CHAPTER 5

# ARITHMETIC PROGRESSIONS

## 5.1 INTRODUCTION

In earlier classes, you might have come across various patterns of numbers like

$$1, \quad 3, \quad 5, \quad 7, \quad 9, \quad \ldots\ldots\ldots$$

$$0, \quad -2, \quad -4, \quad -6, \quad -8, \quad \ldots\ldots\ldots$$

$$1, \quad 4, \quad 9, \quad 16, \quad 25, \quad \ldots\ldots\ldots$$

etc.

These patterns are generally known as sequences. In this chapter, we intend to study a particular type of sequences which are known as arithmetic progressions.

## 5.2 SEQUENCES

As mentioned above that an arrangement of numbers of which one number is designated as the first, another as the second, another as the third and so on is known as a sequence.

Consider the following arrangement of numbers:

$$1, \quad 8, \quad 27, \quad 64, \quad 125, \quad \ldots\ldots\ldots$$

$$1, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad \frac{1}{4}, \quad \frac{1}{5}, \quad \ldots\ldots\ldots$$

$$2, \quad 4, \quad 6, \quad 8, \quad 10, \quad \ldots\ldots\ldots$$

In each of the above arrangements numbers are arranged in a definite order according to some rule. In the first arrangement the numbers are cubes of natural numbers and in the second arrangement the numbers are reciprocals of natural numbers whereas in the third arrangement the numbers are even natural number. Each of the above arrangements is a sequence. Thus, we may define a sequence formally as follows:

**DEFINITION** *A sequence is an arrangement of numbers in a definite order according to some rule.*

The various numbers occurring in a sequence are called its terms. We denote the terms of a sequence by $a_1, a_2, a_3, \ldots$ etc. or $x_1, x_2, x_3, \ldots$ etc. Here, the subscripts denote the positions of the terms. First number or the number at first place is called its first term of the sequence and is denoted by $a_1$. The number at the second place is called the second term and is denoted by $a_2$ and so on. In general, the number at the $n$th place is called the $n$th term of the sequence and is denoted by $a_n$. The $n$th term is also called the *general term* of the sequence.

For example, 2, 4, 6, 8, 10, ... is a sequence whose

first term is 2 i.e., $a_1 = 2$; second term is 4 i.e., $a_2 = 4$

third term is 6 i.e., $a_3 = 6$; fourth term is 8 i.e., $a_4 = 8$

and so on.

Similarly, 1, 4, 9, 16, 25, ... is a sequence such that

$a_1 = 1, a_2 = 4, a_3 = 9, a_4 = 16, a_5 = 25$ and so on.

Often, it is possible to express the rule which generates the various terms of a sequence in terms of an algebraic formula. For example, consider the sequence of even natural numbers i.e., 2, 4, 6, 8, 10, ....

We have,

$a_1$ = First term = 2 = $2 \times 1$, $a_2$ = Second term = 4 = $2 \times 2$

$a_3$ = Third term = 6 = $2 \times 3$, $a_4$ = Fourth term = 8 = $2 \times 4$

$a_5$ = Fifth term = 10 = $2 \times 5$, $a_6$ = sixth term = 12 = $2 \times 6$

and so on.

It is evident from this that

$a_n$ = $n$th term = $2 \times n = 2n$

Let us now consider the sequence of squares of natural numbers i.e., 1, 4, 9, 16, 25,....
Here, we have

$a_1 = 1 = 1^2, \quad a_2 = 4 = 2^2, a_3 = 9 = 3^2, a_4 = 16 = 4^2 \; a_5 = 25 = 5^2, \; a_6 = 36 = 6^2$

and so on.

It follows from this that

$a_n$ = $n$th term = $n^2$

Similarly, consider the sequence of odd natural numbers i.e., 1, 3, 5, 7, 9, 11, ...
We find that

$a_1 = 1 = 2 \times 1 - 1, \quad a_2 = 3 = 2 \times 2 - 1, a_3 = 5 = 2 \times 3 - 1, a_4 = 7 = 2 \times 4 - 1$

and so on.

In general, $a_n = 2 \times n - 1 = 2n - 1$.

It follows from the above discussion that a sequence can be described either by listing its first few terms till the rule for writing down the other terms becomes clear or, by writing the algebraic formula for the $n$th term of the sequence.

For example, the sequence of even natural numbers i.e., 2, 4, 6, 8, 10,... can be described as

$a_n = 2n$, where $n = 1, 2, 3, ....$

Similarly, the sequence of odd natural numbers i.e., 1, 3, 5, 7, 9, .... can be described as

$a_n = 2n - 1$, where $n = 1, 2, 3, 4, ....$

The sequence, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$ can be described as

$a_n = \frac{1}{n}$, where $n = 1, 2, 3, \cdots$

The sequence 1, 4, 9, 16,... can be described as

$a_n = n^2$, where $n = 1, 2, 3, 4, ...$

In the above discussion, we have seen that a sequence can be described by listing its first few terms till the rule for writing down the other terms becomes clear. We can also describe a sequence by writing the algebraic formula for its $n$th term or general term. In some cases, the terms of the sequence do not follow some fixed pattern but they are generated by some recursive relation.

Consider for instance, the sequence, 1, 1, 2, 3, 5, 8, ...

Here, we have

$$a_1 = 1, a_2 = 1$$
$$a_3 = 2 = 1 + 1 = a_1 + a_2$$
$$a_4 = 3 = 1 + 2 = a_2 + a_3$$
$$a_5 = 5 = 2 + 3 = a_3 + a_4$$
$$a_6 = 8 = 3 + 5 = a_4 + a_5 \text{ and so on}$$

In general

$$a_n = a_{n-1} + a_{n-2} \text{ for } n > 2.$$

Thus, the above sequence is described as

$$a_1 = 1, a_2 = 1 \text{ and } a_n = a_{n-1} + a_{n-2} \text{ for all } n > 2.$$

Let us now discuss some examples to illustrate the applications of what we have discussed so far.

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

EXAMPLE 1 *Write the first three terms in each of the sequence defined by the following:*

(i) $a_n = 3n + 2$ (ii) $a_n = n^2 + 1$

SOLUTION (i) We have, $a_n = 3n + 2$

Putting $n = 1, 2$, and 3, we get

$$a_1 = 3 \times 1 + 2 = 3 + 2 = 5,$$
$$a_2 = 3 \times 2 + 2 = 6 + 2 = 8,$$
$$a_3 = 3 \times 3 + 2 = 9 + 2 = 11$$

Thus, the required first three terms of the sequence defined by $a_n = 3n + 2$ are 5, 8, and 11.

(ii) We have, $a_n = n^2 + 1$

Putting $n = 1, 2$, and 3, we get

$$a_1 = 1^2 + 1 = 1 + 1 = 2,$$
$$a_2 = 2^2 + 1 = 4 + 1 = 5,$$

and, $a_3 = 3^2 + 1 = 9 + 1 = 10.$

Thus, the first three terms of the sequence defined by $a_n = n^2 + 1$ are 2, 5, and 10.

EXAMPLE 2 *Write the first five terms of the sequence defined by* $a_n = (-1)^{n-1} \cdot 2^n$

SOLUTION We have, $a_n = (-1)^{n-1} \cdot 2^n$

Putting $n = 1, 2, 3, 4$, and 5, we get

$$a_1 = (-1)^{1-1} \times 2^1 = (-1)^0 \times 2 = 2$$
$$a_2 = (-1)^{2-1} \times 2^2 = (-1)^1 \times 4 = -4$$
$$a_3 = (-1)^{3-1} \times 2^3 = (-1)^2 \times 8 = 8$$
$$a_4 = (-1)^{4-1} \times 2^4 = (-1)^3 \times 16 = -16$$

and, $a_5 = (-1)^{5-1} \times 2^5 = (-1)^4 \times 32 = 32.$

**EXAMPLE 3** *What is 18th term of the sequence defined by* $a_n = \dfrac{n(n-3)}{n+4}$

SOLUTION We have, $a_n = \dfrac{n(n-3)}{n+4}$

Putting $n = 18$, we get

$$a_{18} = \frac{18 \times (18-3)}{18+4} = \frac{18 \times 15}{22} = \frac{135}{11}$$

**EXAMPLE 4** *A sequence is defined by* $a_n = n^3 - 6n^2 + 11n - 6$. *Show that the first three terms of the sequence are zero and all other terms are positive.*

SOLUTION We have, $a_n = n^3 - 6n^2 + 11n - 6$

Putting $n = 1, 2, 3$, we get

$$a_1 = (1)^3 - 6 \times 1^2 + 11 \times 1 - 6 = 1 - 6 + 11 - 6 = 12 - 12 = 0$$

$$a_2 = 2^3 - 6 \times 2^2 + 11 \times 2 - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0$$

and, $$a_3 = 3^3 - 6 \times 3^2 + 11 \times 3 - 6 = 27 - 54 + 33 - 6 = 60 - 60 = 0$$

Thus, we have

$$a_1 = a_2 = a_3 = 0$$

We observe that $a_n = n^3 - 6n^2 + 11n - 6$ is a cubic polynomial in $n$ and it vanishes for $n = 1$, 2, and 3. Therefore, by factor theorem $(n-1)$, $(n-2)$ and $(n-3)$ are factors of $a_n$. Thus, we have

$$a_n = (n-1)(n-2)(n-3)$$

In this expression, if we substitute any value of $n$ which is greater than 3, then each factor on the RHS is positive. Therefore,

$$a_n > 0 \quad \text{for all } n > 3.$$

Hence, first three terms of the sequence are zero and all other terms are positive.

**LEVEL-2**

**EXAMPLE 5** *Let a sequence be defined by* $a_1 = 3$, $a_n = 3a_{n-1} + 1$ *for all* $n > 1$. *Find the first four terms of the sequence.*

SOLUTION We have, $a_1 = 3$

and, $a_n = 3a_{n-1} + 1$ for all $n > 1$.

Putting $n = 2, 3$, and 4, we get

$$a_2 = 3a_1 + 1 = 3 \times 3 + 1 = 10$$

$$a_3 = 3a_2 + 1 = 3 \times 10 + 1 = 31$$

and, $$a_4 = 3a_3 + 1 = 3 \times 31 + 1 = 94.$$

Hence, the first four terms of the sequence are 3, 10, 31 and 94.

**EXAMPLE 6** *Let a sequence be defined by* $a_1 = 1$, $a_2 = 1$ *and,* $a_n = a_{n-1} + a_{n-2}$ *for all* $n > 2$. *Find* $\dfrac{a_{n+1}}{a_n}$ *for* $n = 1, 2, 3, 4$.

SOLUTION We have, $a_1 = 1,\ a_2 = 1$

and, $a_n = a_{n-1} + a_{n-2}$ for all $n > 2$

Putting $n = 3, 4$, and 5, we get

$$a_3 = a_2 + a_1 = 1 + 1 = 2$$

$$a_4 = a_3 + a_2 = 2 + 1 = 3$$

and, $a_5 = a_4 + a_3 = 3 + 2 = 5$

Thus, we have

$$a_1 = 1,\ a_2 = 1,\ a_3 = 2,\ a_4 = 3 \text{ and } a_5 = 5$$

Now, putting $n = 1, 2, 3$ and 4 in $\frac{a_{n+1}}{a_n}$, we get

$$\frac{a_2}{a_1} = \frac{1}{1} = 1 \qquad [\because a_1 = a_2 = 1]$$

$$\frac{a_3}{a_2} = \frac{2}{1} = 2 \qquad [\because a_2 = 1 \text{ and } a_3 = 2]$$

$$\frac{a_4}{a_3} = \frac{3}{2} \qquad [\because a_3 = 2 \text{ and } a_4 = 3]$$

$$\frac{a_5}{a_4} = \frac{5}{3} \qquad [\because a_4 = 3 \text{ and } a_5 = 5]$$

## EXERCISE 5.1

### LEVEL-1

1. Write the first five terms of each of the following sequences whose $n$th terms are:

 (i) $a_n = 3n + 2$ (ii) $a_n = \frac{n-2}{3}$ (iii) $a_n = 3^n$

 (iv) $a_n = \frac{3n-2}{5}$ (v) $a_n = (-1)^n \cdot 2^n$ (vi) $a_n = \frac{n(n-2)}{2}$

 (vii) $a_n = n^2 - n + 1$ (viii) $a_n = 2n^2 - 3n + 1$ (ix) $a_n = \frac{2n-3}{6}$

2. Find the indicated terms in each of the following sequences whose $n$th terms are:

 (i) $a_n = 5n - 4;\ a_{12}$ and $a_{15}$ (ii) $a_n = \frac{3n-2}{4n+5};\ a_7$ and $a_8$

 (iii) $a_n = n(n-1)(n-2);\ a_5$ and $a_8$ (iv) $a_n = (n-1)(2-n)(3+n);\ a_1, a_2, a_3$

 (v) $a_n = (-1)^n n;\ a_3, a_5, a_8$

3. Find the next five terms of each of the following sequences given by:

 (i) $a_1 = 1,\ a_n = a_{n-1} + 2,\ n \geq 2$ (ii) $a_1 = a_2 = 2,\ a_n = a_{n-1} - 3,\ n > 2$

 (iii) $a_1 = -1,\ a_n = \frac{a_{n-1}}{n},\ n \geq 2$ (iv) $a_1 = 4,\ a_n = 4a_{n-1} + 3,\ n > 1.$

### ANSWERS

1. (i) $a_1 = 5, a_2 = 8, a_3 = 11, a_4 = 14, a_5 = 17$ (ii) $a_1 = -\frac{1}{3}, a_2 = 0, a_3 = \frac{1}{3}, a_4 = \frac{2}{3}, a_5 = 1$

   (iii) $a_1 = 3, a_2 = 9, a_3 = 27, a_4 = 81, a_5 = 243$ (iv) $a_1 = \frac{1}{5}, a_2 = \frac{4}{5}, a_3 = \frac{7}{5}, a_4 = 2, a_5 = \frac{13}{5}$

   (v) $a_1 = -2, a_2 = 4, a_3 = -8, a_4 = 16, a_5 = -32$ (vi) $a_1 = \frac{-1}{2}, a_2 = 0, a_3 = \frac{3}{2}, a_4 = 4, a_5 = \frac{15}{2}$

   (vii) $a_1 = 1, a_2 = 3, a_3 = 7, a_4 = 13, a_5 = 21$ (viii) $a_1 = 0, a_2 = 3, a_3 = 10, a_4 = 21, a_5 = 36$

   (ix) $a_1 = -\frac{1}{6}, a_2 = \frac{1}{6}, a_3 = \frac{1}{2}, a_4 = \frac{5}{6}, a_5 = \frac{7}{6}$

2. (i) $a_{12} = 56, a_{15} = 71$ (ii) $a_7 = \frac{19}{33}, a_8 = \frac{22}{37}$ (iii) $a_5 = 60, a_8 = 336$

   (iv) $a_1 = 0, a_2 = 0, a_3 = -12$ (v) $a_3 = -3, a_5 = -5, a_8 = 8$

3. (i) $a_2 = 3, a_3 = 5, a_4 = 7, a_5 = 9, \ a_6 = 11$

   (ii) $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

   (iii) $a_2 = -\frac{1}{2}, a_3 = -\frac{1}{6}, a_4 = -\frac{1}{24}, a_5 = -\frac{1}{120}, a_6 = -\frac{1}{720}$

   (iv) $a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$

## 5.3 ARITHMETIC PROGRESSION (A.P.)

In this section, we shall discuss a particular type of sequences in which each term, except the first, progresses in a definite manner. Consider for instance, the following sequences

(i) 1, 4, 7, 10, 13, .... (ii) 12, 7, 2, – 3, – 8, ...

(iii) – 9, – 7, – 5, – 3, – 2, 1, 3, ....

In each of these sequences every term except the first is obtained by adding a fixed number (positive or negative) to the preceding term. For example, in the sequence given in (i), each term is obtained by adding 3 to the preceding term. In the sequence given in (ii) each term is – 5 more than the preceding term and in the sequence given in (iii) each term is obtained by adding 2 to the preceding term.

All these sequences are called arithmetic sequences or arithmetic progressions abbreviated as A.P. Thus, we may define an arithmetic sequence as follows:

**ARITHMETIC PROGRESSION (A.P.)** *A sequence* $a_1, a_2, a_3, \ldots, a_n, \ldots$ *is called an arithmetic progression, if there exists a constant number d such that*

$$
\begin{aligned}
a_2 &= a_1 + d \\
a_3 &= a_2 + d \\
a_4 &= a_3 + d \\
&\vdots \\
a_n &= a_{n-1} + d \text{ and so on.}
\end{aligned}
$$

*The constant 'd' is called the common difference of the A.P.*

Thus, if the first term is $a$ and the common difference is $d$, then

$$a, a+d, a+2d, a+3d, a+4d, \ldots\ldots\ldots\ldots$$

is an arithmetic progression.

In other words, a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an arithmetic progression if the difference of a term and the preceding term is always constant. This constant is called the common difference of the A.P.

Thus, if $a_1, a_2, a_3, \ldots, a_n, \ldots$ is an A.P. with common difference '$d$', then,

$$a_2 - a_1 = d$$

$$a_3 - a_2 = d$$

$$a_4 - a_3 = d$$

$$\vdots \qquad \vdots$$

$$a_n - a_{n-1} = d \text{ and so on.}$$

**ILLUSTRATION 1** *The sequence 1, 4, 7, 10, 13, ... is an A.P. whose first term is 1 and the common difference is equal to 3.*

**ILLUSTRATION 2** *The sequence 11, 7, 3, – 1, ... is an A.P. whose first term is 11 and the common difference is equal to – 4.*

*It follows from the above discussion that the sequence $a_1, a_2, a_3, \ldots, a_n, a_{n+1}, \ldots$ is an A.P. with common difference 'd' if and only if*

$$a_{n+1} - a_n = d \text{ for } n = 1, 2, 3, 4, \ldots$$

This suggests us the following algorithm to determine whether a sequence is an A.P. or not when we are given an algebraic formula for the general term of the sequence.

**ALGORITHM**

STEP I *Obtain $a_n$*

STEP II *Replace n by $(n+1)$ in $a_n$ to get $a_{n+1}$*

STEP III *Calculate $a_{n+1} - a_n$*

STEP IV *Check the value of $a_{n+1} - a_n$. If $a_{n+1} - a_n$ is independent of n, then the given sequence is an A.P. Otherwise it is not an A.P.*

**ILLUSTRATION 3** *Show that the sequence defined by $a_n = 4n + 5$ is an A.P. Also, find its common difference.*

SOLUTION We have, $a_n = 4n + 5$

Replacing $n$ by $(n+1)$, we get

$$a_{n+1} = 4(n+1) + 5 = 4n + 9$$

Now, $\quad a_{n+1} - a_n = (4n + 9) - (4n + 5) = 4$

Clearly, $\quad a_{n+1} - a_n$ is independent of $n$ and is equal to 4.

So, the given sequence is an A.P. with common difference 4.

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

ILLUSTRATION 1 *Show that the sequence defined by $a_n = 2n^2 + 1$ is not an A.P.*

SOLUTION We have, $a_n = 2n^2 + 1$

Replacing $n$ by $(n+1)$ in $a_n$, we obtain

$$a_{n+1} = 2(n+1)^2 + 1 = 2n^2 + 4n + 3$$

Now, $a_{n+1} - a_n = (2n^2 + 4n + 3) - (2n^2 + 1) = 4n + 2$

Clearly, $a_{n+1} - a_n$ is not independent of $n$ and is therefore not constant. So, the given sequence is not an A.P.

ILLUSTRATION 2 *Show that a sequence is an A.P. if its nth term is a linear expression in n and in such a case the common difference is equal to the coefficient of n.*

SOLUTION Let there be a sequence whose $n$th term is a linear expression in $n$

i.e. $a_n = An + B$, where $A, B$ are constants.

$\Rightarrow$ $a_{n+1} = A(n+1) + B$

$\therefore$ $a_{n+1} - a_n = \{A(n+1) + B\} - \{An + B\} = A$

Clearly, $a_{n+1} - a_n$ is independent of $n$ and is therefore a constant. So, the sequence is an A.P. with common difference $A$.

NOTE *Readers may use the above statement as a standard result.*

ILLUSTRATION 3 *The nth term of a sequence is $3n - 2$. Is the sequence an A.P.? If so, find its 10th term.*

SOLUTION We have, $a_n = 3n - 2$.

Clearly, $a_n$ is a linear expression in $n$. So, the given sequence is an A.P. with common difference 3.

