...(v)}$$

$$130u + 240v = \frac{43}{10} \quad \text{...(vi)}$$

Subtracting equation (vi) from equation (v), we get

$$370u = 8 - \frac{43}{10} \Rightarrow 370u = \frac{37}{10} \Rightarrow u = \frac{1}{100}$$

Putting $u = \frac{1}{100}$ in equation (v), we get

$$5 + 240v = 8 \Rightarrow 240v = 3 \Rightarrow v = \frac{1}{80}$$

Now, $u = \frac{1}{100}$ and $v = \frac{1}{80}$

$\Rightarrow$ $\frac{1}{x} = \frac{1}{100}$ and $\frac{1}{y} = \frac{1}{80}$

$\Rightarrow$ $x = 100$ and $y = 80$.

Hence, Speed of the train = 100 km/hr

Speed of the car = 80 km/hr.

EXAMPLE 4 *A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.*

SOLUTION Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr. Then,

Speed upstream = $(x - y)$ km/hr

Speed downstream = $(x + y)$ km/hr

Now, Time taken to cover 32 km upstream $= \frac{32}{x-y}$ hrs

Time taken to cover 36 km downstream $= \frac{36}{x+y}$ hrs

But, total time of journey is 7 hours.

$$\therefore \quad \frac{32}{x-y} + \frac{36}{x+y} = 7 \quad \text{...(i)}$$

Time taken to cover 40 km upstream $= \frac{40}{x-y}$

Time taken to cover 48 km downstream $= \frac{48}{x+y}$

In this case, total time of journey is given to be 9 hours.

$$\therefore \quad \frac{40}{x-y} + \frac{48}{x+y} = 9 \quad \text{...(ii)}$$

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

$$32u + 36v = 7 \Rightarrow 32u + 36v - 7 = 0 \quad \text{...(iii)}$$

$$40u + 48v = 9 \Rightarrow 40u + 48v - 9 = 0 \quad \ldots(iv)$$

Solving these equations by cross-multiplication, we get

$$\frac{u}{36 \times -9 - 48 \times -7} = \frac{-v}{32 \times -9 - 40 \times -7} = \frac{1}{32 \times 48 - 40 \times 36}$$

$$\Rightarrow \quad \frac{u}{-324 + 336} = \frac{-v}{-288 + 280} = \frac{1}{1536 - 1440}$$

$$\Rightarrow \quad \frac{u}{12} = \frac{v}{8} = \frac{1}{96}$$

$$\Rightarrow \quad u = \frac{12}{96} \text{ and } v = \frac{8}{96}$$

$$\Rightarrow \quad u = \frac{1}{8} \text{ and } v = \frac{1}{12}$$

Now, $u = \frac{1}{8} \Rightarrow \frac{1}{x - y} = \frac{1}{8} \Rightarrow x - y = 8$ ... (v)

and, $v = \frac{1}{12} \Rightarrow \frac{1}{x + y} = \frac{1}{12} \Rightarrow x + y = 12$ ... (vi)

Solving equations (v) and (vi), we get $x = 10$ and $y = 2$.
Hence, Speed of the boat in still water = 10 km/hr
Speed of the stream = 2 km/hr.

LEVEL-2

EXAMPLE 5 *X takes 3 hours more than Y to walk 30 km. But, if X doubles his pace, he is ahead of Y by $1\frac{1}{2}$ hours. Find their speed of walking.*

SOLUTION Let the speed of X and Y be x km/hr and y km/hr respectively. Then,

Time taken by X to cover 30 km $= \frac{30}{x}$ hrs,

and, Time taken by Y to cover 30 km $= \frac{30}{y}$ hrs

By the given conditions, we have

$$\frac{30}{x} - \frac{30}{y} = 3 \Rightarrow \frac{10}{x} - \frac{10}{y} = 1 \quad \ldots(i)$$

If X doubles his pace, then speed of X is $2x$ km/hr

∴ Times taken by X to cover 30 km $= \frac{30}{2x}$ hrs.

Times taken by Y to cover 30 km $= \frac{30}{y}$ hrs

According to the given conditions, we have

$$\frac{30}{y} - \frac{30}{2x} = 1\frac{1}{2}$$

$$\Rightarrow \quad \frac{30}{y} - \frac{30}{2x} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{10}{y} - \frac{5}{x} = \frac{1}{2}$$

$$\Rightarrow \quad -\frac{10}{x} + \frac{20}{y} = 1 \qquad \ldots(ii)$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equations (i) and (ii) we get

$$10u - 10v = 1 \Rightarrow 10u - 10v - 1 = 0 \qquad \ldots(iii)$$

$$-10u + 20v = 1 \Rightarrow -10u + 20v - 1 = 0 \qquad \ldots(iv)$$

Adding equations (iii) and (iv), we get

$$10v - 2 = 0 \Rightarrow v = \frac{1}{5}$$

Putting $v = \frac{1}{5}$ in equation (iii), we get

$$10u - 3 = 0 \Rightarrow u = \frac{3}{10}$$

Now, $u = \frac{3}{10} \Rightarrow \frac{1}{x} = \frac{3}{10} \Rightarrow x = \frac{10}{3}$ and, $v = \frac{1}{5} \Rightarrow \frac{1}{y} = \frac{1}{5} \Rightarrow y = 5$

Hence, X's speed $= \frac{10}{3}$ km/hr and, Y's speed = 5 km/hr.

EXAMPLE 6 *After covering a distance of 30 km with a uniform speed there is some defect in a train engine and therefore, its speed is reduced to 4/5 of its original speed. Consequently, the train reaches its destination late by 45 minutes. Had it happened after covering 18 kilometres more, the train would have reached 9 minutes earlier. Find the speed of the train and the distance of journey.*

SOLUTION Let the original speed of the train be x km/hr and the length of the journey be y km. Then,

Time taken = (y/x) hrs.

