CHAPTER 8

CIRCLES

8.1 SECANT AND TANGENT

In Class IX, we have studied that a circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the centre and the constant distance is known as the radius. We have also studied various terms related to a circle like chord, segment, sector, arc etc. Let us now see, how many different positions a line can take with respect to a given circle.

Consider a circle $C(O, r)$ and a line l in a plane. We find that there are three different situations as shown in Fig. 8.1.

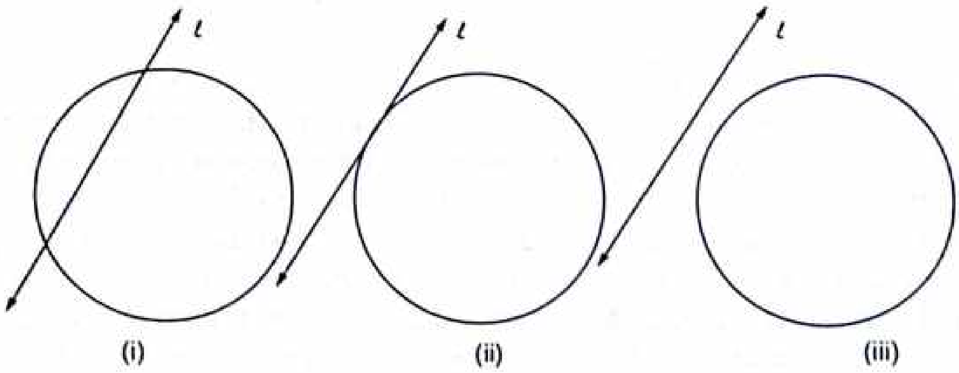

Fig. 8.1

(i) *When the line l intersects the circle in two distinct points* : Since a circle cannot pass through three collinear points. So, the line l intersects the circle in two points only. In such a case the line l is called a *secant* of the circle. Thus, we have the following definition:

SECANT *A line which intersects a circle in two distinct points is called a secant of the circle.*

(ii) *When the line l intersects the circle in only one point* : In this case the line is said to be a tangent to the circle and point is called the point of contact of the tangent. The concept of tangent can also be understood as follows:

Consider a secant PQ of a circle intersecting it in two points P and Q. If we rotate the secant about the point P so that the point Q comes nearer and nearer to the point P, we find that ultimately the point Q will coincide with P. In this case the two points of intersection will coincide and the secant will touch the circle at P. Thus, we have the following definition:

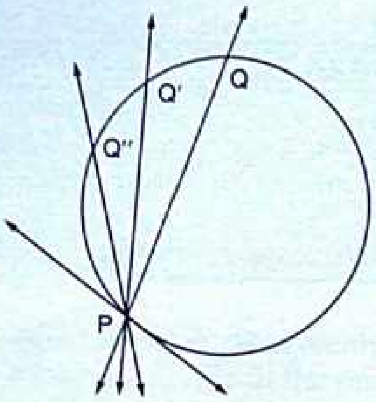

Fig. 8.2

TANGENT *A tangent to a circle is a line that intersects the circle in exactly one point.*

The point is called the point of contact of the tangent and the line is said to touch the circle at this point.

The word tangent is originated from the Latin word TANGERE which means 'to touch'.

NOTE *The point of contact is the only point which is common to the tangent and the circle and every other point on the tangent lies outside the circle. Thus, of all the points on a tangent to a circle, the point of contact is nearest to the centre of the circle.*

In order to understand the existence of the tangent to a circle at a point let us perform the following activities:

ACTIVITY 1 Draw a circle and a secant AB of the circle on a paper. Now, draw various lines parallel to the secant AB on both sides of it. You will find that the lengths of chords cut by the lines will decrease as we go away from the secant AB as shown in Fig. 8.3. You will also notice that on two sides of secant AB, $A_1 B_1$ and $A_2 B_2$ are the limiting positions of secants when the lengths of the chords cut become zero. These are the tangents to the circle parallel to the secant AB. It is evident from the above activity that there cannot be more than two tangents parallel to the given secant or in general, to a given line. It also establishes that a tangent is the secant when both of the end points of the corresponding chord coincide.

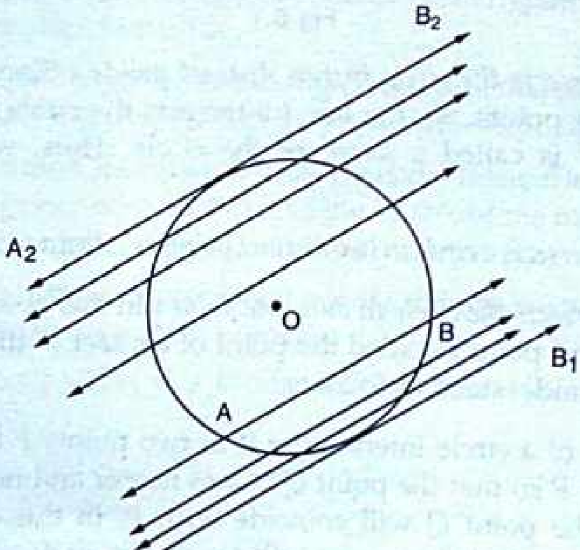

Fig. 8.3

ACTIVITY 2 Take a circular wire and a straight wire AB. Attach the straight wire AB at a point P of the circular wire in such a way that AB can rotate about the point P in a plane. Put the system on a table and rotate wire AB about point P to get different positions of the wire as shown in Fig. 8.4. You will see that the wire AB intersects the circular wire at P and at another point Q_1 or Q_2 etc. when it is rotated in clockwise sense about point P. And in one position $A_1\ B_1$ it will intersect the circle just at point P only. In fact, it becomes tangent to the circular wire at P. This shows that a tangent exists at a point P on the circle.

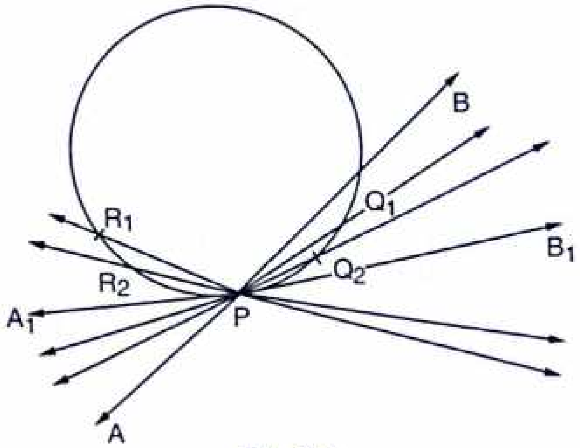

Fig. 8.4

Now, if we rotate wire AB in anticlockwise sense about point P, it will intersect the circle at P and an another point R_1 or R_2 etc. In position $A_1\ B_1$ it intersects the circle at P only. Thus, there is only one tangent at point P.

This activity establishes that the tangent at a point P to a circle is the limiting position of secants PQ_1, PQ_2, PR_1, PR_2 etc. through P when Q_1, Q_2, R_1, R_2 etc. coincide with P.

8.2 SOME PROPERTIES OF TANGENT TO A CIRCLE

If you look at the wheels of a bicycle you will find that its all spokes are along its radii. When a bicycle runs, its wheels move along the lines which are tangents at the points where they touch the ground. You will also notice that in all positions, the radii through the points of contact with the ground appear to be at right angle to the tangent. We shall now prove this property and some other properties of tangents to a circle as theorems.

THEOREM 1 *A tangent to a circle is perpendicular to the radius through the point of contact.*
[NCERT, CBSE 2009, CBSE 2012, 2014, 2015, 2016]

GIVEN A circle $C(O, r)$ and a tangent AB at a point P.

TO PROVE $OP \perp AB$.

CONSTRUCTION Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

PROOF We know that among all line segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Cleraly, $OP = OR$ [Radii of the same circle]

Now, $OQ = OR + RQ$

$\Rightarrow$ $OQ > OR$

$\Rightarrow$ $OQ > OP$ $[\because OP = OR]$

$\Rightarrow$ $OP < OQ$

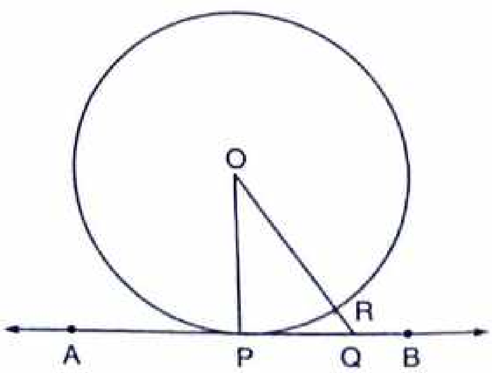

Fig. 8.5

Thus, OP is shorter than any other segment joining O to any point of AB.
Hence, $OP \perp AB$.

Q.E.D.

REMARK *The converse of the above theorem is also true as given below.*

THEOREM 2 *A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.* **[NCERT, CBSE 2012, 2013]**

GIVEN A radius OP of a circle $C(O, r)$ and a line APB, perpendicular to OP.

TO PROVE AB is a tangent to the circle at the point P.

PROOF Take a point Q, different from P, on the line AB.

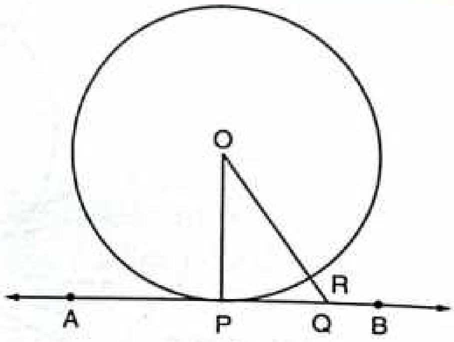

Fig. 8.6

Now, $OP \perp AB$.

$\Rightarrow$ Among all the line segments joining O to a point on AB, OP is the shortest.

$\Rightarrow$ $OP < OQ$

$\Rightarrow$ $OQ > OP$

$\Rightarrow$ Q lies outside the circle.

Thus, every point on AB, other than P, lies outside the circle. This shows that AB meets the circle only at the point P.

Hence, AB is a tangent to the circle at P.

Q. E. D.

REMARK *The above theorem provides us a method of constructing a tangent at a given point to a given circle. For this we draw a line through the given point perpendicular to the radius at the given point.*

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length of PQ.* **[NCERT]**

SOLUTION Since tangent at a point is perpendicular to the radius through that point. Therefore, OP is perpendicular to PQ. In right triangle OPQ, we have

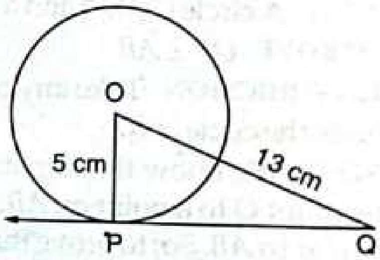

Fig. 8.7

$$OQ^2 = OP^2 + PQ^2 \quad \text{[Using Pythagoras theorem]}$$

$$\Rightarrow \quad 13^2 = 5^2 + PQ^2$$

$$\Rightarrow \quad PQ^2 = 169 - 25 = 144$$

$$\Rightarrow \quad PQ = 12 \text{cm}.$$

EXAMPLE 2 *A line through the centre O of a circle of radius 7 cm cuts the tangent, at a point P on the circle, at Q such that PQ = 24 cm. Find OQ.*

SOLUTION Since tangent at a point on a circle is perpendicular to the radius through the point. Therefore, OP is perpendicular to PQ.

In right triangle OPQ, we have

$$OQ^2 = OP^2 + PQ^2$$

$\Rightarrow$ $OQ^2 = 7^2 + 24^2$

$\Rightarrow$ $OQ^2 = 49 + 576 \Rightarrow OQ^2 = 625$

$\Rightarrow$ $OQ = 25$ cm.

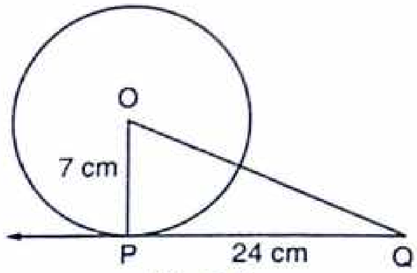

Fig. 8.8

EXERCISE 8.1

LEVEL-1

1. Fill in the blanks:
   (i) The common point of a tangent and the circle is called .... [NCERT]
   (ii) A circle may have .... parallel tangents. [NCERT]
   (iii) A tangent to a circle intersects it in .... point(s). [NCERT]
   (iv) A line intersecting a circle in two points is called a .... [NCERT]
   (v) The angle between tangent at a point on a circle and the radius through the point is ....
2. How many tangents can a circle have? [NCERT]
3. O is the centre of a circle of radius 8 cm. The tangent at a point A on the circle cuts a line through O at B such that $AB = 15$ cm. Find OB.
4. If the tangent at a point P to a circle with centre O cuts a line through O at Q such that $PQ = 24$ cm and $OQ = 25$ cm. Find the radius of the circle.

ANSWERS

1. (i) Point of contact (ii) two (iii) one (iv) secant (v) 90°.
2. Infinitely many 3. 17 cm. 4. 7 cm.

8.3 TANGENT FROM A POINT ON A CIRCLE

In this section, we will learn about the number of tangents drawn from a point to a circle. We will also learn some properties of these tangents. In order to have an idea of the number of tangents drawn from a point to a circle, let us perform the following activity:

ACTIVITY Draw a circle on a paper and take a point P inside it. Let us now try to draw a tangent to the circle through this point P. We observe that all the lines through point P intersect the circle in two distinct points. Therefore, none of them can be a tangent to the circle as shown in Fig. 8.9.

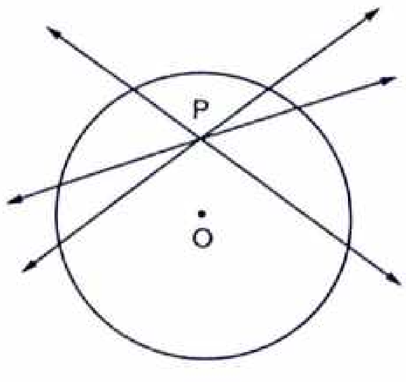

Fig. 8.9

Now, take a point P on the circle. We have discussed in section 8.2 that there exists one and only one tangent to a circle at a given point on it. So, there is only one tangent to the circle at point P as shown in Fig. 8.10(i).

Finally, let us take a point P outside the circle and try to draw tangents to the circle from point P. We observe that we can draw exactly two tangents to the circle through point P as shown in Fig. 8.10(ii).

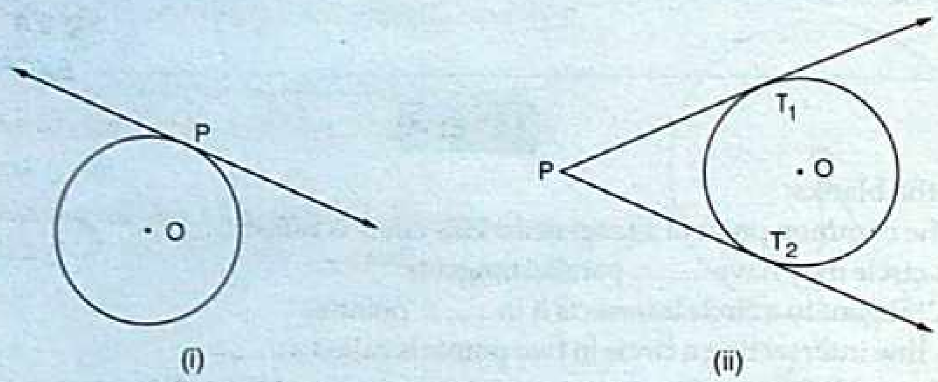

Fig. 8.10

These facts can be summarized as follows:

(i) No tangent can be drawn to a circle from a point lying inside it.

(ii) One and only one tangent can be drawn to a circle at a point on the circle.

(iii) Two tangents can be drawn to a circle from a point lying outside it.

In Fig. 8.10(ii), PT_1 and PT_2 are two tangents drawn from a point P lying outside the circle. These tangents touch the circle at T_1 and T_2 respectively. So, T_1 and T_2 are known as the points of contact of tangents PT_1 and PT_2 respectively.

LENGTH OF TANGENT *The length of the segment of the tangent between the point and the given point of contact with the circle is called the length of the tangent from the point to the circle.*

In Fig. 8.10(i), PT_1 and PT_2 are the lengths of tangents from point P to the circle.

In the following theorem, we will prove that these two lengths are equal.

THEOREM 3 *The lengths of two tangents drawn from an external point to a circle are equal.*

[NCERT, CBSE 2008, 2009, 2010, 2013, 2014, 2015, 2016, 2017, 2018]

GIVEN AP and AQ are two tangents from a point A to a circle $C(O, r)$.

TO PROVE $AP = AQ$

CONSTRUCTION Join OP, OQ and OA.

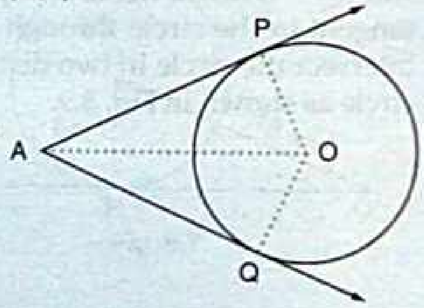

Fig. 8.11

PROOF In order to prove that $AP = AQ$, we shall first prove that $\Delta OPA \cong \Delta OQA$.

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \quad OP \perp AP$ and $OQ \perp AQ$.

$\Rightarrow \quad \angle OPA = \angle OQA = 90^\circ$ ...(i)

Now, in right triangles OPA and OQA, we have

$OP = OQ$ [Radii of a circle]

$\angle OPA = \angle OQA$ [From (i)]

and, $OA = OA$ [Common]

So, by *RHS*-criterion of congruence, we get

$$\Delta OPA \cong \Delta OQA$$

$\Rightarrow$ $AP = AQ$

Q. E. D.

THEOREM 4 *If two tangents are drawn to a circle from an external point, then:*

(i) *they subtend equal angles at the centre,*

(ii) *they are equally inclined to the segment, joining the centre to that point.*

GIVEN A circle $C(O, r)$ and a point A outside the circle such that AP and AQ are the tangents drawn to the circle from point A.

TO PROVE (i) $\angle AOP = \angle AOQ$ (ii) $\angle OAP = \angle OAQ$.

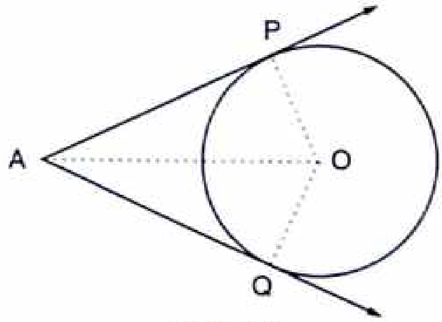

Fig. 8.12

PROOF In right triangles OAP and OAQ, we have

$AP = AQ$ [Tangents from an external point are equal]

$OP = OQ$ [Radii of a circle]

and, $OA = OA$ [Common]

So, by *SSS*-criterion of congruence, we have

$$\Delta OAP \cong \Delta OAQ$$

$\Rightarrow$ $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

Q.E.D.