Putting $n = 10$, we get

$$a_{10} = 3 \times 10 - 2 = 28.$$

REMARK *It is evident from the above examples that a sequence is not an A.P. if its nth term is not a linear expression in n.*

## EXERCISE 5.2

### LEVEL-1

1. Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.
2. Show that the sequence defined by $a_n = 3n^2 - 5$ is not an A.P.
3. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15th term and the common difference.
4. Write the sequence with $n$th term:

   (i) $a_n = 3 + 4n$ (ii) $a_n = 5 + 2n$ (iii) $a_n = 6 - n$ (iv) $a_n = 9 - 5n$

   Show that all of the above sequences form A.P.
5. The $n^{th}$ term of an A.P. is $6n + 2$. Find the common difference. [CBSE 2008]

6. Justify whether it is true to say that the sequence having following $n^{th}$ term is an A.P.
   (i) $a_n = 2n - 1$ (ii) $a_n = 3n^2 + 5$ (iii) $a_n = 1 + n + n^2$

### ANSWERS

1. 5
3. $a_{15} = -45$, Common difference $= -4$
4. (i) 7, 11, 15, 19,... (ii) 7, 9, 11, 13,... (iii) 5, 4, 3, 2, 1, 0, – 1,... (iv) 4, – 1, – 6, – 11,...

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

**EXAMPLE 1** *Write an A.P. whose first term is 10 and common difference is 3.*

SOLUTION We know that if $a$ is the first term and $d$ is the common difference, then the arithmetic progression is

$$a, a + d, a + 2d, a + 3d, \dots$$

Here, $a = 10$ and $d = 3$.

So, the arithmetic progression is 10, 13, 16, 19, 22, ...

**EXAMPLE 2** *Write an A.P. having 4 as the first term and –3 as the common difference.*

SOLUTION The arithmetic progression with first term $a$ and common difference $d$ is given by

$$a, a + d, a + 2d, a + 3d, \dots$$

i.e., each term is obtained by adding '$d$' to the preceding term.

Here, $a = 4$ and $d = -3$.

So, the arithmetic progression is

$$4, 4 + (-3), 4 + 2 \times (-3), 4 + 3(-3), 4 + 4(-3), \dots$$

or, $4, 1, -2, -5, -8, \dots$

**EXAMPLE 3** *Write an A.P. whose first term and common difference are – 1.25 and – 0.25 respectively.*

SOLUTION Here, $a = -1.25$ and $d = -0.25$

So, the arithmetic progression is

$$-1.25, -1.25 + (-0.25), -1.25 + 2(-0.25), -1.25 + 3(-0.25), \dots$$

or, $-1.25, -1.50, -1.75, -2, \dots$

**EXAMPLE 4** *For the following arithmetic progressions write the first term and common difference*

(i) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ (ii) 0.6, 1.7, 2.8, 3.9, ...

SOLUTION (i) We have,

$$\frac{5}{3} - \frac{1}{3} = \frac{4}{3}, \frac{9}{3} - \frac{5}{3} = \frac{4}{3}, \frac{13}{3} - \frac{9}{3} = \frac{4}{3}$$

Clearly, the difference between a term and the preceding term is same and is equal to $\frac{4}{3}$.

So, the given sequence is an A.P. with first term $\frac{1}{3}$ and common difference $\frac{4}{3}$.

(ii) We have,

$$1.7 - 0.6 = 1.1,\ 2.8 - 1.7 = 1.1,\ 3.9 - 2.8 = 1.1$$

So, the given sequence is an A.P. with first term 0.6 and common difference 1.1.

**EXAMPLE 5** *In which of the following situations, the sequence formed will form an A.P.?*

(i) *Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.*

(ii) *The amount of money in the account every year when ₹100 are deposited annually to accumulate at compound interest at 4% per annum.*

SOLUTION (i) We have,

Total strength of students in the auditorium = 1000

Number of students left in the auditorium when first batch of 25 students leaves the auditorium = 1000 – 25 = 975

Number of students left in the auditorium when second batch of 25 students leaves the auditorium = 975 – 25 = 950

Number of students left in the auditorium when third batch of 25 students leaves the audiotirum = 950 – 25 = 925 and so on.

Thus, the number of students left in the auditorium at different stages are

1000, 975, 950, 925, ....

Clearly, it is an A.P. with first term 1000 and common difference – 25.

(ii) We know that if $P$ is the principal and $r$% per annum is the rate of interest, compound annually, then the amount $A_n$ at the end of $n$ years is given by

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Here, $P$ = ₹ 100 and $r = 4$.

$$\therefore \quad A_n = 100\left(1 + \frac{4}{100}\right)^n = 100 \times \left(\frac{26}{25}\right)^n = 100 \times (1.04)^n$$

Thus, the amount of money in the account at the end of different years is given by

₹ $100 \times 1.04$, ₹ $100 \times (1.04)^2$, ₹ $100 \times (1.04)^3, \ldots$

or, ₹ 104, ₹ 108.16, ₹ 112.48, ...

Clearly, it is not forming an A.P.

**EXAMPLE 6** *Find the common difference and write the next three terms of the A.P.* 3, –2, –7, –12,...

SOLUTION We have,

Second term – First term = – 2 – (3) = – 5

Third term – Second term = – 7 – (– 2) = – 5

So, the given sequence is an A.P. with common difference – 5.

Since, each term of an A.P. is obtained by adding common difference to the preceding term.

$$\therefore \quad a_5 = a_4 + (-5) = -12 + (-5) = -17$$

$$a_6 = a_5 + (-5) = -17 + (-5) = -22$$

and, $$a_7 = a_6 + (-5) = -22 + (-5) = -27$$

## EXERCISE 5.3

### LEVEL-1

1. For the following arithmetic progressions write the first term $a$ and the common difference $d$:

   (i) $-5, -1, 3, 7, \ldots$ [NCERT] (ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots$

   (iii) $0.3, 0.55, 0.80, 1.05, \ldots$ (iv) $-1.1, -3.1, -5.1, -7.1, \ldots$

2. Write the arithmetic progression when first term $a$ and common difference $d$ are as follows:

   (i) $a = 4, \quad d = -3$ [NCERT] (ii) $a = -1, \quad d = \frac{1}{2}$ [NCERT]

   (iii) $a = -1.5, \quad d = -0.5$

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

   (i) The cost of digging a well for the first metre is ₹ 150 and rises by ₹ 20 for each succeeding metre. [NCERT]

   (ii) The amount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder. [NCERT]

   (iii) Divya deposited ₹ 1000 at compound interest at the rate of 10% per annum. The amount at the end of first year, second year, third year, ..., and so on. [NCERT EXEMPLAR]

4. Find the common difference and write the next four terms of each of the following arithmetic progressions :

   (i) $1, -2, -5, -8, \ldots$ (ii) $0, -3, -6, -9, \ldots$

   (iii) $-1, \frac{1}{4}, \frac{3}{2}, \ldots$ (iv) $-1, -\frac{5}{6}, -\frac{2}{3}, \ldots$

5. Prove that no matter what the real numbers $a$ and $b$ are, the sequence with $n$th term $a + nb$ is always an A.P. What is the common difference?

6. Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common difference.

   (i) $3, 6, 12, 24, \ldots$ (ii) $0, -4, -8, -12, \ldots$

   (iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$ (iv) $12, 2, -8, -18, \ldots$

   (v) $3, 3, 3, 3, \ldots$ (vi) $p, p+90, p+180, p+270, \ldots$ where $p = (999)^{999}$

   (vii) $1.0, 1.7, 2.4, 3.1, \ldots$ (viii) $-225, -425, -625, -825, \ldots$

   (ix) $10, 10+2^5, 10+2^6, 10+2^7, \ldots$

   (x) $a+b, (a+1)+b, (a+1)+(b+1), (a+2)+(b+1), (a+2)+(b+2), \ldots$

   (xi) $1^2, 3^2, 5^2, 7^2, \ldots$ (xii) $1^2, 5^2, 7^2, 73, \ldots$

7. Find the common difference of the A.P. and write the next two terms:

   (i) $51, 59, 67, 75, \ldots$ (ii) $75, 67, 59, 51, \ldots$

   (iii) $1.8, 2.0, 2.2, 2.4, \ldots$ (iv) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

   (v) $119, 136, 153, 170, \ldots$

### ANSWERS

1. (i) $a=-5, d=4$ (ii) $a=\frac{1}{5}, d=\frac{2}{5}$ (iii) $a=0.3, d=0.25$ (iv) $a=-1.1, d=-2$
2. (i) $4, 1, -2, -5, -8, \ldots$ (ii) $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots$ (iii) $-1.5, -2, -2.5, -3, \ldots$
3. (i) A.P. (ii) Does not form an A.P. (iii) A.P.
4. (i) $-3; a_5=-11, a_6=-14, a_7=-17, a_8=-20$

   (ii) $-3; a_5=-12, a_6=-15, a_7=-18, a_8=-21$

   (iii) $\frac{5}{4}; a_4=\frac{11}{4}, a_5=\frac{16}{4}, a_6=\frac{21}{4}, a_7=\frac{26}{4}$ (iv) $\frac{1}{6}; a_4=-\frac{1}{2}, a_5=-\frac{1}{3}, a_6=-\frac{1}{6}, a_7=0$
5. (i) $b$
6. (i) No (ii) Yes, $d=-4$ (iii) No

   (iv) Yes, $d=-104$ (v) Yes, $d=0$ (vi) Yes, $d=90$

   (vii) Yes, $d=0.7$ (viii) Yes, $d=-200$ (ix) No

   (x) Yes, $d=1$ (xi) No (xii) Yes, $d=24$
7. (i) $d=8, a_5=83, a_6=91$ (ii) $d=-8, a_5=43, a_6=35$

   (iii) $d=0.2, a_5=2.6, a_6=2.8$ (iv) $d=\frac{1}{4}; a_5=1, a_6=\frac{5}{4}$

   (v) $d=17; a_5=187, a_6=204$

## 5.4 GENERAL TERM OF AN A.P.

In this section, we shall find the formula for the $n$ th term or general term of an A.P. in terms of its first term and the common difference. The same will be used to solve some problems on A.P.

**THEOREM** *Let a be the first term and d be the common difference of an A.P. Then, its nth term or general term is given by*

$$a_n = a + (n-1)d.$$

PROOF Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ be the given A.P. Then,

$$a_1 = a$$

$$\Rightarrow \quad a_1 = a + (1-1)d \quad \text{...(i)}$$

Since each term of an A.P. is obtained by adding common difference to the preceding term. Therefore,

$$a_2 = a + d$$

$$\Rightarrow \quad a_2 = a + (2-1)d \quad \text{...(ii)}$$

Similarly, we have

$$a_3 = a_2 + d$$

$$\Rightarrow \quad a_3 = (a+d) + d$$

$$\Rightarrow \quad a_3 = a + 2d$$

$$\Rightarrow \quad a_3 = a + (3-1)d \quad \text{...(iii)}$$

and, $a_4 = a_3 + d$

$\Rightarrow$ $a_4 = (a + 2d) + d$

$\Rightarrow$ $a_4 = a + 3d$

$\Rightarrow$ $a_4 = a + (4 - 1)\,d$ ...(iv)

Observing the pattern in equation (i), (ii), (iii) and (iv), we find that

$$a_n = a + (n - 1)\,d$$

**Q.E.D.**

REMARK *It is evident from the above theorem that*
*General term of an A.P. = First term + (Term number − 1) × (Common difference)*

### 5.4.1 $n^{th}$ TERM OF AN A.P. FROM THE END

Let there be an A.P. with first term $a$ and common difference $d$. If there are $m$ terms in the A.P., then

$n$th term from the end $= (m - n + 1)$th term the beginning

$\Rightarrow$ $n$th term from the end $= a_{m-n+1}$

$\Rightarrow$ $n$th term from the end $= a + (m - n + 1 - 1)\,d$

$\Rightarrow$ $n$th term from the end $= a + (m - n)\,d$

Also, if $l$ is the last term of the A.P., then $n$th term from the end is the $n$th term of an A.P. whose first term is $l$ and common difference is $-d$.

$\therefore$ $n$th term from the end = Last term $+ (n - 1)(-d)$

$\Rightarrow$ $n$th term from the end $= l - (n - 1)\,d$

ILLUSTRATION *Find the 6th term from the end of the A.P. 17, 14, 11, ..., −40* **[CBSE 2005]**

SOLUTION We have,

$l$ = Last term = −40 and, $d$ = Common difference = −3

$\therefore$ 6th term from the end $= l - (6 - 1)\,d = -40 - 5 \times -3 = -25$

### 5.4.2 MIDDLE TERM(S) OF A FINITE A.P.

Let there be a finite A.P. with first term $a$, common difference $d$ and number of terms $n$.

If $n$ is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term is the middle term and is given by $a + \left(\frac{n+1}{2} - 1\right)d$.

If $n$ is even, then $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2} + 1\right)^{th}$ are middle terms given by $a + \left(\frac{n}{2} - 1\right)d$ and $a + \left(\frac{n}{2} + 1 - 1\right)d = a + \frac{n}{2}d$ respectively.

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

EXAMPLE 1 *Find the 12th, 24th and nth term of the A.P. given by 9, 13, 17, 21, 25,...*

SOLUTION We have, $a$ = First term = 9

and, $d$ = Common difference = 4 [$\because 13 - 9 = 4, 17 - 13 = 4, 21 - 17 = 4$ etc.]

We know that the $n$th term of an A.P. with first term $a$ and common difference $d$ is given by

$$a_n = a + (n - 1)\,d$$

$\therefore$ $a_{12} = a + (12 - 1)\,d = a + 11d = 9 + 11 \times 4 = 53$

$a_{24} = a + (24 - 1)d = a + 23d = 9 + 23 \times 4 = 101$

and, $a_n = a + (n - 1)d = 9 + (n - 1) \times 4 = 4n + 5$

Thus, we have

$a_{12} = 53, a_{24} = 101$ and $a_n = 4n + 5$

**EXAMPLE 2** *Show that the sequence 9, 12, 15, 18, ... is an A.P. Find its 16th term and the general term.*

SOLUTION We have,

$(12 - 9) = (15 - 12) = (18 - 15) = 3$

Therefore, the given sequence is an A.P. with common difference 3.

$a$ = First term = 9

$\therefore$ 16th term $= a_{16} = a + (16 - 1)d = a + 15d$ $\quad [\because a_n = a + (n-1)d]$

$\Rightarrow a_{16} = 9 + 15 \times 3 = 54$

$\because$ General term = $n$th term $= a + (n - 1)d$

$\therefore a_n = 9 + (n - 1) \times 3 = 3n + 6$

**EXAMPLE 3** *The first term of an A.P. is –7 and the common difference 5. Find its 18th term and the general term.*

SOLUTION We have,

$a$ = First term = – 7 and, $d$ = Common difference = 5

$\therefore a_{18} = a + (18 - 1)d$ $\quad [\because a_n = a + (n-1)d]$

$\Rightarrow a_{18} = a + 17d = -7 + 17 \times 5 = 78$

and, $a_n = a + (n - 1) \times 5 = -7 + (n - 1) \times 5$

$\Rightarrow a_n = -7 + 5n - 5 = 5n - 12$

**EXAMPLE 4** *Determine the 10th term from the end of the A.P. 4, 9, 14, ..., 254.*

SOLUTION We have,

$l$ = Last term = 254 and, $d$ = Common difference = 5.

$\therefore$ 10th term from the end $= l - (10 - 1)d = l - 9d = 254 - 9 \times 5 = 209$

**EXAMPLE 5** *Which term of the sequence –1, 3, 7, 11, ... is 95 ?*

SOLUTION Clearly, the given sequence is an A.P such that

$a$ = First term = – 1 and, $d$ = Common difference = 4

Let 95 be the $n$th term of the given A.P. Then,

$a_n = 95$

$\Rightarrow a + (n - 1)d = 95$

$\Rightarrow -1 + (n - 1) \times 4 = 95$

$\Rightarrow -1 + 4n - 4 = 95$

$\Rightarrow 4n - 5 = 95 \Rightarrow 4n = 100 \Rightarrow n = 25$

Thus, 95 is 25th term of the given sequence.

**EXAMPLE 6** *Which term of the sequence 4, 9, 14, 19,... is 124 ?*

SOLUTION Clearly, the given sequence is an A.P. with first term $a$ (= 4) and common difference $d$ (= 5)

Let 124 be the $n$th term of the given sequence. Then,

$$a_n = 124 \Rightarrow a+(n-1)d = 124 \Rightarrow 4+(n-1)\times 5 = 124 \Rightarrow 5n-1 = 124 \Rightarrow 5n = 125 \Rightarrow n = 25$$

Hence, 25th term of the given sequence is 124.

**EXAMPLE 7** *How many terms are there in the sequence 3, 6, 9, 12, ..., 111?*

SOLUTION Clearly, the given sequence is an A.P. with first term $a = 3$ and common difference $d = 3$. Let there be $n$ terms in the given sequence. Then,

$$n\text{th term} = 111$$
$$\Rightarrow \quad a + (n-1)d = 111$$
$$\Rightarrow \quad 3 + (n-1)\times 3 = 111 \Rightarrow n = 37$$

Thus, the given sequence contains 37 terms.

**EXAMPLE 8** *Find the middle term of the A.P. 6, 13, 20, ..., 216.* [CBSE 2015]

SOLUTION Clearly, 6, 13, 20, ..., 216 is an A.P. with first term $a = 6$ and common difference $d = 7$. Let there be $n$ terms in the given A.P. Then,

$$a_n = 216$$
$$\Rightarrow \quad a + (n-1)d = 216$$
$$\Rightarrow \quad 6 + 7(n-1) = 216 \qquad [\because a = 6 \text{ and } d = 7]$$
$$\Rightarrow \quad 7n = 217 \Rightarrow n = 31$$

Here, $n$ is odd so $\left(\frac{n+1}{2}\right)^{th}$ *i.e.* $\left(\frac{31+1}{2}\right)^{th} = 16^{th}$ term is the middle term and is given by

$$a_{16} = a + (16-1)d = a + 15d = 6 + 15\times 7 = 111$$

**EXAMPLE 9** *Find the middle term(s) of the A.P. 7, 13, 19, ..., 241.*

SOLUTION Clearly, 7, 13, 19, ..., 241 is an A.P. with first term $a = 7$ and common difference $d = 6$. Let there be $n$ terms in the A.P. Then,

$$a_n = 241$$
$$\Rightarrow \quad a + (n-1)d = 241$$
$$\Rightarrow \quad 7 + 6(n-1) = 241$$
$$\Rightarrow \quad 6n = 240 \Rightarrow n = 40$$

Clearly, $n$ is even. So, $\left(\frac{n}{2}\right)^{th} = 20^{th}$ and $\left(\frac{n}{2}+1\right)^{th} = 21^{th}$ are middle terms and are given by

$$a_{20} = a + (20-1)d = a + 19d = 7 + 19\times 6 = 121$$

and, $$a_{21} = a + (21-1)d = a + 20d = 7 + 20\times 6 = 127$$

**EXAMPLE 10** *Consider the A.P. 2, 5, 8, 11, ..., 302. Show that twice of the middle term of the above A.P. is equal to the sum of its first and last term.*

SOLUTION Clearly, 2, 5, 8, 11, ..., 302 is an A.P. with first term $a = 2$ and common difference $d = 3$. Let there be $n$ terms in the given A.P.. Then,

$$n^{th} \text{ term} = 302$$
$$\Rightarrow \quad a + (n-1)d = 302$$
$$\Rightarrow \quad 2 + 3(n-1) = 302$$
$$\Rightarrow \quad 3n = 303$$

$\Rightarrow \quad n = 101$

Clearly, $n$ is odd. Therefore, $\left(\frac{n+1}{2}\right)^{th}$ *i.e.* $51^{st}$ term is the middle term.

Now,

$$\text{Middle term} = a_{51} = a + 50d = 2 + 50 \times 3 = 152$$

$$\text{First term} + \text{Last term} = 2 + 302 = 304$$

Clearly, twice the middle term is equal to the sum of the first and last term.

**EXAMPLE 11** *In the A.P. 1, 7, 13, 19, ..., 415, prove that the sum of the middle terms is equal to the sum of first and last terms.*

**SOLUTION** We observe that 1, 7, 13, 19, ..., 415 is an A.P. with first term $a = 1$ and common difference $d = 6$. Let there be $n$ terms in the given A.P. Then,

$$n^{th} \text{ term} = 415$$

$\Rightarrow \quad a + (n-1)d = 415$

$\Rightarrow \quad 1 + 6(n-1) = 415$

$\Rightarrow \quad 6n = 420$

$\Rightarrow \quad n = 70$

So, there are 70 terms in the given A.P. Therefore, $\left(\frac{70}{2}\right)^{th} = 35^{th}$ and $\left(\frac{70}{2}+1\right)^{th} = 36^{th}$ are the middle terms.

Now,

$$a_{35} = a + 34d = 1 + 34 \times 6 = 205 \text{ and, } a_{36} = a + 35d = 1 + 35 \times 6 = 211$$

$\therefore \quad a_{35} + a_{36} = 205 + 211 = 416$

Also, $\quad a_1 + a_{70} = 1 + 415 = 416$

Clearly, $a_{35} + a_{36} = a_1 + a_{70}$.

