1 Real Numbers

In class IX we studied about real numbers, especially about irrational numbers. In this chapter, we shall continue our discussion on real numbers. We begin with two important results, namely Euclid's division lemma and the fundamental theorem of arithmetic.

LEMMA *A lemma is a proven statement used for proving another statement.*

EUCLID'S DIVISION LEMMA

For any two given positive integers a and b, there exist unique whole numbers q and r such that

$$a = bq + r, \text{ where } 0 \le r < b.$$ [CBSE 2009C]

Here, we call a as *dividend*, b as *divisor*, q as *quotient* and r as *remainder*.

Dividend = (*divisor* × *quotient*) + *remainder*.

Example Suppose we divide 117 by 14. Then, we get 8 as quotient and 5 as remainder.

Here *dividend* = 117, *divisor* = 14, *quotient* = 8 and remainder = 5.

Clearly, $117 = (14 \times 8) + 5$.

```
14) 117 (8
   -112
   ----
      5
   ----
```

EXAMPLE 1 *A number when divided by 73 gives 34 as quotient and 23 as remainder. Find the number.*

SOLUTION Here divisor = 73, quotient = 34 and remainder = 23.

By Euclid's division lemma, we have

dividend = (divisor × quotient) + remainder

$= (73 \times 34) + 23$

$= (2482 + 23) = 2505.$

Hence, the required number is 2505.

ALGORITHM *An algorithm is a series of well-defined steps which gives a method for solving a certain type of problem.*

EUCLID'S DIVISION ALGORITHM *It is a technique to compute the HCF of two given positive integers, say a and b with a > b, in the following steps.*

Step 1. *On dividing a by b, we get the quotient q and remainder r such that $a = bq + r$, where $0 \le r < b$.*

Step 2. If $r = 0$ then $\text{HCF}(a, b) = b$.

If $r \neq 0$ then apply the division lemma to b and r.

Step 3. Continue the process till the remainder is 0.

The last divisor will be the required HCF.

EXAMPLE 2 *Use Euclid's algorithm to find the HCF of 272 and 1032.*

SOLUTION We find HCF(272, 1032) using the following steps.

Step 1. Since 1032 > 272, we divide 1032 by 272 to get 3 as quotient and 216 as remainder.

So, by Euclid's division lemma, we get

$1032 = 272 \times 3 + 216.$

272) 1032 (3
816
216) 272 (1
216
56) 216 (3
168
48) 56 (1
48
8) 48 (6
48
0

Step 2. Since the remainder $216 \neq 0$, we divide 272 by 216 to get 1 as quotient and 56 as remainder.

$\therefore$ by Euclid's division lemma, we get

$272 = 216 \times 1 + 56.$

Step 3. Since the remainder $56 \neq 0$, we divide 216 by 56 to get 3 as quotient and 48 as remainder.

$\therefore$ by Euclid's division lemma, we get

$216 = 56 \times 3 + 48.$

Step 4. Since the remainder $48 \neq 0$, we divide 56 by 48 to get 1 as quotient and 8 as remainder.

$\therefore$ by Euclid's division lemma, we get

$56 = 48 \times 1 + 8.$

Step 5. Since the remainder $8 \neq 0$, we divide 48 by 8 to get 6 as quotient and 0 as remainder.

$\therefore$ by Euclid's division lemma, we get

$48 = 8 \times 6 + 0.$

The remainder has now become 0, so our procedure stops.

Hence, HCF(272, 1032) = 8.

Note 8 = HCF(48, 8) = HCF(56, 48) = HCF(216, 56) = HCF(272, 216) = HCF(1032, 272).

REMARK This method is also known as **successive division method.**

EXAMPLE 3 *Use Euclid's algorithm to find HCF*(196, 38220).

SOLUTION We find HCF(196, 38220), using the following steps.

Since 38220 > 196, we divide 38220 by 196 to get 195 as quotient and 0 as remainder.

$$\begin{array}{r} 196\,)\,38220\,(\,195 \\ \underline{196} \\ 1862 \\ \underline{1764} \\ 980 \\ \underline{980} \\ \underline{0} \end{array}$$

$\therefore$ by Euclid's division lemma, we get

$38220 = 196 \times 195 + 0.$

Since the remainder is 0, so our procedure stops.

$\therefore$ HCF(196, 38220) = 196.

EXAMPLE 4 *Use Euclid's algorithm to find HCF of* 1651 *and* 2032. *Express the HCF in the form* $1651m + 2032n$.