REMARK *It follows from the above theorem that the centre of the circle lies on the angle bisector of $\angle PAQ$. This fact can be used in drawing circles touching two intersecting lines. In particular, a circle can be drawn to touch all the three sides of a triangle as discussed below:*

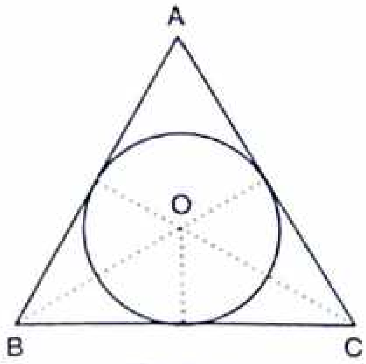

Fig. 8.13

Let ABC be a triangle. Draw angle bisectors of any two angles, say $\angle B$ and $\angle C$. Suppose they intersect at O. Then, as O lies on the angle bisectors of $\angle B$ and $\angle C$, a circle can be drawn with *centre O to touch AB, BC and AC. The radius of the circle will be the length of the perpendicular from O on any side. This circle is called the incircle of ΔABC and its centre is called the incentre of ΔABC.*

ILLUSTRATIVE EXAMPLES

LEVEL-1

Type I ON FINDING THE LENGTH OF THE TANGENT OR RADIUS OF A CIRCLE

EXAMPLE 1 *A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12 cm. Find the radius of the circle.*

SOLUTION Since tangent to a circle is perpendicular to the radius through the point of contact.

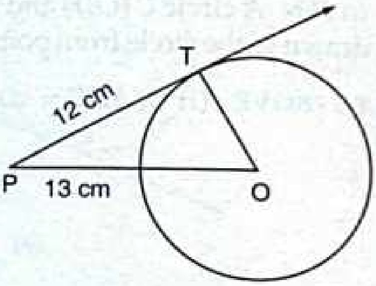

Fig. 8.14

$\therefore \quad \angle OTP = 90°$

In right triangle OTP, we have

$$OP^2 = OT^2 + PT^2$$

$$\Rightarrow \quad 13^2 = OT^2 + 12^2$$

$$\Rightarrow \quad OT^2 = 13^2 - 12^2 = (13-12)(13+12) = 25$$

$$\Rightarrow \quad OT = 5.$$

Hence, radius of the circle is 5 cm.

EXAMPLE 2 *Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that the radius of the circle is 7 cm.*

SOLUTION Let P be the given point, O be the centre of the circle and PT be the length of tangent from P. Then, $OP = 25$ cm and $OT = 7$ cm.

Since tangent to a circle is always perpendicular to the radius through the point of contact.

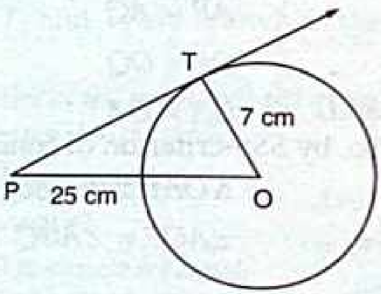

Fig. 8.15

$\therefore \quad \angle OTP = 90°$

in right triangle OTP, we have

$$OP^2 = OT^2 + PT^2$$

$$\Rightarrow \quad 25^2 = 7^2 + PT^2$$

$$\Rightarrow \quad PT^2 = 25^2 - 7^2 = (25-7)(25+7) = 576$$

$$\Rightarrow \quad PT = 24 \text{ cm}.$$

Hence, length of tangent from $P = 24$ cm.

Type II BASED ON THE RESULT THAT THE TANGENTS DRAWN FROM AN EXTERIOR POINT TO A CIRCLE ARE EQUAL IN LENGTH.

EXAMPLE 3 *In Fig. 8.16, if AB = AC, prove that BE = EC.*

OR

ABC is an isosceles triangle in which AB = AC, circumscribed about a circle, as shown in Fig. 8.16. Prove that the base is bisected by the point of contact.

[CBSE 2008, 2012, 2014]

SOLUTION Since tangents from an exterior point to a circle are equal in length.

$\therefore \quad AD = AF$ [Tangents from A] ...(i)

$BD = BE$ [Tangents from B] ...(ii)

$CE = CF$ [Tangents from C] ...(iii)

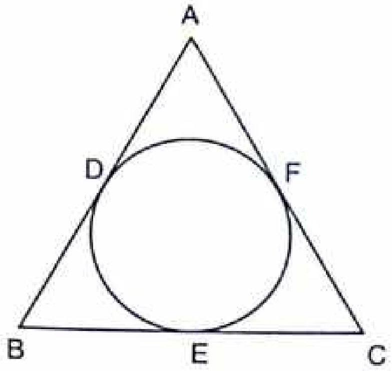

Fig. 8.16

Now,

$$AB = AC$$

$\Rightarrow \quad AB - AD = AC - AD$ [Subtracting AD from both sides]

$\Rightarrow \quad AB - AD = AC - AF$ [Using (i)]

$\Rightarrow \quad BD = CF$

$\Rightarrow \quad BE = CF$ [Using (ii)]

$\Rightarrow \quad BE = CE$ [Using (iii)]

EXAMPLE 4 *A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that*

$$AQ = \frac{1}{2} \text{ (Perimeter of } \Delta ABC\text{)}.$$ **[CBSE 2000, 2001, 2002, NCERT EXEMPLAR]**

SOLUTION Since tangents from an exterior point to a circle are equal in length.

$\therefore \quad BP = BQ$ [From B] ...(i)

$CP = CR$ [From C] ...(ii)

and, $AQ = AR$ [From A] ...(iii)

From (iii), we have

$$AQ = AR$$

$\Rightarrow \quad AB + BQ = AC + CR$

$\Rightarrow \quad AB + BP = AC + CP$ [Using (i) and (ii)] ...(iv)

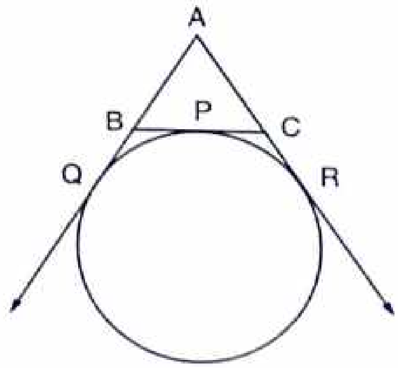

Fig. 8.17

Now,

Perimeter of $\Delta ABC = AB + BC + AC$

$\Rightarrow$ Perimeter of $\Delta ABC = AB + (BP + PC) + AC$

$\Rightarrow$ Perimeter of $\Delta ABC = (AB + BP) + (AC + PC)$

$\Rightarrow$ Perimeter of $\Delta ABC = 2(AB + BP)$ [Using (iv)]

$\Rightarrow$ Perimeter of $\Delta ABC = 2(AB + BQ)$ [Using (i)]

$\Rightarrow$ Perimeter of $\Delta ABC = 2AQ$

$\therefore \quad AQ = \frac{1}{2}$ (Perimeter of ΔABC)

EXAMPLE 5 *In Fig. 8.18, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that, $XA + AR = XB + BR$.* **[CBSE 2014]**

SOLUTION Since lengths of tangents from an exterior point to a circle are equal.

$\therefore$ $XP = XQ$ [From X] ...(i)

$AP = AR$ [From A] ...(ii)

$BQ = BR$ [From B] ...(iii)

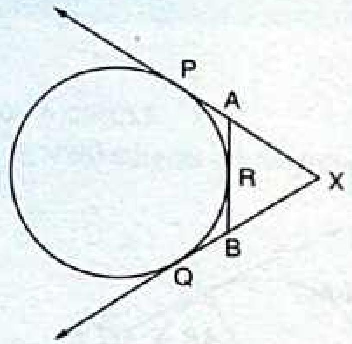

Fig. 8.18

Now, $XP = XQ$

$\Rightarrow$ $XA + AP = XB + BQ$

$\Rightarrow$ $XA + AR = XB + BR$ [Using (i) and (ii)]

EXAMPLE 6 *In Fig. 8.19, the incircle of ΔABC touches the sides BC, CA and AB at D, E and F respectively. Show that*

$$AF + BD + CE = AE + BF + CD = \frac{1}{2}(\text{Perimeter of } \Delta ABC)$$

SOLUTION Since lengths of the tangents from an exterior point to a circle are equal.

$\therefore$ $AF = AE$ [From A] ...(i)

$BD = BF$ [From B] ...(ii)

and, $CE = CD$ [From C] ...(iii)

Adding equations (i), (ii) and (iii), we get

$$AF + BD + CE = AE + BF + CD$$

Now,

$$\text{Perimeter of } \Delta ABC = AB + BC + AC$$

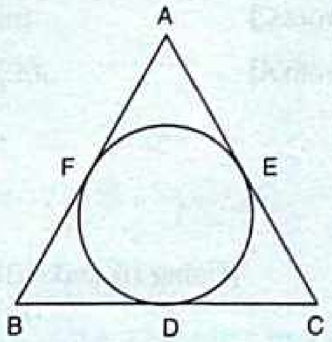

Fig. 8.19

$\Rightarrow$ $\text{Perimeter of } \Delta ABC = (AF + FB) + (BD + CD) + (AE + EC)$

$\Rightarrow$ $\text{Perimeter of } \Delta ABC = (AF + AE) + (BF + BD) + (CD + CE)$

$\Rightarrow$ $\text{Perimeter of } \Delta ABC = 2\,AF + 2\,BD + 2\,CE$

$\Rightarrow$ $\text{Perimeter of } \Delta ABC = 2(AF + BD + CE)$ $\left[\begin{array}{l}\text{From (i), (ii) and (iii), we get}\\ AE = AF, BD = BF \text{ and } CD = CE\end{array}\right]$

$\Rightarrow$ $AF + BD + CE = \frac{1}{2}(\text{Perimeter of } \Delta ABC)$

Hence, $AF + BD + CE = AE + BF + CD = \frac{1}{2}(\text{Perimeter of } \Delta ABC)$

EXAMPLE 7 *In Fig. 8.20, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.*

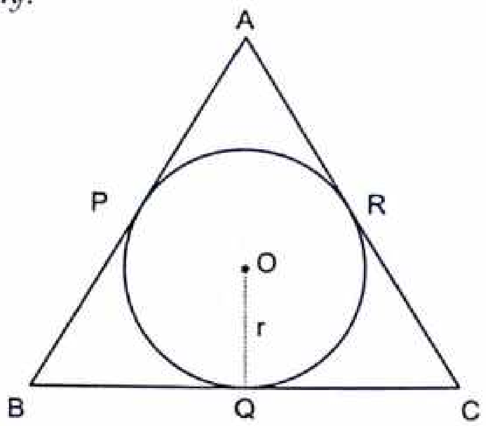

Fig. 8.20

Prove that:

(i) $AB + CQ = AC + BQ$

(ii) $Area\,(\Delta ABC) = \frac{1}{2}(Perimeter\ of\ \Delta ABC) \times r$ **[CBSE 2013]**

SOLUTION We know that lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $AP = AR, BP = BQ$ and $CQ = CR$

(i) $AB + CQ = AP + PB + CQ$

$= AR + BQ + CQ$ $[\because AP = AR \text{ and } PB = BQ]$

$= (AR + CR) + BQ$ $[\because CQ = CR]$

$= AC + BQ$ $[\because AR + CR = AC]$

(ii) Area (ΔABC) = Area (ΔOBC) + Area (ΔOAB) + Area (ΔOAC)

$$= \frac{1}{2}(BC \times OQ) + \frac{1}{2}(AB \times OP) + \frac{1}{2}(AC \times OR)$$

$$= \frac{1}{2}(BC \times r) + \frac{1}{2}(AB \times r) + \frac{1}{2}(AC \times r)$$

$$= \frac{1}{2}(BC + AB + AC) \times r$$

$$= \frac{1}{2}(\text{Perimeter of } \Delta ABC) \times r$$

EXAMPLE 8 *In Fig. 8.21, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.*

[CBSE 2013]

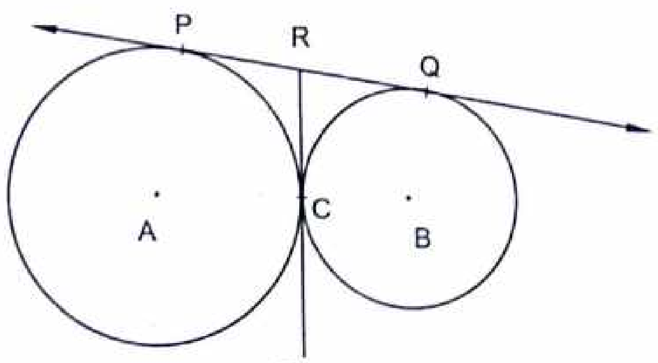

Fig. 8.21

SOLUTION We know that the tangents drawn from an external point to a circle are equal.

$\therefore \quad RP = RC$ and $RC = RQ$

$\Rightarrow \quad RP = RQ$

$\Rightarrow \quad R$ is the mid-point of PQ.

EXAMPLE 9 *A circle touches all the four sides of a quadrilateral ABCD. Prove that:*

$AB + CD = BC + DA$. **[NCERT, CBSE 2008, 2009, 2012, 2013, 2014, 2015, 2017]**

SOLUTION Since tangents drawn from an exterior point to a circle are equal in length.

$\therefore \quad AP = AS$ [From A] ...(i)

$BP = BQ$ [From B] ...(ii)

$CR = CQ$ [From C] ...(iii)

and, $DR = DS$ [From D] ...(iv)

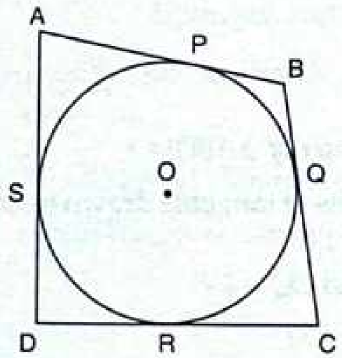

Fig. 8.22

Adding (i), (ii), (iii) and (iv), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow \quad (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow \quad AB + CD = AD + BC$$

Hence, $AB + CD = BC + DA$

EXAMPLE 10 *If a hexagon ABCDEF circumscribes a circle, prove that*

$AB + CD + EF = BC + DE + FA$. **[NCERT EXEMPLAR]**

SOLUTION Let O be the centre of the circle touching sides AB, BC, CD, DE, EF and FA at P, Q, R, S, T and U respectively. The lengths of tangents drawn from an external point to a circle are equal.

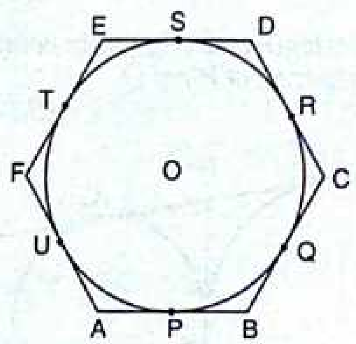

Fig. 8.23

$\therefore \quad AP = AU, BP = BQ, CQ = CR, DR = DS, ES = ET$ and $FU = FT$

Now,

$$AB + CD + EF$$
$$= (AP + PB) + (CR + DR) + (ET + TF)$$
$$= (AU + PB) + (CQ + DS) + (ES + FU) \quad \left[\begin{array}{l}\because AP = AU, PB = BQ, CR = CQ, \\ DR = DS, ET = ES, FT = FU\end{array}\right]$$
$$= (AU + FU) + (BQ + CQ) + (DS + ES)$$
$$= AF + BC + DE = BC + DE + FA$$

EXAMPLE 11 *Let s denote the semi perimeter of a triangle ABC in which BC = a, CA = b and AB = c. If a circle touches the sides BC, CA, AB at D, E, F respectively, prove that*

$$AF = AE = s - a, BD = BF = s - b \text{ and } CD = CE = s - c.$$ **[NCERT EXEMPLAR]**

SOLUTION We have,

$$s = \frac{AB + BC + CA}{2} = \frac{a + b + c}{2}$$

$\Rightarrow$ $a + b + c = 2s$

$\Rightarrow$ $b + c = 2s - a, c + a = 2s - b$ and $a + b = 2s - c$

$\Rightarrow$ $b + c - a = 2(s - a), c + a - b = 2(s - b)$ and $a + b - c = 2(s - c)$

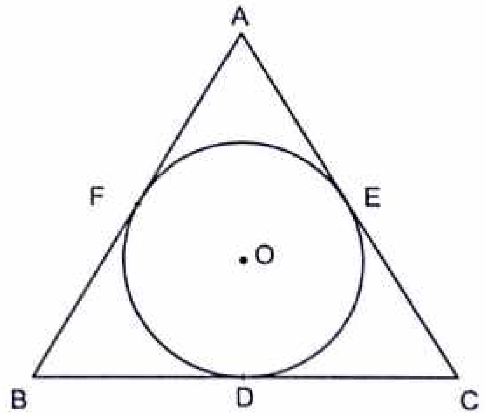

Fig. 8.24

The lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $AF = AE, BD = BF$ and $CD = CE$

Now,

$2s = BC + CA + AB$

$\Rightarrow$ $2s = (BD + DC) + (CE + AE) + (AF + BF)$

$\Rightarrow$ $2s = (BD + DC) + (CD + AF) + (AF + BD)$

$\Rightarrow$ $2s = 2(BD + DC) + 2AF$

$\Rightarrow$ $2s = 2BC + 2AF$

$\Rightarrow$ $2s = 2a + 2AF$

$\Rightarrow$ $AF = s - a \Rightarrow AF = AE = s - a$

Again,

$2s = BC + CA + AB$

$\Rightarrow$ $2s = (BD + CD) + (CE + AE) + (AF + FB)$

$\Rightarrow \quad 2s = (BF + CE) + (CE + AE) + (AE + FB)$

$\Rightarrow \quad 2s = 2BF + 2(AE + CE)$

$\Rightarrow \quad 2s = 2BF + 2AC$

$\Rightarrow \quad 2s = 2BF + 2b$

$\Rightarrow \quad BF = s - b$

$\Rightarrow \quad BD = BF = s - b$

Similarly, we can prove that $CD = CE = s - c$.

EXAMPLE 12 *If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by* $r = \frac{a + b - c}{2}$ *or, $r = s - c$, where s is the semi-perimeter of the triangle.* **[NCERT EXEMPLAR]**

SOLUTION Let the circle touches the sides BC, CA and AB of the right triangle ABC at D, E and F respectively.

We have,

$$BC = a, CA = b \text{ and } AB = c$$

It is given that the triangle ABC is right angled at C.

$\therefore \quad AB^2 = BC^2 + AC^2$

$\Rightarrow \quad c^2 = a^2 + b^2$

The length of tangents drawn from a point to a circle are equal.