**EXAMPLE 12** *For what value of n are the $n^{th}$ terms of the following two A.P's the same?*
*(i) 1, 7, 13, 19, ...* *(ii) 69, 68, 67, ...* **[CBSE 2006C]**

**SOLUTION** Clearly, 1, 7, 13, 19, .... forms an A.P. with first term 1 and common difference 6. Therefore, its $n^{th}$ term is given by

$$a_n = 1 + (n-1) \times 6 = 6n - 5$$

Also, 69, 68, 67, 66, ... forms an A.P. with first term 69 and common difference –1.

So, its $n^{th}$ term is given by

$$a_n' = 69 + (n-1) \times (-1) = -n + 70$$

The two A.Ps will have identical $n$th terms, if

$$a_n = a_n'$$

$\Rightarrow \quad 6n - 5 = -n + 70$

$\Rightarrow \quad 7n = 75$

$\Rightarrow \quad n = \frac{75}{7}$, which is not a natural number.

Hence, there is no value of $n$ for which the two A.Ps will have identical terms.

**EXAMPLE 13** *If the $8^{th}$ term of an A.P. is 31 and the $15^{th}$ term is 16 more than the $11^{th}$ term, find the A.P.* **[CBSE 2006C]**

**SOLUTION** Let $a$ be the first term and $d$ be the common difference of the A.P.

We have,

$a_8 = 31$ and $a_{15} = 16 + a_{11}$

$\Rightarrow \quad a + 7d = 31$ and $a + 14d = 16 + a + 10d$

$\Rightarrow \quad a + 7d = 31$ and $4d = 16$

$\Rightarrow \quad a + 7d = 31$ and $d = 4$

$\Rightarrow \quad a + 7 \times 4 = 31 \Rightarrow a + 28 = 31 \Rightarrow a = 3$

Hence, the A.P. is $a, a + d, a + 2d, a + 3d, \ldots$ i.e., $3, 7, 11, 15, 19, \ldots$

**EXAMPLE 14** *Which term of the arithmetic progression 5, 15, 25, ... will be 130 more than its 31st term?* [CBSE 2006C, 2017]

SOLUTION We have, $a = 5$ and $d = 10$

$\therefore \quad a_{31} = a + 30d = 5 + 30 \times 10 = 305$

Let $n^{th}$ term of the given A.P. be 130 more than its 31st term. Then,

$a_n = 130 + a_{31}$

$\therefore \quad a + (n-1)d = 130 + 305$

$\Rightarrow \quad 5 + 10(n-1) = 435$

$\Rightarrow \quad 10(n-1) = 430$

$\Rightarrow \quad n - 1 = 43$

$\Rightarrow \quad n = 44$

Hence, 44th term of the given A.P. is 130 more than its 31st term.

**EXAMPLE 15** *If the 10th term of an A.P. is 52 and 17th term is 20 more than the 13th term, find the A.P.* [CBSE 2006C]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the A.P.

We have,

$a_{10} = 52$ and $a_{17} = a_{13} + 20$

$\Rightarrow \quad a + 9d = 52$ and $a + 16d = a + 12d + 20$

$\Rightarrow \quad a + 9d = 52$ and $4d = 20$

$\Rightarrow \quad a + 9d = 52$ and $d = 5$

$\Rightarrow \quad a + 45 = 52$ and $d = 5$

$\Rightarrow \quad a = 7$ and $d = 5$

Hence, the A.P. is $a, a + d, a + 2d, a + 3d, \ldots$ i.e., $7, 12, 17, 22, \ldots$

**EXAMPLE 16** *Is 184 a term of the sequence 3, 7, 11, ...?*

SOLUTION Clearly, the given sequence is an A.P. with first term $a$ (= 3) and common difference $d$ (= 4).

Let the $n$th term of the given sequence be 184. Then,

$a_n = 184$

$\Rightarrow \quad a + (n-1)d = 184$

$\Rightarrow \quad 3 + (n-1) \times 4 = 184 \Rightarrow 4n = 185 \Rightarrow n = 46\frac{1}{4}$

Since $n$ is not a natural number. So, 184 is not a term of the given sequence.

**EXAMPLE 17** *The 10th term of an A.P. is 52 and 16th term is 82. Find the 32nd term and the general term.*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Let the A.P. be $a_1, a_2, a_3, ..., a_n, ...$
It is given that

$$a_{10} = 52 \text{ and, } a_{16} = 82$$

$\Rightarrow$ $a + (10-1)d = 52$ and, $a + (16-1)d = 82$

$\Rightarrow$ $a + 9d = 52$ ...(i)

and, $a + 15d = 82$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$$-6d = -30 \Rightarrow d = 5$$

Putting $d = 5$ in equation (i), we get

$$a + 45 = 52 \Rightarrow a = 7$$

$\therefore$ $a_{32} = a + (32-1)d = 7 + 31 \times 5 = 162$

and, $a_n = a + (n-1)d = 7 + (n-1) \times 5 = 5n + 2$

Hence, $a_{32} = 162$ and $a_n = 5n + 2$

EXAMPLE 18 *The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.* [CBSE 2009]

SOLUTION Let $a$ be the first term and '$d$' be the common difference of the A.P. It is given that

$$a_5 + a_9 = 72 \text{ and, } a_7 + a_{12} = 97$$

$\Rightarrow$ $(a + 4d) + (a + 8d) = 72$ and, $(a + 6d) + (a + 11d) = 97$

Thus, we have

$\Rightarrow$ $2a + 12d = 72$ ...(i)

$\Rightarrow$ $2a + 17d = 97$ ...(ii)

Subtracting (i) from (ii), we get

$$5d = 25 \Rightarrow d = 5$$

Putting $d = 5$ in (i), we get

$$2a + 60 = 72 \Rightarrow 2a = 12 \Rightarrow a = 6$$

$\therefore$ $a = 6$ and $d = 5$

Hence, the A.P. is 6, 11, 16, 21, 26, ...

EXAMPLE 19 *Determine the general term of an A.P. whose 7th term is –1 and 16th term 17.*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Let the A.P. be $a_1, a_2, a_3, ..., a_n, ...$
It is given that

$$a_7 = -1 \text{ and } a_{16} = 17$$

$\Rightarrow$ $a + (7-1)d = -1$ and, $a + (16-1)d = 17$

$\Rightarrow$ $a + 6d = -1$ ... (i)

and, $a + 15d = 17$ ... (ii)

Subtracting equation (i) from equation (ii), we get

$$9d = 18 \Rightarrow d = 2$$

Putting $d = 2$ in equation (i), we get

$$a + 12 = -1 \Rightarrow a = -13$$

Hence, General term $= a_n = a + (n-1)d = -13 + (n-1) \times 2 = 2n - 15$

**EXAMPLE 20** *If five times the fifth term of an A.P. is equal to 8 times its eighth term, show that its $13^{th}$ term is zero.*

SOLUTION Let $a_1, a_2, a_3, ..., a_n, ...$ be the A.P. with its first term $a$ and common difference $d$. It is given that

$$5a_5 = 8a_8$$
$$\Rightarrow \quad 5(a + 4d) = 8(a + 7d)$$
$$\Rightarrow \quad 5a + 20d = 8a + 56d$$
$$\Rightarrow \quad 3a + 36d = 0$$
$$\Rightarrow \quad 3(a + 12d) = 0$$
$$\Rightarrow \quad a + 12d = 0 \Rightarrow a + (13 - 1)d = 0 \Rightarrow a_{13} = 0$$

Hence, 13th term is zero.

**EXAMPLE 21** *How many numbers of two digits are divisible by 7?*

SOLUTION We observe that 14 is the first two digit number divisible by 7 and 98 is the last two digit number divisible by 7. Thus, we have to determine the number of terms in the sequence

$$14, 21, 28, ..., 98$$

Clearly, it is an A.P. with first term = 14 and common difference = 7 i.e. $a = 14$ and $d = 7$.

Let there be $n$ terms in this A.P. Then,

$$n\text{th term} = 98$$
$$\Rightarrow \quad 14 + (n - 1) \times 7 = 98$$
$$\Rightarrow \quad 14 + 7n - 7 = 98$$
$$\Rightarrow \quad 7n = 91 \Rightarrow n = 13$$

Hence, there are 13 numbers of two digits which are divisible by 7.

**EXAMPLE 22** *Find the number of integers between 50 and 500 which are divisible by 7.*

SOLUTION We observe that 56 is the first integer between 50 and 500 which is divisible by 7. Also, when we divide 500 by 7 the remainder is 3. Therefore, $500 - 3 = 497$ is the largest integer divisible by 7 and lying between 50 and 500. Thus, we have to find the number of terms in an A.P. with first term = 56, last term = 497 and common difference = 7 (as the numbers are divisible by 7).

Let there be $n$ terms in the A.P. Then,

$$a_n = 497$$
$$\Rightarrow \quad a + (n - 1)d = 497$$
$$\Rightarrow \quad 56 + (n - 1) \times 7 = 497 \qquad [\because a = 56 \text{ and } d = 7]$$
$$\Rightarrow \quad 7n + 49 = 497$$
$$\Rightarrow \quad 7n = 448 \Rightarrow n = 64$$

Thus, there are 64 integers between 50 and 500 which are divisible by 7.

**EXAMPLE 23** *Which term of the A.P. 3, 15, 27, 39,... will be 132 more than its 54th term?* [NCERT]

SOLUTION Given A.P. is 3, 15, 27, 39,...

Clearly, its

First term = 3 and, Common difference = 12.

Let $n^{th}$ term of the A.P. be 132 more than its 54th term

i.e., $$a_n = 132 + a_{54}$$
$$\Rightarrow \quad a + (n - 1)d = 132 + (a + 53d)$$

$\Rightarrow \qquad 3 + 12(n-1) = 132 + (3 + 53 \times 12)$

$\Rightarrow \qquad 12n - 9 = 771$

$\Rightarrow \qquad 12n = 780 \Rightarrow n = 65$

Hence, 65th term of the given A.P. is 132 more than its 54th term.

**EXAMPLE 24** *Two A.P's have the same common difference. The first term of one of these is 3, and that of the other is 8. What is the difference between their*

(i) *2nd terms ?* (ii) *4th terms ?* (iii) *10th terms ?* (iv) *30th terms ?*

**SOLUTION** Let the common difference of the two A.P's be $d$. Then, their $n^{th}$ terms are

$$a_n = 3 + (n-1)d \text{ and } b_n = 8 + (n-1)d$$

$\Rightarrow \qquad a_n - b_n = [3 + (n-1)d] - [8 + (n-1)d]$

$\Rightarrow \qquad a_n - b_n = -5$ for all $n \in N$.

Hence, $\quad a_2 - b_2 = -5, a_4 - b_4 = -5, a_{10} - b_{10} = -5$ and $a_{30} - b_{30} = -5$.

**EXAMPLE 25** *A sum of ₹1000 is invested at 8% simple interest per annum. Calculate the interest at the end of 1, 2, 3,... years. Is the sequence of interests an A.P.? Find the interest at the end of 30 years.* [NCERT]

**SOLUTION** Let $P$ be the principle, $R$ rate of interest and $I_n$ be the interest at the end of $n$ years. We know that

$$I_n = \frac{PRn}{100} \qquad \left[\text{Using : Interest} = \frac{PRT}{100}\right]$$

Here, we have

$$P = ₹\,1000, \text{ and } R = 8\% \text{ per annum}$$

$\therefore \qquad I_n = ₹\left(\frac{1000 \times 8 \times n}{100}\right) = ₹\,80\,n$

Putting $\quad n = 1, 2, 3, \ldots$, we have

$$I_1 = ₹\,80, I_2 = ₹\,160, I_3 = ₹\,240 \text{ and so on.}$$

Since, $I_n$ is a linear expression in $n$. Therefore, the sequence of interest forms an A.P. with common difference 80.

Also, $\quad$ Interest at the end of 30 years $= I_{30} = ₹\,(80 \times 30) = ₹\,2400$

**EXAMPLE 26** *In a flower bed there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there in the flower bed?* [NCERT]

**SOLUTION** The number of rose plants in first, second third, ..., and last row are respectively 23, 21, 19, ..., 5.

Let the number of rows of rose plants be $n$.

The sequence 23, 21, 19, ..., 5 is an A.P. with first term $a$ (= 23), common difference $d$ (= −2) and $n$th term (= 5).

$\therefore \qquad a_n = a + (n-1)d$

$\Rightarrow \qquad 5 = 23 + (n-1) \times -2$

$\Rightarrow \qquad 5 = 23 - 2n + 2 \Rightarrow 20 = 2n \Rightarrow n = 10$

Hence, there are 10 rows of rose plants.

EXAMPLE 27 *Suba Rao started work in 1995 at an annual salary of ₹ 5000 and received a ₹200 raise each year. In what year did his annual salary will reach ₹7000?* [NCERT]

SOLUTION Annual salary received by Suba Rao in 1995, 1996, 1997, ... is

₹ 5000, ₹ 5200, ₹ 5400, ...

Clearly, it is an arithmetic progression with first term $a = 5000$ and common difference $d = 200$.

Suppose Suba Rao's annual salary reaches to ₹ 7000 in $n$th years. Then,

$n$th term of the above A.P. = ₹ 7000

$$\Rightarrow \quad a + (n-1)d = 7000$$

$$\Rightarrow \quad 5000 + (n-1) \times 200 = 7000$$

$$\Rightarrow \quad (n-1) \times 200 = 2000$$

$$\Rightarrow \quad n - 1 = \frac{2000}{200} \Rightarrow n - 1 = 10 \Rightarrow n = 11$$

Thus, 11th annual salary received by Suba Rao will be ₹ 7000. This means that after 10 years i.e., in the year 2005 his annual salary will reach to ₹ 7000.

EXAMPLE 28 *Jasleen saved ₹5 in the first week of the year and then increased her weekly savings by ₹1.75 each week. In what week will her weekly savings be ₹20.75?* [NCERT]

SOLUTION Suppose Jasleen's weekly savings will be ₹ 20.75 in the $n^{th}$ week.

Clearly, Jasleen's weekly savings form an A.P. with first term $a = 5$ and common difference $d = 1.75$.

$$\therefore \quad n\text{th term} = 20.75$$

$$\Rightarrow \quad a + (n-1)d = 20.75$$

$$\Rightarrow \quad 5 + (n-1) \times 1.75 = 20.75$$

$$\Rightarrow \quad (n-1) \times 1.75 = 15.75$$

$$\Rightarrow \quad n - 1 = \frac{15.75}{1.75} \Rightarrow n - 1 = 9 \Rightarrow n = 10$$

Hence, Jasleen's weekly savings will be ₹ 20.75 in 10th week.

## LEVEL-2

EXAMPLE 29 *If the $m^{th}$ term of an A.P. be $1/n$ and $n^{th}$ term be $1/m$, then show that its $(mn)^{th}$ term is 1.* [CBSE 2017]

SOLUTION Let $a$ and $d$ be the first term and common difference respectively of the given A.P. Then,

$$\frac{1}{n} = m\text{th term} \Rightarrow \frac{1}{n} = a + (m-1)d \quad \text{...(i)}$$

$$\frac{1}{m} = n\text{th term} \Rightarrow \frac{1}{m} = a + (n-1)d \quad \text{...(ii)}$$

On subtracting equation (ii) from equation (i), we get

$$\frac{1}{n} - \frac{1}{m} = (m-n)d \Rightarrow \frac{m-n}{mn} = (m-n)d \Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in equation (i), we get

$$\frac{1}{n} = a + \frac{(m-1)}{mn} \Rightarrow \frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn} \Rightarrow a = \frac{1}{mn}$$

$\therefore$ $(mn)$ th term $= a + (mn-1)d = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$ $\left[\because a = \frac{1}{mn} = d\right]$

EXAMPLE 30 *If the $p^{th}$ term of an A.P. is $q$ and the $q^{th}$ term is $p$, prove that its $n^{th}$ term is $(p+q-n)$.* [CBSE 2008, 2017]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$p$th term $= q \Rightarrow a + (p-1)d = q$ ...(i)

$q$th term $= p \Rightarrow a + (q-1)d = p$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$$(p-q)d = (q-p) \Rightarrow d = -1$$

Putting $d = -1$ in equation (i), we get

$$a + (p-1)\times(-1) = q \Rightarrow a = (p+q-1)$$

$\therefore$ $n$ th term $= a + (n-1)d = (p+q-1) + (n-1)\times(-1) = (p+q-n)$

EXAMPLE 31 *If $m$ times the $m^{th}$ term of an A.P. is equal to $n$ times its $n^{th}$ term, show that the $(m+n)^{th}$ term of the A.P. is zero.* [CBSE 2008]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

($m$ times $m^{th}$ term) = ($n$ times $n^{th}$ term)

$\Rightarrow$ $m\, a_m = n\, a_n$

$\Rightarrow$ $m\{a + (m-1)d\} = n\{a + (n-1)d\}$

$\Rightarrow$ $m\{a + (m-1)d\} - n\{a + (n-1)d\} = 0$

$\Rightarrow$ $a(m-n) + \{m(m-1) - n(n-1)\}d = 0$

$\Rightarrow$ $a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$

$\Rightarrow$ $a(m-n) + (m-n)(m+n-1)d = 0$

$\Rightarrow$ $(m-n)\{a + (m+n-1)d\} = 0$

$\Rightarrow$ $a + (m+n-1)d = 0$ $[\because m \neq n]$

$\Rightarrow$ $a_{m+n} = 0$

Hence, $(m+n)^{th}$ term of the given A.P. is zero.

EXAMPLE 32 *If $p^{th}$, $q^{th}$ and $r^{th}$ terms of an A.P. are $a$, $b$, $c$ respectively, then show that*

(i) $a(q-r) + b(r-p) + c(p-q) = 0$ [CBSE 2016]

(ii) $(a-b)r + (b-c)p + (c-a)q = 0$

SOLUTION (i) Let $A$ be the first term and $D$ be the common difference of the given A.P. Then,

$a = p$th term $\Rightarrow a = A + (p-1)D$ ... (i)

$b = q$th term $\Rightarrow b = A + (q-1)D$ ... (ii)

$c = r$th term $\Rightarrow c = A + (r-1)D$ ... (iii)

We have,

$a(q-r) + b(r-p) + c(p-q)$

$= \{A + (p-1)D\}(q-r) + \{A + (q-1)D\}(r-p) + \{A + (r-1)D\}(p-q)$

[Using (i), (ii) and (iii)]

$= A\{(q-r)+(r-p)+(p-q)\} + D\{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\}$

$= A \times 0 + D\{p(q-r)+q(r-p)+r(p-q)-(q-r)-(r-p)-(p-q)\}$

$= A \times 0 + D \times 0 = 0$

(ii) On subtracting equation (ii) from equation (i), equation (iii) from equation (ii) and equation (i) from equation (iii), we get

$a-b=(p-q)D, (b-c)=(q-r)D$ and $c-a=(r-p)D$

$\therefore \quad (a-b)r+(b-c)p+(c-a)q$

$= (p-q)Dr+(q-r)Dp+(r-p)Dq$

$= D\{(p-q)r+(q-r)p+(r-p)q\} = D \times 0 = 0$

EXAMPLE 33 *Which term of the sequence* $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ *is the first negative term?*

SOLUTION The given sequence is an A.P. in which first term $a$ (= 20) and common difference $d$ (= – 3/4). Let the $n^{th}$ term of the given A.P. be the first negative term. Then,

$a_n < 0$

$\Rightarrow \quad a+(n-1)d < 0$

$\Rightarrow \quad 20+(n-1)\times(-3/4) < 0$

$\Rightarrow \quad \frac{83}{4} - \frac{3n}{4} < 0$

$\Rightarrow \quad 83 - 3n < 0$

$\Rightarrow \quad 3n > 83 \Rightarrow n > 27\frac{2}{3} \Rightarrow n \geq 28$ [$\because$ $n$ is a natural number]

Thus, $28^{th}$ term of the given sequence is the first negative term.

EXAMPLE 34 *Two A.P's have the same common difference. The difference between their $100^{th}$ terms is 111 222 333. What is the difference between their Millionth terms?*

SOLUTION Let the two A.P's be $a_1, a_2, a_3, \ldots, a_n, \ldots$ and $b_1, b_2, b_3, \ldots, b_n, \ldots$

Also, let $d$ be the common difference of two A.P's. Then,

$a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1)d$

$\Rightarrow \quad a_n - b_n = \{a_1 + (n-1)d\} - \{b_1 + (n-1)d\}$

$\Rightarrow \quad a_n - b_n = a_1 - b_1$

Clearly, $a_n - b_n$ is independent of $n$ and is equal to $a_1 - b_1$. In other words

$a_n - b_n = a_1 - b_1$ for all $n \in N$.

$\Rightarrow \quad a_{100} - b_{100} = a_1 - b_1$

and, $\quad a_k - b_k = a_1 - b_1$, where $k = 10,00,000$.

But, $\quad a_{100} - b_{100} = 111\,222\,333$

$\therefore \quad a_1 - b_1 = 111\,222\,333$

$\Rightarrow \quad a_k - b_k = a_1 - b_1 = 111\,222\,333$, where $k = 10,00,000$.