SOLUTION We find HCF(1651, 2032) using the following steps.

$$\begin{array}{l} 1651\,)\,2032\,(\,1 \\ \underline{1651} \\ 381\,)\,1651\,(\,4 \\ \underline{1524} \\ 127\,)\,381\,(\,3 \\ \underline{381} \\ \underline{0} \end{array}$$

Step 1. Since $2032 > 1651$, we divide 2032 by 1651 to get 1 as quotient and 381 as remainder.

$\therefore$ by Euclid's division lemma, we get

$2032 = 1651 \times 1 + 381.$ … (i)

Step 2. Since the remainder $381 \neq 0$, we divide 1651 by 381 to 4 as quotient and 127 as remainder.

$\therefore$ by Euclid's division lemma, we get

$1651 = 381 \times 4 + 127.$ … (ii)

Step 3. Since the remainder $127 \neq 0$, we divide 381 by 127 to get 3 as quotient and 0 as remainder.

$\therefore$ by Euclid's division lemma, we get

$381 = 127 \times 3 + 0.$ … (iii)

The remainder is now 0, so our procedure stops.

$\therefore$ HCF(1651, 2032) = 127.

Now, from (ii), we get

$1651 = 381 \times 4 + 127$

$\Rightarrow \quad 127 = 1651 - 381 \times 4$

$\Rightarrow \quad 127 = 1651 - (2032 - 1651 \times 1) \times 4$ [from (i)]

$\Rightarrow \quad 127 = 1651 - 2032 \times 4 + 1651 \times 4$

$\Rightarrow \quad 127 = 1651 \times 5 + 2032 \times (-4).$

Hence, $m = 5$, $n = -4$.

SOME APPLICATIONS OF EUCLID'S DIVISION LEMMA

EXAMPLE 1 *Show that every positive even integer is of the form* $2m$ *and that every positive odd integer is of the form* $(2m + 1)$, *where m is some integer.*

SOLUTION Let n be an arbitrary positive integer.

On dividing n by 2, let m be the quotient and r be the remainder.

Then, by Euclid's division lemma, we have

$n = 2m + r$, where $0 \le r < 2$.

$\therefore \quad n = 2m$ or $(2m + 1)$, for some integer m.

Case I *When* $n = 2m$.

In this case, n is clearly even.

Case II *When* $n = 2m + 1$.

In this case, n is clearly odd.

Hence, every positive even integer is of the form $2m$ and every positive odd integer is of the form $(2m + 1)$ for some integer m.

EXAMPLE 2 *Show that any positive integer is of the form* $3m$ *or* $(3m + 1)$ *or* $(3m + 2)$ *for some integer m.*

SOLUTION Let n be an arbitrary positive integer.

On dividing n by 3, let m be the quotient and r be the remainder. Then, by Euclid's division lemma, we have

$n = 3m + r$, where $0 \le r < 3$.

$\therefore \quad n = 3m$ or $(3m + 1)$ or $(3m + 2)$, for some integer m.

Thus, any positive integer is of the form $3m$ or $(3m + 1)$ or $(3m + 2)$ for some integer m.

EXAMPLE 3 *Show that any positive odd integer is of the form* $(4m + 1)$ *or* $(4m + 3)$ *for some integer m.*

SOLUTION Let n be an arbitrary odd positive integer.

On dividing n by 4, let m be the quotient and r be the remainder. So, by Euclid's division lemma, we have

$$n = 4m + r, \text{ where } 0 \le r < 4.$$

$$\therefore \quad n = 4m \text{ or } (4m+1) \text{ or } (4m+2) \text{ or } (4m+3).$$

Clearly, $4m$ and $(4m + 2)$ are even and since n is odd, so $n \neq 4m$ and $n \neq (4m+2)$.

$$\therefore \quad n = (4m+1) \text{ or } (4m+3), \text{ for some integer } m.$$

Hence, any positive odd integer is of the form $(4m+1)$ or $(4m+3)$ for some integer m.

EXAMPLE 4 *Show that every positive odd integer is of the form* $(6m+1)$ *or* $(6m+3)$ *or* $(6m+5)$ *for some integer m.*

SOLUTION Let n be a given positive odd integer.

On dividing n by 6, let m be the quotient and r be the remainder. Then, by Euclid's division lemma, we have

$$n = 6m + r, \text{ where } 0 \le r < 6$$

$$\Rightarrow \quad n = 6m + r, \text{ where } r = 0, 1, 2, 3, 4, 5$$

$$\Rightarrow \quad n = 6m \text{ or } (6m+1) \text{ or } (6m+2) \text{ or } (6m+3) \text{ or } (6m+4) \text{ or } (6m+5).$$

But, $n = 6m, (6m+2), (6m+4)$ give even values of n.