$\therefore \quad AE = AF, CD = CE \text{ and } BD = BF$

We observe that $CD = OE$ and $CE = OD$

$\therefore \quad CD = r \text{ and } CE = r$

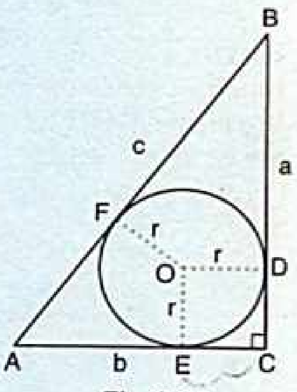

Fig. 8.25

Now,

$AF = AE \quad \text{and} \quad BD = BF$

$\Rightarrow \quad AF = AC - CE \quad \text{and} \quad BF = BC - CD$

$\Rightarrow \quad AF = b - r \quad \text{and} \quad BF = a - r$

$\Rightarrow \quad AF + BF = (b - r) + (a - r)$

$\Rightarrow \quad AB = a + b - 2r$

$\Rightarrow \quad c = a + b - 2r$

$\Rightarrow \quad r = \frac{a + b - c}{2}$

ALITER From example 11, we obtain

$CD = CE = s - c$

$\Rightarrow \quad r = s - c$

$\Rightarrow \quad r = \frac{a + b + c}{2} - c = \frac{a + b - c}{2}$

EXAMPLE 13 *In Fig. 8.26, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD.* **[NCERT EXEMPLAR]**

SOLUTION Produce AB and CD to meet at P. The lengths of tangents drawn from P to the two circles are equal.

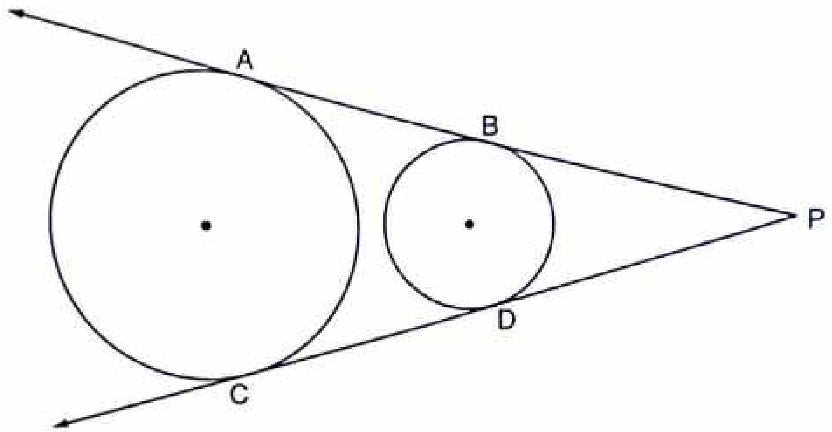

Fig. 8.26

$\therefore$ $PA = PC$ and $PB = PD$

$\Rightarrow$ $PA - PB = PC - PD$

$\Rightarrow$ $AB = CD$

EXAMPLE 14 *If all the side of a parallelogram touch a circle, show that the parallelogram is a rhombus.*

OR

Prove that a parallelogram circumscribing a circle is a rhombus.
[NCERT, CBSE 2000C, 2002, 2008, 2012, 2013, 2014]

SOLUTION Let $ABCD$ be a parallelogram such that its sides touch a circle with centre O.
We know that the tangents to a circle from an exterior point are equal in length.

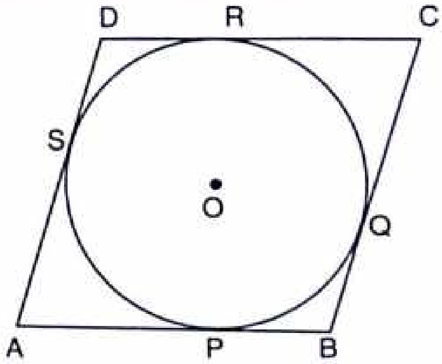

Fig. 8.27

$\therefore$ $AP = AS$ [From A] ...(i)

$BP = BQ$ [From B] ...(ii)

$CR = CQ$ [From C] ...(iii)

and, $DR = DS$ [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow$ $(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow$ $AB + CD = AD + BC$

$\Rightarrow$ $2\,AB = 2\,BC$ $[\because ABCD$ is a parallelogram $\therefore AB = CD$ and $BC = AD]$

$\Rightarrow$ $AB = BC$

Thus, $AB = BC = CD = AD$

Hence, $ABCD$ is a rhombus.

EXAMPLE 15 *PA and PB are tangents from P to the circle with centre O. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN = AK + BN.*

SOLUTION We know that the tangents drawn from an external point to a circle are equal in length.

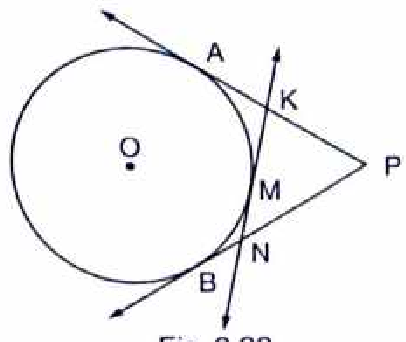

Fig. 8.28

$\therefore$ $PA = PB$ [From P] ...(i)

$KA = KM$ [From K] ...(ii)

and, $NB = NM$ [From N] ...(iii)

Adding (ii) and (iii), we get

$KA + NB = KM + NM$

$\Rightarrow \quad AK + BN = KM + MN$

$\Rightarrow \quad AK + BN = KN$

EXAMPLE 16 *ABCD is a quadrilateral such that $\angle D = 90°$. A circle C (O, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find r.*

SOLUTION Since tangents to a circle is perpendicular to the radius through the point.

$\therefore \quad \angle ORD = \angle OSD = 90°$

It is given that $\angle D = 90°$. Also, $OR = OS$. Therefore, $ORDS$ is a square.

Since tangents from an exterior point to a circle are equal in length.

$\therefore \quad BP = BQ$

$CQ = CR$

and, $\quad DR = DS.$

Now,

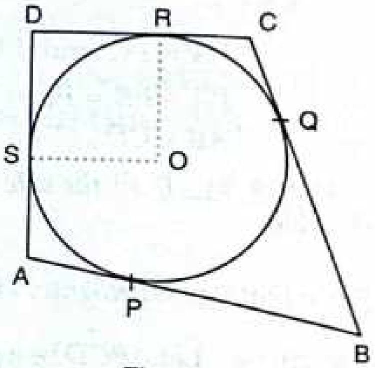

Fig. 8.29

$BP = BQ$

$\Rightarrow \quad BQ = 27 \qquad [\because BP = 27 \text{ cm (Given)}]$

$\Rightarrow \quad BC - CQ = 27$

$\Rightarrow \quad 38 - CQ = 27 \qquad [\because BC = 38 \text{ cm}]$

$\Rightarrow \quad CQ = 11 \text{ cm}$

$\Rightarrow \quad CR = 11 \text{ cm} \qquad [\because CR = CQ]$

$\Rightarrow \quad CD - DR = 11$

$\Rightarrow \quad 25 - DR = 11 \qquad [\because CD = 25 \text{ cm}]$

$\Rightarrow \quad DR = 14 \text{ cm}$

But, $ORDS$ is a square. Therefore, $OR = DR = 14$ cm.

Hence, $r = 14$ cm.

EXAMPLE 17 *Prove that the tangents at the extremities of any chord make equal angles with the chord.* **[NCERT EXEMPLAR]**

SOLUTION Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose the tangents meet at P. Join OP. Suppose OP meets AB at C. We have to prove that $\angle PAC = \angle PBC$.

In triangles PCA and PCB, we have

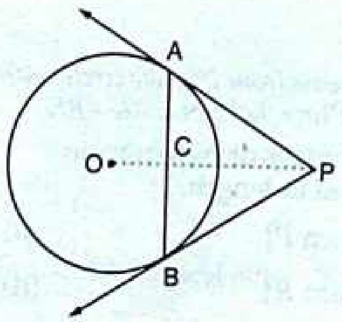

Fig. 8.30

$$PA = PB \quad [\because \text{ Tangents from an external point are equal}]$$

$$\angle APC = \angle BPC \quad [\because PA \text{ and } PB \text{ are equally inclined to } OP]$$

and, $PC = PC$ [Common]

So, by *SAS*-criterion of congruence, we obtain

$$\Delta PAC \cong \Delta PBC$$

$$\Rightarrow \quad \angle PAC = \angle PBC$$

EXAMPLE 18 *From an external point P, two tangents PA and PB are drawn to the circle with centre O. Prove that OP is the perpendicular bisector of AB.*

[CBSE 2015]

SOLUTION Suppose OP intersects AB at C.
In triangles PAC and PBC, we have

$$PA = PB \quad [\because \text{ Tangents from an external point are equal}]$$

$$\angle APC = \angle BPC \quad [\because PA \text{ and } PB \text{ are equally inclined to } OP]$$

and, $PC = PC$ [Common]

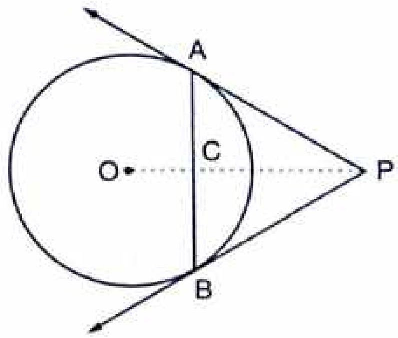

Fig. 8.31

So, by *SAS*-criterion of similaritry, we obtain

$$\Delta PAC \cong \Delta PBC$$

$$\Rightarrow \quad AC = BC \text{ and } \angle ACP = \angle BCP$$

But, $\angle ACP + \angle BCP = 180°$

$$\therefore \quad \angle ACP = \angle BCP = 90°$$

Hence, $OP \perp AB$.

EXAMPLE 19 *Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that* $\angle PTQ = 2\angle OPQ$. **[NCERT, CBSE 2009, 2017]**

SOLUTION We know that lengths of tangents drawn from an external point to a circle are equal.

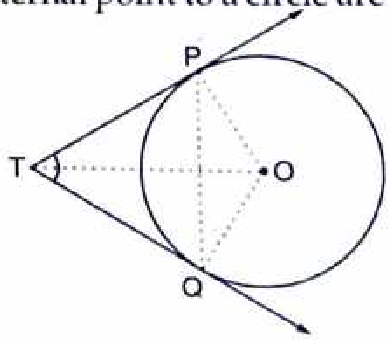

Fig. 8.32

$$\therefore \quad TP = TQ$$

$$\Rightarrow \quad \Delta TPQ \text{ is an isosceles triangle.}$$

$$\Rightarrow \quad \angle TPQ = \angle TQP$$

In ΔTPQ, we have

$$\angle TPQ + \angle TQP + \angle PTQ = 180°$$

$$\Rightarrow \quad 2\angle TPQ = 180° - \angle PTQ$$

$$\Rightarrow \quad \angle TPQ = 90° - \frac{1}{2}\angle PTQ$$

$$\Rightarrow \quad \frac{1}{2}\angle PTQ = 90° - \angle TPQ \quad \ldots (i)$$

Since, $OP \perp TP$.

$$\therefore \quad \angle OPT = 90°$$

$\Rightarrow \quad \angle OPQ + \angle TPQ = 90°$

$\Rightarrow \quad \angle OPQ = 90° - \angle TPQ$ ...(ii)

From (i) and (ii), we get

$$\frac{1}{2}\angle PTQ = \angle OPQ \Rightarrow \angle PTQ = 2\angle OPQ$$

EXAMPLE 20 *PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.* **[CBSE 2014, NCERT, CBSE 2016]**

SOLUTION Let $TR = y$.

Since OT is perpendicular bisector of PQ.

$\therefore \quad PR = QR = 4$ cm ...(i)

In right triangle ORP, we have

$OP^2 = OR^2 + PR^2$

$\Rightarrow \quad OR^2 = OP^2 - PR^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad OR = 3$ cm.

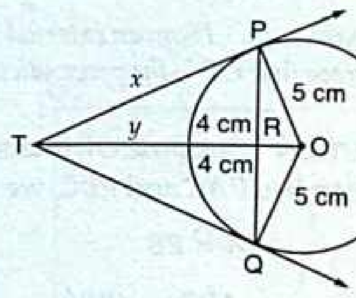

Fig. 8.33

In right triangles PRT and OPT, we have

$TP^2 = TR^2 + PR^2$

and, $\quad OT^2 = TP^2 + OP^2$ ...(ii)

$\Rightarrow \quad OT^2 = (TR^2 + PR^2) + OP^2$ [Substituting the value of TP^2]

$\Rightarrow \quad (y+3)^2 = y^2 + 16 + 25$

$\Rightarrow \quad 6y = 32$

$\Rightarrow \quad y = \frac{16}{3}$

$\Rightarrow \quad TR = \frac{16}{3}$

$\therefore \quad TP^2 = TR^2 + PR^2$

$\Rightarrow \quad TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16 = \frac{400}{9}$ [Using (i) and (ii)]

$\Rightarrow \quad TP = \frac{20}{3}$ cm

ALITER Since ΔTPQ is isosceles and TO is the angle bisector of $\angle PTQ$. Therefore, $OT \perp PQ$ and OT bisects PQ.

$\therefore \quad PR = QR = 4$ cm.

In right triangle ORP, we have

$OP^2 = OR^2 + PR^2$

$\Rightarrow \quad OR^2 = OP^2 - PR^2 = 25 - 16 = 9$

$\Rightarrow \quad OR = 3$ cm.

In triangles TRP and PRO, we have

$\angle TPR + \angle PTR = 90°$ and $\angle TPR + \angle RPO = 90°$

$\Rightarrow \quad \angle TPR + \angle PTR = \angle TPR + \angle RPO$

$\Rightarrow \quad \angle PTR = \angle RPO$

Also, $\angle TRP = \angle ORP = 90°$

$PR = PR$ [Common]

$\therefore \quad \Delta TRP \sim \Delta PRO$

$$\Rightarrow \quad \frac{TP}{PO} = \frac{RP}{RO} \Rightarrow \frac{TP}{5} = \frac{4}{3} \Rightarrow TP = \frac{20}{3} \text{cm}$$

EXAMPLE 21 *In Fig. 8.34, l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between l and m. Prove that* $\angle DFE = 90°$. **[NCERT, CBSE 2000, 2013]**

SOLUTION Since tangents drawn from an external point to a circle are equal. Therefore, $DA = DC$.

Thus, in triangles ADF and DFC, we have

$DA = DC$

$DF = DF$ Common]

$AF = CF$ [Radii of the same circle]

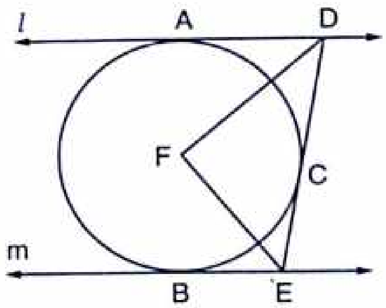

Fig. 8.34

So, by SSS-criterion of congruence, we obtain

$\Delta ADF \cong \Delta DFC$

$\Rightarrow \quad \angle ADF = \angle CDF$

$\Rightarrow \quad \angle ADC = 2\angle CDF$ ...(i)

Similarly, we can prove that

$\angle BEF = \angle CEF$

$\Rightarrow \quad \angle CEB = 2\angle CEF$ ...(ii)

Now, $\angle ADC + \angle CEB = 180°$ [Sum of the interior angles on the same side of transversal is 180°]

$\Rightarrow \quad 2\angle CDF + 2\angle CEF = 180°$ [Using equations (i) and (ii)]

$\Rightarrow \quad \angle CDF + \angle CEF = 90°$

$\Rightarrow \quad 180° - \angle DFE = 90°$ [$\because \angle CDF, \angle CEF$ and $\angle DFE$ are angles of a triangle $\therefore \angle CDF + \angle CEF + \angle DFE = 180°$]

$\Rightarrow \quad \angle DFE = 90°$

EXAMPLE 22 *Prove that the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.* **[NCERT]**

SOLUTION Let PA and PB be two tangents drawn from an external point P to a circle with centre O. We have to prove that angles $\angle AOB$ and $\angle APB$ are supplementary i.e. $\angle AOB + \angle APB = 180°$.

In right triangles OAP and OBP, we have

$PA = PB$ [Tangents drawn from an external point are equal]

$OA = OB$ [Each equal to radius]

and, $OP = OP$ [Common]

So, by SSS-criterion of congruence, we obtain

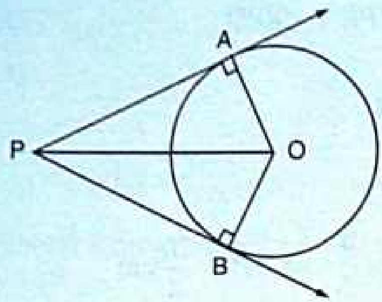

Fig. 8.35

$\Delta OAP \cong \Delta OBP$

$\Rightarrow$ $\angle OPA = \angle OPB$ and $\angle AOP = \angle BOP$

$\Rightarrow$ $\angle APB = 2\angle OPA$ and $\angle AOB = 2\angle AOP$ ...(i)

But, $\angle AOP = 90° - \angle OPA$ [$\because$ ΔOAP is right triangle]

$\therefore$ $2\angle AOP = 180° - 2\angle OPA$

$\Rightarrow$ $\angle AOB = 180° - \angle APB$ [Using (i)]

$\Rightarrow$ $\angle AOB + \angle APB = 180°$

Type III **BASED ON THE RESULT THAT THE TANGENT TO A CIRCLE AT A POINT IS PERPENDICULAR TO THE RADIUS THROUGH THE POINT**

EXAMPLE 23 *Show that tangent lines at the end points of a diameter of a circle are parallel.*

[CBSE 2014, 2017, NCERT]

SOLUTION Let AB be a diameter of a given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively. Since tangent at a point to a circle is perpendicular to the radius through the point.

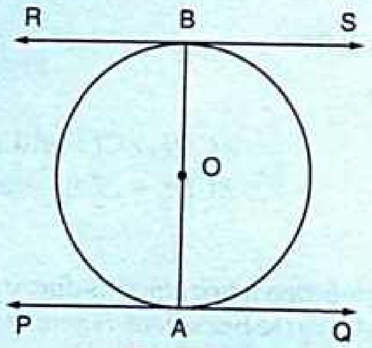

Fig. 8.36

$\therefore$ $AB \perp PQ$ and $AB \perp RS$

$\Rightarrow$ $\angle PAB = 90°$ and $\angle ABS = 90°$

$\Rightarrow$ $\angle PAB = \angle ABS$

$\Rightarrow$ $PQ \parallel RS$ [$\because$ $\angle PAB$ and $\angle ABS$ are alternate angles]

EXAMPLE 24 *In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.*

[CBSE 2012]

SOLUTION Let O be the common centre of two con-centric circles, and let AB be a chord of the larger circle touching the smaller circle at P.

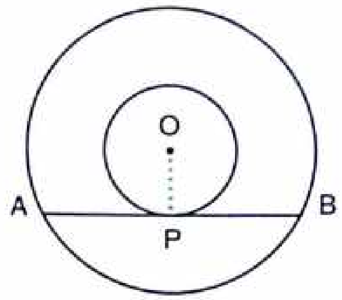

Fig. 8.37

Join OP.

Since OP is the radius of the smaller circle and AB is a tangent to this circle at a point P.

$\therefore \quad OP \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle, bisects the chord. So,

$$OP \perp AB$$

$\Rightarrow \quad AP = BP$

Hence, AB is bisected at P.