Hence, the difference between millionth terms is same as the difference between 100th terms i.e., 111 222 333.

EXAMPLE 35 *Find a, b and c if it is given that the numbers a, 7, b, 23, c. are in A.P:*

[NCERT EXEMPLAR]

SOLUTION Let $d$ be the common difference of the A.P formed by the numbers $a, 7, b, 23, c$. Then,

$$7 = a + d, b = a + 2d, 23 = a + 3d \text{ and } c = a + 4d$$

$$\Rightarrow \quad d = 7 - a, b = a + 2d, 23 = a + 3d \text{ and } c = a + 4d$$

Putting $d = 7 - a$ in $23 = a + 3d$, we obtain

$$23 = a + 3(7 - a) \Rightarrow 2a = -2 \Rightarrow a = -1$$

$$\therefore \quad d = 7 - a \Rightarrow d = 7 + 1 = 8$$

Thus, $b = a + 2d = -1 + 16 = 15$ and $c = a + 4d = -1 + 4 \times 8 = 31$

Hence, $a = -1, b = 15$ and $c = 31$.

## EXERCISE 5.4

**LEVEL-1**

1. Find:
   (i) $10^{th}$ term of the A.P. 1, 4, 7, 10,...
   (ii) $18^{th}$ term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$
   (iii) $n^{th}$ term of the A.P. 13, 8, 3, −2,...
   (iv) $10^{th}$ term of the A.P. −40, −15, 10, 35, ...
   (v) $8^{th}$ term of the A.P. 117, 104, 91, 78,...
   (vi) $11^{th}$ term of the A.P. 10.0, 10.5, 11.0, 11.5, ...
   (vii) $9^{th}$ term of the A.P. $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$
2. (i) Which term of the A.P. 3, 8, 13, ... is 248?
   (ii) Which term of the A.P. 84, 80, 76,... is 0?
   (iii) Which term of the A.P. 4, 9, 14, ... is 254?
   (iv) Which term of the A.P. 21, 42, 63, 84, ... is 420?
   (v) Which term of the A.P. 121, 117, 113, ... is its first negative term?
3. (i) Is 68 a term of the A.P. 7, 10, 13,...?
   (ii) Is 302 a term of the A.P. 3, 8, 13,...?
   (iii) Is −150 a term of the A.P. 11, 8, 5, 2, ...? [CBSE 2017]
4. How many terms are there in the A.P.?
   (i) 7, 10, 13, ... 43.
   (ii) $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$.
   (iii) 7, 13, 19, ..., 205.
   (iv) $18, 15\frac{1}{2}, 13, \ldots, -47$.
5. The first term of an A.P. is 5, the common difference is 3 and the last term is 80; find the number of terms.
6. The $6^{th}$ and $17^{th}$ terms of an A.P. are 19 and 41 respectively, find the $40^{th}$ term.
7. If $9^{th}$ term of an A.P. is zero, prove that its $29^{th}$ term is double the $19^{th}$ term.
8. If 10 times the $10^{th}$ term of an A.P. is equal to 15 times the $15^{th}$ term, show that $25^{th}$ term of the A.P. is zero.
9. The $10^{th}$ and $18^{th}$ terms of an A.P. are 41 and 73 respectively. Find $26^{th}$ term.
10. In a certain A.P. the $24^{th}$ term is twice the $10^{th}$ term. Prove that the 72nd term is twice the $34^{th}$ term.

11. The $26^{th}$, $11^{th}$ and last term of an A.P. are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms. [NCERT EXEMPLAR]

12. If the $n^{th}$ term of the A.P. 9, 7, 5, ... is same as the $n^{th}$ term of the A.P. 15, 12, 9, ... find $n$.

13. Find the $12^{th}$ term from the end of the following arithmetic progressions:
    (i) 3, 5, 7, 9, ... 201 (ii) 3, 8, 13, ..., 253 [NCERT] (iii) 1, 4, 7, 10, ..., 88

14. The $4^{th}$ term of an A.P. is three times the first and the $7^{th}$ term exceeds twice the third term by 1. Find the first term and the common difference.

15. Find the second term and $n^{th}$ term of an A.P. whose $6^{th}$ term is 12 and the $8^{th}$ term is 22.

16. How many numbers of two digit are divisible by 3 ?

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

18. The sum of $4^{th}$ and $8^{th}$ terms of an A.P. is 24 and the sum of the $6^{th}$ and $10^{th}$ terms is 34. Find the first term and the common difference of the A.P. [NCERT]

19. The first term of an A.P. is 5 and its $100^{th}$ term is $-292$. Find the $50^{th}$ term of this A.P.

20. Find $a_{30} - a_{20}$ for the A.P.
    (i) $-9, -14, -19, -24, \ldots$ (ii) $a, a+d, a+2d, a+3d, \ldots$

21. Write the expression $a_n - a_k$ for the A.P. $a, a+d, a+2d, \ldots$
    Hence, find the common difference of the A.P. for which
    (i) $11^{th}$ term is 5 and $13^{th}$ term is 79.
    (ii) $a_{10} - a_5 = 200$
    (iii) $20^{th}$ term is 10 more than the $18^{th}$ term.

22. Find $n$ if the given value of $x$ is the $n^{th}$ term of the given A.P.
    (i) $25, 50, 75, 100, \ldots; x = 1000$ (ii) $-1, -3, -5, -7, \ldots; x = -151$
    (iii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots; x = 550$ (iv) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots, x = \frac{171}{11}$

23. The eighth term of an A.P. is half of its second term and the eleventh term exceeds one third of its fourth term by 1. Find the $15^{th}$ term.

24. Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12. [NCERT]

25. The $7^{th}$ term of an A.P. is 32 and its $13^{th}$ term is 62. Find the A.P. [CBSE 2004]

26. Which term of the A.P. 3, 10, 17, ... will be 84 more than its $13^{th}$ term? [CBSE 2004]

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms? [NCERT]

28. For what value of $n$, the $n$th terms of the arithmetic progressions 63, 65, 67, ... and 3, 10, 17, ... are equal? [CBSE 2008]

29. How many multiples of 4 lie between 10 and 250? [NCERT]

30. How many three digit numbers are divisible by 7? [CBSE 2013, NCERT]

31. Which term of the arithmetic progression 8, 14, 20, 26, ... will be 72 more than its $41^{st}$ term? [CBSE 2006C]

32. Find the term of the arithmetic progression 9, 12, 15, 18, ... which is 39 more than its $36^{th}$ term. [CBSE 2006C]
33. Find the $8^{th}$ term from the end of the A.P. 7, 10, 13, ..., 184 [CBSE 2005]
34. Find the $10^{th}$ term from the end of the A.P. 8, 10, 12,..., 126. [CBSE 2006]
35. The sum of $4^{th}$ and $8^{th}$ terms of an A.P. is 24 and the sum of $6^{th}$ and $10^{th}$ terms is 44. Find the A.P. [CBSE 2009]
36. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its $21^{st}$ term? [CBSE 2009]
37. The $17^{th}$ term of an A.P. is 5 more than twice its $8^{th}$ term. If the $11^{th}$ term of the A.P. is 43, find the $n^{th}$ term. [CBSE 2012]
38. Find the number of all three digit natural numbers which are divisible by 9. [CBSE 2013]
39. The $19^{th}$ term of an A.P. is equal to three times its sixth term. If its 9th term is 19, find the A.P. [CBSE 2013]
40. The $9^{th}$ term of an A.P. is equal to 6 times its second term. If its $5^{th}$ term is 22, find the A.P. [CBSE 2013]
41. The $24^{th}$ term of an A.P. is twice its $10^{th}$ term. Show that its $72^{nd}$ term is 4 times its $15^{th}$ term. [CBSE 2013]
42. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5. [CBSE 2014]
43. If the seventh term of an A.P. is 1/9 and its ninth term is 1/7, find its $(63)^{rd}$ term. [CBSE 2014]
44. The sum of $5^{th}$ and $9^{th}$ terms of an A.P. is 30. If its $25^{th}$ term is three times its $8^{th}$ term, find the AP. [CBSE 2014]
45. Find whether 0 (zero) is a term of the A.P. 40, 37, 34, 31,... . [CBSE 2014]
46. Find the middle term of the A.P. 213, 205, 197, ..., 37. [CBSE 2015]
47. If the $5^{th}$ term of an A.P. is 31 and $25^{th}$ term is 140 more than the $5^{th}$ term, find the A.P. [CBSE 2015]

## LEVEL-2

48. Find the sum of two middle terms of the A.P.: $-\frac{4}{3}, -1, \frac{-2}{3}, -\frac{1}{3}, ..., 4\frac{1}{3}$. [NCERT EXEMPLAR]
49. If $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term, prove that $(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term.
50. If an A.P. consists of $n$ terms with first term $a$ and $n^{th}$ term $l$ show that the sum of the $m^{th}$ term from the beginning and the $m^{th}$ term from the end is $(a+l)$.
51. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3? [NCERT EXEMPLAR]
52. Find the $12^{th}$ term from the end of the A.P. $-2, -4, -6, ..., -100$. [NCERT EXEMPLAR]
53. For the A.P.: $-3, -7, -11, ...$, can we find $a_{30} - a_{20}$ without actually finding $a_{30}$ and $a_{20}$? Give reasons for your answer. [NCERT EXEMPLAR]
54. Two A.P.s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. The difference between their $10^{th}$ terms is the same as the difference between their $21^{st}$ terms, which is the same as the difference between any two corresponding terms. Why? [NCERT EXEMPLAR]

## ANSWERS

1. (i) 28 (ii) $35\sqrt{2}$ (iii) $-5n+18$ (iv) 185
(v) 26 (vi) 15 (vii) $\frac{19}{4}$

2. (i) 50 (ii) 22 (iii) 51 (iv) $20^{th}$ term
(v) $32^{nd}$

3. (i) No (ii) No (iii) No

4. (i) 13 (ii) 27 (iii) 34 (iv) 27

5. (i) 26

6. 87

9. 105

11. $d=-\frac{1}{5}, n=27$

12. 7

13. (i) 179 (ii) 198 (iii) 55

14. First term = 3, Common difference = 2

15. $a_2=-8, a_n=5n-18$

16. 30

17. 69

18. $-\frac{1}{2}, \frac{5}{2}$

19. $-142$

20. (i) $-50$ (ii) $10d$ (iii) 5 (iv) 17

21. (i) $(n-k)d, 37$ (ii) 40

22. (i) 40 (ii) 76 (iii) 100

23. 3

24. 4, 10, 16, 22, ...

25. 2, 7, 12, 17,....

26. $25^{th}$

27. 100

28. 13

29. 60

30. 128

31. $53^{rd}$

32. $49^{th}$

33. 163

34. 108

35. $-13, -8, -3, 2, 7, \ldots$

36. $31^{st}$

37. $4n-1$

38. 100

39. 3, 5, 7, 9, ....

40. 2, 7, 12, 17, ...

42. 89

43. 1

44. 3, 5, 7, 9, 11,...

45. No

46. 125

47. 3, 10, 17, 24, ....

48. 3

51. 73

52. $-78$

53. Yes, $a_{30}-a_{20}=(30-20)d=-40$

54. The difference between any two corresponding terms of such A.P's is the same as the difference between their first term.

## 5.5 SELECTION OF TERMS IN AN A.P.

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

| *Number of terms* | *Terms* | *Common difference* |
|---|---|---|
| 3 | $a-d, a, a+d$ | $d$ |
| 4 | $a-3d, a-d, a+d, a+3d$ | $2d$ |
| 5 | $a-2d, a-d, a, a+d, a+2d$ | $d$ |
| 6 | $a-5d, a-3d, a-d, a+d, a+3d, a+5d$ | $2d$ |

It should be noted that in case of an odd number of terms, the middle term is $a$ and the common difference is $d$ while in case of an even number of terms the middle terms are $a-d, a+d$ and the common differences is $2d$.

REMARK 1 *If three numbers a, b, c in order are in A.P. Then,*

$$b-a = \text{Common difference} = c-b$$

$$\Rightarrow \quad b-a=c-b$$

$$\Rightarrow \quad 2b=a+c$$

*Thus, a, b, c are in A.P. if and only if* $2b=a+c$.

REMARK 2 *If a, b, c are in A.P., then b is known as the arithmetic mean (AM) between a and c.*

REMARK 3 *If a, c, b are in A.P. Then,*

$$2c=a+b \Rightarrow c=\frac{a+b}{2}$$

*Thus,* A.M. *between a and b is* $\frac{a+b}{2}$.

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

EXAMPLE 1 *If $2x, x+10, 3x+2$ are in A.P., find the value of x.*

SOLUTION Since, $2x, x+10, 3x+2$ are in A.P.

$\therefore \quad 2(x+10) = 2x + (3x+2)$

$\Rightarrow \quad 2x + 20 = 5x + 2$

$\Rightarrow \quad 3x = 18 \Rightarrow x = 6$

EXAMPLE 2 *The sum of three numbers in A.P. is −3, and their product is 8. Find the numbers.*

SOLUTION Let the numbers be $(a-d), a, (a+d)$. It is given that the sum of the numbers is −3.

$\therefore \quad (a-d) + a + (a+d) = -3 \Rightarrow 3a = -3 \Rightarrow a = -1$

It is also given that the product of the product of the numbers is 8.

$\therefore \quad (a-d)(a)(a+d) = 8$

$\Rightarrow \quad a(a^2 - d^2) = 8$

$\Rightarrow \quad (-1)(1-d^2) = 8 \qquad [\because a = -1]$

$\Rightarrow \quad d^2 = 9 \Rightarrow d = \pm 3$

If $d = 3$, the numbers are $-4, -1, 2$. If $d = -3$, the numbers are $2, -1, -4$.
Thus, the numbers are $-4, -1, 2$, or $2, -1, -4$.

EXAMPLE 3 *Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.*

SOLUTION Let the numbers be $(a-3d), (a-d), (a+d), (a+3d)$. Then,

Sum of numbers = 20

$\Rightarrow \quad (a-3d) + (a-d) + (a+d) + (a+3d) = 20 \Rightarrow 4a = 20 \Rightarrow a = 5$

It is given that, sum of the squares = 120

$\Rightarrow \quad (a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$

$\Rightarrow \quad 4a^2 + 20d^2 = 120$

$\Rightarrow \quad a^2 + 5d^2 = 30$

$\Rightarrow \quad 25 + 5d^2 = 30 \qquad [\because a = 5]$

$\Rightarrow \quad 5d^2 = 5 \Rightarrow d = \pm 1$

If $d = 1$, then the numbers are 2, 4, 6, 8. If $d = -1$, then the numbers are 8, 6, 4, 2.
Thus, the numbers are 2, 4, 6, 8 or 8, 6, 4, 2.

EXAMPLE 4 *Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7:15.*

SOLUTION Let the four parts be $(a-3d), (a-d), (a+d)$ and $(a+3d)$. Then,

Sum of the numbers = 32

$\Rightarrow \quad (a-3d) + (a-d) + (a+d) + (a+3d) = 32 \Rightarrow 4a = 32 \Rightarrow a = 8$

It is given that

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\Rightarrow \quad \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$\Rightarrow \quad \frac{64-9d^2}{64-d^2} = \frac{7}{15} \Rightarrow 128d^2 = 512 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

Thus, the four parts are $a-3d, a-d, a+d$ and $a+3d$ i.e., 2, 6, 10, 14.

**LEVEL-2**

EXAMPLE 5 *If the numbers a, b, c, d, e form an A.P., then find the value of* $a-4b+6c-4d+e$.

SOLUTION Let $D$ be the common difference of the given A.P. Then,

$$b = a+D, c = a+2D, d = a+3D \text{ and } e = a+4D$$

$$\therefore \quad a-4b+6c-4d+e = a-4(a+D)+6(a+2D)-4(a+3D)+(a+4D)$$

$$\Rightarrow \quad a-4b+6c-4d+e = a-4a-4D+6a+12D-4a-12D+a+4D$$

$$\Rightarrow \quad a-4b+6c-4d+e = a-4a+6a-4a+a-4D+12D-12D+4D$$

$$\Rightarrow \quad a-4b+6c-4d+e = 0$$

EXAMPLE 6 *The sum of the first three terms of an A.P. is 33. If the product of first and the third term exceeds the second term by 29, find the AP.* [NCERT EXEMPLAR]

SOLUTION Let the first three terms in A.P. be $a-d, a, a+d$. It is given that the sum of these terms is 33.

$$\therefore \quad a-d+a+a+d = 33 \Rightarrow 3a = 33 \Rightarrow a = 11$$

It is given that

$$(a-d)(a+d) = a+29$$

$$\Rightarrow \quad a^2-d^2 = a+29$$

$$\Rightarrow \quad 121-d^2 = 11+29$$

$$\Rightarrow \quad d^2 = 81$$

$$\Rightarrow \quad d = \pm 9$$

Thus, we have $a = 11, d = 9$ or $a = 11$ and $d = -9$.
Hence, the two AP's are 2, 11, 20, 29, ... and 20, 11, 2, ... .

EXAMPLE 7 *Determine k so that* $k^2+4k+8, 2k^2+3k+6, 3k^2+4k+4$ *are three conse- cutive terms of an A.P.* [NCERT EXEMPLAR]

SOLUTION We know that if $a, b, c$ are three consecutive terms of an A.P., then

$$b-a = c-b \text{ i.e. } 2b = a+c$$

Thus, if $k^2+4k+8, 2k^2+3k+6$ and $3k^2+4k+4$ are three consecutive terms of an A.P., then

$$2(2k^2+3k+6) = (k^2+4k+8)+(3k^2+4k+4)$$

$$\Rightarrow \quad 4k^2+6k+12 = 4k^2+8k+12$$

$$\Rightarrow \quad 2k = 0 \Rightarrow k = 0.$$

EXAMPLE 8 *If* $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ *is the A.M. between a and b. Then, find the value of n.*

SOLUTION We know that the A.M. between $a$ and $b$ is $\frac{a+b}{2}$.

It is given that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between $a$ and $b$.

$$\therefore \quad \frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$$

$$\Rightarrow \quad 2(a^{n+1}+b^{n+1})=(a^n+b^n)(a+b)$$

$$\Rightarrow \quad 2a^{n+1}+2b^{n+1}=a^{n+1}+ab^n+a^nb+b^{n+1}$$

$$\Rightarrow \quad a^{n+1}+b^{n+1}=ab^n+a^nb$$

$$\Rightarrow \quad a^{n+1}-a^nb=ab^n-b^{n+1}$$

$$\Rightarrow \quad a^n(a-b)=b^n(a-b)$$

$$\Rightarrow \quad a^n=b^n \Rightarrow \frac{a^n}{b^n}=1 \Rightarrow \left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^0 \Rightarrow n=0$$

## EXERCISE 5.5

### LEVEL-1

1. Find the value of $x$ for which $(8x+4)$, $(6x-2)$ and $(2x+7)$ are in A.P.
2. If $x+1$, $3x$ and $4x+2$ are in A.P., find the value of $x$.
3. Show that $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$ are in A.P.
4. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.
5. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.
6. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.
7. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers. [CBSE 2016]
8. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes to the product of their means is 5 : 6. [CBSE 2016]

### LEVEL-2

9. The angles of a quadrilateral are in A.P. whose common difference is $10^\circ$. Find the angles.
10. Split 207 into three parts such that these are in A.P. and the product of the two smaller parts is 4623. [NCERT EXEMPLAR]
11. The angles of a triangle are in A.P. The greatest angle is twice the least. Find all the angles. [NCERT EXEMPLAR]
12. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the number. [NCERT EXEMPLAR, CBSE 2018]

### ANSWERS

1. 15/2
2. 3
4. 1, 7, 13
5. 6, 9, 12
6. 5, 10, 15, 20
7. 2, 4, 6 or 6, 4, 2
8. 8, 12, 16, 20
9. $75^\circ, 85^\circ, 95^\circ, 105^\circ$
10. 67, 69, 71
11. $40^\circ, 60^\circ, 80^\circ$
12. 2, 6, 10, 14

## 5.6 SUM TO $n$ TERMS OF AN A.P.

**THEOREM** *The sum $S_n$ of $n$ terms of an A.P. with first term 'a' and common difference 'd' is*

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

or, $$S_n = \frac{n}{2}\{a + l\}, \text{ where } l = \text{last term} = a + (n-1)d$$

PROOF Let $a_1, a_2, a_3, \ldots$ be an A.P. with first term $a$ and common difference $d$. Then,

$$a_1 = a, a_2 = a + d, a_3 = a + 2d, a_4 = a + 3d, \ldots, a_n = a + (n-1)d$$

Now, $S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$

$\Rightarrow$ $S_n = a + (a + d) + (a + 2d) + \ldots + (a + (n-2)d) + \{a + (n-1)d\}$ ...(i)

Writing the above series in a reverse order, we get

$S_n = \{a + (n-1)d\} + \{a + (n-2)d\} + \ldots + (a + d) + a$ ...(ii)

Adding the corresponding terms of equations (i) and (ii), we get

$2S_n = \{2a + (n-1)d\} + \{2a + (n-1)d\} + \ldots + \{2a + (n-1)d\}$

$\Rightarrow$ $2S_n = n\{2a + (n-1)d\}$ [$\because 2a + (n-1)d$ repeats $n$ times]

$\Rightarrow$ $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Now, $l$ = last term = $n^{th}$ term = $a + (n-1)d$

$\therefore$ $S_n = \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}[a + \{a + (n-1)d\}] = \frac{n}{2}\{a + l\}$

Q.E.D.