Thus, when n is odd, it is of the form $(6m+1)$ or $(6m+3)$ or $(6m+5)$ for some integer m.

EXAMPLE 5 *Using Euclid's division lemma, show that the square of any positive integer is either of the form* $3m$ *or* $(3m+1)$ *for some integer m.*

[CBSE 2008]

SOLUTION Let n be an arbitrary positive integer.

On dividing n by 3, let q be the quotient and r be the remainder. Then, by Euclid's division lemma, we have

$$n = 3q + r, \text{ where } 0 \le r < 3.$$

$$\therefore \quad n^2 = 9q^2 + r^2 + 6qr \qquad \ldots \text{(i), where } 0 \le r < 3.$$

Case I *When* $r = 0$.

Putting $r = 0$ in (i), we get

$$n^2 = 9q^2 = 3(3q^2) = 3m, \text{ where } m = 3q^2 \text{ is an integer.}$$

Case II *When* $r = 1$.

Putting $r = 1$ in (i), we get

$$n^2 = (9q^2 + 1 + 6q) = 3(3q^2 + 2q) + 1 = 3(3q^2 + 2q) + 1$$
$$= 3m + 1, \text{ where } m = (3q^2 + 2q) \text{ is an integer.}$$

Case III *When* $r = 2$.

Putting $r = 2$ in (i), we get

$$n^2 = (9q^2 + 4 + 12q) = 3(3q^2 + 4q + 1) + 1$$
$$= 3m + 1, \text{ where } m = (3q^2 + 4q + 1) \text{ is an integer.}$$

Hence, the square of any positive integer is of the form $3m$ or $(3m + 1)$ for some integer m.

EXAMPLE 6 *Using Euclid's division lemma, show that the cube of any positive integer is of the form* $9m$ *or* $(9m + 1)$ *or* $(9m + 8)$ *for some integer* m.

[CBSE 2009C]

SOLUTION Let n be an arbitrary positive integer.

On dividing n by 3, let q be the quotient and r be the remainder. So, by Euclid's division lemma, we have

$$n = 3q + r, \text{ where } 0 \le r < 3.$$

$$\therefore \quad n^3 = (3q + r)^3 = 27q^3 + r^3 + 9qr(3q + r)$$
$$= (27q^3 + 27q^2 r + 9qr^2) + r^3 \qquad \ldots \text{(i), where } 0 \le r < 3.$$

Case I *When* $r = 0$.

Putting $r = 0$ in (i), we get

$$n^3 = 27q^3 = 9(3q^3) = 9m, \text{ where } m = 3q^3 \text{ is an integer.}$$

Case II *When* $r = 1$.

Putting $r = 1$ in (i), we get

$$n^3 = (27q^3 + 27q^2 + 9q) + 1 = 9q(3q^2 + 3q + 1) + 1$$
$$= 9m + 1, \text{ where } m = q(3q^2 + 3q + 1) \text{ is an integer.}$$

Case III *When* $r = 2$.

Putting $r = 2$ in (i), we get

$$n^3 = (27q^3 + 54q^2 + 36q) + 8 = 9q(3q^2 + 6q + 4) + 8$$
$$= 9m + 8, \text{ where } m = q(3q^2 + 6q + 4) \text{ is an integer.}$$

Hence, the cube of any positive integer is of the form $9m$ or $(9m + 1)$ or $(9m + 8)$ for some integer m.

EXAMPLE 7 *Show that one and only one out of* $n, (n+1)$ *and* $(n+2)$ *is divisible by* 3, *where n is any positive integer.*

SOLUTION On dividing n by 3, let q be the quotient and r be the remainder.

Then, $n = 3q + r$, where $0 \le r < 3$

$\Rightarrow \quad n = 3q + r$, where $r = 0, 1$ or 2

$\Rightarrow \quad n = 3q$ or $n = (3q+1)$ or $n = (3q+2)$.

Case I If $n = 3q$ then n is clearly divisible by 3.

Case II If $n = (3q+1)$ then $(n+2) = (3q+3) = 3(q+1)$, which is clearly divisible by 3.

In this case, $(n+2)$ is divisible by 3.

Case III If $n = (3q+2)$ then $(n+1) = (3q+3) = 3(q+1)$, which is clearly divisible by 3.

In this case, $(n+1)$ is divisible by 3.