CASE I *When defect in the engine occurs after covering a distance of 30 km.*

We have,

Speed for first 30 km = x km/hr

and, Speed for the remaining $(y - 30)$ km $= \frac{4}{5}x$ km/hrs

$\therefore$ Time taken to cover 30 km $= \frac{30}{x}$ hrs

Time taken to cover $(y - 30)$ km $= \frac{y - 30}{(4x/5)}$ hrs. $= \frac{5}{4x}(y - 30)$ hrs.

According to the given condition, we have,

$$\frac{30}{x} + \frac{5}{4x}(y - 30) = \frac{y}{x} + \frac{45}{60}$$

$$\Rightarrow \quad \frac{30}{x} + \frac{5y - 150}{4x} = \frac{y}{x} + \frac{3}{4}$$

$\Rightarrow \quad \frac{120 + 5y - 150}{4x} = \frac{4y + 3x}{4x}$

$\Rightarrow \quad 5y - 30 = 4y + 3x$

$\Rightarrow \quad 3x - y + 30 = 0 \qquad \ldots(i)$

CASE II *When defect in the engine occurs after covering a distance of 48 km.*

Speed for first 48 km = x km/hr.

Speed for the remaining $(y - 48)$ km $= \frac{4x}{5}$ km/hr

$\therefore$ Time taken to cover 48 km $= \frac{48}{x}$ hrs.

Time taken to cover $(y - 48)$ km $= \left(\frac{y - 48}{4x/5}\right)$ hr $= \left\{\frac{5(y - 48)}{4x}\right\}$ hr

According to the given condition, the train now reaches 9 minutes earlier i.e., 36 minutes later.

$$\frac{48}{x} + \frac{5(y - 48)}{4x} = \frac{y}{x} + \frac{36}{60}$$

$\Rightarrow \quad \frac{48}{x} + \frac{5y - 240}{4x} = \frac{y}{x} + \frac{3}{5}$

$\Rightarrow \quad \frac{192 + 5y - 240}{4x} = \frac{5y + 3x}{5x}$

$\Rightarrow \quad \frac{5y - 48}{4} = \frac{5y + 3x}{5}$

$\Rightarrow \quad 25y - 240 = 20y + 12x$

$\Rightarrow \quad 12x - 5y + 240 = 0 \qquad \ldots(ii)$

Thus, we have the following system of simultaneous equations:

$$3x - y + 30 = 0$$

$$12x - 5y + 240 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-240 + 150} = \frac{-y}{720 - 360} = \frac{1}{-15 + 12}$$

$\Rightarrow \quad \frac{x}{-90} = \frac{-y}{360} = \frac{1}{-3}$

$\Rightarrow \quad x = \frac{-90}{-3} = 30$ and $y = \frac{-360}{-3} = 120$

Hence, the original speed of the train is 30 km/hr and the length of the journey is 120 km.

EXAMPLE 7 *A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey.*

SOLUTION Let the actual speed of the train be x km/hr and the actual time taken be y hours. Then,

$$\text{Distance covered} = (xy) \text{ km} \quad \ldots \text{(i)} \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$$

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is $(x+6)$ km/hr, time of journey is $(y-4)$ hours.

$\therefore$ Distance covered $= (x+6)(y-4)$

$\Rightarrow$ $xy = (x+6)(y-4)$ [Using (i)]

$\Rightarrow$ $-4x + 6y - 24 = 0$

$\Rightarrow$ $-2x + 3y - 12 = 0$ ... (ii)

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours i.e., when speed is $(x-6)$ km/hr, time of journey is $(y-6)$ hours.

$\therefore$ Distance covered $= (x-6)(y+6)$

$\Rightarrow$ $xy = (x-6)(y+6)$ [Using (i)]

$\Rightarrow$ $6x - 6y - 36 = 0$

$\Rightarrow$ $x - y - 6 = 0$ ... (iii)

Thus, we obtain the following system of equations:

$$-2x + 3y - 12 = 0$$
$$x - y - 6 = 0$$

By using cross-multiplication, we have,

$$\frac{x}{3 \times -6 - (-1) \times -12} = \frac{-y}{-2 \times -6 - 1 \times -12} = \frac{1}{-2 \times -1 - 1 \times 3}$$

$$\Rightarrow \quad \frac{x}{-30} = \frac{-y}{24} = \frac{1}{-1}$$

$$\Rightarrow \quad x = 30 \text{ and } y = 24.$$

Putting the values of x and y in equation (i), we obtain

$$\text{Distance} = (30 \times 24) \text{ km} = 720 \text{ km}.$$

Hence, the length of the journey is 720 km.

EXERCISE 3.10

LEVEL-1

1. Points A and B are 70 km. a part on a highway. A car starts from A and another car starts from B simulataneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars. **[CBSE 2002]**

2. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

3. The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

4. A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

5. A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

6. A person rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream. [NCERT EXEMPLAR]

7. Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km. by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car. Find the speed of the train and car respectively.

8. A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car. [CBSE 2001]

9. Places A and B are 80 km apart from each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find the speeds of the cars. [CBSE 2002]

10. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

11. Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

12. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current. [NCERT]

13. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream. [NCERT EXEMPLAR]

14. Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi. [CBSE 2006C]

15. A train covered a certain distance at a uniform speed. If the train could have been 10 km/hr. faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train. [NCERT]

16. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of two cars? [NCERT, CBSE 2009]

LEVEL-2

17. While covering a distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking.
18. A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours. Find the speeds of A and B.