NOTE 1 *In the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, there are four quantities viz. $S_n$, $a$, $n$ and $d$. If any three of these are known, the fourth can be determined. Sometimes two of these quantities are given, in such cases remaining two quantities are provided by some other relation.*

NOTE 2 *In the sum $S_n$ of $n$ terms of a sequence is given, then $n^{th}$ term $a_n$ of the sequence can be determined by using the following formula*

$$a_n = S_n - S_{n-1}$$

## ILLUSTRATIVE EXAMPLES

### LEVEL-1

*Type I* **ON FINDING THE SUM OF AN A.P.**

EXAMPLE 1 *Find the sum of 20 terms of the A.P. 1, 4, 7, 10 ...*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then, we have

$a = 1$ and $d = 3$.

We have to find the sum of 20 terms of the given A.P.

Putting $a = 1, d = 3, n = 20$ in $S_n = \frac{n}{2}\{2a + (n-1)d\}$, we get

$$S_{20} = \frac{20}{2}\{2 \times 1 + (20-1) \times 3\} = 10 \times 59 = 590$$

EXAMPLE 2 *If the $n^{th}$ term of an A.P. is $(2n + 1)$, find the sum of first $n$ terms of the A.P.* [CBSE 2005]

SOLUTION We have,

$$a_n = (2n+1) \Rightarrow a_1 = 2 \times 1 + 1 = 3$$

So, the given sequence is an A.P. with first term $a = a_1 = 3$ and last term $l = a_n = 2n + 1$. Therefore, the sum of $n$ terms is given by

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}\{3 + (2n+1)\} = \frac{n}{2}(2n+4) = n(n+2)$$

**EXAMPLE 3** *Find the sum of first 30 terms of an A.P. whose second term is 2 and seventh term is 22.*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$a_2 = 2 \text{ and } a_7 = 22$$

$$\Rightarrow \quad a + d = 2 \text{ and } a + 6d = 22$$

Solving these two equations, we get $a = -2$ and $d = 4$. Putting $n = 30, a = -2$ and $d = 4$ in $S_n = \frac{n}{2}\{2a + (n-1)d\}$, we obtain

$$\therefore \quad S_{30} = \frac{30}{2}\{2 \times (-2) + (30-1) \times 4\}$$

$$\Rightarrow \quad S_{30} = 15(-4 + 116) = 15 \times 112 = 1680$$

Hence, the sum of first 30 terms is 1680.

**EXAMPLE 4** *Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$a_3 = 7 \text{ and } a_7 = 3a_3 + 2 \quad \text{[Given]}$$

$$\Rightarrow \quad a + 2d = 7 \text{ and } a + 6d = 3(a + 2d) + 2$$

$$\Rightarrow \quad a + 2d = 7 \text{ and } a + 6d = 3a + 6d + 2$$

$$\Rightarrow \quad a + 2d = 7 \text{ and } a = -1$$

$$\Rightarrow \quad a = -1, d = 4$$

Putting $n = 20, a = -1$ and $d = 4$ in $S_n = \frac{n}{2}\{2a + (n-1)d\}$, we get

$$S_{20} = \frac{20}{2}\{2 \times -1 + (20-1) \times 4\} = \frac{20}{2}(-2 + 76) = 740$$

**EXAMPLE 5** *Find the sum of first n natural numbers.*

SOLUTION First $n$ natural numbers are $1, 2, 3, 4, \ldots, (n-1), n$.

Clearly, it is an A.P. with first term 1 and common difference 1.

Let $S_n$ denote the required. Then,

$$S_n = \frac{n}{2}(1 + n) \quad \left[\text{Using} : S_n = \frac{n}{2}(a + l)\right]$$

$$\Rightarrow \quad S_n = \frac{n(n+1)}{2}$$

**EXAMPLE 6** *The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P.* **[CBSE 2009]**

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$S_6 = 42 \Rightarrow \frac{6}{2}\{2a + (6-1)d\} = 42 \Rightarrow 2a + 5d = 14 \quad \ldots(i)$$

It is given that

$$a_{10} : a_{30} = 1 : 3$$

$$\Rightarrow \quad \frac{a + 9d}{a + 29d} = \frac{1}{3}$$

$$\Rightarrow \quad 3a + 27d = a + 29d$$

$$\Rightarrow \quad 2a - 2d = 0$$

$$\Rightarrow \quad a = d \quad \ldots(ii)$$

Solving (i) and (ii), we get $a = d = 2$

$\therefore \quad a_{13} = a + 12d = 2 + 2 \times 12 = 26$

Hence, first term = 2 and thirteenth term = 26.

EXAMPLE 7 *Find the sum of all three digit natural numbers, which are divisible by 7.* [CBSE 2006C]

SOLUTION The smallest and the largest numbers of three digits, which are divisible by 7 are 105 and 994 respectively. So, the sequence of three digit numbers which are divisible by 7 are 105, 112, 119, ..., 994. Clearly, it is an A.P. with first term $a = 105$ and common difference $d = 7$. Let there be $n$ terms in this sequence. Then,

$$a_n = 994 \Rightarrow a + (n-1)d = 994 \Rightarrow 105 + (n-1) \times 7 = 994 \Rightarrow n = 128$$

Now,

$$\text{Required sum} = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow \quad \text{Required sum} = \frac{128}{2}\{2 \times 105 + (128 - 1) \times 7\} = 70336$$

EXAMPLE 8 *Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.*

SOLUTION Clearly, the numbers between 250 and 1000 which are divisible by 3 are

252, 255, 258, ..., 999.

This is an A.P. with first term $a = 252$, Common difference = 3 and last term = 999.

Let there be $n$ terms in this A.P. Then,

$$a_n = 999$$

$$\Rightarrow \quad a + (n-1)d = 999$$

$$\Rightarrow \quad 252 + (n-1) \times 3 = 999$$

$$\Rightarrow \quad n = 250$$

$$\therefore \quad \text{Required sum} = S_n = \frac{n}{2}(a + l) = \frac{250}{2}(252 + 999) = 156375$$

EXAMPLE 9 *Find the sum of all odd integers between 2 and 100 divisible by 3.*

SOLUTION The odd integers between 2 and 100 which are divisible by 3 are

3, 9, 15, 21, ..., 99.

Clearly, it is an A.P. with first term $a = 3$ and, common difference $d = 6$.

Let there be $n$ terms in this sequence. Then,

$$a_n = 99$$

$\Rightarrow$ $a + (n-1)d = 99$

$\Rightarrow$ $3 + (n-1) \times 6 = 99 \Rightarrow n = 17$

$\therefore$ Required sum $= S_n = \frac{n}{2}(a+l) = \frac{17}{2}(3+99) = 867$

EXAMPLE 10 *If the ratio of the 11th term of an A.P. to its 18th term is 2 : 3, find the ratio of the sum of first five terms to the the sum of its first 10 terms.* [CBSE 2017]

SOLUTION Let '$a$' be the first term and '$d$' be the common difference of the given A.P.
It is given that

$$\frac{a_{11}}{a_{18}} = \frac{2}{3} \Rightarrow \frac{a+10}{a+17d} = \frac{2}{3} \Rightarrow 3a+30d = 2a+34d \Rightarrow a = 4d$$

$$\therefore \quad \frac{S_5}{S_{10}} = \frac{\frac{5}{2}\{2a+(5-1)d\}}{\frac{10}{2}\{2a+(10-1)d\}} = \frac{1}{2}\left\{\frac{2a+4d}{2a+9d}\right\} = \frac{1}{2}\left\{\frac{2\times 4d+4d}{2\times 4d+9d}\right\} = \frac{1}{2}\left(\frac{12d}{17d}\right) = \frac{6}{17}$$

Hence, $S_5 : S_{10} = 6:17$

*Type II* **ON FIDING THE NUMBER OF TERMS IN AN A.P. WHEN THEIR SUM IS GIVEN**

EXAMPLE 11 *How many terms of the series 54, 51, 48,... be taken so that their sum is 513? Explain the double answer.* [CBSE 2005]

SOLUTION Clearly, the given sequence is an A.P. with first term $a$ (= 54) and common differenced $d$ (= – 3). Let the sum of $n$ terms be 513. Then,

$$S_n = 513$$

$\Rightarrow$ $\frac{n}{2}\{2a + (n-1)d\} = 513$

$\Rightarrow$ $\frac{n}{2}[108 + (n-1) \times -3] = 513$

$\Rightarrow$ $n^2 - 37n + 342 = 0$

$\Rightarrow$ $(n-18)(n-19) = 0 \Rightarrow n = 18$ or, 19

Here, the common difference is negative. So, 19th term is given by

$$a_{19} = 54 + (19-1) \times -3 = 0$$

Thus, the sum of 18 terms as well as that of 19 terms is 513.

EXAMPLE 12 *Find the number of terms in the series* $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots$ *of which the sum is 300, explain the double answer.*

SOLUTION The given series is an arithmetic series with first term $a$ (= 20) and the common difference $d\left(= -\frac{2}{3}\right)$. Let the sum of $n$ terms be 300. Then,

$$S_n = 300$$

$\Rightarrow$ $\frac{n}{2}\{2a + (n-1)d\} = 300$

$\Rightarrow \quad \frac{n}{2}\{2\times 20+(n-1)(-2/3)\}=300$

$\Rightarrow \quad n^2-61n+900=0 \Rightarrow (n-25)(n-36)=0 \Rightarrow n=25$ or, 36

So, sum of 25 terms = sum of 36 terms = 300.

Here, the common difference is negative therefore terms go on diminishing and 31st term becomes zero. All terms after 31st term are negative. These negative terms when added to positive terms from 26th term to 30th term, they cancel out each other and the sum remains same.

Hence, the sum of 25 terms as well as that of 36 terms is 300.

*Type III* **ON FIDING THE DESIRED TERM WHEN THE SUM OF $n$ TERMS OF AN A.P. IS GIVEN**

**EXAMPLE 13** *If $S_n$, the sum of first n terms of an A.P., is given by $S_n=5n^2+3n$, then find its $n^{th}$ term.* [CBSE 2009]

SOLUTION Let $a_n$ be the $n^{th}$ term of the A.P. Then,

$a_n=S_n-S_{n-1}$

$\Rightarrow \quad a_n=(5n^2+3n)-\{5(n-1)^2+3(n-1)\}$ $\left[\begin{array}{l}\text{Replacing } n \text{ by } (n-1) \text{ in } S_n\\ \text{to get } S_{n-1}=5(n-1)^2+3(n-1)\end{array}\right]$

$\Rightarrow \quad a_n=(5n^2+3n)-(5n^2-7n+2)$

$\Rightarrow \quad a_n=10n-2$

**EXAMPLE 14** *In an A.P., the sum of first n terms is $\frac{3n^2}{2}+\frac{5n}{2}$. Find its 25th term.* [CBSE 2006C]

SOLUTION Let $S_n$ denote the sum of $n$ terms of an A.P. whose $n$th term is $a_n$.
We have,

$$S_n=\frac{3n^2}{2}+\frac{5n}{2}$$

$\Rightarrow \quad S_{n-1}=\frac{3}{2}(n-1)^2+\frac{5}{2}(n-1)$ [Replacing $n$ by $(n-1)$]

$\therefore \quad S_n-S_{n-1}=\left\{\frac{3n^2}{2}+\frac{5n}{2}\right\}-\left\{\frac{3}{2}(n-1)^2+\frac{5}{2}(n-1)\right\}$

$\Rightarrow \quad a_n=\frac{3}{2}\{n^2-(n-1)^2\}+\frac{5}{2}\{n-(n-1)\}$ [$\because a_n=S_n-S_{n-1}$]

$\Rightarrow \quad a_n=\frac{3}{2}(2n-1)+\frac{5}{2}$

$\Rightarrow \quad a_{25}=\frac{3}{2}(2\times 25-1)+\frac{5}{2}=\frac{3}{2}\times 49+\frac{5}{2}=76$ [Replacing $n$ by 25]

**EXAMPLE 15** *A manufacturer of TV sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in (i) the first year (ii) the 10th year (iii) 7 years.* [CBSE 2015, NCERT]

SOLUTION Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P.

Let $a$ be the first term and $d$ be the common difference of the A.P. formed i.e., '$a$' denotes the production in the first year and $d$ denotes the number of units by which the production increases every year.

We have,

$$a_3 = 600 \text{ and } a_7 = 700 \Rightarrow a + 2d = 600 \text{ and } a + 6d = 700$$

Solving these equations, we get: $a = 550$ and $d = 25$

(i) We have, $a = 550$

$\therefore$ Production in the first year is of 550 TV sets.

(ii) The production in the 10th term is given by $a_{10}$.

$\therefore$ Production in the 10th year $= a_{10} = a + 9d = 550 + 9 \times 25 = 775$

So, production in 10th year is of 775 TV sets.

(iii) Total production in 7 years

= Sum of 7 terms of the A.P. with first term $a$ (=550) and common difference $d$ (= 25).

$$= \frac{7}{2}\{2 \times 550 + (7-1) \times 25\} = \frac{7}{2}(1100 + 150) = 4375$$

Thus, the total production in 7 years is of 4375 TV sets.

EXAMPLE 16 *A sum of ₹280 is to be used to award four prizes. If each prize after the first is ₹20 less than its preceding prize, find the value of each of the prizes.* [CBSE 2014, NCERT]

SOLUTION The values of four prizes form an A.P. with common difference $d = -20$ the sum of whose terms is 280. Let the value of first prize be ₹ $a$. Then,

$$\text{Sum} = 280$$

$$\Rightarrow \quad \frac{4}{2}\{2a + (4-1) \times -20\} = 280$$

$$\Rightarrow \quad 2(2a - 60) = 280 \Rightarrow a - 30 = 70 \Rightarrow a = 100$$

Hence, the values of 4 prizes are ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

EXAMPLE 17 *In a school, students thought of planting trees in and around the school to reduce noise polution and air polution. It was decided that the number of trees that each section of each class will plant be the same as the class in which they are studying e.g. – a section of I class will plant 1 tree, a section of II class will plant 2 trees and so on a section of class XII will plant 12 trees. There are three sections of each class. How many trees will be planted by the students?*

[CBSE 2014, NCERT]

SOLUTION Since each section of each class plants the same number of trees as the class number and there are three sections of each class.

$\therefore$ Total number of trees planted by the students

$$= 3[1 + 2 + 3 + \ldots + 12]$$

$$= 3\left[\frac{12}{2}\{2 \times 1 + (12-1) \times 1\}\right] = 3\{6(2 + 11)\} = 18 \times 13 = 234$$

## LEVEL-2

EXAMPLE 18 *An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P.* [NCERT EXEMPLAR]

SOLUTION In 38 terms of the A.P., we find that 19th term is the middle term and 18th 19th and 20th terms are three middle most terms.

Let $a$ be the first term and $d$ be the common difference of the A.P. Then,

$$a_{18} + a_{19} + a_{20} = 225 \text{ and, } a_{35} + a_{36} + a_{37} = 429 \quad \text{[Given]}$$

$$\Rightarrow \quad (a + 17d) + (a + 18d) + (a + 19d) = 225 \text{ and, } (a + 34d) + (a + 35d) + (a + 36d) = 429$$

$$\Rightarrow \quad 3a + 54d = 225 \text{ and } 3a + 105d = 429$$

$\Rightarrow \quad a + 18d = 75$ and $a + 35d = 143$

$\Rightarrow \quad a = 3$ and $d = 4$

Hence, the A.P. is 3, 7, 11, 15, ..... .

EXAMPLE 19 *Find the sum of all 11 terms of an A.P. whose middle most term is 30.* [NCERT EXEMPLAR]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Clearly, in an A.P. consisting of 11 terms, $\left(\frac{11+1}{2}\right)^{th}$ *i.e.* 6th term is the middle most term. It is given that the middle most term is 30.

$$\therefore \quad a + 5d = 30 \qquad \text{...(i)}$$

$$\therefore \quad S_{11} = \frac{11}{2}\{2a + (11-1)d\} = 11(a+5d) = 11 \times 30 = 330 \qquad \text{[Using (i)]}$$

EXAMPLE 20 *If the $m^{th}$ term of an A.P. is $\frac{1}{n}$ and the $n^{th}$ term is $\frac{1}{m}$, show that the sum of $mn$ terms is $\frac{1}{2}(mn+1)$.* [CBSE 2015, 2017]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \qquad \text{...(i)}$$

and, $$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \qquad \text{...(ii)}$$

Subtracting equation (ii) from equation (i), we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow (m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in equation (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

$$\therefore \quad S_{mn} = \frac{mn}{2}\{2a + (mn-1)d\}$$

$$\Rightarrow \quad S_{mn} = \frac{mn}{2}\left\{\frac{2}{mn} + (mn-1) \times \frac{1}{mn}\right\} = \frac{1}{2}(mn+1)$$

EXAMPLE 21 *The sum of $n$, $2n$, $3n$ terms of an A.P. are $S_1, S_2, S_3$ respectively. Prove that*

$$S_3 = 3(S_2 - S_1)$$

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$S_1 = \text{Sum of } n \text{ terms} \Rightarrow S_1 = \frac{n}{2}\{2a + (n-1)d\} \qquad \text{...(i)}$$

$$S_2 = \text{Sum of } 2n \text{ terms} \Rightarrow S_2 = \frac{2n}{2}\{2a + (2n-1)d\} \qquad \text{...(ii)}$$

and, $$S_3 = \text{Sum of } 3n \text{ terms} \Rightarrow S_3 = \frac{3n}{2}\{2a + (3n-1)d\} \qquad \text{...(iii)}$$

Now,

$$S_2 - S_1 = \frac{2n}{2}\{2a + (2n-1)d\} - \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow \quad S_2 - S_1 = \frac{n}{2}[2\{2a+(2n-1)d\} - \{2a+(n-1)d\}] = \frac{n}{2}\{2a+(3n-1)d\}$$

$$\therefore \quad 3(S_2 - S_1) = \frac{3n}{2}\{2a + (3n-1)d\} = S_3 \quad \text{[Using (iii)]}$$

Hence, $S_3 = 3(S_2 - S_1)$.

**EXAMPLE 22** *The sums of n terms of three arithmetical progressions are $S_1$, $S_2$ and $S_3$. The first term of each is unity and the common differences are 1, 2 and 3 respectively. Prove that* $S_1 + S_3 = 2S_2$.