EXAMPLE 25 *In two concentric circles, a chord of length 24 cm of larger circle becomes a tangent to the smaller circle whose radius is 5 cm. Find the radius of the larger circle.*

SOLUTION Let O be the centre of concentric circles and APB be the chord of length 24 cm, of the larger circle touching the smaller circle at P. Then, $OP \perp AB$ and P is the mid-point of AB.

$\therefore \quad AP = PB = 12$ cm

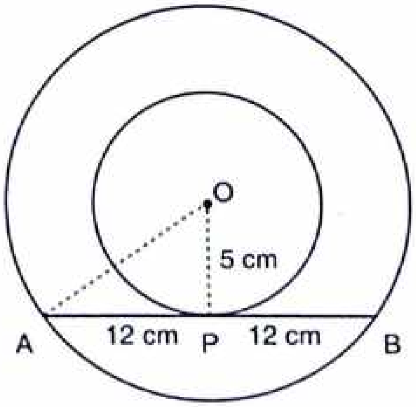

Fig. 8.38

In ΔOPA, we have

$$OA^2 = OP^2 + AP^2 \quad \text{[By Pythagoras Theorem]}$$

$\Rightarrow \quad OA^2 = 5^2 + 12^2 = 169$

$\Rightarrow \quad OA = 13$ cm

Hence, the radius of the smaller circle is 13 cm.

EXAMPLE 26 *Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.* [NCERT]

SOLUTION Let O be the centre of the concentric circles of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at P. Then

$$AP = PB \text{ and } OP \perp AB$$

Applying Pythagoras theorem in ΔOPA, we have

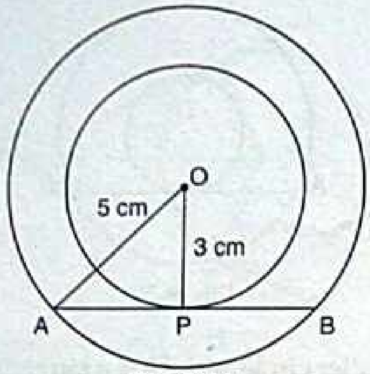

Fig. 8.39

$OA^2 = OP^2 + AP^2$

$\Rightarrow \quad 25 = 9 + AP^2$

$\Rightarrow \quad AP^2 = 16 \Rightarrow AP = 4$ cm

$\therefore \quad AB = 2AP = 8$ cm

EXAMPLE 27 *The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D. Find the length AD.*

SOLUTION Produce BD to meet the bigger circle at E. Join AE. Then,

$\angle AEB = 90$ [Angle in a semicircle]

$OD \perp BE$ [$\because$ BE is tangent to the smaller circle at D and OD is its radius]

and, $BD = DE$ [$\because$ BE is a chord of the circle and $OD \perp BE$]

$\therefore \quad OD \parallel AE$ [$\because \angle AEB = \angle ODB = 90°$]

In ΔAEB, O and D are mid-points of AB and BE. Therefore, by mid-point theorem, we have

$OD = \frac{1}{2} AE$

$\Rightarrow \quad AE = 2 \times 8 = 16$ cm [$\because OD = 8$ cm]

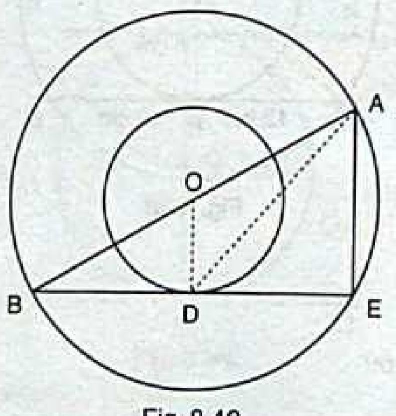

Fig. 8.40

In ΔODB, we have

$OB^2 = OD^2 + BD^2$ [By Pythagoras Theorem]

$\Rightarrow \quad 13^2 = 8^2 + BD^2$

$\Rightarrow \quad BD^2 = 169 - 64 = 105$

$\Rightarrow \quad BD = \sqrt{105}$ cm

$\Rightarrow \quad DE = \sqrt{105}$ cm [$\because BD = DE$]

In ΔAED, we have

$$AD^2 = AE^2 + ED^2 \qquad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow \quad AD^2 = 16^2 + (\sqrt{105})^2 = 256 + 105 = 361$$

$$\Rightarrow \quad AD = 19 \text{ cm}$$

EXAMPLE 28 *In Fig. 8.41, O is the centre of the circle. PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.* **[NCERT EXEMPLAR]**

SOLUTION Since tangent at a point to a circle is perpendi-cular to the radius through the point.

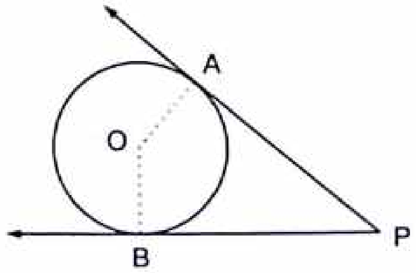

Fig. 8.41

$$\therefore \quad OA \perp AP \text{ and } OB \perp BP$$

$$\Rightarrow \quad \angle OAP = 90^\circ \text{ and } \angle OBP = 90^\circ$$

$$\Rightarrow \quad \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ \qquad \ldots(i)$$

In quadrilateral $OAPB$, we have

$$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360^\circ$$

$$\Rightarrow \quad (\angle APB + \angle AOB) + (\angle OAP + \angle OBP) = 360^\circ$$

$$\Rightarrow \quad \angle APB + \angle AOB + 180^\circ = 360^\circ \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \angle APB + \angle AOB = 180^\circ \qquad \ldots(ii)$$

From (i) and (ii), we can say that the quadrilateral $AOBP$ is cyclic.

Type IV BASED ON THE RESULT THAT THE TANGENTS DRAWN FROM AN EXTERNAL POINT OF A CIRCLE SUBTEND EQUAL ANGLES AT THE CENTRE

EXAMPLE 29 *A circle touches the sides of a quadrilateral ABCD at P, Q, R, S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.*

[NCERT, CBSE 2012, 2014]

GIVEN A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral $ABCD$ at the points P, Q, R and S respectively.

TO PROVE $\angle AOB + \angle COD = 180^\circ$ and, $\angle AOD + \angle BOC = 180^\circ$

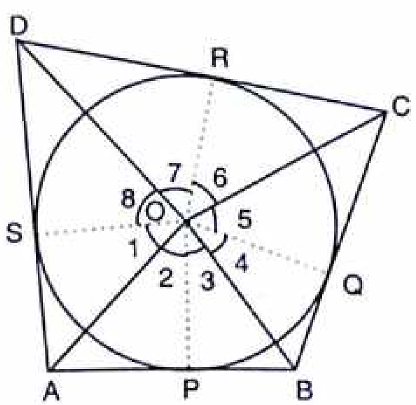

Fig. 8.42

CONSTRUCTION Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [Sum of all the angles subtended at a point is 360°]

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

$\Rightarrow \quad \angle AOB + \angle COD = 180°$ [$\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD$, $\angle 1 + \angle 8 = \angle AOD$ and $\angle 4 + \angle 5 = \angle BOC$]

and, $\angle AOD + \angle BOC = 180°$

Type V MISCELLANEOUS PROBLEMS

EXAMPLE 30 *Prove that the segment joining the points of contact of two parallel tangents passes through the centre.* **[CBSE 2014]**

SOLUTION Let PAQ and RBS be two parallel tangents to a circle with centre O. Join OA and OB. Draw $OC \parallel PQ$.

Now, $PA \parallel CO$

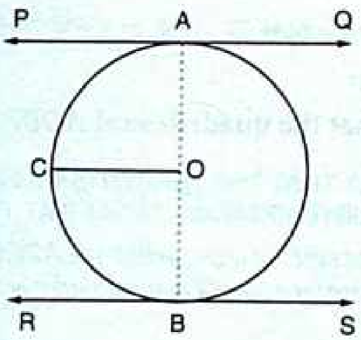

Fig. 8.43

$\Rightarrow \quad \angle PAO + \angle COA = 180°$ [Sum of the angles on the same side of a transversal is 180°]

$\Rightarrow \quad 90° + \angle COA = 180°$ [$\because \angle PAO$ = angle between a tangent and radius = 90°]

$\Rightarrow \quad \angle COA = 90°$

Similarly, $\angle COB = 90°$

$\therefore \quad \angle COA + \angle COB = 90° + 90° = 180°$

Hence, AOB is a straight line passing through O.

EXAMPLE 31 *O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find length of AB.*

[CBSE 2016, NCERT EXEMPLAR]

SOLUTION Clearly $\angle OPT = 90°$

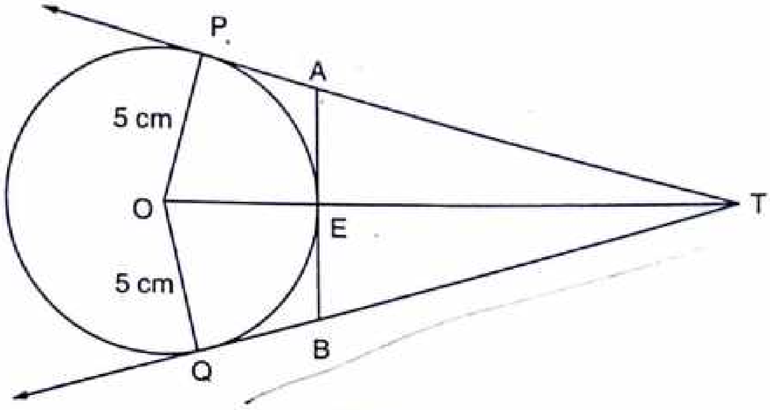

Fig. 8.44

Applying Pythagoras in ΔOPT, we have

$$OT^2 = OP^2 + PT^2$$
$$\Rightarrow \quad 13^2 = 5^2 + PT^2$$
$$\Rightarrow \quad PT^2 = 169 - 25 = 144$$
$$\Rightarrow \quad PT = 12 \text{ cm}$$

Since lengths of tangents drawn from a point to a circle are equal. Therefore,

$$AP = AE = x \text{ (say)}$$
$$\Rightarrow \quad AT = PT - AP = (12 - x) \text{ cm}$$

Since AB is the tangent to the circle E. Therefore, $OE \perp AB$

$$\Rightarrow \quad \angle OEA = 90°$$
$$\Rightarrow \quad \angle AET = 90°$$
$$\Rightarrow \quad AT^2 = AE^2 + ET^2 \quad \text{[Applying Pythagoras Theorem in } \Delta AET\text{]}$$
$$\Rightarrow \quad (12 - x)^2 = x^2 + (13 - 5)^2$$
$$\Rightarrow \quad 144 - 24x + x^2 = x^2 + 64$$
$$\Rightarrow \quad 24x = 80$$
$$\Rightarrow \quad x = \frac{10}{3} \text{ cm}$$

Similarly, $BE = \frac{10}{3}$ cm

$$\therefore \quad AB = AE + BE = \left(\frac{10}{3} + \frac{10}{3}\right) \text{cm} = \frac{20}{3} \text{ cm}$$

LEVEL-2

EXAMPLE 32 *The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.*

[CBSE 2014, NCERT]

SOLUTION Let I be the incentre of ΔABC such that in-radius $= IL = IM = IN = 4$ cm.

Also, $AM = 6$ cm, and $CM = 8$ cm.

Let $BL = BN = x$ cm

We have,

$AM = 6$ cm and $AM = AN$

$\therefore$ $AN = 6$ cm

Similarly, $CL = CM = 8$ cm

$\therefore$ $a = BC = BL + CL = (x+8)$ cm

$b = AC = AM + CM = (6+8)$ cm $= 14$ cm

and, $c = AB = AN + BN = (x+6)$ cm

$\therefore$ $2s = a + b + c$

$\Rightarrow$ $2s = x + 8 + 14 + x + 6$

$\Rightarrow$ $s = x + 14$

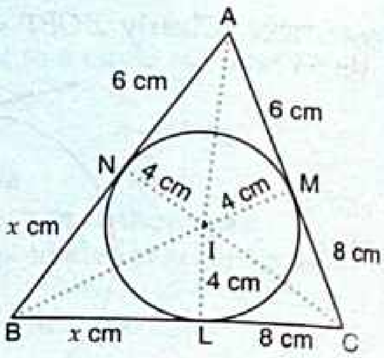

Fig. 8.45

Now,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)(x+14-x-8)(x+14-14)(x+14-x-6)}$$

$$= \sqrt{(x+14)\times(6)\times(x)\times 8}$$

$$= \sqrt{48x(x+14)} \qquad \text{...(i)}$$

Also,

Area of ΔABC = Area of ΔIBC + Area of ΔICA + Area of ΔIAB

$$= \frac{1}{2}\times BC\times IL + \frac{1}{2}\times CA\times IM + \frac{1}{2}\times AB\times IN$$

$$= \frac{1}{2}\times(x+8)\times 4 + \frac{1}{2}\times 14\times 4 + \frac{1}{2}\times(x+6)\times 4$$

$$= 2(x+8) + 28 + 2(x+6)\text{ cm}^2 \quad = 4x + 56 \text{ cm}^2 \qquad \text{...(ii)}$$

From (i) and (ii), we get

$\sqrt{48x(x+14)} = (4x+56)$

$\Rightarrow$ $48x(x+14) = (4x+56)^2$

$\Rightarrow$ $48x(x+14) = 16(x+14)^2$

$\Rightarrow$ $3x(x+14) = (x+14)^2$

$\Rightarrow$ $3x(x+14) - (x+14)^2 = 0$

$\Rightarrow$ $(x+14)(3x-x-14) = 0$

$\Rightarrow$ $2(x+14)(x-7) = 0$

$\Rightarrow$ $x - 7 = 0$ $\quad [\because x > 0 \therefore x + 14 \neq 0]$

$\Rightarrow$ $x = 7$

$\therefore$ $BC = (x+8)$ cm $= 15$cm and $AB = (x+6)$ cm $= 13$ cm.

EXAMPLE 33 *A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in Fig. 8.46. Find AD, BE and CF.* **[CBSE 2001, 2013, 2015, 2016]**

SOLUTION We know that the tangents drawn from an external point to a circle are equal. Therefore,

$AD = AF = x$, say

$BD = BE = y$, say

and, $CE = CF = z$, say

Now,

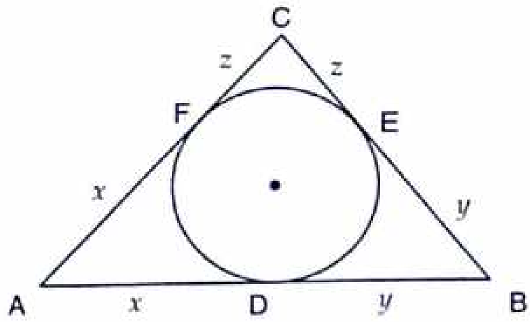

Fig. 8.46

$AB = 12\text{cm}, BC = 8\text{cm}$ and, $CA = 10\text{cm}$

$\Rightarrow \quad x + y = 12, y + z = 8$ and $z + x = 10$

$\Rightarrow \quad (x+y)+(y+z)+(z+x) = 12+8+10$

$\Rightarrow \quad 2(x+y+z) = 30$

$\Rightarrow \quad x+y+z = 15$

Now,

$x + y = 12$ and $x + y + z = 15 \Rightarrow 12 + z = 15 \Rightarrow z = 3.$

$y + z = 8$ and $x + y + z = 15 \Rightarrow x + 8 = 15 \Rightarrow x = 7$

and, $z + x = 10$ and $x + y + z = 15 \Rightarrow y + 10 = 15 \Rightarrow y = 5.$

Hence, $AD = x = 7$ cm, $BE = y = 5$ cm and $CF = z = 3$ cm.

EXAMPLE 34 *Find the locus of the centres of circles which touch a given line at a given point.*

SOLUTION Let APB be the given line, and let a circle with centre O touch APB at P. Then, $\angle OPB = 90°$. Let there be another circle with centre O' which touches the line APB at P. Then, $\angle O'PB = 90°$.

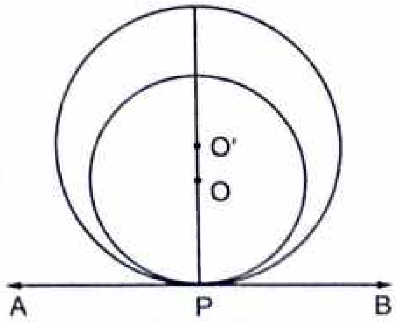

Fig. 8.47

This is possible only when O and O' lie on the same line $O'OP$. Hence, the required locus is a line perpendicular to the given line at the point of contact.

EXAMPLE 35 *In Fig. 8.48, circles C (O, r) and C (O' r/2) touch internally at a point A and AB is a chord of the circle C (O, r) intersecting C (O', r/2) at C. Prove that AC = CB.*

SOLUTION Join OA, OC and OB. Clearly, $\angle OCA$ is the angle in a semi-circle.

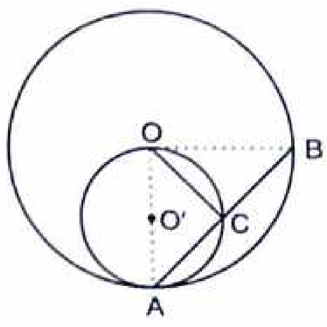

Fig. 8.48

$\therefore \quad \angle OCA = 90^\circ$

In right triangles OCA and OCB, we have

$$OA = OB = r$$

$$\angle OCA = \angle OCB = 90^\circ$$

and, $\quad OC = OC$

So, by *RHS*-criterion of congruence, we get

$$\Delta OCA \cong \Delta OCB$$

$\Rightarrow \quad AC = CB$

EXAMPLE 36 *In two concentric circles, prove that all chords of the outer circle which touch the inner are of equal length.*

SOLUTION Let AB and CD be two chords of the circle which touch the inner circle at M and N respectively.

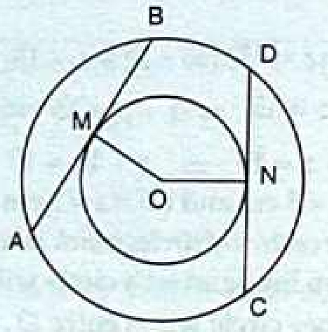

Fig. 8.49

Then, we have to prove that $AB = CD$.

Since AB and CD are tangents to the smaller circle.

$\therefore \quad OM = ON =$ Radius of the smaller circle.

Thus, AB and CD are two chords of the larger circle such that they are equidistant from the centre.

Hence, $\quad AB = CD$.

EXAMPLE 37 *Find the locus of centres of circles which touch two intersecting lines.*

SOLUTION Let l_1 and l_2 be two intersecting lines which intersect at point P. Let O be the centre of the circle which touches both l_1 and l_2.