ANSWERS

1. Speed of car starting from point $A = 40$ km/hr.
   Speed of car starting from point $B = 30$ km/hr.
2. Speed of sailor = 10 km/hr, speed of current = 2 km/hr.
3. Speed of stream = 3 km/hr, Speed of boat = 8 km/hr.
4. Speed of stream = 4 km/hr, Speed of boat = 10 km/hr.
5. Distance = 36 km, Original speed = 4 km/hr.
6. 2.5 km/hr
7. Train; 80 km/hr, car: 100 km/hr.
8. 100 km/hr, 80 km/hr.
9. 35 km/hr, 25 km/hr.
10. 6 km/hr, 2 km/hr.
11. 60 km/hr, 80 km/hr.
12. 6 km/hr, 4 km/hr.
13. 10 km/hr, 4 km/hr.
14. 100 km/hr, 80 km/hr.
15. 600 km.
16. 60 km/hr, 40 km/hr
17. Ajit's speed = 5 km/hr, Amit's speed = 7.5 km/hr.
18. $\frac{10}{3}$ km/hr, 5 km/hr.

HINT TO SELECTED PROBLEM

5. Let the original speed be y km/hr and total time taken be x hrs. Then,
   Distance = (xy) km.

   Since distance covered in each case is same.

$$\therefore \quad \left(y + \frac{1}{2}\right)(x - 1) = xy \text{ and, } (y - 1)(x + 3) = xy$$

$$\Rightarrow \quad \frac{x}{2} - y - \frac{1}{2} = 0 \text{ and } -x + 3y - 3 = 0.$$

3.7.6 APPLICATIONS TO MISCELLANEOUS PROBLEMS

Following examples will illustrate the applications.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹75 and for a journey of 15 km the charge paid is ₹110. What will a person have to pay for travelling a distance of 25 km?*

[NCERT, CBSE 2000]

SOLUTION Let the fixed charges of taxi be ₹ x and the running charges be ₹ y km/hr.

According to the given condition, we have

$$x + 10y = 75 \quad \dots (i)$$

$$x + 15y = 110 \quad \dots (ii)$$

Subtracting equation (ii) from equation (i), we get

$$-5y = -35 \Rightarrow y = 7$$

Putting $y = 7$ in equation (i), we get $x = 5$.

∴ Total charges from travelling a distance of 25 km $= x + 25y$

$$= ₹\,(5 + 25 \times 7) = ₹\,180.$$

EXAMPLE 2 *The total expenditure per month of a household consists of a fixed rent of the house and mess charges depending upon the number of people sharing the house. The total monthly expenditure is ₹ 3900 for 2 people and ₹ 7500 for 5 people. Find the rent of the house and the mess charges per head per month.*

SOLUTION Let the monthly rent of the house be ₹ x and the mess charges per head per month be ₹ y.

According to the given conditions, we have,

$$x + 2y = 3900 \quad \dots (i)$$

$$x + 5y = 7500 \quad \dots (ii)$$

Subtracting equation (ii) from equation (i), we get

$$-3y = -3600 \Rightarrow y = 1200$$

Putting $y = 1200$ in equation (i), we get $x = 1500$

Hence, monthly rent = ₹ 1500 and mess charges per head per month = ₹ 1200.

EXAMPLE 3 *The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle.* **[NCERT]**

SOLUTION Let the length and breadth of the rectangle be x and y units respectively. Then,

Area = xy sq. units.

If length is reduced by 5 units and the breadth is increased by 3 units, then area is reduced by 9 square units.

$$\therefore \quad xy - 9 = (x - 5)(y + 3)$$

$$\Rightarrow \quad xy - 9 = xy + 3x - 5y - 15$$

$$\Rightarrow \quad 3x - 5y - 6 = 0 \quad \dots (i)$$

When length is increased by 3 units and breadth by 2 units, the area is increased by 67 sq. units.

$$\therefore \quad xy + 67 = (x + 3)(y + 2)$$

$$\Rightarrow \quad xy + 67 = xy + 2x + 3y + 6$$

$$\Rightarrow \quad 2x + 3y - 61 = 0 \quad \dots (ii)$$

Thus, we get the following system of linear equations:

$$3x - 5y - 6 = 0$$

$$2x + 3y - 61 = 0$$

By using cross-multiplication, we have

$$\frac{x}{305+18} = \frac{-y}{-183+12} = \frac{1}{9+10}$$

$$\Rightarrow \quad x = \frac{323}{19} = 17 \text{ and } y = \frac{171}{19} = 9$$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

EXAMPLE 4 *A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹1500 after 4 year of service and ₹1800 after 10 years of service, what was his starting salary and what is the annual increment?*

SOLUTION Let the starting salary of the man be ₹ x and the fixed annual increment be ₹ y. Then,

Salary after 4 years of service = ₹ $(x + 4y)$

Salary after 10 years of service = ₹ $(x + 10y)$

$$\therefore \quad x + 4y = 1500 \quad \ldots \text{(i)}$$

$$x + 10y = 1800 \quad \ldots \text{(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$6y = 300 \Rightarrow y = 50$$

Putting $y = 50$ in equation (i), we get $x = 1300$.

Hence the starting salary was ₹ 1300 and annual increment is ₹ 50.

EXAMPLE 5 *A person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But, if he had interchanged the amounts invested, he would have received ₹4 more as interest. How much amount did he invest at different rates?*

SOLUTION Suppose the person invested ₹ x at the rate of 12% simple interest and ₹ y at the rate of 10% simple interest. Then,

$$\text{Yearly interest} = \frac{12x}{100} + \frac{10y}{100}$$

$$\therefore \quad \frac{12x}{100} + \frac{10y}{100} = 130$$

$$\Rightarrow \quad 12x + 10y = 13000$$

$$\Rightarrow \quad 6x + 5y = 6500 \quad \ldots \text{(i)}$$

If the invested amounts are interchanged, then yearly interest increases by ₹ 4.

$$\therefore \quad \frac{10x}{100} + \frac{12y}{100} = 134$$

$$\Rightarrow \quad 10x + 12y = 13400$$

$$\Rightarrow \quad 5x + 6y = 6700 \quad \ldots \text{(ii)}$$

Subtracting equation (ii) from equation (i), we get

$$x - y = -200 \quad \ldots \text{(iii)}$$

Adding equation (ii) and (i), we get

$$11x + 11y = 13200$$

$$\Rightarrow \quad x + y = 1200 \quad \ldots \text{(iv)}$$

Adding equations (iii) and (iv), we get

$$2x = 1000 \Rightarrow x = 500$$

Putting $x = 500$ in equation (iii), we get $y = 700$

Thus, the person invested ₹ 500 at the rate of 12% per year and ₹ 700 at the rate of 10% per year.