SOLUTION We have,

$S_1$ = Sum of $n$ terms of an A.P. with first term 1 and common difference 1

$$\Rightarrow \quad S_1 = \frac{n}{2}\{2 \times 1 + (n-1) \times 1\} = \frac{n}{2}(n+1)$$

$S_2$ = Sum of $n$ terms of an A.P. with first term 1 and common difference 2

$$\Rightarrow \quad S_2 = \frac{n}{2}\{2 \times 1 + (n-1) \times 2\} = n^2$$

$S_3$ = Sum of $n$ terms of an A.P. with first term 1 and common difference 3

$$\Rightarrow \quad S_3 = \frac{n}{2}\{2 \times 1 + (n-1) \times 3\} = \frac{n}{2}(3n-1)$$

Now, $S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1) = 2n^2$ and $S_2 = n^2$

Hence, $S_1 + S_3 = 2S_2$

**EXAMPLE 23** *The sum of the third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the A.P.

We have,

$a_3 + a_7 = 6$ and $a_3 a_7 = 8$

$\Rightarrow \quad (a + 2d) + (a + 6d) = 6$ and $(a + 2d)(a + 6d) = 8$

$\Rightarrow \quad 2a + 8d = 6$ and $(a + 2d)(a + 6d) = 8$

$\Rightarrow \quad a + 4d = 3$ and $(a + 2d)(a + 6d) = 8$

$\Rightarrow \quad a = 3 - 4d$ and $(a + 2d)(a + 6d) = 8$

$\Rightarrow \quad (3 - 4d + 2d)(3 - 4d + 6d) = 8$ [Putting $a = 3 - 4d$ in the second equation]

$\Rightarrow \quad (3 - 2d)(3 + 2d) = 8$

$$\Rightarrow \quad 9 - 4d^2 = 8 \Rightarrow 4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE I *When* $d = \frac{1}{2}$: Putting $d = \frac{1}{2}$ in $a = 3 - 4d$, we get

$$a = 3 - 4 \times \frac{1}{2} = 3 - 2 = 1$$

$$\therefore \quad S_{16} = \frac{16}{2}\{2a + (16-1)d\} = 8\left\{2 \times 1 + 15 \times \frac{1}{2}\right\} = 8 \times \frac{19}{2} = 76$$

CASE II When $d = -\frac{1}{2}$: Putting $d = -\frac{1}{2}$ in $a = 3 - 4d$, we get $a = 3 + 2 = 5$

$$\therefore \quad S_{16} = \frac{16}{2}\{2a + (16-1)d\} = 8\left\{10 + 15 \times -\frac{1}{2}\right\} = 8 \times \frac{5}{2} = 20$$

EXAMPLE 24 *If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then prove that the sum of (m + n) terms is – (m + n).*

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$S_m = n$$

$$\Rightarrow \quad \frac{m}{2}\{2a + (m-1)d\} = n$$

$$\Rightarrow \quad 2am + m(m-1)d = 2n \qquad \text{...(i)}$$

and, $$S_n = m$$

$$\Rightarrow \quad \frac{n}{2}\{2a + (n-1)d\} = m$$

$$\Rightarrow \quad 2an + n(n-1)d = 2m \qquad \text{...(ii)}$$

Subtracting equation (ii) from equation (i), we get

$$2a(m-n) + \{m(m-1) - n(n-1)\}d = 2n - 2m$$

$$\Rightarrow \quad 2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = -2(m-n)$$

$$\Rightarrow \quad 2a + (m+n-1)d = -2 \qquad \text{[On dividing both sides by } (m-n)] \quad \text{...(iii)}$$

Now,

$$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$$

$$\Rightarrow \quad S_{m+n} = \frac{(m+n)}{2}(-2) \qquad \text{[Using (iii)]}$$

$$\Rightarrow \quad S_{m+n} = -(m+n)$$

EXAMPLE 25 *If the sum of m terms of an A.P. is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.* [CBSE 2017]

SOLUTION Let $a$ be the first term and $d$ be the common difference of the given A.P. Then,

$$S_m = S_n$$

$$\Rightarrow \quad \frac{m}{2}\{2a + (m-1)d\} = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow \quad 2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$$

$$\Rightarrow \quad 2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$$

$$\Rightarrow \quad (m-n)\{2a + (m+n-1)d\} = 0$$

$$\Rightarrow \quad 2a + (m+n-1)d = 0 \qquad [\because m - n \neq 0] \quad \text{...(i)}$$

Now,

$$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\} = \frac{m+n}{2} \times 0 = 0 \qquad \text{[Using (i)]}$$

**EXAMPLE 26** *The sum of the first p, q, r terms of an A.P. are a, b, c respectively. Show that*

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

SOLUTION Let $A$ be the first term and $D$ be the common difference of the given A.P. Then,

$$a = \text{Sum of } p \text{ terms}$$

$$\Rightarrow \quad a=\frac{p}{2}\{2A+(p-1)D\}$$

$$\Rightarrow \quad \frac{2a}{p}=\{2A+(p-1)D\} \quad ...(i)$$

$$b = \text{Sum of } q \text{ terms}$$

$$\Rightarrow \quad b=\frac{q}{2}\{2A+(q-1)D\}$$

$$\Rightarrow \quad \frac{2b}{q}=\{2A+(q-1)D\} \quad ...(ii)$$

and, $c$ = Sum of $r$ terms

$$\Rightarrow \quad c=\frac{r}{2}\{2A+(r-1)D\}$$

$$\Rightarrow \quad \frac{2c}{r}=\{2A+(r-1)D\} \quad ...(iii)$$

Multiplying equation (i), (ii) and (iii) by $(q-r),(r-p)$ and $(p-q)$ respectively and adding, we get

$$\frac{2a}{p}(q-r)+\frac{2b}{q}(r-p)+\frac{2c}{r}(p-q)$$

$$=\{2A+(p-1)D\}(q-r)+\{2A+(q-1)D\}(r-p)+\{2A+(r-1)D\}(p-q)$$

$$=2A(q-r+r-p+p-q)+D\{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\}$$

$$=2A\times 0+D\times 0=0$$

**EXAMPLE 27** *The ratio of the sum of n terms of two A.P's is (7n + 1) : (4n + 27). Find the ratio of their $m^{th}$ terms.*

SOLUTION Let $a_1, a_2$ be the first terms and $d_1, d_2$ the common differences of the two given A.P's. Then, the sums of their $n$ terms are given by

$$S_n=\frac{n}{2}\{2a_1+(n-1)d_1\} \text{ and, } S_n'=\frac{n}{2}\{2a_2+(n-1)d_2\}$$

$$\therefore \quad \frac{S_n}{S_n'}=\frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}}=\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}$$

It is given that

$$\frac{S_n}{S_n'}=\frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{2a_1 + (n-1) d_1}{2a_2 + (n-1) d_2} = \frac{7n+1}{4n+27} \quad \text{...(i)}$$

To find the ratio of the $m^{th}$ terms of the two given A.P's, we replace $n$ by $(2m - 1)$ in equation (i).

Replacing $n$ by $(2m-1)$ in equation (i), we get

$$\therefore \quad \frac{2a_1 + (2m-2) d_1}{2a_2 + (2m-2) d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \quad \frac{a_1 + (m-1) d_1}{a_2 + (m-1) d_2} = \frac{14m-6}{8m+23}$$

Hence, the ratio of the $m^{th}$ terms of the two A.P's is $(14m-6) : (8m+23)$.

EXAMPLE 28 *The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the $m^{th}$ and $n^{th}$ terms is $(2m-1) : (2n-1)$.* [CBSE 2016, 2017]

SOLUTION Let $a$ be the first term and $d$ the common difference of the given A.P. Then, the sums of $m$ and $n$ terms are given by

$$S_m = \frac{m}{2}\{2a + (m-1) d\} \text{ and, } S_n = \frac{n}{2}\{2a + (n-1) d\} \text{ respectively.}$$

Then,

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{\frac{m}{2}\{2a + (m-1) d\}}{\frac{n}{2}\{2a + (n-1) d\}} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{2a + (m-1) d}{2a + (n-1) d} = \frac{m}{n}$$

$$\Rightarrow \quad \{2a + (m-1) d\} n = \{2a + (n-1) d\} m$$

$$\Rightarrow \quad 2a(n-m) = d\{(n-1) m - (m-1) n\}$$

$$\Rightarrow \quad 2a(n-m) = d(n-m)$$

$$\Rightarrow \quad d = 2a$$

$$\therefore \quad \frac{T_m}{T_n} = \frac{a + (m-1) d}{a + (n-1) d} = \frac{a + (m-1) 2a}{a + (n-1) 2a} = \frac{2m-1}{2n-1}$$

EXAMPLE 29 *If there are $(2n+1)$ terms in A.P., then prove that the ratio of the sum of odd terms and the sum of even terms is $(n+1) : n$.*

SOLUTION Let $a$ and $d$ be the first term and common difference respectively of the given A.P. Let $a_k$ denote the $k^{th}$ terms of the given A.P. Then,

$$a_k = a + (k-1) d$$

Now, $S_1$ = Sum of odd terms

$$\Rightarrow \quad S_1 = a_1 + a_3 + a_5 + \dots + a_{2n+1}$$

$$\Rightarrow \quad S_1 = \frac{n+1}{2}\{a_1 + a_{2n+1}\}$$

$$\Rightarrow \quad S_1 = \frac{n+1}{2}\{a + a + (2n+1-1)d\} \qquad [\because a_{2n+1} = a + (2n+1-1)d]$$

$$\Rightarrow \quad S_1 = (n+1)(a+nd)$$

and, $\quad S_2$ = Sum of even terms

$$\Rightarrow \quad S_2 = a_2 + a_4 + a_6 + \ldots + a_{2n}$$

$$\Rightarrow \quad S_2 = \frac{n}{2}[a_2 + a_{2n}]$$

$$\Rightarrow \quad S_2 = \frac{n}{2}[(a+d) + \{a + (2n-1)d\}] \qquad [\because a_{2n} = a + (2n-1)d]$$

$$\Rightarrow \quad S_2 = n(a+nd)$$

$$\therefore \quad S_1 : S_2 = (n+1)(a+nd) : n(a+nd) = (n+1) : n$$

**EXAMPLE 30** *Show that the sum of an A.P. whose first term is a, the second term is b and the last term is c, is equal to* $\frac{(a+c)(b+c-2a)}{2(b-a)}$. [NCERT EXEMPLAR]

SOLUTION Let there be $n$ terms in the given A.P.
We have,

First term $= a$, second term $= b$

$\therefore$ Common difference $d = b - a$

It is given that the last term is $c$ i.e. $n$th term $= c$.

$$\therefore \quad c = a + (n-1)d$$

$$\Rightarrow \quad c = a + (n-1)(b-a)$$

$$\Rightarrow \quad n - 1 = \frac{c-a}{b-a} \Rightarrow n = \frac{b+c-2a}{b-a} \qquad \ldots(i)$$

Let $S_n$ be the sum of $n$ terms of the A.P. Then,

$$S_n = \frac{n}{2}(a+c) = \frac{(b+c-2a)(a+c)}{2(b-a)} \qquad \text{[Using (i)]}$$

**EXAMPLE 31** *Solve the equation:* $1 + 4 + 7 + 10 + \cdots + x = 287$. [NCERT EXEMPLAR]

SOLUTION Here, 1, 4, 7, 10, ..., $x$, is an A.P. with first term $a = 1$ and common difference $d = 3$. Let there be $n$ terms in the A.P. Then,

$$x = n\text{th term} \Rightarrow x = 1 + (n-1) \times 3 = 3n - 2 \qquad \ldots(i)$$

Now,

$$1 + 4 + 7 + 10 + \cdots + x = 287$$

$$\Rightarrow \quad \frac{n}{2}(1+x) = 287 \qquad \left[\text{Using } S_n = \frac{n}{2}(a+l)\right]$$

$$\Rightarrow \quad \frac{n}{2}(1 + 3n - 2) = 287 \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad 3n^2 - n = 574 \Rightarrow 3n^2 - n - 574 = 0 \Rightarrow 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow \quad 3n(n-14) + 41(n-14) = 0$$

$\Rightarrow \quad (n-14)(3n+41)=0 \Rightarrow n-14=0$ $\quad [\because 3n+4 \neq 0]$

$\Rightarrow \quad n=14$

Putting $n = 14$ in (i), we get $x = 3 \times 14 - 2 = 40$.

EXAMPLE 32 *A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc; the penalty for each succeeding day being ₹50 more than for the preceding day. How much does a delay of 30 days cost the contractor?* [NCERT]

SOLUTION Since the penalty for each succeeding day is ₹ 50 more than for the preceding day. Therefore, amount of penalty for different days forms an A.P. with first term $a$ (= 200) and common difference $d$ (= 50). We have to find how much does a delay of 30 days cost the contractor? In other words, we have to find the sum of 30 terms of the A.P.

$\therefore \quad \text{Required sum} = \frac{30}{2}\{2 \times 200 + (30-1) \times 50\}$ $\quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad \text{Required sum} = 15(400 + 29 \times 50)$

$\Rightarrow \quad \text{Required sum} = 15(400 + 1450)$

$\Rightarrow \quad \text{Required sum} = 15 \times 1850 = 27750$

Thus, a delay of 30 days will cost the contractor of ₹ 27750.

EXAMPLE 33 *The digits of a positive integer, having three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.*

SOLUTION Let the digits at ones, tens and hundreds place be $(a-d)$, $a$ and $(a+d)$ respectively. Then, the number is

$$(a+d) \times 100 + a \times 10 + (a-d) = 111a + 99d$$

The number obtained by reversing the digits is

$$(a-d) \times 100 + a \times 10 + (a+d) = 111a - 99d$$

It is given that the sum of the digits is 15.

$$(a-d) + a + (a+d) = 15 \quad \ldots(i)$$

Also, it is given that the number obtained by reversing the digits is 594 less than the original number.

$\therefore \quad 111a - 99d = 111a + 99d - 594$ $\quad \ldots(ii)$

$\Rightarrow \quad 3a = 15$ and $198d = 594$

$\Rightarrow \quad a = 5$ and $d = 3$

So, the number is $111 \times 5 + 99 \times 3 = 852$.

EXAMPLE 34 *A man repays a loan of ₹3250 by paying ₹20 in the first month and then increases the payment by ₹15 every month. How long will it take him to clear the loan?*

SOLUTION Suppose the loan is cleared in $n$ months. Clearly, the amounts form an A.P. with first term 20 and the common difference 15.

$\therefore \quad \text{Sum of the amounts} = 3250$

$\Rightarrow \quad \frac{n}{2}\{2 \times 20 + (n-1) \times 15\} = 3250$

$\Rightarrow \quad \frac{n}{2}(40 + 15n - 15) = 3250$

$\Rightarrow \quad n(15n+25)=6500$

$\Rightarrow \quad 15n^2+25n-6500=0$

$\Rightarrow \quad 3n^2+5n-1300=0$

$\Rightarrow \quad (n-20)(3n+65)=0$

$\Rightarrow \quad n=20 \text{ or}, n=-\frac{65}{3} \Rightarrow n=20 \qquad \left[\because n \neq -\frac{65}{3}\right]$

Thus, the loan is cleared in 20 months.

EXAMPLE 35 *A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See Fig. 5.1). Calculate the total volume of concrete required to build the terrace.* [NCERT]

SOLUTION We observe that the length and width of each step are 50 m and $\frac{1}{2}$ m respectively. Also, we have

Height of first step $=\frac{1}{4}$ m.

Height of second step $=\left(\frac{1}{4}+\frac{1}{4}\right)$ m $=\left(2\times\frac{1}{4}\right)$ m

Height of third step $=\frac{3}{4}$ m and so on.

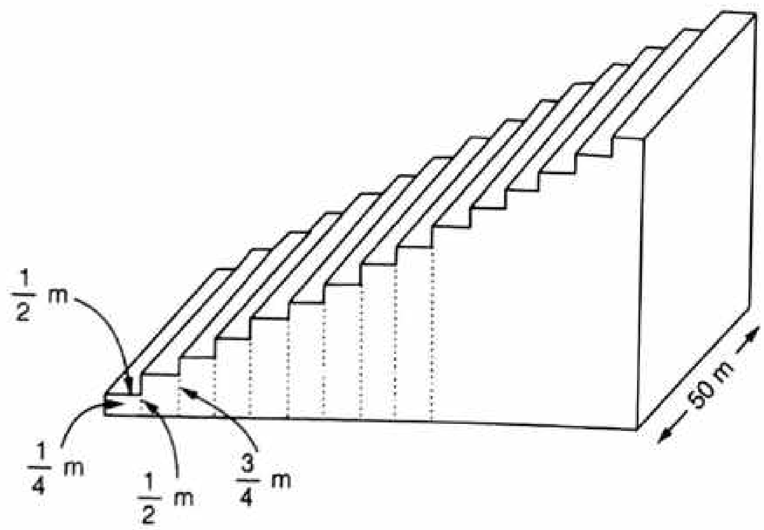


Fig. 5.1

Let $V_1, V_2, V_3, \dots, V_{15}$ denote respectively the volumes of the concrete required to build the first, second, third, ..., fifteenth step. Then,

$$V_1=\left(50\times\frac{1}{2}\times\frac{1}{4}\right)\text{m}^3,\ V_2=\left\{50\times\frac{1}{2}\times\left(2\times\frac{1}{4}\right)\right\}\text{m}^3,\ V_3=\left\{50\times\frac{1}{2}\times\left(3\times\frac{1}{4}\right)\right\}\text{m}^3,$$

$$V_4=\left(50\times\frac{1}{2}\times 1\right)\text{m}^3 \text{ and so on.}$$

$\therefore$ Total volume of the concrete $= V_1 + V_2 + V_3 + \cdots + V_{15}$

$$= \left\{50 \times \frac{1}{2} \times \frac{1}{4}\right\} + \left\{50 \times \frac{1}{2} \times \left(2 \times \frac{1}{4}\right)\right\} + \left\{50 \times \frac{1}{2}\left(3 \times \frac{1}{4}\right)\right\} + \cdots + \left\{50 \times \frac{1}{2} \times \left(15 \times \frac{1}{4}\right)\right\} \text{m}^3$$

$$= \left(50 \times \frac{1}{2}\right) \left\{\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \cdots + \frac{15}{4}\right\} \text{m}^3$$

$$= 25 \left\{\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \cdots + \frac{15}{4}\right\} \text{m}^3$$

$$= \frac{25}{4} \{1 + 2 + 3 + \cdots + 15\} \text{ m}^3 = \frac{25}{4} \times \frac{15}{2}(1 + 15) \text{ m}^3 = \frac{25}{4} \times \frac{15}{2} \times 16 \text{ m}^3 = 750 \text{ m}^3$$

**EXAMPLE 36** *200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 5.2). In how many rows 200 logs are placed and how many logs are in the top row?* [NCERT]

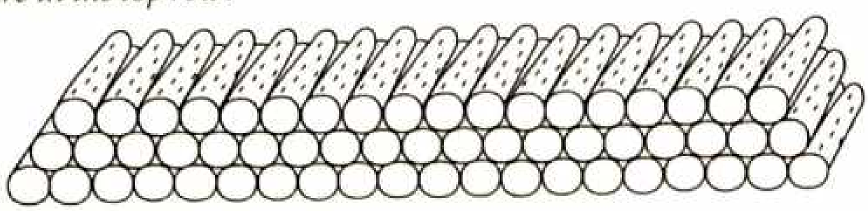

Fig. 5.2

SOLUTION Suppose 200 logs are stacked in $n$ rows.

There are 20 logs in the first row and the number of logs in a row is one less than the number logs in the preceding row. So, number of logs in various rows form an A.P. with first term $a$ (= 20) and common difference $d$ (= – 1). As there are 200 logs in all rows.

$\therefore$ (Sum of $n$ terms of an A.P. with $a = 20$ and $d = -1$) = 200

$\Rightarrow \quad \frac{n}{2}\{2a + (n-1)d\} = 200$

$\Rightarrow \quad \frac{n}{2}\{2 \times 20 + (n-1) \times -1\} = 200$

$\Rightarrow \quad \frac{n}{2}(40 - n + 1) = 200$

$\Rightarrow \quad n(41 - n) = 400$

$\Rightarrow \quad n^2 - 41n + 400 = 0$

$\Rightarrow \quad (n - 25)(n - 16) = 0 \Rightarrow n = 16 \text{ or, } n = 25$

Now,

If $n = 25$, then number of logs in $25^{th}$ row is equal to $25^{th}$ terms of an A.P. with first term 20 and common difference – 1.

$\therefore$ Number of logs in $25^{th}$ row $= a + 24d = 20 - 24 = -4$

Clearly, this is not meaningful.

$\therefore \quad n = 16$

Thus, logs are placed in 16 rows.

Number of logs in top row

= Number of logs in $16^{th}$ row

$= 16^{th}$ term of an A.P. with $a = 20$ and $d = -1$

$= a + 15d$

$= 20 + 15 \times -1 = 5$

Hence, there are 5 logs in the top row.

EXAMPLE 37 *Raghav buys a shop for ₹1, 20,000. He pays half of the amount in cash and agrees to pay the balance in 12 annual instalments of ₹5000 each. If the rate of interest is 12% and he pays with the instalment the interest due on the unpaid amount, find the total cost of the shop.*

SOLUTION Raghav pays half of ₹ 1,20,000 i.e. ₹ 60,000 in cash and the balance ₹ 60,000 in 12 annual instalments of ₹ 5000 each. With each instalment he pays interest on the unpaid amount at the rate of 12% per annum.

$\therefore$ Amount of first instalment = ₹ 5000 + Interest on unpaid amount of ₹ 60000

$$= ₹\ 5000 + ₹\left(\frac{12}{100} \times 60000\right) = ₹\ 5000 + ₹\ 7200 = ₹\ 12200$$

$\therefore$ Amount of second instalment = ₹ 5000 + Interest on unpaid amount of ₹ 55000

$$= ₹\ 5000 + ₹\left(\frac{12}{100} \times 55000\right) = ₹\ 5000 + ₹\ 6600 = ₹\ 11600$$

$\therefore$ Amount of third instalment = ₹ 5000 + Interest on unpaid amount of ₹ 50000

$$= ₹\ 5000 + ₹\left(\frac{12}{100} \times 50000\right) = ₹\ 5000 + ₹\ 6000 = ₹\ 11000$$

Clearly, amount of various instalments form an A.P. with first term ₹ 12200 and common difference – 600

$\therefore$ Total cost of the shop = ₹ [60,000 + Sum of 12 instalments]

$$= ₹\left[60,000 + \frac{12}{2}\{2 \times 12200 + (12-1) \times (-600)\}\right]$$

$$= ₹\ [60,000 + 6\,(24,400 - 6,600)]$$

$$= ₹\ [60,000 + 1,06,800] = ₹\ 1,66,800$$

EXAMPLE 38 *Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km in each succeeding hour. After how many hours will the second car overtake the first car if both cars go non-stop?*

SOLUTION Suppose the second car overtakes the first car after $t$ hours. Then, the two cars travel the same distance in $t$ hours.