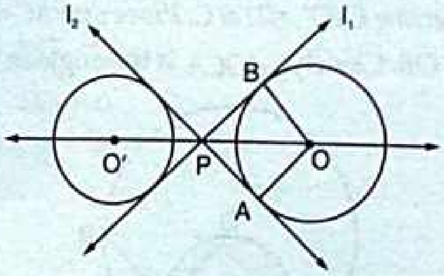

Fig. 8.50

In triangles OAP and OBP, we obtain

$OA = OB$ [Each equal to radius]

$PA = PB$ [Tangents drawn from an external point to a circle are equal]

and, $OP = OP$ [Common]

So, by SSS-congruence criterion, we obtain

$\Delta OAP \cong \Delta OBP$

$\Rightarrow$ $\angle APO = \angle BPO$

$\Rightarrow$ OP is the bisector of $\angle APB$

$\Rightarrow$ O lies on the bisector of the angle between l_1 and l_2.

Hence, the required locus is the line bisecting the angle between the given lines.

EXAMPLE 38 *Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C, respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.*

SOLUTION In order to prove that P is the circumcentre of ΔABC, it is sufficient to show that P is the point of intersection of perpendicular bisectors of the sides of

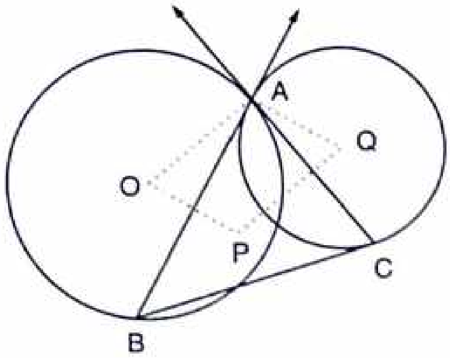

Fig. 8.51

ΔABC, i. e. OP and PQ are perpendicular bisectors of sides AB and AC respectively. Now, AC is tangent at A to the circle with centre at O and OA is its radius.

$\therefore$ $OA \perp AC$

$\Rightarrow$ $PQ \perp AC$ [$\because$ $OAQP$ is a parallelogram $\therefore$ $OA \parallel PQ$]

$\Rightarrow$ PQ is the perpendicular bisector of AC. [$\because$ Q is the centre of the circle]

Similarly, BA is the tangent to the circle at A and AQ is its radius through A.

$\therefore$ $BA \perp AQ$

$\therefore$ $BA \perp OP$ [$\because$ $AQPO$ is parallelogram $\therefore$ $OP \parallel AQ$]

$\Rightarrow$ OP is the perpendicular bisector of AB.

Thus, P is the point of intersection of perpendicular bisectors PQ and PO of sides AC and AB respectively

Hence, P is the circumcentre of ΔABC.

EXAMPLE 39 *Two circles with centres A and B of radii 3 cm and 4 cm respectively intersect at two points C and D such that AC and BC are tangents to the two circles. Find the length of the common chord CD.* **[NCERT EXEMPLAR]**

SOLUTION Since tangent at a point to a circle is perpendicular to the radius through the point of contact. Therefore, $\angle ACB = 90°$

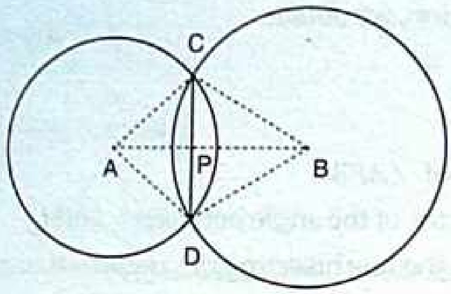

Fig. 8.52

In ΔACB, we have

$$AB^2 = AC^2 + BC^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow \quad AB^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow \quad AB = 5 \text{ cm}$$

Since the line joining the centres of two intersecting circles is perpendicular bisector of their common chord.

$$\therefore \quad AP \perp CD \text{ and } CP = PD$$

Let $AP = x$. Then, $BP = 5 - x$ $[\because AB = 5\text{ cm}]$

Further, let $CP = DP = y$ cm.

In ΔAPC and ΔBPC applying Pythagoras theorem, we have

$$AC^2 = AP^2 + PC^2, \; BC^2 = PB^2 + PC^2$$

$$\Rightarrow \quad 3^2 = x^2 + y^2 \text{ and } 4^2 = (5-x)^2 + y^2$$

$$\Rightarrow \quad 4^2 - 3^2 = \{(5-x)^2 + y^2\} - \{x^2 + y^2\} \quad \text{[On subtracting first from second]}$$

$$\Rightarrow \quad 7 = 25 - 10x$$

$$\Rightarrow \quad 10x = 18 \Rightarrow x = 1.8 \text{ cm}$$

$$\therefore \quad 3^2 = x^2 + y^2 \Rightarrow y = \sqrt{9 - (1.8)^2} = \sqrt{5.76} = 2.4 \text{ cm}$$

Hence, $CD = 2CP = 2y = 4.8$ cm

EXAMPLE 40 *If an isosceles triangle ABC in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.* **[NCERT EXEMPLAR]**

SOLUTION Let O be the centre of the circle and let P be the mid-point of BC. Then, $OP \perp BC$. Since ΔABC is isosceles and P is the mid-point of BC. Therefore, $AP \perp BC$ as median from the vertex in an isosceles triangle is perpendicular to the base.

Let $AP = x$ and $PB = CP = y$.

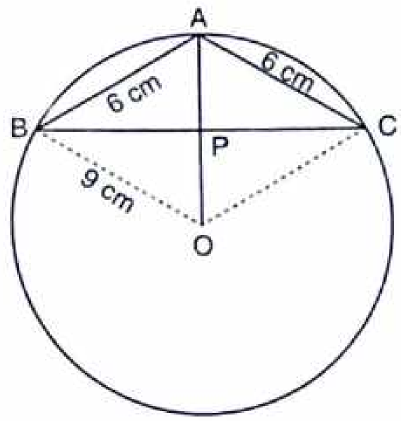

Fig. 8.53

Applying Pythagoras theorem in Δ's APB and OPB, we have

$$AB^2 = BP^2 + AP^2 \text{ and } OB^2 = OP^2 + BP^2$$

$\Rightarrow \quad 36 = y^2 + x^2 \quad \ldots\text{(i)} \quad \text{and, } 81 = (9-x)^2 + y^2 \quad \ldots\text{(ii)}$

$\Rightarrow \quad 81 - 36 = \{(9-x)^2 + y^2\} - \{y^2 + x^2\}$ [Subtracting (i) from (ii)]

$\Rightarrow \quad 45 = 81 - 18x$

$\Rightarrow \quad x = 2 \text{ cm}$

Putting $x = 2$ in (i), we get

$$36 = y^2 + 4 \Rightarrow y^2 = 32 \Rightarrow y = 4\sqrt{2} \text{ cm}$$

$\therefore \quad BC = 2BP = 2y = 8\sqrt{2} \text{ cm}$

Hence, Area of $\Delta ABC = \frac{1}{2}(BC \times AP = \frac{1}{2} \times 8\sqrt{2} \times 2 \text{ cm}^2 = 8\sqrt{2} \text{ cm}^2$

EXAMPLE 41 *AB is a diameter of a circle. P is a point on the semi-circle APB. AH and BK are perpendiculars from A and B respectively to the tangent at P. Prove that AH + BK = AB.*

SOLUTION Clearly, $\angle MPO = 90°$

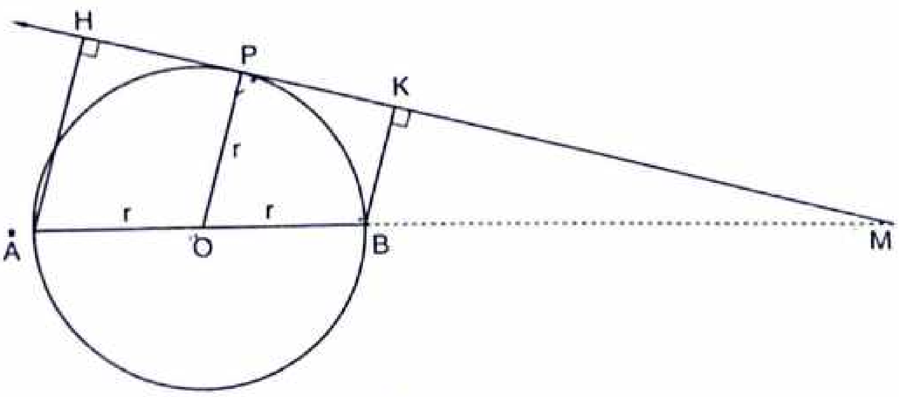

Fig. 8.54

Since $BK \perp HM$, $AH \perp HM$ and $OP \perp HM$. Therefore, $AH \parallel OP \parallel BK$.
Let $AH = x$, $BK = y$ and $OP = r$. Further, let $BM = z$.

In ΔMKB and ΔMHA, we have

$$\angle MKB = \angle MHA = 90^\circ$$

and $\angle BMK = \angle AMH$ [Common]

So, by AA criterion of similarity, we have

$$\Delta MKB \sim \Delta MHA$$

$$\Rightarrow \quad \frac{BK}{AH} = \frac{MB}{MA} \Rightarrow \frac{y}{x} = \frac{z}{2r+z} \Rightarrow 2ry + yz = zx \Rightarrow z = \frac{2ry}{x-y} \quad \ldots(i)$$

In ΔMKB and ΔMPO, we have

$$\angle MKB = \angle MPO = 90^\circ$$

$$\angle BMK = \angle OMP$$ [Common]

So, by AA criterion of similarity, we obtain

$$\Delta MKB \sim \Delta MPO$$

$$\Rightarrow \quad \frac{BK}{OP} = \frac{BM}{OM} \Rightarrow \frac{y}{r} = \frac{z}{r+z} \Rightarrow ry + yz = rz \Rightarrow z = \frac{ry}{r-y} \quad \ldots(ii)$$

From (i) and (ii), we get

$$\frac{ry}{r-y} = \frac{2ry}{x-y} \Rightarrow 2r - 2y = x - y \Rightarrow 2r = x + y \Rightarrow AB = AH + BK$$

EXAMPLE 42 *From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that*

(i) $PA \cdot PB = PN^2 - AN^2$ (ii) $PN^2 - AN^2 = OP^2 - OT^2$ (iii) $PA \cdot PB = PT^2$

[NCERT EXEMPLAR]

SOLUTION (i) $PA \cdot PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ $\left[\begin{array}{l} \because ON \perp AB \\ \therefore N \text{ is the mid-point of } AB \\ \Rightarrow AN = BN \end{array}\right]$

$= PN^2 - AN^2$

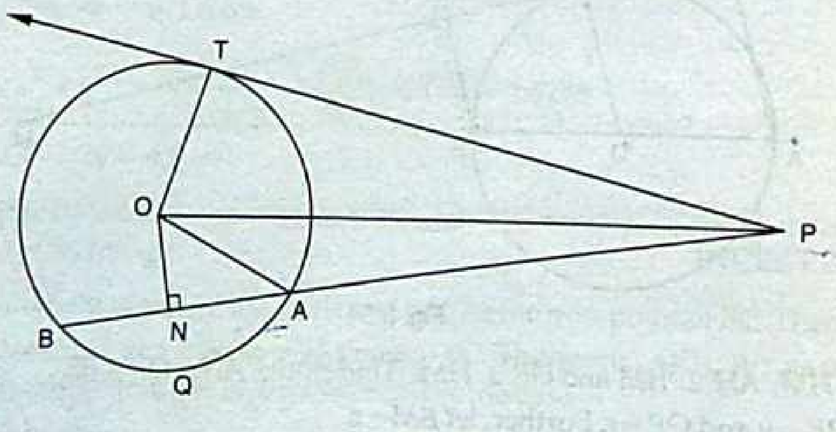

Fig. 8.55

(ii) Applying Pythagoras theorem in right triangle PNO, we obtain

$$OP^2 = ON^2 + PN^2 \Rightarrow PN^2 = OP^2 - ON^2$$

$$\therefore \quad PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$$

$$= OP^2 - (ON^2 + AN^2)$$

$$= OP^2 - OA^2 \quad \text{[Using Pythagoras theorem in } \Delta ONA\text{]}$$

$$= OP^2 - OT^2 \quad [\because OA = OT = \text{radius}]$$

(iii) From (i) and (ii), we obtain

$$PA \cdot PB = PN^2 - AN^2 \text{ and } PN^2 - AN^2 = OP^2 - OT^2$$

$$\Rightarrow \quad PA \cdot PB = OP^2 - OT^2$$

Applying Pythagoras theorem in $\Delta\, OTP$, we obtain

$$OP^2 = OT^2 + PT^2 \Rightarrow OP^2 - OT^2 = PT^2$$

Thus, we obtain

$$PA \,.\, PB = OP^2 - OT^2 \text{ and } OP^2 - OT^2 = PT^2$$

Hence, $PA \,.\, PB = PT^2$.

EXERCISE 8.2

LEVEL-1

1. If PT is a tangent at T to a circle whose centre is O and $OP = 17$ cm, $OT = 8$ cm, Find the length of the tangent segment PT.
2. Find the length of a tangent drawn to a circle with radius 5 cm, from a point 13 cm from the centre of the circle.
3. A point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 10 cm. Find the radius of the circle.
4. If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.
5. If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair.
6. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle. [NCERT EXEMPLAR]
7. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ. [NCERT EXEMPLAR]
8. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A. [NCERT EXEMPLAR]
9. If AB, AC, PQ are tangents in Fig. 8.56 and $AB = 5$ cm, find the perimeter of ΔAPQ. [CBSE 2000]

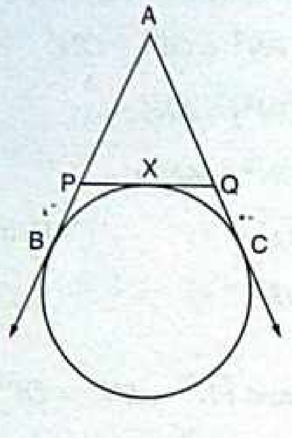

Fig. 8.56

10. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

11. In Fig. 8.57, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30°$, find $m\angle PRS$.

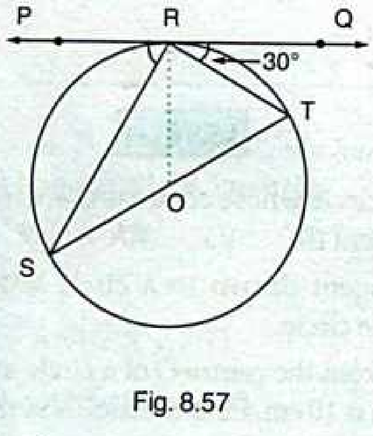

Fig. 8.57

12. If PA and PB are tangents from an outside point P. such that $PA = 10$ cm and $\angle APB = 60°$. Find the length of chord AB. **[CBSE 2016]**

13. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC. **[NCERT EXEMPLAR]**

14. From an external point P, tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn, which intersects PA and PB at C and D respectively. If $PA = 14$ cm, find the perimeter of ΔPCD.

[NCERT EXEMPLAR]

15. In Fig. 8.58, ABC is a right triangle right-angled at B such that $BC = 6$ cm and $AB = 8$ cm. Find the radius of its incircle. **[CBSE 2002]**

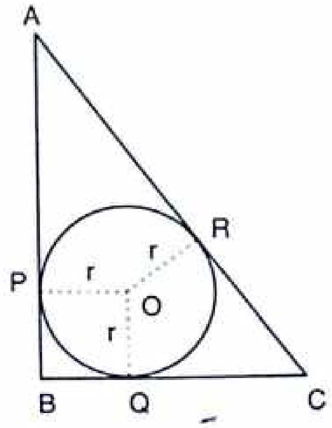

Fig. 8.58

16. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. **[NCERT EXEMPLAR]**

17. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that ΔAPB is equilateral.

18. Two tangent segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120°$. Prove that $OP = 2\,AP$. **[CBSE 2014]**

19. If ΔABC is isosceles with $AB = AC$ and $C(O, r)$ is the incircle of the ΔABC touching BC at L, prove that L bisects BC.

20. AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30°$. The tangent at C intersects AB at a point D. Prove that $BC = BD$. **[NCERT EXEMPLAR]**

21. In Fig. 8.59, a circle touches all the four sides of a quadrilateral $ABCD$ with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD. **[CBSE 2002]**

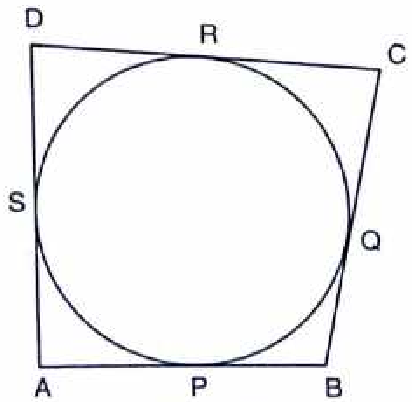

Fig. 8.59

22. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

23. Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that $TQ = TR$.

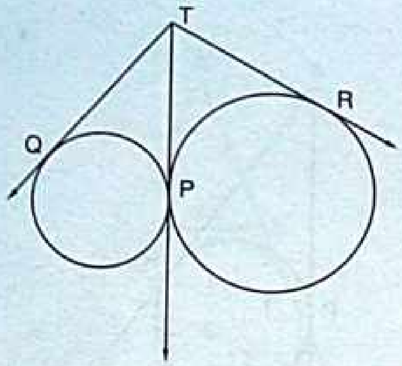

Fig. 8.60

24. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC. **[NCERT EXEMPLAR]**

25. In Fig. 8.61, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90^\circ$. If $AD = 23$ cm, $AB = 29$ cm and $DS = 5$ cm, find the radius r of the circle.

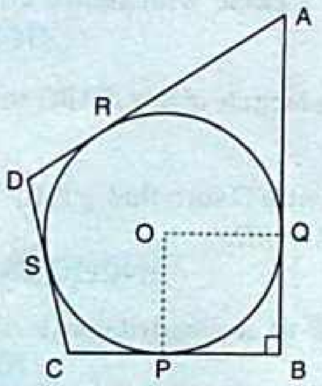

Fig. 8.61

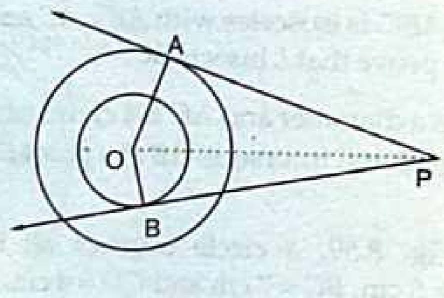

Fig. 8.62

26. In Fig. 8.62, there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If $AP = 12$ cm, find the length of BP. **[CBSE 2010, 2012, 2016]**

27. In Fig. 8.63, AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA. **[CBSE 2010]**

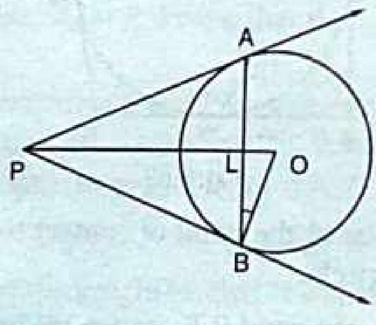

Fig. 8.63

28. In Fig. 8.64, PA and PB are tangents from an external point P to a circle with centre O. LN touches the circle at M. Prove that $PL + LM = PN + MN$. **[CBSE 2010]**

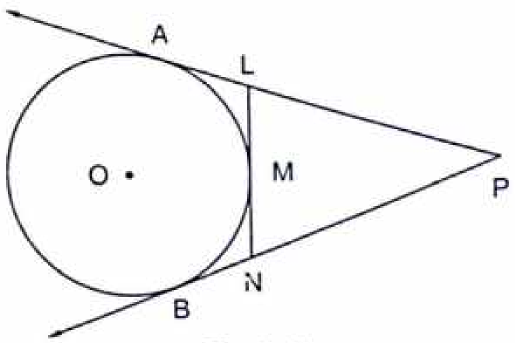

Fig. 8.64

29. In Fig. 8.65, BDC is a tangent to the given circle at point D such that $BD = 30$ cm and $CD = 7$ cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle.
Calculate (i) AF (ii) radius of the circle.