EXAMPLE 6 *The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves ₹200 per month, find their monthly incomes.* **[NCERT]**

SOLUTION Let the income of first person be ₹ $9x$ and the income of second person be ₹ $7x$. Further, let the expenditures of first and second person be $4y$ and $3y$ respectively. Then,

Saving of first person $= 9x - 4y$

Saving of second person $= 7x - 3y$

$\therefore \quad 9x - 4y = 200 \Rightarrow 9x - 4y - 200 = 0$ ... (i)

and, $\quad 7x - 3y = 200 \Rightarrow 7x - 3y - 200 = 0$ ... (ii)

Solving equation (i) and (ii) by cross-multiplication, we have

$$\frac{x}{800 - 600} = \frac{-y}{-1800 + 1400} = \frac{1}{-27 + 28}$$

$\Rightarrow \quad x = 200$ and $y = 400$.

Thus, monthly income of first person $=$ ₹ $9x =$ ₹ $(9 \times 200) =$ ₹ 1800

and, monthly income of second person $=$ ₹ $7x =$ ₹ $(7 \times 200) =$ ₹ 1400

EXAMPLE 7 *In a* ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. *Find the three angles.* **[NCERT]**

SOLUTION Let $\angle A = x°$, $\angle B = y°$. Then,

$$\angle C = 3\angle B \Rightarrow \angle C = 3y°$$

$\therefore \quad 3\angle B = 2(\angle A + \angle B)$

$\Rightarrow \quad 3y = 2(x + y) \Rightarrow y = 2x \Rightarrow 2x - y = 0$ ... (i)

Since $\angle A$, $\angle B$ and $\angle C$ are angles of a triangle.

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad x + y + 3y = 180 \Rightarrow x + 4y = 180$ ... (ii)

Putting $y = 2x$ in equation (ii), we get

$$x + 8x = 180 \Rightarrow 9x = 180 \Rightarrow x = 20°$$

Putting the value of x in equation (i), we get $y = 40°$

Hence, $\angle A = 20° \angle B = 40°$ and $\angle C = 3y° = (3 \times 40°) = 120°$

EXAMPLE 8 *Find the four angles of a cyclic quadrilateral ABCD in which* $\angle A = (2x - 1)°$, $\angle B = (y + 5)°$ $\angle C = (2y + 15)°$ *and* $\angle D = (4x - 7)°$.

SOLUTION We know that the sum of the opposite angles of a cyclic quadrilateral is 180°. In the cyclic quadrilateral $ABCD$, angles A and C and and angles B and D form pairs of opposite angles.

$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow \quad 2x - 1 + 2y + 15 = 180$ and $y + 5 + 4x - 7 = 180$

$\Rightarrow \quad 2x + 2y = 166$ and $4x + y = 182$

$\Rightarrow \quad x + y = 83$ ... (i)

and, $\quad 4x + y = 182$ ... (ii)

Subtracting equation (i) from equation (ii), we get

$$3x = 99 \Rightarrow x = 33$$

Substituting $x = 33$ in equation (i), we get $y = 50$

Hence, $\angle A = (2 \times 33 - 1)^\circ = 65^\circ$, $\angle B = (y + 5)^\circ = (50 + 5)^\circ = 55^\circ$

$\angle C = (2y + 15)^\circ = (2 \times 50 + 15)^\circ = 115^\circ$ and $\angle D = (4 \times 33 - 7)^\circ = 125^\circ$

LEVEL-2

EXAMPLE 9 *A man sold a chair and a table together for ₹1520 thereby making a profit of 25% on the chair and 10% on table. By selling them together for ₹1535 he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.*

SOLUTION Let the cost price of one chair be ₹ x and that of one table be ₹ y. Profit on a chair = 25%.

$\therefore$ Selling price of one chair $= x + \frac{25}{100}x = \frac{125}{100}x$

Profit on a table = 10%

$\therefore$ Selling price of one table $= y + \frac{10y}{100} = \frac{110}{100}y$

According to the given condition, we have

$$\frac{125}{100}x + \frac{110}{100}y = 1520 \Rightarrow 125x + 110y = 152000 \Rightarrow 25x + 22y = 30400 \quad \text{... (i)}$$

If profit on a chair is 10% and on a table is 25%, then total selling price is ₹ 1535.

$$\therefore \quad \left(x + \frac{10}{100}x\right) + \left(y + \frac{25}{100}y\right) = 1535$$

$$\Rightarrow \quad \frac{110}{100}x + \frac{125}{100}y = 1535$$

$$\Rightarrow \quad 110x + 125y = 153500$$

$$\Rightarrow \quad 22x + 25y = 30700 \quad \text{... (ii)}$$

Subtracting equation (ii) from equation (i), we get

$$3x - 3y = -300 \Rightarrow x - y = -100 \quad \text{... (iii)}$$

Adding equation (ii) and (i), we get

$$47x + 47y = 61100 \Rightarrow x + y = 1300 \quad \text{... (iv)}$$

Solving equations (iii) and (iv), we get

$$x = 600 \text{ and } y = 700$$

Hence, the cost price of a chair is ₹ 600 and that of a table is ₹ 700.