Distance travelled by the first car in $t$ hours = $10\,t$ km.

Distance travelled by the second car in $t$ hours.

= Sum of $t$ terms of an A.P. with first term 8 and common difference 1/2.

$$= \frac{t}{2}\left\{2 \times 8 + (t-1) \times \frac{1}{2}\right\} = \frac{t\,(t+31)}{4}$$

When the second car overtakes the first car, we have

$$10t = \frac{t\,(t+31)}{4} \Rightarrow 40t = t^2 + 31t \Rightarrow t^2 - 9t = 0 \Rightarrow t\,(t-9) = 0 \Rightarrow t = 9 \qquad [\because t \neq 0]$$

Thus, the second car will overtake the first car in 9 hours.

**EXAMPLE 39** *150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.*

SOLUTION Suppose the work is completed in $n$ days when the workers started dropping. Since 4 workers are dropped on every day except the first day. Therefore, the total number of workers who worked all the $n$ days is the sum of $n$ terms of an A.P. with first term 150 and common difference $-4$

i.e., $$\frac{n}{2}\{2 \times 150 + (n-1) \times -4\} = n(152 - 2n)$$

Had the workers not dropped then the work would have finished it in $(n-8)$ days with 150 workers working on each day. Therefore, the total number of workers who would have worked all the $n$ days is $150(n-8)$.

$\therefore \quad n(152 - 2n) = 150(n-8)$

$\Rightarrow \quad 152n - 2n^2 = 150n - 1200$

$\Rightarrow \quad 2n^2 - 2n - 1200 = 0$

$\Rightarrow \quad n^2 - n - 600 = 0$

$\Rightarrow \quad (n-25)(n+24) = 0 \qquad [\because n > 0]$

$\Rightarrow \quad n = 25$

Hence, the work is completed in 25 days.

**EXAMPLE 40** *Along a road lie an odd number of stones placed at intervals of 10 metres. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried the job with one of the end stones by carrying them in succession. In carrying all the stones he covered a distance of 3 km. Find the number of stones.*

SOLUTION Let there be $(2n+1)$ stones. Clearly, one stone lies in the middle and $n$ stones on each side of it in a row. Let $P$ be the mid-stone and let $A$ and $B$ be the end stones on the left and right of $P$ respectively.

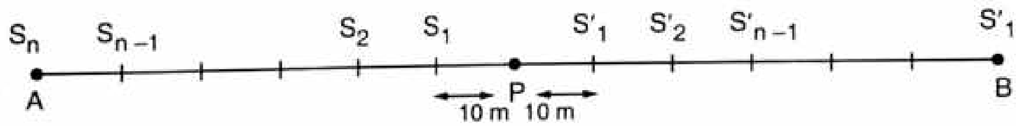


Fig. 5.3

Clearly, there are $n$ intervals each of length 10 metres on both the sides of $P$. Now, suppose the man starts from $A$. He picks up the end stone on the left of mid-stone and goes to the mid-stone, drops it and goes to $(n-1)$ th stone on left, picks it up, goes to the mid-stone and drops it. This process is repeated till he collects all stones on the left of the mid-stone at the mid-stone. So, distance covered in collecting stones on the left of the mid-stones is

$$10 \times n + 2[10 \times (n-1) + 10 \times (n-2) + \ldots + 10 \times 2 + 10 \times 1].$$

After collecting all stones on left of the mid-stone the man goes to the stone $B$ on the right side of the mid-stone, picks it up, goes to the mid-stone and drops it. Then, he goes to $(n-1)^{th}$ stone on the right and the process is repeated till he collects all stones at the mid-stone.

Distance covered in collecting the stones on the right side of the mid-stone.

$$= 2\left[10 \times n + 10 \times (n-1) + 10 \times (n-2) + \ldots + 10 \times 2 + 10 \times 1\right]$$

$\therefore$ Total distance covered

$$= 10 \times n + 2\left[10 \times (n-1) + 10 \times (n-2) + \ldots + 10 \times 2 + 10 \times 1\right] + 2\left[10 \times n + 10 \times (n-1) + \ldots + 10 \times 2 + 10 \times 1\right]$$

$$= 4\left[10 \times n + 10 \times (n-1) + \ldots + 10 \times 2 + 10 \times 1\right] - 10 \times n$$

$$= 40\{1+2+3+\ldots+n\} - 10n = 40\left\{\frac{n}{2}(1+n)\right\} - 10n = 20n(n+1) - 10n = 20n^2 + 10n$$

But, the total distance covered is 3 km i.e. 3000 m

$\therefore \quad 20n^2 + 10n = 3000$

$\Rightarrow \quad 2n^2 + n - 300 = 0$

$\Rightarrow \quad (n-12)(2n+25) = 0 \qquad [\because 2n + 25 \neq 0]$

$\Rightarrow \quad n = 12$

Hence, the number of stones $= 2n + 1 = 25$

EXAMPLE 41 *The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.* [NCERT]

SOLUTION Let there be a value of $x$ such that the sum of the numbers of the houses preceding the house numbered $x$ is equal to the sum of the numbers of the houses following it i.e.,

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| House: | $H_1$ | $H_2$ | $H_3$ | $\cdots$ | $H_{x-1}$ | $H_x$ | $H_{x+1}$ | $\cdots$ | $H_{49}$ |
| House No. | 1 | 2 | 3 | | $(x-1)$ | $x$ | $(x+1)$ | $\cdots$ | 49 |

$1 + 2 + 3 + \cdots + (x-1) = (x+1) + (x+2) + \cdots + 49$

$\Rightarrow \quad 1+2+3+\cdots+(x-1) = \{1+2+3+\cdots+x+(x+1)+\cdots+49\} - (1+2+3+\cdots+x)$

$\Rightarrow \quad \frac{x-1}{2}\{1+(x-1)\} = \frac{49}{2}(1+49) - \frac{x}{2}(1+x) \qquad \left[\text{Using: } S_n = \frac{n}{2}(a+l)\right]$

$\Rightarrow \quad \frac{x(x-1)}{2} = \frac{49 \times 50}{2} - \frac{x(x+1)}{2}$

$\Rightarrow \quad x(x-1) = 49 \times 50 - x(x+1) \qquad$ [Multiplying both sides by 2]

$\Rightarrow \quad (x^2 - x) + (x^2 + x) = 49 \times 50$

$\Rightarrow \quad 2x^2 = 49 \times 50$

$\Rightarrow \quad x^2 = 49 \times 25$

$\Rightarrow \quad x^2 = 7^2 \times 5^2$

$\Rightarrow \quad x = 7 \times 5 = 35$

Since, $x$ is not a fraction. Hence, the value of $x$ satisfying the given condition exists and is equal to 35.

EXAMPLE 42 *A ladder has rungs 25 cm apart (See Fig 5.4). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are 2.5 metre apart, what is the length of the wood required for the rungs?* [NCERT]

SOLUTION It is given that the gap between two consecutive rungs is 25 cm and the top and bottom rungs are 2.5 metre i.e., 250 cm apart.

$\therefore$ Number of rungs $= \frac{250}{25} + 1 = 11.$

It is given that the rungs are decreasing uniformly in length from 45 cm at the bottom to 25 cm at the top. Therefore, lengths of the rungs form an A.P. with first term $a = 45$ cm and $11^{th}$ term $= 25$ cm.

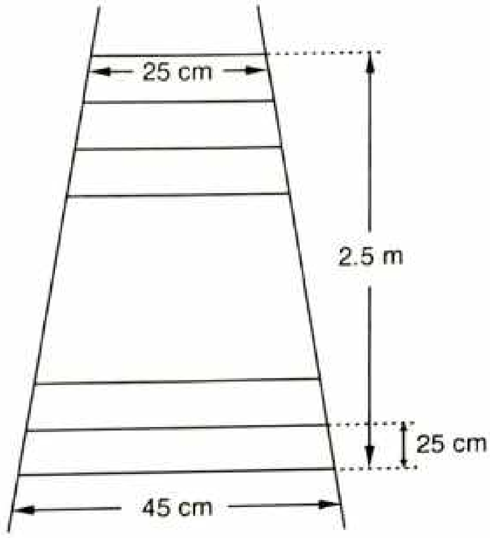


Fig. 5.4

$\therefore$ Length of the wood required for rungs

= Sum of 11 terms of an A.P. with first term 45 cm and last term = 25 cm

$= \frac{11}{2}(45 + 25)$ cm $\left[\because S_n = \frac{n}{2}(a + l)\right]$

$= 385$ cm $= 3.85$ metres

EXAMPLE 43 *A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, .... as shown in Fig. 5.5 what is the total length of such a spiral made up of thirteen consecutive semi-circles?* $\left(\text{Take } \pi = \frac{22}{7}\right)$. [NCERT]

SOLUTION Let $l_1, l_2, l_3, l_4, \ldots, l_{13}$ be the lengths (circumferences) of semi-circles of radii $r_1 = 0.5$ cm, $r_2 = 1.0$ cm, $r_3 = 1.5$ cm, $r_4 = 2.0$ cm, $r_5 = 2.5$ cm, ... respectively. Then,

$$l_1 = \pi r_1 = \pi \times 0.5 \text{ cm} = \frac{\pi}{2} \text{ cm}$$

$$l_2 = \pi r_2 = \pi \times 1 \text{ cm} = 2\left(\frac{\pi}{2}\right) \text{ cm}$$

$$l_3 = \pi r_3 = \pi \times \frac{3}{2} \text{ cm} = 3\left(\frac{\pi}{2}\right) \text{ cm}$$

$$l_4 = \pi r_4 = \pi \times 2 \text{ cm} = 4\left(\frac{\pi}{2}\right) \text{ cm}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$l_{13} = \pi r_{13} = \pi \times \frac{13}{2} \text{ cm} = 13\left(\frac{\pi}{2}\right) \text{ cm}$$

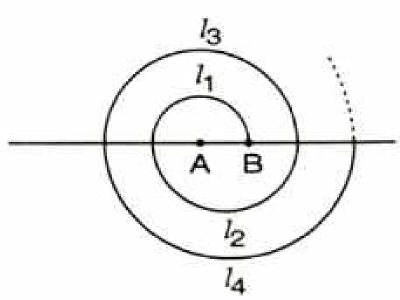


Fig. 5.5

$\therefore$ Total length of the spiral $= l_1 + l_2 + l_3 + \cdots + l_{13}$

$$= \left\{\frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) + 3\left(\frac{\pi}{2}\right) + \cdots + 13\left(\frac{\pi}{2}\right)\right\} \text{cm}$$

$$= \frac{\pi}{2}(1 + 2 + 3 + \cdots + 13)\text{ cm}$$

$$= \frac{\pi}{2} \times \frac{13}{2}(1 + 13)\text{ cm} \quad \left[\text{Using } S_n = \frac{n}{2}(a + l)\right]$$

$$= \frac{\pi}{2} \times \frac{13}{2} \times 14\text{ cm} = \frac{1}{2} \times \frac{22}{7} \times 13 \times 7\text{ cm} = 143\text{ cm}$$

**EXAMPLE 44** *In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are n potatoes in the line (See Fig. 5.6). Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in the bucket, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?* [NCERT


Fig. 5.6

SOLUTION We have,

$d_1$ = Distance run by the competitor to pick up first potato $= 2 \times 5$ m
$d_2$ = Distance run by the competitor to pick up second potato $= 2\,(5 + 3)$ m
$d_3$ = Distance run by the competitor to pick up third potato $= 2\,(5 + 2 \times 3)$ m
$d_4$ = Distance run by the competitor to pick up fourth potato $= 2\,(5 + 3 \times 3)$ m
$\vdots$ $\vdots$
$d_n$ = Distance run by the competitor to pick up $n^{th}$ potato $= 2\,\{5 + (n-1) \times 3\}$ m

$\therefore$ Total distance run by the competitor to pick up $n$ potatoes

$$= d_1 + d_2 + d_3 + \cdots + d_n$$

$$= 2 \times 5 + 2(5 + 3) + 2(5 + 2 \times 3) + 2(5 + 3 \times 3) + \cdots + 2\{5 + (n-1) \times 3\} \text{ metres}$$

$$= 2[5 + \{5 + 3\} + \{5 + (2 \times 3)\} + \{5 + (3 \times 3\} + \cdots + \{5 + (n-1) \times 3\}]$$

$$= 2\Big[(\underbrace{5 + 5 + \cdots + 5}_{n-\text{times}}) + \{3 + (2 \times 3) + (3 \times 3) + \cdots + (n-1) \times 3\}\Big]$$

$$= 2[5n + 3\{1 + 2 + 3 + \cdots + (n-1)\}]$$

$$= 2\left[5n + 3\left(\frac{n-1}{2}\right)\{1 + (n-1)\}\right] \quad \left[\text{Using: } S_n = \frac{n}{2}(a + l)\right]$$

$$= 2\left\{5n + \frac{3n(n-1)}{2}\right\} = [10n + 3n(n-1)] = 3n^2 + 7n = n(3n + 7) \text{ metres}$$

## EXERCISE 5.6

### LEVEL-1

1. Find the sum of the following arithmetic progressions:
   (i) 50, 46, 42, ... to 10 terms (ii) 1, 3, 5, 7, ... to 12 terms

(iii) $3, 9/2, 6, 15/2, \ldots$ to 25 terms (iv) $41, 36, 31, \ldots$ to 12 terms

(v) $a+b, a-b, a-3b, \ldots$ to 22 terms (vi) $(x-y)^2, (x^2+y^2), (x+y)^2 \ldots$, to $n$ terms

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots$ to $n$ terms

(viii) $-26, -24, -22, \ldots$ to 36 terms.

2. Find the sum to $n$ term of the A.P. $5, 2, -1, -4, -7, \ldots,$
3. Find the sum of $n$ terms of an A.P. whose $n^{th}$ terms is given by $a_n = 5 - 6n$.
4. Find the sum of last ten terms of the A.P.: $8, 10, 12, 14, \ldots, 126$. [NCERT EXEMPLAR]
5. Find the sum of the first 15 terms of each of the following sequences having $n^{th}$ term as
   (i) $a_n = 3 + 4n$ (ii) $b_n = 5 + 2n$ (iii) $x_n = 6 - n$ (iv) $y_n = 9 - 5n$ [NCERT]
6. Find the sum of first 20 terms of the sequence whose $n^{th}$ term is $a_n = An + B$.
7. Find the sum of the first 25 terms of an A.P. whose $n^{th}$ term is given by $a_n = 2 - 3n$. [CBSE 2004]
8. Find the sum of the first 25 terms of an A.P. whose $n^{th}$ term is given by $a_n = 7 - 3n$. [CBSE 2004]
9. If the sum of a certain number of terms starting from first term of an A.P. is $25, 22, 19, \ldots,$ is 116. Find the last term.
10. (i) How many terms of the sequence $18, 16, 14, \ldots$ should be taken so that their sum is zero?
    (ii) How many terms are there in the A.P. whose first and fifth terms are $-14$ and 2 respectively and the sum of the terms is 40 ?
    (iii) How many terms of the A.P. $9, 17, 25, \ldots$ must be taken so that their sum is 636? [NCERT]
    (iv) How many terms of the A.P. $63, 60, 57, \ldots$ must be taken so that their sum is 693? [CBSE 2005]
    (v) How many terms of the A.P. $27, 24, 21 \ldots$ should be taken so that their sum is zero? [CBSE 2016]
11. Find the sum of the first
    (i) 11 terms of the A.P : $2, 6, 10, 14, \ldots$
    (ii) 13 terms of the A.P : $-6, 0, 6, 12, \ldots$
    (iii) 51 terms of the A.P : whose second term is 2 and fourth term is 8.
12. Find the sum of
    (i) the first 15 multiples of 8 [NCERT, CBSE 2017]
    (ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6. [NCERT]
    (iii) all 3 – digit natural numbers which are divisible by 13. [CBSE 2006C]
    (iv) all 3-digit natural numbers, which are multiples of 11. [CBSE 2012]
    (v) all 2-digit natural numbers divisible by 4. [CBSE 2017]
    (vi) first 8 multiples of 3. [CBSE 2018]
13. Find the sum:
    (i) $2 + 4 + 6 + \ldots + 200$ (ii) $3 + 11 + 19 + \ldots + 803$
    (iii) $(-5) + (-8) + (-11) + \ldots + (-230)$ (iv) $1 + 3 + 5 + 7 + \ldots + 199$
    (v) $7 + 10\frac{1}{2} + 14 + \ldots + 84$ (vi) $34 + 32 + 30 + \ldots + 10$
    (vii) $25 + 28 + 31 + \ldots + 100$ [CBSE 2006C]
    (viii) $18 + 15\frac{1}{2} + 13 + \cdots + \left(-49\frac{1}{2}\right)$ [CBSE 2013]

14. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
15. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.
16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.
17. If $12^{th}$ term of an A.P. is –13 and the sum of the first four terms is 24, what is the sum of first 10 terms ?
18. Find the sum of $n$ terms of the series $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots$ [CBSE 2017]
19. In an A.P., if the first term is 22, the common difference is – 4 and the sum to $n$ terms is 64, find $n$.
20. In an A.P., if the $5^{th}$ and $12^{th}$ terms are 30 and 65 respectively, what is the sum of first 20 terms?
21. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively. [NCERT]
22. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of $n$ terms. **[CBSE 2013, 2016, NCERT]**
23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. [NCERT]
24. In an A.P. the first term is 8, $n$th term is 33 and the sum to first $n$ terms is 123. Find $n$ and $d$, the common differences. [CBSE 2008]
25. In an A.P., the first term is 22, $n$th term is –11 and the sum to first $n$ terms is 66. Find $n$ and $d$, the common difference. [CBSE 2008]
26. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find its common difference. [CBSE 2014]
27. The first and the last terms of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference. [CBSE 2014]
28. The sum of first $q$ terms of an A.P. is 162. The ratio of its $6^{th}$ term to its $13^{th}$ term is $1:2$. Find the first and $15^{th}$ term of the A.P. [CBSE 2015]
29. If the 10th term of an A.P. is 21 and the sum of its first ten terms is 120, find its $n^{th}$ term. [CBSE 2014]
30. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the $28^{th}$ term of this A.P. [CBSE 2014]
31. The sum of first seven terms of an A.P. is 182. If its 4th and the 17th terms are in the ratio $1:5$, find the A.P. [CBSE 2014]
32. The $n$th term of an A.P. is given by $(-4n+15)$. Find the sum of first 20 terms of this A.P. [CBSE 2013]
33. In an A.P., the sum of first ten terms is –150 and the sum of its next ten terms is –550. Find the A.P. [CBSE 2010]
34. Sum of the first 14 terms of an A.P. is 1505 and its first term is 10. Find its $25^{th}$ term. [CBSE 2012]
35. In an A.P., the first term is 2, the last term is 29 and the sum of the terms is 155. Find the common difference of the A.P. [CBSE 2010]

36. The first and the last term of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum? [NCERT]

37. Find the number of terms of the A.P. $-12, -9, -6, \ldots, 21$. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained. [CBSE 2013]

38. The sum of the first $n$ terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P. [CBSE 2014]

39. The sum of first $n$ terms of an A.P. is $5n - n^2$. Find the $n^{th}$ term of this A.P. [CBSE 2014]

40. The sum of the first $n$ terms of an A.P. is $4n^2 + 2n$. Find the $n^{th}$ term of this A.P. [CBSE 2014]

41. The sum of first $n$ terms of an A.P. is $3n^2 + 4n$. Find the 25th term of this A.P. [CBSE 2013]

42. The sum of first $n$ terms of an A.P. is $5n^2 + 3n$. If its $m$th term is 168, find the value of $m$. Also, find the 20th term of this A.P. [CBSE 2013]

43. The sum of first $q$ terms of an A.P. is $63q - 3q^2$. If its $p$th term is $-60$, find the value of $p$. Also, find the 11th term of this A.P. [CBSE 2013]

44. The sum of first $m$ terms of an A.P. is $4m^2 - m$. If its $n$th term is 107, find the value of $n$. Also, find the 21st term of this A.P. [CBSE 2013]

45. If the sum of the first $n$ terms of an A.P. is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the third, the tenth and the $n$th terms. [NCERT]

46. If the sum of first $n$ terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$, then find its $n^{th}$ term. Hence write its $20^{th}$ term. [CBSE 2015]

47. In an A.P., the sum of first $n$ terms is $\frac{3n^2}{2} + \frac{13}{2}n$. Find its $25^{th}$ term. [CBSE 2006C]

48. Find the sum of all natural numbers between 1 and 100, which are divisible by 3.

49. Find the sum of first $n$ odd natural numbers.

50. Find the sum of all odd numbers between
    (i) 0 and 50 [CBSE 2017] (ii) 100 and 200.

51. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

52. Find the sum of all integers between 84 and 719, which are multiples of 5.

53. Find the sum of all integers between 50 and 500, which are divisible by 7.

54. Find the sum of all even integers between 101 and 999.

55. (i) Find the sum of all integers between 100 and 550, which are divisible by 9.
    (ii) all integers between 100 and 550 which are not divisible by 9.
    (iii) all integers between 1 and 500 which are multiplies of 2 as well as of 5.
    (iv) all integers from 1 to 500 which are multiplies 2 as well as of 5.
    (v) all integers from 1 to 500 which are multiples of 2 or 5.