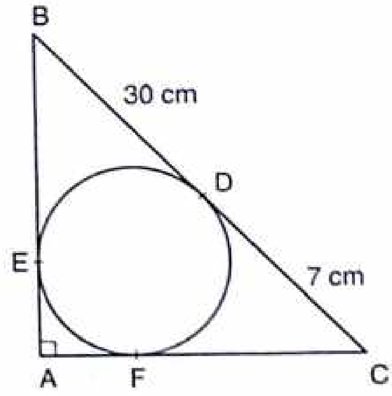

Fig. 8.65

30. If d_1, d_2 $(d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.
[NCERT EXEMPLAR]

31. In Fig. 8.66, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.
[CBSE 2015, NCERT EXEMPLAR]

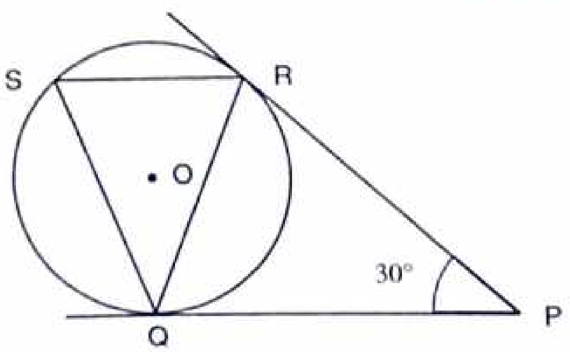

Fig. 8.66

32. From an external point P, tangents $PA = PB$ are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$. **[CBSE 2016]**

33. In Fig. 8.67, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120°$. Prove that $OA = 2AB$. **[CBSE 2016]**

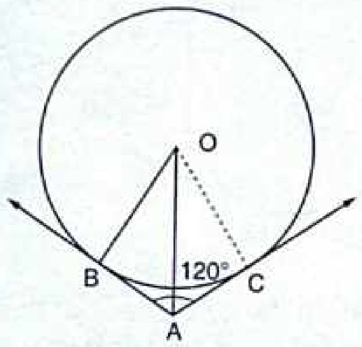

Fig. 8.67

34. The lengths of three consecutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm, and 7 cm respectively. Determine the length of the fourth side.

35. The common tangents AB and CD to two circles with centres O and O' intersect at E between their centres. Prove that the points O, E and O' are collinear. **[NCERT EXEMPLAR]**

36. In Fig. 8.68, common tangents PQ and RS to two circles intersect at A. Prove that $PQ = RS$. **[CBSE 2014, NCERT EXEMPLAR]**

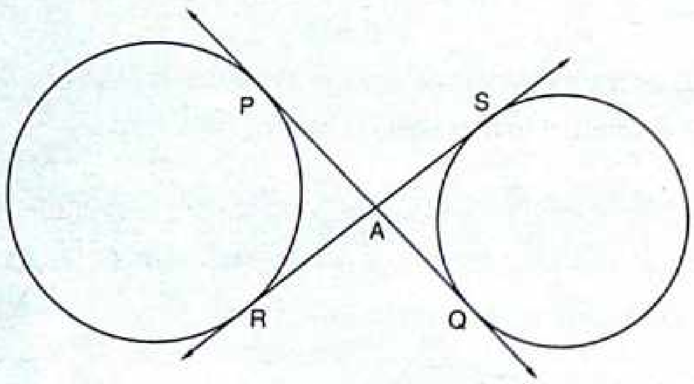

Fig. 8.68

37. Two concentric circles are of diameters 30 cm and 18 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[CBSE 2014]**

38. AB and CD are common tangents to two circles of equal radii. Prove that $AB = CD$. **[NCERT EXEMPLAR]**

39. A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of lengths 14 cm and 16 cm respectively. If area of ΔPQR is 336 cm^2, find the sides PQ and PR. **[CBSE 2014]**

40. In Fig. 8.69, the tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110°$, find $\angle CBA$. **[NCERT EXEMPLAR]**

[Hint: Join CO.]

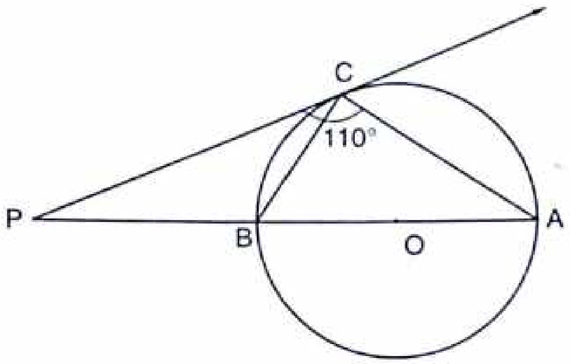

Fig. 8.69

41. AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig. 8.70. Prove that $\angle BAT = \angle ACB$. **[NCERT EXEMPLAR]**

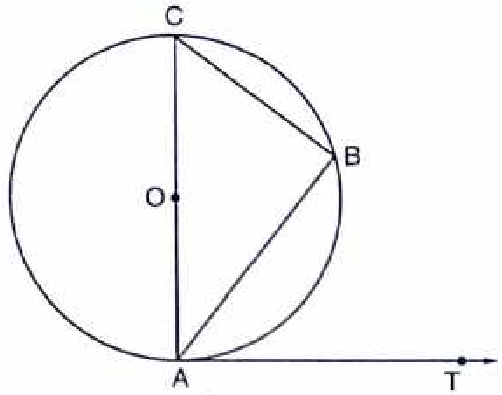

Fig. 8.70

LEVEL-2

42. In Fig. 8.71, a ΔABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the lengths of sides AB and AC, when area of ΔABC is 84 cm^2. **[CBSE 2015]**

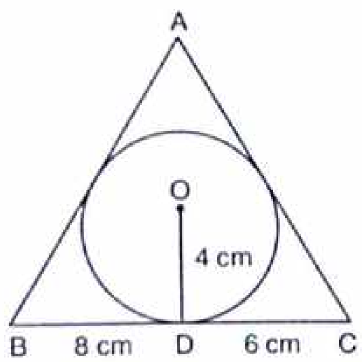

Fig. 8.71

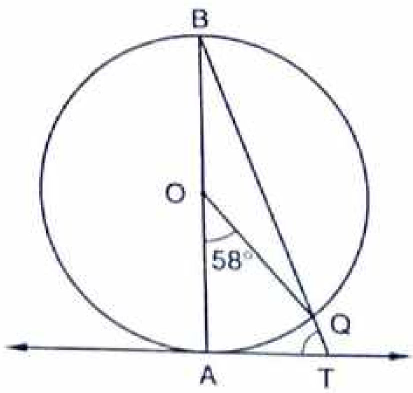

Fig. 8.72

43. In Fig. 8.72, AB is a diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. **[CBSE 2015]**

44. In Fig. 8.73, $OQ:PQ=3:4$ and perimeter of $\Delta POQ = 60$ cm. Determine PQ, QR and OP.

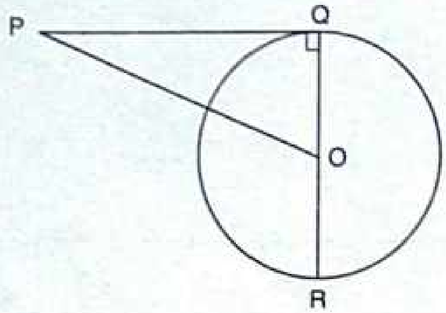

Fig. 8.73

45. Equal circles with centres O and O' touch each other at X. OO' produced to meet a circle with centre O', at A. AC is a tangent to the circle whose centre is O. $O'D$ is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.

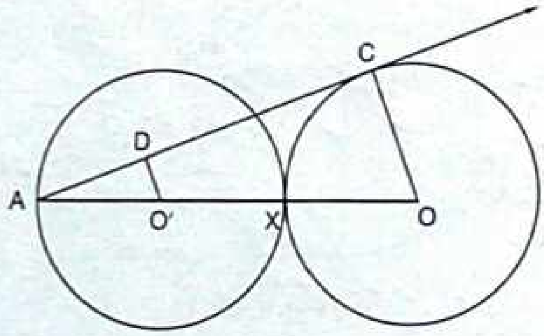

Fig. 8.74

46. In Fig. 8.75, BC is a tangent to the circle with centre O. OE bisects AP. Prove that $\Delta AEO \sim \Delta ABC$.

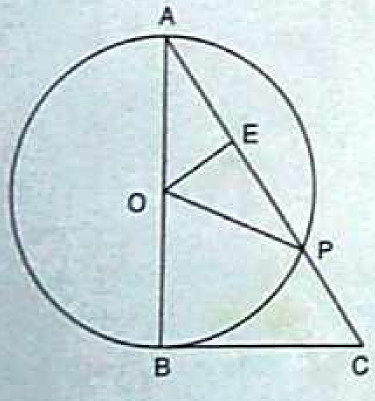

Fig. 8.75

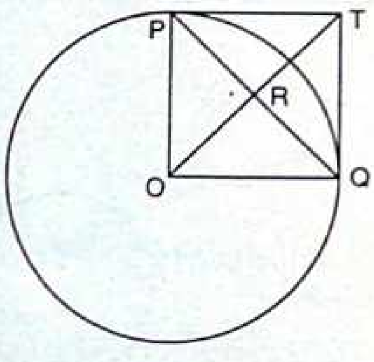

Fig. 8.76

47. In Fig. 8.76, $PO \perp QO$. The tangents to the circle at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.

48. In Fig. 8.77, O is the centre of the circle and BCD is tangent to it at C. Prove that $\angle BAC + \angle ACD = 90°$.

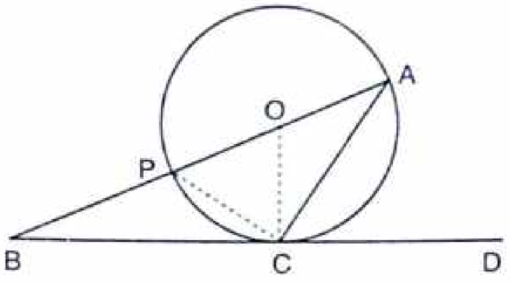

Fig. 8.77

49. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines. **[NCERT EXEMPLAR]**

50. In Fig. 8.78, there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm, find the length of PS. **[CBSE 2017]**

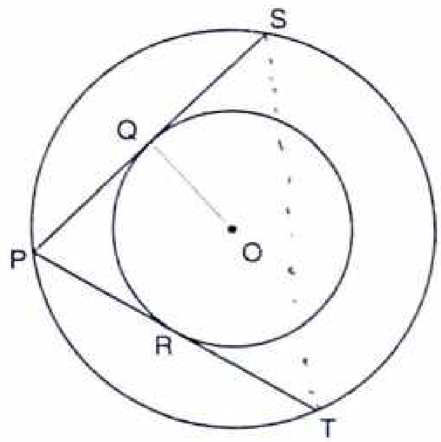

Fig. 8.78

51. In Fig. 8.79, PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and QOR is a diameter. If $\angle POR = 130°$ and S is a point on the circle, find $\angle 1 + \angle 2$. **[CBSE 2017]**

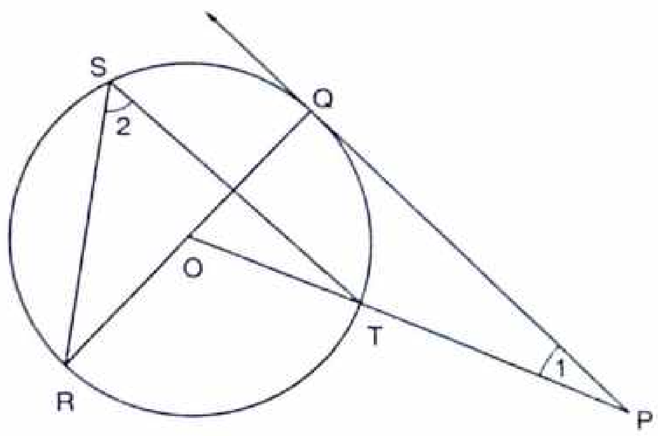

Fig. 8.79

52. In Fig. 8.80, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$. [CBSE 2017]

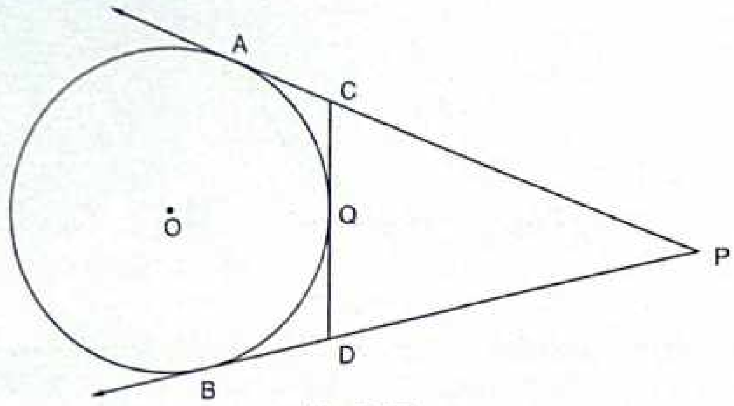

Fig. 8.80

ANSWERS

1. 15 cm	2. 12 cm	3. 24 cm	6. 3 cm	9. 10 cm	11. 60°
12. 10 cm	14. 28 cm	15. 2 cm	21. 3 cm	24. 24 cm	25. 11. cm
26. $4\sqrt{10}$ cm	27. $\frac{40}{3}$ cm	29. (i) 5 cm (ii) 5 cm		31. 75°	32. 100°
34. 6 cm	37. 24 cm	40. 70	42. 13 cm, 15 cm		43. 61°
44. $PQ = 20$ cm, $QR = 30$ cm, $OP = 25$ cm		45. $\frac{1}{3}$		50. 10 cm	52. 18 cm

HINTS TO SELECTED PROBLEMS

1. Since ΔOTP is a right triangle. Therefore,

$$OT^2 + PT^2 = OP^2 \Rightarrow PT^2 = OP^2 - OT^2 \Rightarrow PT^2 = 17^2 - 8^2 \Rightarrow PT = 15 \text{ cm}.$$

9. We have, $PB = PX$, $QC = QX$ and $AB = AC$

$\therefore$ Perimeter of $\Delta APQ = AP + PQ + AQ$

$$= AP + (PX + XQ) + AQ$$
$$= (AP + PX) + (AQ + XQ)$$
$$= (AP + PB) + (AQ + QC) = AB + AC = 10 \text{ cm}.$$

11. We have, $\angle TRQ = 30°$. Since ST is a diameter and angle in a semi-circle is a right angle. Therefore, $\angle SRT = 90°$.

Now, $\angle TRQ + \angle SRT + \angle PRS = 180° \Rightarrow 30° + 90° + \angle PRS = 180° \Rightarrow \angle PRS = 60°$

12. Tangents from an external point are equal in length. Therefore,

$PA = PB \Rightarrow \Delta PAB$ is isosceles $\Rightarrow \angle PAB = \angle PBA = 60° \Rightarrow \Delta PAB$ is equilateral
Hence, $AB = 10$ cm.

15. We have, $AR = AP = AB - BP = (8 - r)$ cm and, $CR = CQ = CB - BQ = (6 - r)$ cm.

$\therefore\ AC = AR + CR = (8 - r + 6 - r)\text{cm} = (14 - 2r)$ cm

Now, $AC^2 = AB^2 + BC^2 \Rightarrow (14 - 2r)^2 = 8^2 + 6^2 \Rightarrow r = 2$ cm

17. Join OP. Suppose OP meets the circle at Q. join AQ.
Now,

OP = Diameter

$\Rightarrow \quad OQ + PQ =$ Diameter

$\Rightarrow \quad PQ =$ Diameter – Radius [$\because OQ =$ radius]

$\Rightarrow \quad PQ =$ Radius.

$\therefore \quad OQ = PQ =$ Radius.

Thus, OP is the hypotenuse of right triangle OAP and Q is the mid-point of OP. The mid-point of hypotenuse of a right triangle is equidistant from the vertices

$\therefore \quad OA = AQ = OQ$

$\Rightarrow \quad \Delta OAQ$ is equilateral

$\Rightarrow \quad \angle AOQ = 60°$

So, $\angle APO = 30°$.

$\therefore \quad \angle APB = 2\angle APO = 60°$

Also, $PA = PB \Rightarrow \angle PAB = \angle PBA$.

But, $\angle APB = 60°$. Therefore, $\angle PAB = \angle PBA = 60°$

Hence, ΔAPB is equilateral

21. In example 9 on page 8.12, we have proved that if a circle touches all the four sides of quadrilateral $ABCD$. Then,

$$AB + CD = AD + BC \Rightarrow 6 + 4 = AD + 7 \Rightarrow AD = 3\text{cm}.$$

23. The tangents drawn from an external point to a circle are equal.

$\therefore \quad TP = TQ$ and $TP = TR \Rightarrow TQ = TR$

25. Proceed as in Example 16.

26. Join OA, OB and OP. In ΔOAP, we have

$$OP^2 = OA^2 + AP^2 \Rightarrow OP^2 = 5^2 + 12^2 \Rightarrow OP = 13\text{ cm}$$

In ΔOBP, we have

$$OP^2 = OB^2 + BP^2 \Rightarrow 13^2 = 3^2 + BP^2 \Rightarrow BP^2 = 169 - 9 = 160 \Rightarrow BP = \sqrt{160}\text{ cm} = 4\sqrt{10}\text{ cm}$$

27. We have, $AB = 16$ cm. Therefore, $AL = BL = 8$ cm.

In ΔOLB, we have

$$OB^2 = OL^2 + LB^2 \Rightarrow 10^2 = OL^2 + 8^2 \Rightarrow OL^2 = 100 - 64 = 36 \Rightarrow OL = 6\text{ cm}$$

Let $PL = x$ and $PB = y$. Then, $OP = (x + 6)$ cm.