EXAMPLE 10 *Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.*

SOLUTION Let the number of students be x and the number of rows be y. Then,

Number of students in each row $= x/y$

When one student is extra in each row, there are 2 rows less i.e., when each row has $\left(\frac{x}{y}+1\right)$ students, the number of rows is $(y-2)$.

$\therefore$ Total number of students = No. of rows × No. of students in each row

$$\Rightarrow \quad x=\left(\frac{x}{y}+1\right)(y-2)$$

$$\Rightarrow \quad x=x-\frac{2x}{y}+y-2 \Rightarrow -\frac{2x}{y}+y-2=0 \qquad \ldots\text{(i)}$$

If one student is less in each row, then there are 3 rows more i.e., when each row has $\left(\frac{x}{y}-1\right)$ students, the number of rows is $(y+3)$

$\therefore$ Total number of students = No. of rows × No. of students in each row

$$\Rightarrow \quad x=\left(\frac{x}{y}-1\right)(y+3) \Rightarrow x=x+\frac{3x}{y}-y-3 \Rightarrow \frac{3x}{y}-y-3=0 \qquad \ldots\text{(ii)}$$

Putting $\frac{x}{y}=u$ in (i) and (ii), we get

$$\Rightarrow \quad -2u+y-2=0 \qquad \ldots\text{(iii)}$$

$$\Rightarrow \quad 3u-y-3=0 \qquad \ldots\text{(iv)}$$

Adding (iii) and (iv), we get

$$\Rightarrow \quad u-5=0 \Rightarrow u=5$$

Putting $u=5$ in (iii), we get $y=12$

Now, $\quad u=5 \Rightarrow \frac{x}{y}=5 \Rightarrow \frac{x}{12}=5 \Rightarrow x=60$

Hence, the number of students in the class is 60.7

EXAMPLE 11 *8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.*

SOLUTION Suppose that one man alone can finish the work in x days and one boy alone can finish it in y days. Then,

One man's one day's work $=\frac{1}{x}$

One boy's one day's work $=\frac{1}{y}$

$\therefore$ Eight men's one day's work $=\frac{8}{x}$

12 boy's one day's work $=\frac{12}{y}$

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x}+\frac{12}{y}\right)=1 \Rightarrow \frac{80}{x}+\frac{120}{y}=1 \qquad \ldots\text{(i)}$$

Again, 6 men and 8 boys can finish the work in 14 days.

$\therefore \quad 14\left(\frac{6}{x}+\frac{8}{y}\right)=1 \Rightarrow \frac{84}{x}+\frac{112}{y}=1$ ... (ii)

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equations (i) and (ii), we get

$$80u+120v-1=0$$
$$84u+112v-1=0$$

By using cross-multiplication, we have

$$\frac{u}{-120+112}=\frac{-v}{-80+84}=\frac{1}{80\times112-120\times84}$$

$$\Rightarrow \quad \frac{u}{-8}=\frac{v}{-4}=\frac{1}{-1120}$$

$$\Rightarrow \quad u=\frac{-8}{-1120}=\frac{1}{140} \text{ and } v=\frac{-4}{-1120}=\frac{1}{280}$$

Now, $\quad u=\frac{1}{140} \Rightarrow \frac{1}{x}=\frac{1}{140} \Rightarrow x=140$

and, $\quad v=\frac{1}{280} \Rightarrow \frac{1}{y}=\frac{1}{280} \Rightarrow y=280.$

Thus, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

EXAMPLE 12 *On selling a tea-set at 5% loss and a lemon-set at 15% gain, a crockery seller gains ₹ 7. If he sells the tea-set at 5% gain and the lemon-set at 10% gain, he gains ₹13. Find the actual price of the tea-set and the lemon-set.*

SOLUTION Let the cost price of the tea-set and the lemon-set be ₹ x and ₹ y respectively.

CASE I *When tea set is sold at 5% loss and lemon-set at 15% gain.*

Loss on tea-set $=₹\,\frac{5x}{100}=₹\,\frac{x}{20}$

Gain on lemon-set $=₹\,\frac{15y}{100}=₹\,\frac{3y}{20}$

$\therefore \quad$ Net gain $=₹\,\frac{3y}{20}-\frac{x}{20}$

$\Rightarrow \quad \frac{3y}{20}-\frac{x}{20}=7 \Rightarrow 3y-x=140 \Rightarrow x-3y+140=0$ ... (i)

CASE II *When tea-set is sold at 5% gain and the lemon-set at 10% gain.*

Gain on tea-set $=₹\,\frac{5x}{100}=₹\,\frac{x}{20}$

Gain on lemon-set $=₹\,\frac{10y}{100}=₹\,\frac{y}{10}$

$\therefore \quad$ Total gain $=₹\,\frac{x}{20}+\frac{y}{10}$

$$\Rightarrow \quad \frac{x}{20} + \frac{y}{10} = 13 \Rightarrow x + 2y = 260 \Rightarrow x + 2y - 260 = 0 \quad \text{...(ii)}$$

Subtracting equation (ii) from equation (i), we get

$$-5y + 400 = 0 \Rightarrow y = 80$$

Putting $y = 80$ in equation (i), we get

$$x - 240 + 140 = 0 \Rightarrow x = 100$$

Hence, cost prices of tea-set and lemon-set are ₹ 100 and ₹ 80 respectively.

EXAMPLE 13 *It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?* **[NCERT EXEMPLAR]**

SOLUTION Let the time taken by the pipes of larger and smaller diameters alone to fill the pool be x hours and y hours respectively. Let the total volume of the pool be V cubic units.

The pipe of larger diameter fills the pool in x hours. This means that in x hours the volume of water that comes out of the pipe of larger diameter is V cubic units.