56. Let there be an A.P. with first term '$a$', common difference '$d$'. If $a_n$ denotes its $n^{th}$ term and $S_n$ the sum of first $n$ terms, find.
    (i) $n$ and $S_n$, if $a = 5, d = 3$ and $a_n = 50$. (ii) $n$ and $a$, if $a_n = 4, d = 2$ and $S_n = -14$.
    (iii) $d$, if $a = 3, n = 8$ and $S_n = 192$. (iv) $a$, if $a_n = 28, S_n = 144$ and $n = 9$.
    (v) $n$ and $d$, if $a = 8, a_n = 62$ and $S_n = 210$.

(vi) $n$ and $a_n$, if $a = 2, d = 8$ and $S_n = 90$. [NCERT]

(vii) $k$, if $S_n = 3n^2 + 5n$ and $a_k = 164$. (viii) $S_{22}$, if $d = 22$ and $a_{22} = 149$

57. If $S_n$ denotes the sum of first $n$ terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$. [NCERT EXEMPLAR, CBSE 2015]

58. A thief, after committing a theft runs at a uniform speed of 50 m/minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every suceeding minute. After how many minutes, the policeman will catch the thief? [CBSE 2016]

59. The sums of first n terms of three A.P.s are $S_1$, $S_2$ and $S_3$. The first term of each is 5 and their common differences are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$. [CBSE 2016]

60. Resham wanted to save at least ₹ 6500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months ? Will she be able to send her daughter to the school next year? [CBSE 2016]

61. In a school, students decided to plant trees in and around the school to reduce air polution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the studetns. [CBSE 2014]

62. Ramkali would need ₹ 1800 for admission fee and books etc., for her daughter to start going to school from next year. She saved ₹ 50 in the first month of this year and increased her monthly saving by ₹ 20. After a year, how much money will she save? Will she be able to fulfil her dream of sending her daughter to school?

[CBSE 2005, 2014]

63. A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?

64. A man saved ₹ 32 during the first year, ₹ 36 in the second year and in this way he increases his savings by ₹ 4 every year. Find in what time his saving will be ₹ 200.

65. A man arranges to pay off a debt of ₹ 3600 by 40 annual instalments which form an arithmetic series. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid, find the value of the first instalment.

66. There are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

67. A man is employed to count ₹ 10710. He counts at the rate of ₹180 per minute for half an hour. After this he counts at the rate of ₹ 3 less every minute than the preceding minute. Find the time taken by him to count the entire amount.

68. A piece of equipment cost a certain factory ₹ 600,000. If it depreciates in value, 15% the first, 13.5% the next year, 12% the third year, and so on. What will be its value at the end of 10 years, all percentages applying to the original cost?

69. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each prize.

70. If $S_n$ denotes the sum of the first $n$ terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$. [CBSE 2014]

71. Solve the question: $(-4) + (-1) + 2 + 5 + \cdots + x = 437$. [NCERT EXEMPLAR]

72. Which term of the A.P. $-2, -7, -12, \ldots$ will be $-77$? Find the sum of this A.P. upto the term $-77$.

73. The sum of first $n$ terms of an A.P. whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another A.P. whose first term is $-30$ and common difference is 8. Find $n$. [NCERT EXEMPLAR]

74. The students of a school decided to beautify the school on the annual day by fixing colourful on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag? [NCERT EXEMPLAR]

## ANSWERS

1. (i) 320 (ii) 144 (iii) 525 (iv) 162 (v) $22a - 440b$
(vi) $n\{(x-y)^2 + (n-1)xy\}$ (vii) $\frac{n}{2(x+y)}\{n(2x-y) - y\}$ (viii) 324

2. $\frac{n}{2}(13 - 3n)$ 3. $n(2 - 3n)$ 4. 1170 5. (i) 525 (ii) 315 (iii) $-30$ (iv) $-465$

6. $210A + 20B$ 7. $-925$ 8. $-800$ 9. 4

10. (i) 19 (ii) 10 (iii) 12 (iv) 21, 22 (v) 19

11. (i) 242 (ii) 390 (iii) 3774

12. (i) 960 (ii) (a) 2460 (b) 4100 (c) 4920 (iii) 37674 (iv) 44550 (v) 1188 (vi) 108

13. (i) 10100 (ii) 40703 (iii) $-8930$ (iv) 10000 (v) $\frac{2093}{2}$ (vi) 286
(vii) 1625 (viii) $-441$ 14. 38, 6973 15. $-1$, 4, 740

16. 3 17. 0 18. $\frac{1}{2}(7n - 1)$ 19. 4 or 8 20. 1150 21. 5610

22. $n^2$ 23. $n = 16, d = 8/3$ 24. $n = 6, d = 5$ 25. $n = 12, d = -3$

26. 3 27. $d = \frac{8}{3}$ 28. 6, 48 29. $2n + 1$ 30. 57

31. 2, 10, 18, 26, … 32. 760 33. $a = 3, d = -4$ 34. 370

35. 3 36. $n = 38, S = 6973$ 37. 12, 66 38. $6n + 3$ 39. $6 - 2n$

40. $4n - 2$ 41. 151 42. $m = 17, a_{20} = 198$ 43. $p = 21, a_{11} = 0$

44. $n = 14, a_{21} = 163$ 45. $S_1 = 1, S_2 = 4, a_2 = 1, S_3 = 3, a_3 = -1, a_{10} = -15$

46. $a_n = 3n+2, a_{20} = 62$ 47. 80 48. 1683 49. $n^2$

50. (i) 625 (ii) 7500 52. 50800 53. 17696 54. 246950

55. (i) 16425 (ii) 129500 (iii) 12250 (iv) 12750 (v) 75250

56. (i) $n = 16, S_n = 440$ (ii) $n = 7, a = -8$ (iii) $d = 6$ (iv) $a = 4$,
(v) $n = 6,\ d = \frac{54}{5}$ (vi) $n = 5, a_n = 34$ (vii) 27 (viii) – 1804

58. 5 60. ₹ 6720, Yes 61. 312 62. 1920, Yes 63. ₹ 1200

64. 5 years 65. ₹ 51 66. 3500 m 67. 89 minutes

68. ₹ 15000 69. Values of the prizes (in ₹) are: 160, 140, 120, 100, 80, 60, 40

71. 50 72. 16th term, –632 73. 11 74. 728 m, 26 m

## VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

*Answer each of the following questions either in one word or one sentence or as per requirement of the questions:*

1. Define an arithmetic progression.
2. Write the common difference of an A.P. whose $n$th term is $a_n = 3n + 7$.
3. Which term of the sequence 114, 109, 104, .... is the first negative term?
4. Write the value of $a_{30} - a_{10}$ for the A.P. 4, 9, 14, 19,......
5. Write 5th term from the end of the A.P. 3, 5, 7, 9, ...., 201.
6. Write the value of $x$ for which $2x$, $x + 10$ and $3x + 2$ are in A.P.
7. Write the $n$th term of an A.P. the sum of whose $n$ terms is $S_n$.
8. Write the sum of first $n$ odd natural numbers.
9. Write the sum of first $n$ even natural numbers.
10. If the sum of $n$ terms of an A.P. is $S_n = 3n^2 + 5n$. Write its common difference.
11. Write the expression for the common difference of an A.P. whose first term is $a$ and $n$th term is $b$.
12. The first term of an A.P. is $p$ and its common difference is $q$. Find its 10th term. [CBSE 2008]
13. For what value of $p$ are $2p + 1, 13, 5p - 3$ are three consecutive terms of an A.P.? [CBSE 2009]
14. If $\frac{4}{5}$, $a$, 2 are three consecutive terms of an A.P., then find the value of $a$. [CBSE 2009]
15. If the sum of first $p$ term of an A.P. is $ap^2 + bp$, find its common difference. [CBSE 2010]
16. Find the 9th term from the end of the A.P. 5, 9, 13,..., 185. [CBSE 2016]
17. For what value of k will the consecutive terms $2k + 1, 3k + 3$ and $5k - 1$ form on A.P.? [CBSE 2016]
18. Write the $n$th term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, .....$ [CBSE 2017]
19. In an A.P., if the common difference $d = -4$, and the seventh term $a_7$ is 4, then find the first term. [CBSE 2018]

### ANSWERS

2. 3 3. 24th 4. 100 5. 193 6. 6 7. $a_n = S_n - S_{n-1}$ 8. $n^2$

9. $n(n+1)$ 10. 6 11. $\frac{b-a}{n-1}$ 12. $p+9q$ 13. 4 14. $\frac{7}{5}$

15. $2a$ 16. 153 17. 6 18. $\frac{m(n-1)+1}{m}$ 19. 28

## MULTIPLE CHOICE QUESTIONS (MCQs)

*Mark the correct alternative in each of the following:*

1. If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is
   (a) 87 (b) 88 (c) 89 (d) 90
2. If the sum of $P$ terms of an A.P. is $q$ and the sum of $q$ terms is $p$, then the sum of $p+q$ terms will be
   (a) 0 (b) $p-q$ (c) $p+q$ (d) $-(p+q)$
3. If the sum of $n$ terms of an A.P. be $3n^2+n$ and its common difference is 6, then its first term is
   (a) 2 (b) 3 (c) 1 (d) 4
4. The first and last terms of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be
   (a) 5 (b) 6 (c) 7 (d) 8
5. If the sum of $n$ tems of an A.P. is $3n^2+5n$ then which of its terms is 164?
   (a) 26th (b) 27th (c) 28th (d) none of these.
6. If the sum of $n$ terms of an A.P. is $2n^2+5n$, then its $n$th term is
   (a) $4n-3$ (b) $3n-4$ (c) $4n+3$ (d) $3n+4$
7. If the sum of three consecutive terms of an increasing A.P. is 51 and the product of the first and third of these terms is 273, then the third term is
   (a) 13 (b) 9 (c) 21 (d) 17
8. If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are
   (a) 5, 10, 15, 20 (b) 4, 10, 16, 22 (c) 3, 7, 11, 15 (d) none of these
9. Let $S_n$ denote the sum of $n$ terms of an A.P. whose first term is $a$. If the common difference $d$ is given by $d = S_n - kS_{n-1} + S_{n-2}$, then $k =$
   (a) 1 (b) 2 (c) 3 (d) none of these.
10. The first and last term of an A.P. are $a$ and $l$ respectively. If $S$ is the sum of all the terms of the A.P. and the common difference is given by $\frac{l^2-a^2}{k-(l+a)}$, then $k =$
    (a) $S$ (b) $2S$ (c) $3S$ (d) none of these
11. If the sum of first $n$ even natural numbers is equal to $k$ times the sum of first $n$ odd natural numbers, then $k =$
    (a) $\frac{1}{n}$ (b) $\frac{n-1}{n}$ (c) $\frac{n+1}{2n}$ (d) $\frac{n+1}{n}$
12. If the first, second and last term of an A.P. are $a$, $b$ and $2a$ respectively, its sum is
    (a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$ (c) $\frac{3ab}{2(b-a)}$ (d) none of these

13. If $S_1$ is the sum of an arithmetic progression of '$n$' odd number of terms and $S_2$ the sum of the terms of the series in odd places, then $\frac{S_1}{S_2} =$

(a) $\frac{2n}{n+1}$ (b) $\frac{n}{n+1}$ (c) $\frac{n+1}{2n}$ (d) $\frac{n+1}{n}$

14. If in an A.P., $S_n = n^2 p$ and $S_m = m^2 p$, where $S_r$ denotes the sum of $r$ terms of the A.P., then $S_p$ is equal to

(a) $\frac{1}{2}p^3$ (b) $mnp$ (c) $p^3$ (d) $(m+n)p^2$

15. If $S_n$ denote the sum of the first $n$ terms of an A.P. If $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

(a) 4 (b) 6 (c) 8 (d) 10

16. In an AP, $S_p = q$, $S_q = p$ and $S_r$ denotes the sum of first $r$ terms. Then, $S_{p+q}$ is equal to

(a) 0 (b) $-(p+q)$ (c) $p+q$ (d) $pq$

17. If $S_r$ denotes the sum of the first $r$ terms of an A.P. Then, $S_{3n} : (S_{2n} - S_n)$ is

(a) $n$ (b) $3n$ (c) 3 (d) none of these

18. If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 terms is

(a) 3200 (b) 1600 (c) 200 (d) 2800

19. The number of terms of the A.P. 3, 7, 11, 15, ... to be taken so that the sum is 406 is

(a) 5 (b) 10 (c) 12 (d) 14

20. Sum of $n$ terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \cdots$ is

(a) $\frac{n(n+1)}{2}$ (b) $2n(n+1)$ (c) $\frac{n(n+1)}{\sqrt{2}}$ (d) 1

21. The 9th term of an A.P. is 449 and 449th term is 9. The term which is equal to zero is

(a) $501^{th}$ (b) $502^{th}$ (c) $508^{th}$ (d) none of these

22. If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in A.P. Then, $x =$

(a) 5 (b) 3 (c) 1 (d) 2

23. The $n^{th}$ term of an A.P., the sum of whose $n$ terms is $S_n$, is

(a) $S_n + S_{n-1}$ (b) $S_n - S_{n-1}$ (c) $S_n + S_{n+1}$ (d) $S_n - S_{n+1}$

24. The common difference of an A.P., the sum of whose $n$ terms is $S_n$, is

(a) $S_n - 2S_{n-1} + S_{n-2}$ (b) $S_n - 2S_{n-1} - S_{n-2}$

(c) $S_n - S_{n-2}$ (d) $S_n - S_{n-1}$

25. If the sums of $n$ terms of two arithmetic progressions are in the ratio $\frac{3n+5}{5n+7}$, then their $n^{th}$ terms are in the ratio

(a) $\frac{3n-1}{5n-1}$ (b) $\frac{3n+1}{5n+1}$ (c) $\frac{5n+1}{3n+1}$ (d) $\frac{5n-1}{3n-1}$

26. If $S_n$ denote the sum of $n$ terms of an A.P. with first term $a$ and common difference $d$ such that $\frac{S_x}{S_{kx}}$ is independent of $x$, then
(a) $d = a$ (b) $d = 2a$ (c) $a = 2d$ (d) $d = -a$

27. If the first term of an A.P. is $a$ and $n^{th}$ term is $b$, then its common difference is
(a) $\frac{b-a}{n+1}$ (b) $\frac{b-a}{n-1}$ (c) $\frac{b-a}{n}$ (d) $\frac{b+a}{n-1}$

28. The sum of first $n$ odd natural numbers is
(a) $2n-1$ (b) $2n+1$ (c) $n^2$ (d) $n^2-1$

29. Two A.P.'s have the same common difference. The first term of one of these is 8 and that of the other is 3. The difference between their $30^{th}$ terms is
(a) 11 (b) 3 (c) 8 (d) 5

30. If $18, a, b, -3$ are in A.P., the $a + b =$
(a) 19 (b) 7 (c) 11 (d) 15

31. The sum of $n$ terms of two A.P.'s are in the ratio $5n+9 : 9n+6$. Then, the ratio of their $18^{th}$ term is
(a) $\frac{179}{321}$ (b) $\frac{178}{321}$ (c) $\frac{175}{321}$ (d) $\frac{176}{321}$

32. If $\frac{5+9+13+\cdots \text{ to } n \text{ terms}}{7+9+11+\cdots \text{ to } (n+1) \text{ terms}} = \frac{17}{16}$, then $n =$
(a) 8 (b) 7 (c) 10 (d) 11

33. The sum of $n$ terms of an A.P. is $3n^2 + 5n$, then 164 is its
(a) $24^{th}$ term (b) $27^{th}$ term (c) $26^{th}$ term (d) $25^{th}$ term

34. If the $n^{th}$ term of an A.P. is $2n+1$, then the sum of first $n$ terms of the A.P. is
(a) $n(n-2)$ (b) $n(n+2)$ (c) $n(n+1)$ (d) $n(n-1)$

35. If $18^{th}$ and $11^{th}$ term of an A.P. are in the ratio $3:2$, then its $21^{st}$ and $5^{th}$ terms are in the ratio
(a) $3:2$ (b) $3:1$ (c) $1:3$ (d) $2:3$

36. The sum of first 20 odd natural numbers is
(a) 100 (b) 210 (c) 400 (d) 420 **[CBSE 2012]**

37. The common difference of the A.P. is $\frac{1}{2q}, \frac{1-2q}{2q}, \frac{1-4q}{2q}, \cdots$ is
(a) $-1$ (b) 1 (c) $q$ (d) $2q$ **[CBSE 2013]**

38. The common difference of the A.P. $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \cdots$ is
(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $-b$ (d) $b$ **[CBSE 2013]**

39. The common difference of the A.P. $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \cdots$ is
(a) $2b$ (b) $-2b$ (c) 3 (d) $-3$ **[CBSE 2013]**

40. If $k$, $2k-1$ and $2k+1$ are three consecutive terms of an AP, the value of $k$ is
(a) $-2$ (b) $3$ (c) $-3$ (d) $6$ [CBSE 2014]

41. The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$
(a) $\sqrt{70}$ (b) $\sqrt{84}$ (c) $\sqrt{97}$ (d) $\sqrt{112}$ [CBSE 2014]

42. The first three terms of an A.P. respectively are $3y-1$, $3y+5$ and $5y+1$. Then, $y$ equals
(a) $-3$ (b) $4$ (c) $5$ (d) $2$ [CBSE 2014]

## ANSWERS

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (a) | 4. (b) | 5. (b) | 6. (c) |
| 7. (c) | 8. (a) | 9. (b) | 10. (b) | 11. (d) | 12. (c) |
| 13. (a) | 14. (c) | 15. (b) | 16. (b) | 17. (c) | 18. (a) |
| 19. (d) | 20. (c) | 21. (c) | 22. (c) | 23. (b) | 24. (a) |
| 25. (b) | 26. (b) | 27. (b) | 28. (c) | 29. (d) | 30. (d) |
| 31. (a) | 32. (b) | 33. (b) | 34. (b) | 35. (b) | 36. (c) |
| 37. (a) | 38. (c) | 39. (d) | 40. (b) | 41. (d) | 42. (c) |

## SUMMARY

1. A sequence is an arrangement of numbers or objects in a definite order.
2. A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an arithmetic progression, if there exists a constant $d$ such that
$$a_2 - a_1 = d, a_3 - a_2 = d, a_4 - a_3 = d, \ldots, a_{n+1} - a_n = d \text{ and so on.}$$
The constant '$d$' is called the common difference.
3. If '$a$' is the first term and '$d$' the common difference of an AP, then the A.P. is
$$a, a+d, a+2d, a+3d, a+4d, \ldots$$
4. A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is an AP, if $a_{n+1} - a_n$ is independent of $n$.
5. A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is an AP, if and only if its $n^{th}$ term $a_n$ is a linear expression in $n$ and in such a case the coefficient of $n$ is the common difference.
6. The $n^{th}$ term $a_n$ of an A.P. with first term '$a$' and common difference '$d$' is given by
$$a_n = a + (n-1)d.$$
7. Let there be an A.P. with first term '$a$' and common difference $d$. If there are $m$ terms in the AP, then
$$n^{th} \text{ term from the end} = (m-n+1)^{th} \text{ term from the beginning}$$
$$= a + (m-n)d$$
Also,
$$n^{th} \text{ term from the end} = \text{Last term} + (n-1)(-d)$$
$$= l - (n-1)d, \text{ where } l \text{ denotes the last term.}$$
8. Various terms is an A.P. can be chosen in the following manner.

| *Number of terms* | *Terms* | *Common difference* |
|---|---|---|
| 3 | $a-d, a, a+d$ | $d$ |
| 4 | $a-3d, a-d, a+d, a+3d$ | $2d$ |
| 5 | $a-2d, a-d, a, a+d, a+2d$ | $d$ |
| 6 | $a-5d, a-3d, a+d, a+3d, a+5d$ | $2d$ |

9. The sum to $n$ terms of an A.P. with first term '$a$' and common difference '$d$' is given by

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Also, $S_n = \frac{n}{2}\{a + l\}$, where $l$ = last term $= a + (n-1)d$

10. If the ratio of the sums of $n$ terms of two AP's is given, then to find the ratio of their $n^{\text{th}}$ terms, we replace $n$ by $(2n-1)$ in the ratio of the sums of $n$ terms.

11. A sequence is an A.P. if and only if the sum of its $n$ terms is of the form $An^2 + Bn$, where $A$, $B$ are constants. In such a case the common difference is $2A$.