Hence, one and only one out of $n, (n+1)$ and $(n+2)$ is divisible by 3.

EXAMPLE 8 *Show that one and only one out of* $n, n+2, n+4$ *is divisible by* 3, *where n is any positive integer.* [CBSE 2008C]

SOLUTION On dividing n by 3, let q be the quotient and r be the remainder.

Then, $n = 3q + r$, where $0 \le r < 3$

$\Rightarrow \quad n = 3q + r$, where $r = 0, 1, 2$

$\Rightarrow \quad n = 3q$ or $n = 3q + 1$ or $n = 3q + 2$.

Case I If $n = 3q$ then n is divisible by 3.

Case II If $n = 3q+1$ then $(n+2) = 3q+3 = 3(q+1)$, which is divisible by 3.

So, in this case, $(n+2)$ is divisible by 3.

Case III When $n = 3q+2$ then $(n+4) = 3q+6 = 3(q+2)$, which is divisible by 3.

So, in this case, $(n+4)$ is divisible by 3.

Hence, one and only one out of $n, n+2, n+4$ is divisible by 3.

EXAMPLE 9 *If n is an odd integer then show that* $n^2 - 1$ *is divisible by* 8.

SOLUTION We know that every odd integer is of the form $4q+1$ or $4q+3$ for some integer q.

Case I *When* $n = 4q + 1$.

Then, $n^2 - 1 = (4q + 1)^2 - 1$

$= 16q^2 + 8q + 1 - 1$

$= 16q^2 + 8q$

$= 8(q + 2q^2)$, which is divisible by 8.

Case II *When* $n = 4q + 3$.

Then, $n^2 - 1 = (4q + 3)^2 - 1$

$= 16q^2 + 24q + 9 - 1$

$= 16q^2 + 24q + 8$

$= 8(2q^2 + 3q + 1)$, which is divisible by 8.

Hence, if n is an odd integer then $n^2 - 1$ is divisible by 8.

EXERCISE 1A

1. What do you mean by Euclid's division lemma?
2. A number when divided by 61 gives 27 as quotient and 32 as remainder. Find the number.
3. By what number should 1365 be divided to get 31 as quotient and 32 as remainder?
4. Using Euclid's division algorithm, find the HCF of
   (i) 405 and 2520 (ii) 504 and 1188 (iii) 960 and 1575.
5. Show that every positive integer is either even or odd.
6. Show that any positive odd integer is of the form $(6m + 1)$ or $(6m + 3)$ or $(6m + 5)$, where m is some integer.
7. Show that any positive odd integer is of the form $(4m + 1)$ or $(4m + 3)$, where m is some integer.
8. For any positive integer n, prove that $n^3 - n$ is divisible by 6.
9. Prove that if x and y are both odd positive integers then $x^2 + y^2$ is even but not divisible by 4.
10. Use Euclid's algorithm to find HCF of 1190 and 1445. Express the HCF in the form $1190m + 1445n$.

ANSWERS (EXERCISE 1A)

2. 1679 **3.** 43 **4.** (i) 45 (ii) 36 (iii) 15

10. 85; $m = -6$, $n = 5$

HINTS TO SOME SELECTED QUESTIONS

3. Let the required divisor be x.

   Then, dividend = (divisor × quotient) + remainder

   $\Rightarrow \quad 1365 = (x \times 31) + 32$. Find x.

5. Let n be an arbitrary positive integer.

   On dividing n by 2, let m be the quotient and r be the remainder.

   Then, $n = 2m + r$, where $0 \le r < 2$ [by Euclid's division lemma].

   $\therefore \quad n = 2m$ or $n = 2m + 1$ for some integer m.

   So, n is either even or odd.

....

FUNDAMENTAL THEOREM OF ARITHMETIC *Every composite number can be uniquely expressed as a product of primes, except for the order in which these prime factors occurs.*

Examples (i) $12 = 2 \times 2 \times 3$ (ii) $69 = 3 \times 23$ (iii) $105 = 3 \times 5 \times 7$

(iv) $234 = 2 \times 3 \times 3 \times 13$ (v) $462 = 2 \times 3 \times 7 \times 11$

(vi) $651 = 3 \times 7 \times 31$

The above factorisation can easily be verified by actual division.

SOLVED EXAMPLES

EXAMPLE 1 *Show that each of the following is a composite number:*

(i) $5 \times 11 \times 13 + 13$ *(ii)* $6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

SOLUTION We have

(i) $5 \times 11 \times 13 + 13 = 13 \times (5 \times 11 + 1) = (13 \times 56)$.