$\therefore$ In 1 hour volume of the water that comes out of the pipe of larger diameter is $\frac{V}{x}$ cubic units.

So, in four hours, the volume of the water that comes out of the pipe of larger diameter is $\frac{4V}{x}$ cubic units.

Similarly, the volume of the water that comes out of the pipe of smaller diameter in 9 hours is $\frac{9V}{y}$.

It is given that if the pipe of larger diameter is used for 4 hours and that of smaller diameter for 9 hours, only half of the pool can be filled.

$$\therefore \quad \frac{4V}{x} + \frac{9V}{y} = \frac{1}{2}V \Rightarrow \frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad \text{...(i)}$$

If both the pipes are used for 12 hours, they completely fill the tank.

$$\therefore \quad \frac{12V}{x} + \frac{12V}{y} = V \Rightarrow \frac{12}{x} + \frac{12}{y} = 1 \quad \text{...(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in (i) and (ii), we obtain

$$4u + 9v = \frac{1}{2} \quad \text{...(iii)}$$

$$12u + 12v = 1 \quad \text{...(iv)}$$

Multiplying (iii) by 3 and subtracting from (iv), we get

$$-15v = -\frac{1}{2} \Rightarrow v = \frac{1}{30} \Rightarrow \frac{1}{y} = \frac{1}{30} \Rightarrow y = 30$$

Substituting $v = \frac{1}{30}$ in (iii), we get

$$4u + \frac{9}{30} = \frac{1}{2} \Rightarrow 4u = \frac{1}{2} - \frac{9}{30} = \frac{1}{5} \Rightarrow u = \frac{1}{20} \Rightarrow \frac{1}{x} = \frac{1}{20} \Rightarrow x = 20$$

Thus, the pipes of larger and smaller diameters fill the swimming pool alone in 20 hours and 30 hours respectively.

EXERCISE 3.11

LEVEL-1

1. If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.
2. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and breadth is increased by 5 metres. Find the dimensions of the rectangle.
3. In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the rectangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimensions of the rectangle.
4. The incomes of X and Y are in the ratio of $8:7$ and their expenditures are in the ratio $19:16$. If each saves ₹ 1250, find their incomes.
5. A and B each has some money. If A gives ₹ 30 to B, then B will have twice the money left with A. But, if B gives ₹ 10 to A, then A will have thrice as much as is left with B. How much money does each have?
6. $ABCD$ is a cyclic quadrilateral such that $\angle A = (4y + 20)°$, $\angle B = (3y - 5)°$, $\angle C = (4x)°$ and $\angle D = (7x + 5)°$. Find the four angles. **[NCERT]**
7. 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?
8. In a ΔABC, $\angle A = x°$, $\angle B = (3x - 2)°$, $\angle C = y°$. Also, $\angle C - \angle B = 9°$. Find the three angles.
9. In a cyclic quadrilateral $ABCD$, $\angle A = (2x + 4)°$, $\angle B = (y + 3)°$, $\angle C = (2y + 10)°$, $\angle D = (4x - 5)°$. Find the four angles.
10. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test? **[NCERT]**
11. In a ΔABC, $\angle A = x°$, $\angle B = 3x°$ and $\angle C = y°$. If $3y - 5x = 30$, prove that the triangle is right angled.
12. The car hire charges in a city comprise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is ₹ 89 and for a journey of 20 km, the charge paid is ₹ 145. What will a person have to pay for travelling a distance of 30 km? **[CBSE 2000]**

13. A part of monthly hostel charges in a college are fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charge and the cost of food per day. **[NCERT, CBSE 2000]**

14. Half the perimeter of a garden, whose length is 4 more than its width is 36 m. Find the dimensions of the garden.

15. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them. **[NCERT]**

16. 2 Women and 5 men can together finish a piece of embroidery in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the embroidery, and that taken by 1 man alone. **[NCERT]**

17. Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes ₹ 50 and ₹ 100 she received. **[NCERT]**

LEVEL-2

18. There are two examination rooms A and B. If 10 candidates are sent from A to B, the number of students in each room is same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Find the number of students in each room. **[NCERT EXEMPLAR]**

19. A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmedabad costs ₹ 216 and one full and one half reserved first class tickets cost ₹ 327. What is the basic first class full fare and what is the reservation charge? **[NCERT EXEMPLAR]**

20. A wizard having powers of mystic in candations and magical medicines seeing a cock, fight going on, spoke privately to both the owners of cocks. To one he said; if your bird wins, than you give me your stake-money, but if you do not win, I shall give you two third of that'. Going to the other, he promised in the same way to give three fourths. From both of them his gain would be only 12 gold coins. Find the stake of money each of the cock-owners have.

21. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more. Find the number of students in the class.

22. One says, "give me hundred, friend! I shall then become twice as rich as you" The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their respective capital? **[NCERT]**

23. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby getting a sum of ₹ 1008. If she had sold the saree at 10% profit and the sweater at 8% disciount, she would have got ₹ 1028. Find the cost price of the saree and the list price (price before discount) of the sweater. **[NCERT EXEMPLAR]**

24. In a competitive examination, one mark is awarded for each correct answer while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly. **[NCERT EXEMPLAR]**

25. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for 6 days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and charge for each extraday. [NCERT EXEMPLAR]

ANSWERS

1. 253 sq. units
2. Length = 28 m, Breadth = 15 m
3. Length = 28 m, Breadth = 19 m
4. X's income = ₹ 6000, Y's income = ₹ 5250
5. A : ₹ 62, B : ₹ 34
6. $\angle A = 120°, \angle B = 70°, \angle C = 60°, \angle D = 110°$
7. Man: 15 days Boy : 60 days
8. $A = 25°; B = 73°; C = 82°$
9. $A = 70°, B = 53°, C = 110°, D = 127°$
10. 20
12. ₹ 215
13. Fixed charge = ₹ 400; Cost of food per day = ₹ 30
14. Length = 20 m, Width = 16 m
15. 99°, 81°
16. 36 days, 18 days
17. 10, 15
18. 100, 80
19. Fare = ₹ 210, Reservation charge = ₹ 6
20. 42, 40 Gold Coins.
21. 36
22. ₹ 40, ₹ 170
23. ₹ 600, ₹ 400
24. 100
25. ₹ 10, ₹ 3

HINTS TO SELECTED PROBLEMS

10. Let x and y denote the number of right and wrong answers respectively, then
$$3x - y = 40,\ 2x - y = 25$$

19. Suppose basic first class full fare is ₹ x and reservation charge is ₹ y per ticket. Then, $x + y = 216$ and $x + y + (x/2) + y = 327$.