In Δ's PLB and OBP, we have

$$PB^2 = PL^2 + BL^2 \text{ and } OP^2 = OB^2 + PB^2$$

$\Rightarrow \quad y^2 = x^2 + 64$ and $(x+6)^2 = 100 + y^2$ [Substituting the value of y^2 in second equation]

$$\Rightarrow \quad (x + 6)^2 = 100 + x^2 + 64 \Rightarrow 12x = 128 \Rightarrow x = \frac{32}{3}\text{cm}$$

$\therefore \quad y^2 = x^2 + 64 \Rightarrow y^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1600}{9} \Rightarrow y = \frac{40}{3}$ cm

Hence, $PA = PB = \frac{40}{3}$ cm

28. We have,

$PA = PB$ [Tangents drawn from P]

$\Rightarrow PL + AL = PN + BN \Rightarrow PL + LM = PN + MN$ [$\because AL = LM$ and $BN = MN$]

33. In Δ's OAB and OAC, we have

$\angle OBA = \angle OCA = 90^\circ$ and, $OA = OA$ [Common]

So, by RHS congruence criterion, we obtain

$$\Delta OBA \cong \Delta OCA \Rightarrow \angle OAB = \angle OAC = \frac{1}{2} \times 120^\circ = 60^\circ$$

In ΔOBA, we have

$$\cos 60^\circ = \frac{AB}{OA} \Rightarrow \frac{1}{2} = \frac{AB}{OA} \Rightarrow OA = 2AB$$

36. $AP = AR$ and $AS = AQ \Rightarrow AP + AQ = AS + AR \Rightarrow PQ = SR$

39. Since length of tangents drawn from a point to a circle are equal. Therefore, $QS = AT = 14$ cm, $RU = RT = 16$ cm and, $PS = PU = x$.

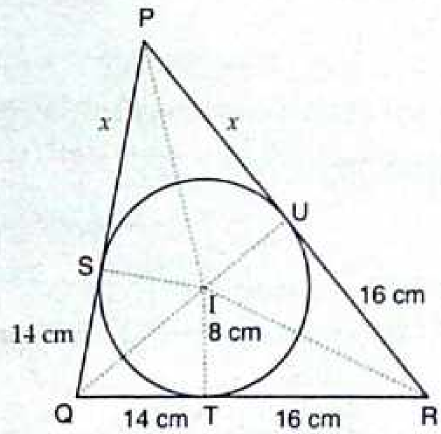

Fig. 8.81

Thus, $PQ = x + 14$, $PR = x + 16$ and $QR = 30$

Now, Area of ΔPQR = Area of ΔIQR + Area of ΔIQP + Area of ΔIPR

$$\Rightarrow \quad 336 = \frac{1}{2}(QR \times 8) + \frac{1}{2}(14 + 2) \times 8 + \frac{1}{2}(16 + x) \times 8$$

$$\Rightarrow \quad 84 = 30 + 14 + x + 16 + x$$

$$\Rightarrow \quad 24 = 2x \Rightarrow x = 12$$

Hence, $PQ = 26$ cm and $PR = 28$ cm

47. In Δ's TPO and TQO, we have

$\angle TPO = \angle TQO$ [Eqch equal to 90°]

So, by RHS congruence criterion, we obtain

$\Delta TPO \cong \Delta TQO \Rightarrow \angle PTO = \angle QTO$

Also, $\Delta PTR \cong \Delta QTR$ [By SAS congruence criterion]

$\therefore \quad PR = QR$ and $\angle TRP = \angle TRQ$

But, $\angle TRP + \angle TRQ = 180°$

$\therefore \quad \angle TRP = \angle TRQ = 90°$

Hence, PQ and OT are right bisectors of each other.

48. $OA = OC$ [Each equal to radius]

$\Rightarrow \quad \angle OAC = \angle OCA$ ...(i)

Clearly, $\angle OCD = 90°$

$\Rightarrow \quad \angle ACD + \angle OCA = 90°$

$\Rightarrow \quad \angle ACD + \angle OAC = 90°$ [From (i)]

$\Rightarrow \quad \angle ACD + \angle BAC = 90°$

VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

Answer each of the following questions either in one word or one sentence or as per requirement of the questions:

1. In Fig. 8.82, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If $PB = 10$ cm and $CQ = 2$ cm, what is the length PC?

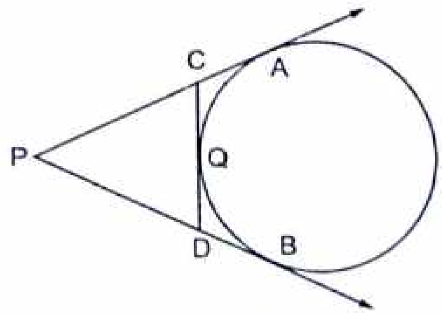

Fig. 8.82

2. What is the distance between two parallel tangents of a circle of radius 4 cm?

3. The length of tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. What is the radius of the circle?

4. Two tangents TP and TQ are drawn from an external point T to a circle with centre O as shown in Fig. 8.83. If they are inclined to each other at an angle of 100°, then what is the value of $\angle POQ$?

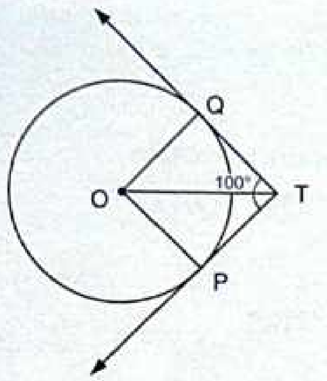

Fig. 8.83

5. What the distance between two parallel tangents to a circle of radius 5 cm?
6. In Q. No. 1, if $PB = 10$ cm, what is the perimeter of ΔPCD?
7. In Fig. 8.84, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If $CP = 11$ cm and $BC = 7$ cm, then find the length of BR. **[CBSE 2009]**

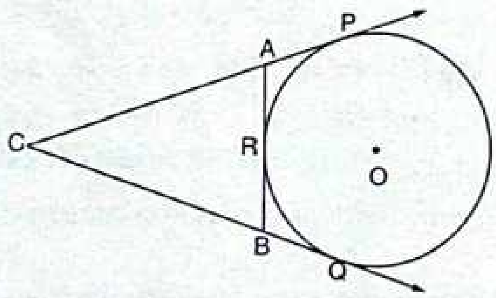

Fig. 8.84

8. In Fig. 8.85, ΔABC is circumscribing a circle. Find the length of BC. **[CBSE 2009]**

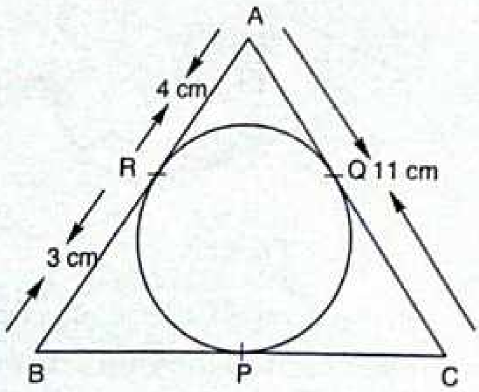

Fig. 8.85

9. In Fig. 8.86, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If $CP = 11$ cm and $BR = 4$ cm, find the length of BC. **[CBSE 2010]**

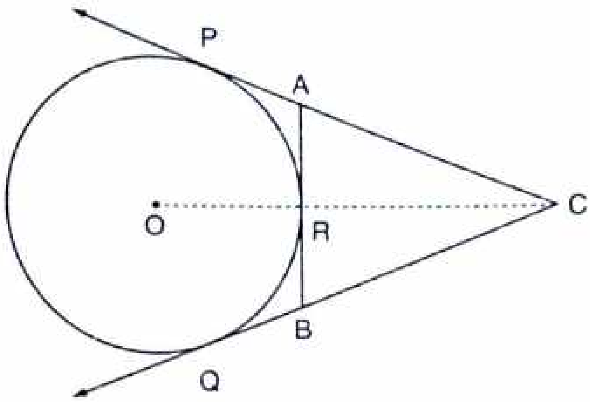

Fig. 8.86

[**Hint:** We have, $CP = 11$ cm

$\therefore \quad CP = CQ \Rightarrow CQ = 11$ cm

Now, $BR = BQ$ [Tangents drawn from B]

$\Rightarrow \quad BQ = 4$ cm

$\therefore \quad BC = CQ - BQ = (11-4)\text{cm} = 7\,\text{cm}$

10. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. In Fig. 8.87, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$. **[CBSE 2015]**

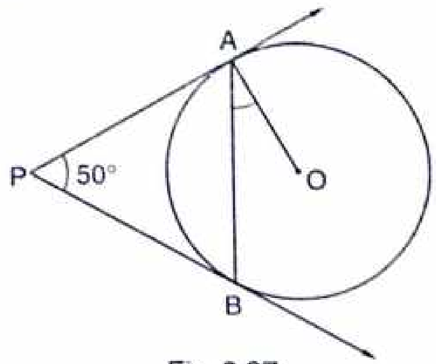

Fig. 8.87

12. In Fig. 8.88, PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60°$. Then, find $\angle PRQ$. **[NCERT EXEMPLAR]**

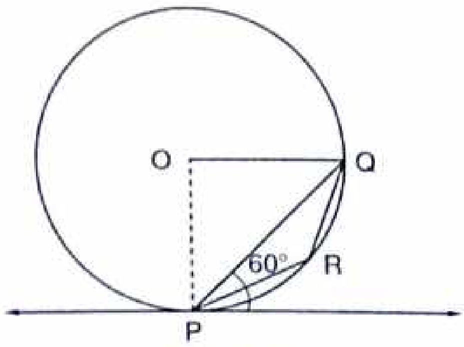

Fig. 8.88

13. In Fig. 8.89, PQL and PRM are tangents to the circle with centre O at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50°$ and $\angle SRM = 60°$. Then, find $\angle QSR$. **[NCERT EXEMPLAR]**

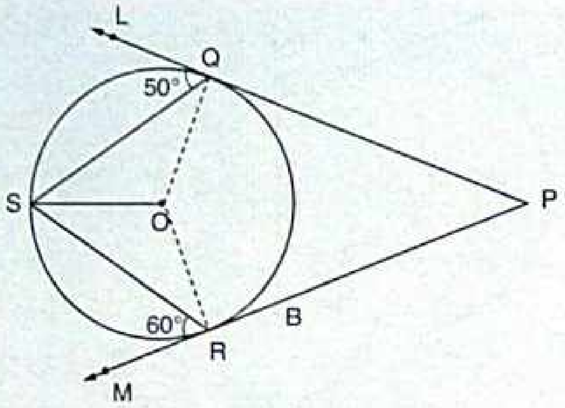

Fig. 8.89

14. In Fig. 8.90, BOA is a diameter of a circle and the tangent at a point P meets BA produced at T. If $\angle PBO = 30°$, then find $\angle PTA$. **[NCERT EXEMPLAR]**

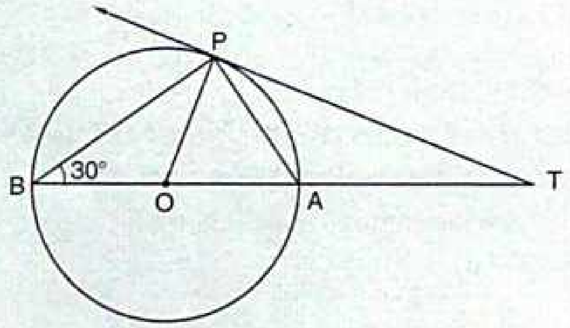

Fig. 8.90

ANSWERS

1. 8 cm	3. 3 cm	4. 80°	5. 10 cm	6. 20 cm	7. 4 cm	8. 10 cm
9. 7 cm	10. 8 cm	11. 25°	12. 120°	13. 70°	14. 30°	

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that $OQ = 12$ cm. Length PQ is
   (a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm
2. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
   (a) 7 cm (b) 12 cm (c) 15 cm (d) 24.5 cm. **[NCERT]**
3. The length of the tangent from a point A at a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is
   (a) $\sqrt{7}$ cm (b) 7 cm (c) 5 cm (d) 25 cm
4. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° then $\angle POA$ is equal to
   (a) 50° (b) 60° (c) 70° (d) 80° **[NCERT]**

5. If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$, then, $\angle PTQ$ is equal to

(a) 60° (b) 70° (c) 80° (d) 90° [NCERT]

6. PQ is a tangent to a circle with centre O at the point P. If ΔOPQ is an isosceles triangle, then $\angle OQP$ is equal to

(a) 30° (b) 45° (c) 60° (d) 90°

7. Two equal circles touch each other externally at C and AB is a common tangent to the circles. Then, $\angle ACB =$

(a) 60° (b) 45° (c) 30° (d) 90°

8. ABC is a right angled triangle, right angled at B such that $BC = 6$ cm and $AB = 8$ cm. A circle with centre O is inscribed in ΔABC. The radius of the circle is

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

9. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120°$, then $\angle OPQ$ is

(a) 60° (b) 45° (c) 30° (d) 90°

10. If four sides of a quadrilateral $ABCD$ are tangential to a circle, then

(a) $AC + AD = BD + CD$ (b) $AB + CD = BC + AD$

(c) $AB + CD = AC + BC$ (d) $AC + AD = BC + DB$

11. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is

(a) $\sqrt{7}$ cm (b) $2\sqrt{7}$ cm (c) 10 cm (d) 5 cm

12. AB and CD are two common tangents to circles which touch each other at C. If D lies on AB such that $CD = 4$ cm, then AB is equal to

(a) 4 cm (b) 6 cm (c) 8 cm (d) 12 cm

13. In Fig. 8.91, if AD, AE and BC are tangents to the circle at D, E and F respectively. Then,

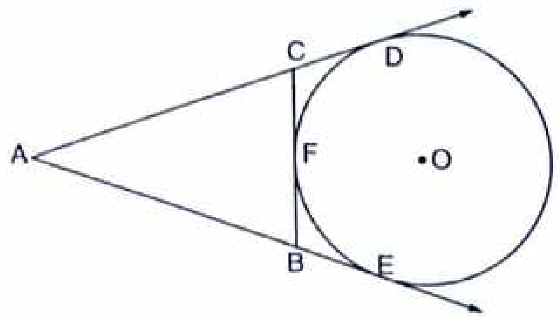

Fig. 8.91

(a) $AD = AB + BC + CA$ (b) $2AD = AB + BC + CA$

(c) $3AD = AB + BC + CA$ (d) $4AD = AB + BC + CA$

14. In Fig. 8.92, RQ is a tangent to the circle with centre O. If $SQ = 6$ cm and $QR = 4$ cm, then $OR =$

(a) 8 cm (b) 3 cm (c) 2.5 cm (d) 5 cm

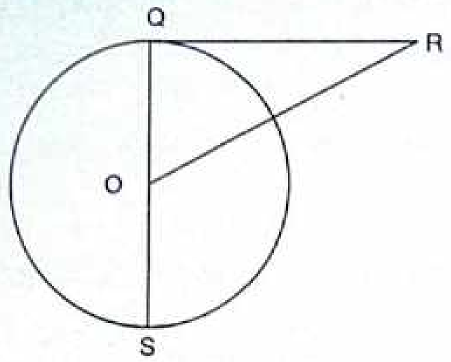

Fig. 8.92

15. In Fig. 8.93, the perimeter of ΔABC is

(a) 30 cm (b) 60 cm (c) 45 cm (d) 15 cm

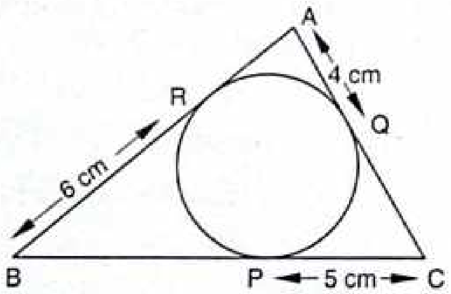

Fig. 8.93

16. In Fig. 8.94, AP is a tangent to the circle with centre O such that $OP = 4$ cm and $\angle OPA = 30°$. Then, $AP =$

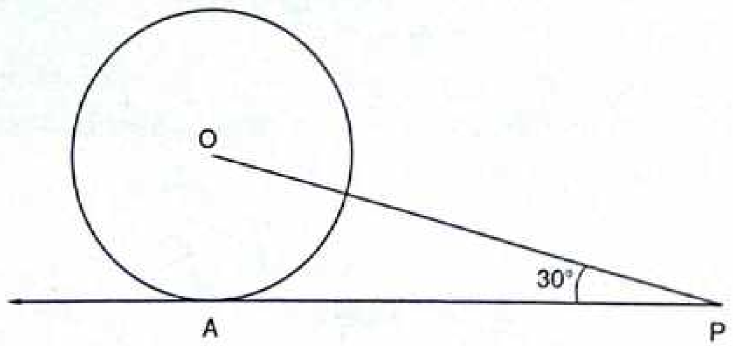

Fig. 8.94

(a) $2\sqrt{2}$ cm (b) 2 cm (c) $2\sqrt{3}$ cm (d) $3\sqrt{2}$ cm

17. AP and PQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If $OA = 15$ cm, then $AP + AQ =$

(a) 12 cm (b) 18 cm (c) 24 cm (d) 36 cm

18. At one end of a diameter PQ of a circle of radius 5 cm, tangent XPY is drawn to the circle. The length of chord AB parallel to XY and at a distance of 8 cm from P is

(a) 5 cm (b) 6 cm (c) 7 cm (d) 8 cm

19. If PT is tangent drawn from a point P to a circle touching it at T and O is the centre of the circle, then $\angle OPT + \angle POT =$