20. Let the stake money of first and second cock-owners be ₹ x and ₹ y respectively. Then, we have
$$x - \frac{3}{4}y = 12 \text{ and } y - \frac{2}{3}x = 12 \Rightarrow \quad 4x - 3y = 48 \text{ and } -2x + 3y = 36$$

22. Let the money with the first person be ₹ x and the money with the second person be ₹ y. Then,
$$x + 100 = 2(y - 100) \text{ and } y + 10 = 6(x - 10).$$

VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

Answer each of the following questions either in one word or one sentence or as per requirement of the questions:

1. Write the value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has no solution.

2. Write the value of k for which the system of equations
$$2x - y = 5$$
$$6x + ky = 15$$
has infinitely many solutions.

3. Write the value of k for which the system of equations $3x - 2y = 0$ and $kx + 5y = 0$ has infinitely many solutions.
4. Write the values of k for which the system of equations $x + ky = 0,\ 2x - y = 0$ has unique solution.
5. Write the set of values of a and b for which the following system of equations has infinitely many solutions.
$$2x + 3y = 7$$
$$2ax + (a + b)\,y = 28$$
6. For what value of k, the following pair of linear equations has infinitely many solutions?
$$10x + 5y - (k - 5) = 0$$
$$20x + 10y - k = 0$$
7. Write the number of solutions of the following pair of linear equations:
$$x + 2y - 8 = 0$$
$$2x + 4y = 16$$ [CBSE 2009]
8. Write the number of solutions of the following pair of linear equations:
$$x + 3y - 4 = 0$$
$$2x + 6y = 7$$

ANSWERS

1. $k = 2$ 2. $k = -3$ 3. $k = \frac{-15}{2}$ 4. $k \neq \frac{-1}{2}$ 5. $a = 4, b = 8$
6. $k = 10$ 7. Infinite 8. 0

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. The value of k for which the system of equations
$kx - y = 2$ and, $6x - 2y = 3$ has a unique solution, is
(a) $= 3$ (b) $\neq 3$ (c) $\neq 0$ (d) $= 0$
2. The value of k for which the system of equations $2x + 3y = 5$ and, $4x + ky = 10$ has infinite number of solutions, is
(a) 1 (b) 3 (c) 6 (d) 0
3. The value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution, is
(a) 10 (b) 6 (c) 3 (d) 1
4. The value of k for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solution, is
(a) 0 (b) 2 (c) 6 (d) 8
5. If the system of equations $2x + 3y = 7$ and, $(a + b)\,x + (2a - b)\,y = 21$ has infinitely many solutions, then
(a) $a = 1, b = 5$ (b) $a = 5, b = 1$ (c) $a = -1, b = 5$ (d) $a = 5, b = -1$
6. If the system of equations $3x + y = 1$ and, $(2k - 1)\,x + (k - 1)\,y = 2k + 1$ is inconsistent, then $k =$
(a) 1 (b) 0 (c) -1 (d) 2

7. If $am \neq bl$, then the system of equations $ax + by = c$ and, $lx + my = n$

(a) has a unique solution
(b) has no solution
(c) has infinitely many solutions
(d) may or may not have a solution.

8. If the system of equations

$2x + 3y = 7$
$2ax + (a + b)y = 28$

has infinitely many solutions, then

(a) $a = 2b$ (b) $b = 2a$ (c) $a + 2b = 0$ (d) $2a + b = 0$

9. The value of k for which the system of equations

$x + 2y = 5$
$3x + ky + 15 = 0$

has no solution is

(a) 6 (b) −6 (c) 3/2 (d) none of these

10. If $2x - 3y = 7$ and $(a + b)x - (a + b - 3)y = 4a + b$ represent coincident lines, then a and b satisfy the equation

(a) $a + 5b = 0$ (b) $5a + b = 0$ (c) $a - 5b = 0$ (d) $5a - b = 0$

11. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are

(a) intersecting
(b) parallel
(c) always coincident
(d) intersecting or coincident

12. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is

(a) ab (b) $2ab$ (c) $\frac{1}{2}ab$ (d) $\frac{1}{4}ab$

13. The area of the triangle formed by the lines $y = x$, $x = 6$ and $y = 0$ is

(a) 36 sq. units (b) 18 sq. units (c) 9 sq. units (d) 72 sq. units

14. If the system of equations $2x + 3y = 5$, $4x + ky = 10$ has infinitely many solutions, then $k =$

(a) 1 (b) $\frac{1}{2}$ (c) 3 (d) 6

15. If the system of equations $kx - 5y = 2$, $6x + 2y = 7$ has no solution, then $k =$

(a) −10 (b) −5 (c) −6 (d) −15

16. The area of the triangle formed by the lines $x = 3$, $y = 4$ and $x = y$ is

(a) 1/2 sq. unit (b) 1 sq. unit (c) 2 sq. unit (d) None of these

17. The area of the triangle formed by the lines $2x + 3y = 12$, $x - y - 1 = 0$ and $x = 0$ (as shown in Fig. 3.24), is

(a) 7 sq. units (b) 7.5 sq. units (c) 6.5 sq. units (d) 6 sq. units

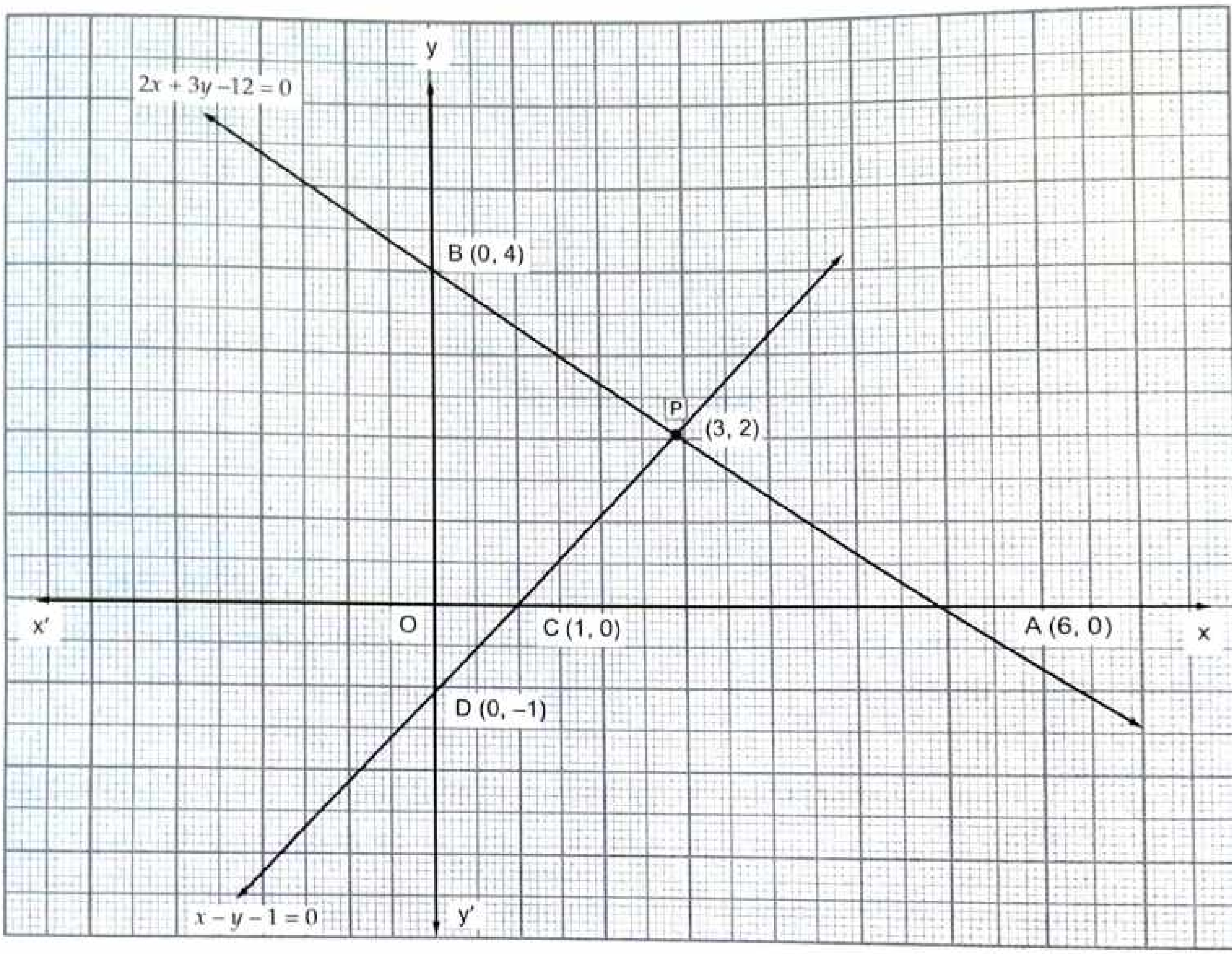

Fig. 3.24

18. The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is

(a) 25 (b) 72 (c) 63 (d) 36

19. If $x = a$, $y = b$ is the solution of the systems of equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(a) 3 and 1 (b) 3 and 5 (c) 5 and 3 (d) – 1 and – 3

20. For what value k, do the equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ represent coincident lines?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) – 2

21. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25

ANSWERS

1. (b)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)
8. (b)	**9.** (a)	**10.** (c)	**11.** (d)	**12.** (c)	**13.** (b)	**14.** (d)
15. (d)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (c)	**21.** (b)

SUMMARY

1. A pair of linear equations in two variables x and y can be represented algebraically as follows:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0,$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that $a_1^2 + b_1^2 \neq 0,\ a_2^2 + b_2^2 \neq 0.$

2. Graphically or geometrically a pair of linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x = b_2y + c_2 = 0$$

in two variables represents a pair of straight lines which are

(i) intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (ii) parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

3. A pair of linear equations in two variables can be solved by the:
(i) Graphical method (ii) Algebraic method.

4. To solve a pair of linear equations in two variables by Graphical method, we first draw the lines represented by them.
   (i) If the pair of lines intersect at a point, then we say that the pair is consistent and the coordinates of the point provide us the unique solution.
   (ii) If the pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations.
   (iii) If the pair of lines are coincident, then it has infinitely many solutions – each point on the line being of solution. In this case, we say that the pair of linear equations is consistent with infinitely many solutions.

5. To solve a pair of linear equations in two variables algebraically, we have following methods:
   (i) Substitution method. (ii) Elimination method.
   (iii) Cross-multiplication method.

6. If $a_1x + b_1y + c_1 = 0$

$$a_2x + b_2y + c_2 = 0$$

is a pair of linear equations in two variables x and y such that

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.