(a) 30° (b) 60° (c) 90° (d) 180°

20. In Fig. 8.95, if $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm, then $AD =$

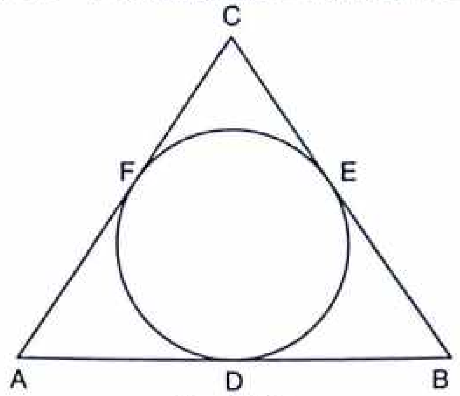

Fig. 8.95

(a) 5 cm (b) 4 cm (c) 6 cm (d) 7 cm

21. In Fig. 8.96, if $AP = PB$, then

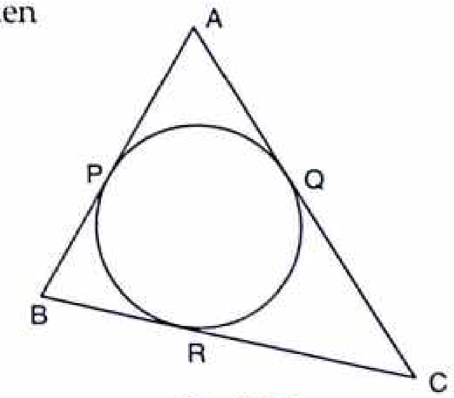

Fig. 8.96

(a) $AC = AB$ (b) $AC = BC$ (c) $AQ = QC$ (d) $AB = BC$

22. In Fig. 8.97, if $AP = 10$ cm, then $BP =$

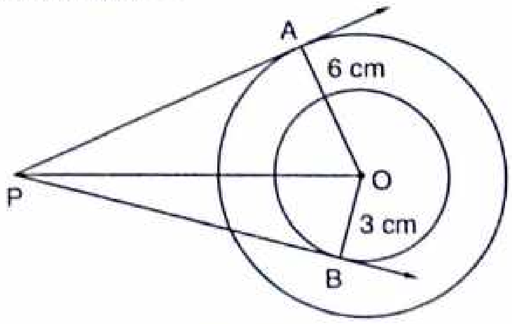

Fig. 8.97

(a) $\sqrt{91}$ cm (b) $\sqrt{127}$ cm (c) $\sqrt{119}$ cm (d) $\sqrt{109}$ cm

23. In Fig. 8.98, if PR is tangent to the circle at P and Q is the centre of the circle, then $\angle POQ =$

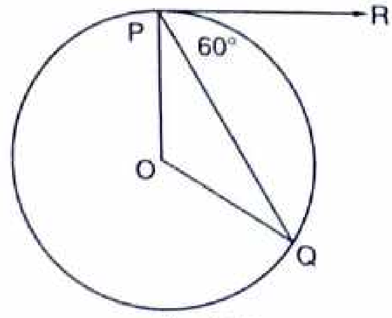

Fig. 8.98

(a) 110° (b) 100° (c) 120° (d) 90°

24. In Fig. 8.99, if quadrilateral $PQRS$ circumscribes a circle, then $PD + QB =$

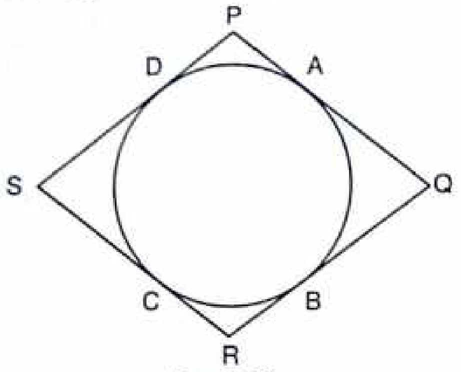

Fig. 8.99

(a) PQ (b) QR (c) PR (d) PS

25. In Fig. 8.100, two equal circles touch each other at T, if $QP = 4.5$ cm, then $QR =$

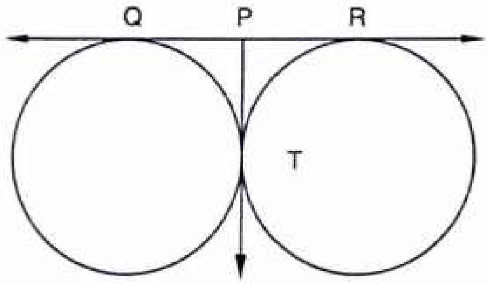

Fig. 8.100

(a) 9 cm (b) 18 cm (c) 15 cm (d) 13.5 cm

26. In Fig. 8.101, APB is a tangent to a circle with centre O at point P. If $\angle QPB = 50°$, then the measure of $\angle POQ$ is

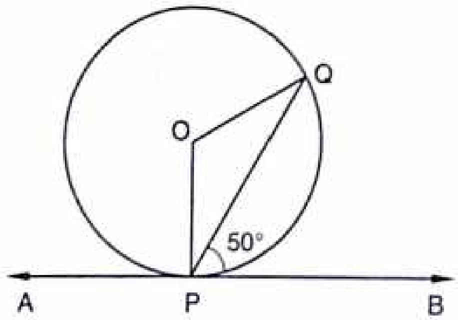

Fig. 8.101

(a) 100° (b) 120° (c) 140° (d) 150°

27. In Fig. 8.102, if tangents PA and PB are drawn to a circle such that $\angle APB = 30°$ and chord AC is drawn parallel to the tangent PB, then $\angle ABC =$ **[NCERT EXEMPLAR]**

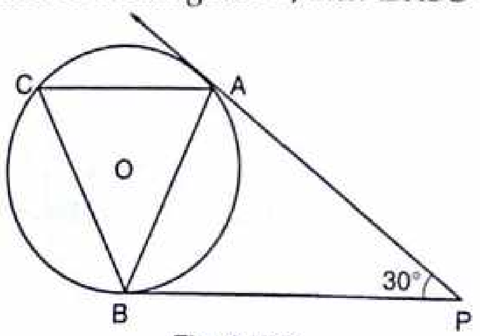

Fig. 8.102

(a) 60° (b) 90° (c) 30° (d) None of these

28. In Fig. 8.103, $PR =$

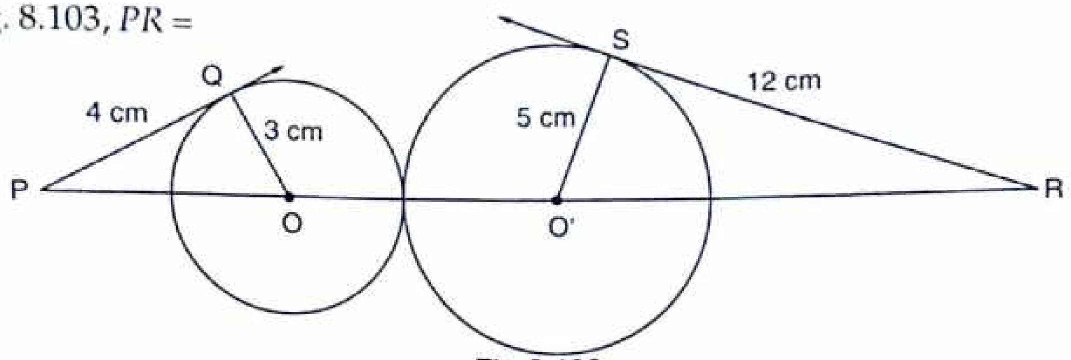

Fig. 8.103

(a) 20 cm (b) 26 cm (c) 24 cm (d) 28 cm

29. Two circles of same radii r and centres O and O' touch each other at P as shown in Fig. 8.104. If OO' is produced to meet the circle $C(O', r)$ at A and AT is a tangent to the circle $C(O, r)$ such that $O'Q \perp AT$. Then $AO : AO' =$

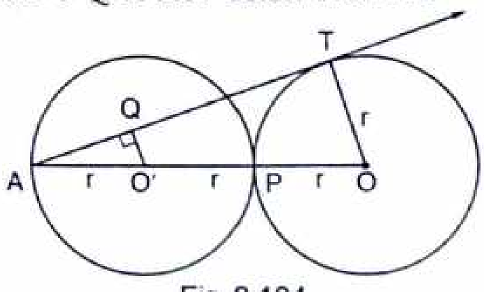

Fig. 8.104

(a) 3/2 (b) 2 (c) 3 (d) 1/4

30. Two concentric circles of radii 3 cm and 5 cm are given. Then length of chord BC which touches the inner circle at P is equal to **[CBSE 2014]**

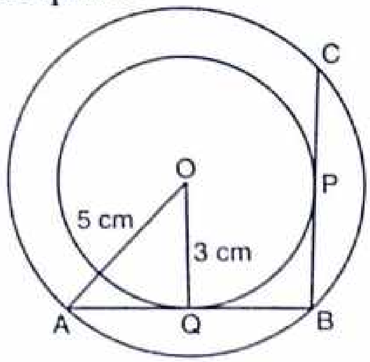

Fig. 8.105

(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

31. In Fig. 8.106, there are two concentric circles with centre O. PR and PQS are tangents to the inner circle from point plying on the outer circle. If $PR = 7.5$ cm, then PS is equal to

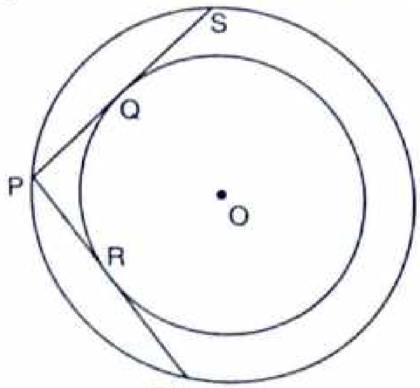

Fig. 8.106

(a) 10 cm (b) 12 cm (c) 15 cm (d) 18 cm

32. In Fig. 8.107, if $AB = 8$ cm and $PE = 3$ cm, then $AE =$

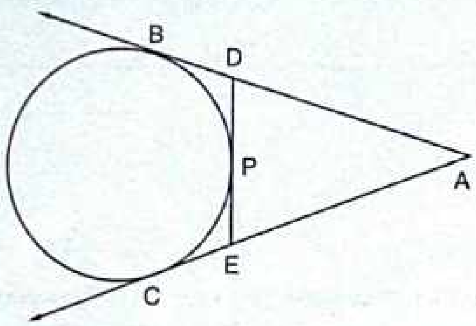

Fig. 8.107

(a) 11 cm (b) 7 cm (c) 5 cm (d) 3 cm

33. In Fig. 8.108, PQ and PR are tangents drawn from P to a circle with centre O. If $\angle OPQ = 35°$, then

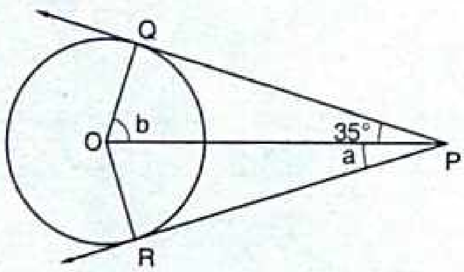

Fig. 8.108

(a) $a = 30°, b = 60°$ (b) $a = 35°, b = 55°$

(c) $a = 40°, b = 50°$ (d) $a = 45°, b = 45°$

34. In Fig. 8.109, if TP and TQ are tangents drawn from an external point T to a circle with centre O such that $\angle TQP = 60°$, then $\angle OPQ =$

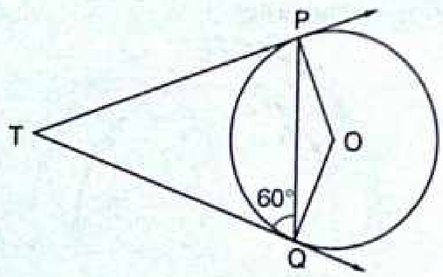

Fig. 8.109

(a) 25° (b) 30° (c) 40° (d) 60°

35. In Fig. 8.110, the sides AB, BC and CA of triangle ABC, touch a circle at P, Q and R respectively. If $PA = 4\text{ cm}, BP = 3\text{ cm}$ and $AC = 11\text{ cm}$, then length of BC is

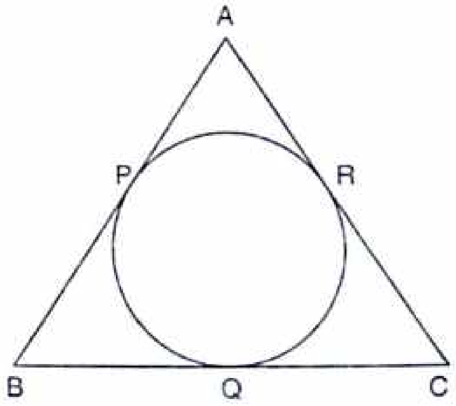

Fig. 8.110

(a) 11 cm (b) 10 cm (c) 14 cm (d) 15 cm [CBSE 2012]

36. In Fig. 8.111, a circle touches the side DF of ΔEDF at H and touches ED and EF produced at K and M respectively. If $EK = 9$ cm, then the perimeter of ΔEDF is

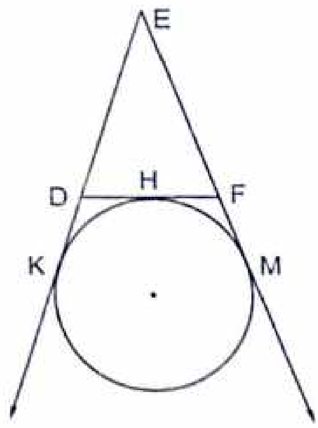

Fig. 8.111

(a) 18 cm (b) 13.5 cm (c) 12 cm (d) 9 cm [CBSE 2012]

37. In Fig. 8.112, DE and DF are tangents from an external point D to a circle with centre A. If $DE = 5$ cm and $DE \perp DF$, then the radius of the circle is

(a) 3 cm (b) 5 cm (c) 4 cm (d) 6 cm [CBSE 2013]

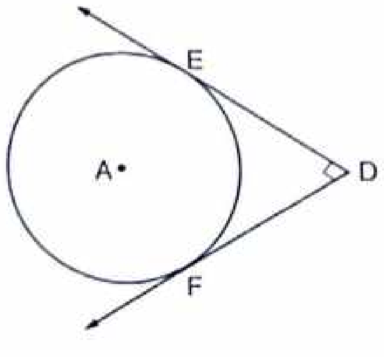

Fig. 8.112

38. In Fig. 8.113, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90°$ and $DS = 5$ cm, then the radius of the circle (in cm) is

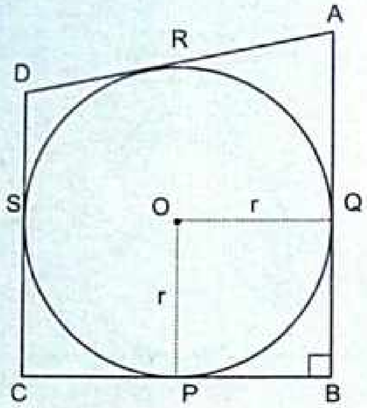

Fig. 8.113

(a) 11 (b) 18 (c) 6 (d) 15 [CBSE 2013]

39. In a right triangle ABC, right angled at B, $BC = 12$ cm and $AB = 5$ cm. The radius of the circle inscribed in the triangle (in cm) is

(a) 4 (b) 3 (c) 2 (d) 1 [CBSE 2014]

40. Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. The value of $\angle APB$ is

(a) 30° (b) 45° (c) 60° (d) 90° [CBSE 2014]

41. In Fig. 8.114, PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46°$, then $\angle QOR$ equals

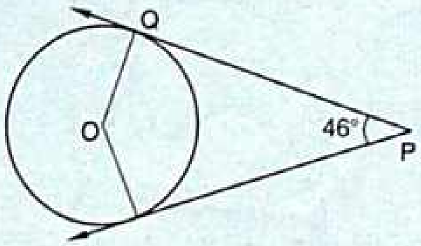

Fig. 8.114

(a) 67° (b) 134° (c) 44° (d) 46° [CBSE 2014]

42. In Fig. 8.115, QR is a common tangent to the given circles touching externally at the point T. The tangent at T meets QR at P. If $PT = 3.8$ cm, then the length of QR (in cm) is

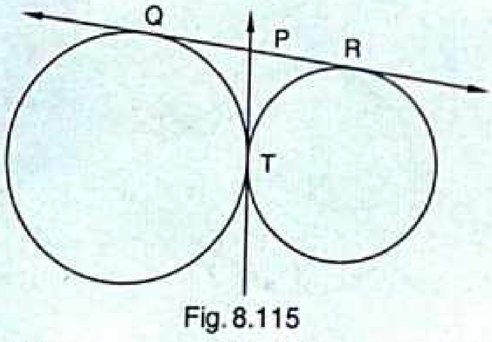

Fig. 8.115

(a) 3.8 (b) 7.6 (c) 5.7 (d) 1.9 [CBSE 2014]

43. In Fig. 8.116, a quadrilateral $ABCD$ is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If $AB = x$ cm, $BC = 7$ cm, $CR = 3$ cm and $AS = 5$ cm, then $x =$

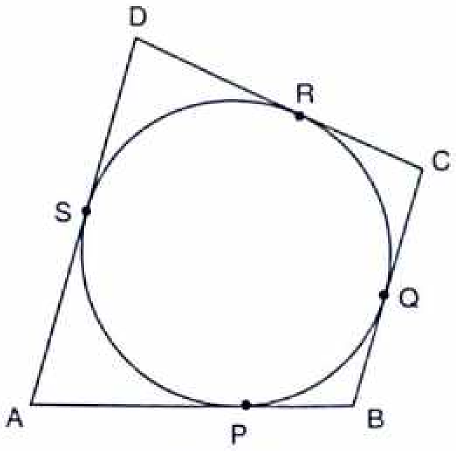

Fig. 8.116

(a) 10 (b) 9 (c) 8 (d) 7 **[CBSE 2014]**

44. If angle between two radii of a circle is 130°, the angle between the tangents at the ends of radii is

(a) 90° (b) 50° (c) 70° (d) 40°

[NCERT EXEMPLAR]

45. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm

[NCERT EXEMPLAR]

46. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is

(a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm

[NCERT EXEMPLAR]

47. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

[NCERT EXEMPLAR]

48. From a point P which is at a distance 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral $PQOR$ is

(a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

[NCERT EXEMPLAR]

49. If PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to

(a) 25° (b) 30° (c) 40° (d) 50°

50. The pair of tangents AP and AQ drawn from an external point to a circle with centre O are prependicular to each other and length of each tangent is 5 cm. The radius of the circle is

(a) 10 cm (b) 7.5 cm (c) 5 cm (d) 2.5 cm

[NCERT EXEMPLAR]

51. In Fig. 8.117, if $\angle AOB = 125°$, then $\angle COD$ is equal to

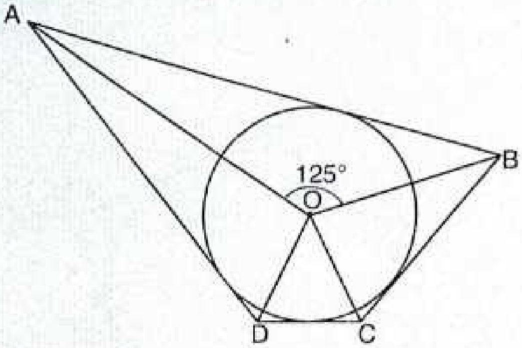

Fig. 8.117

(a) 45° (b) 35° (c) 55° (d) 62½°

[NCERT EXEMPLAR]

52. In Fig. 8.118, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

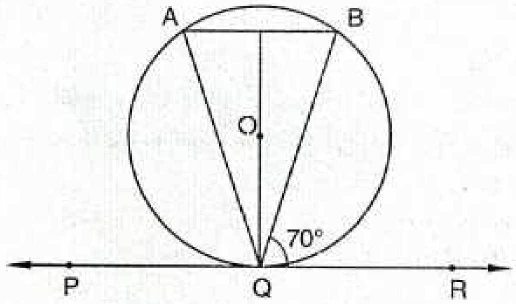

Fig. 8.118

(a) 20° (b) 40° (c) 35° (d) 45°

[NCERT EXEMPLAR]

ANSWERS

1. (d)	2. (a)	3. (c)	4. (a)	5. (b)	6. (b)
7. (d)	8. (b)	9. (c)	10. (b)	11. (b)	12. (c)
13. (b)	14. (d)	15. (a)	16. (c)	17. (c)	18. (d)
19. (c)	20. (d)	21. (b)	22. (b)	23. (c)	24. (a)
25. (a)	26. (a)	27. (c)	28. (b)	29. (c)	30. (c)
31. (c)	32. (c)	33. (b)	34. (b)	35. (b)	36. (a)
38. (a)	39. (c)	40. (d)	41. (b)	42. (b)	43. (b)
44. (b)	45. (d)	46. (b)	47. (d)	48. (a)	49. (a)
50. (c)	51. (c)	52. (b)			

SUMMARY

1. Tangent to a circle at a point is perpendicular to the radius through the point of contact.
2. From a point, lying outside a circle, two and only two tangents can be drawn to it.
3. The lengths of the two tangents drawn from an external point to a circle are equal.