Clearly, it shows that the given number has more than two factors. Hence, it is a composite number.

(ii) $6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= (5 \times 145)$.

Clearly, it shows that the given number has more than two factors. Hence, it is a composite number.

EXAMPLE 2 *Show that any number of the form* 4^n, $n \in N$ *can never end with the digit* 0.

SOLUTION If 4^n ends with 0 then it must have 5 as a factor.

But, $4^n = (2^2)^n = 2^{2n}$ shows that 2 is the only prime factor of 4^n.

Also, we know from the fundamental theorem of arithmetic that the prime factorisation of each number is unique.

So, 5 is not a factor of 4^n.

Hence, 4^n can never end with the digit 0.

EXAMPLE 3 *Show that any number of the form* 6^n, *where* $n \in N$ *can never end with the digit* 0.

SOLUTION If 6^n ends with 0 then it must have 5 as a factor.

But, $6^n = (2 \times 3)^n = (2^n \times 3^n)$ shows that 2 and 3 are the only prime factors of 6^n.

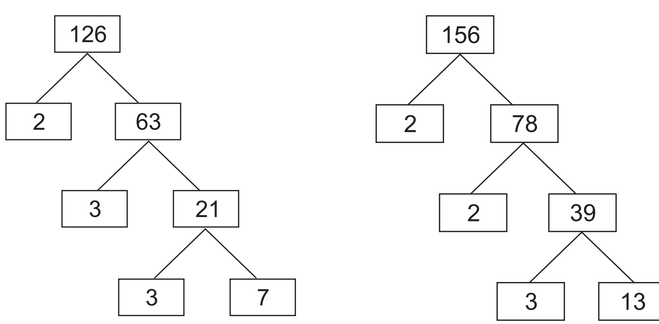
Also, we know from the fundamental theorem of arithmetic that the prime factorisation of each number is unique.

So, 5 is not a factor of 6^n.

Hence, 6^n can never end with the digit 0.

EXAMPLE 4 *Find the HCF and LCM of* 126 *and* 156 *using prime factorisation method.*

SOLUTION We have



$\therefore \quad 126 = (2 \times 3 \times 3 \times 7) = (2 \times 3^2 \times 7)$

and $156 = (2 \times 2 \times 3 \times 13) = (2^2 \times 3 \times 13)$.

$\therefore$ HCF(126, 156) = product of common terms with lowest power

$= (2^1 \times 3^1) = (2 \times 3) = 6$

and LCM(126, 156) = product of prime factors with highest power

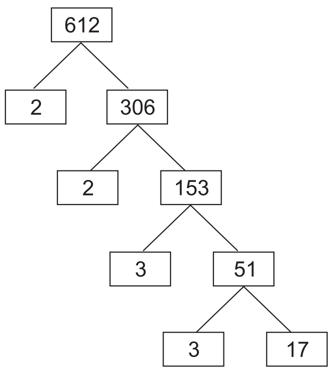
$= (2^2 \times 3^2 \times 7 \times 13) = (4 \times 9 \times 7 \times 13)$

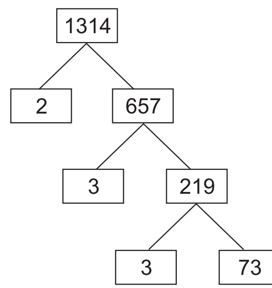
$= 3276.$

$\therefore$ HCF = 6 and LCM = 3276.

EXAMPLE 5 *Find the HCF and LCM of 612 and 1314 using prime factorisation method.*

SOLUTION We have





$\therefore$ $612 = (2 \times 2 \times 3 \times 3 \times 17) = (2^2 \times 3^2 \times 17)$

and $1314 = (2 \times 3 \times 3 \times 73) = (2 \times 3^2 \times 73)$.

$\therefore$ HCF(612, 1314) = product of common terms with lowest power

$= (2 \times 3^2) = (2 \times 9) = 18$

and LCM(612, 1314) = product of prime factors with highest power

$= (2^2 \times 3^2 \times 17 \times 73) = (4 \times 9 \times 17 \times 73)$

$= 44676.$

Hence, HCF = 18 and LCM = 44676.

AN IMPORTANT PROPERTY

Product of two given numbers = product of their HCF and LCM.

Thus, $(a \times b) = \text{HCF}(a, b) \times \text{LCM}(a, b)$.

CAUTION The above result is true for two numbers only.

EXAMPLE 6 *The HCF of two numbers is* 23 *and their LCM is* 1449. *If one of the numbers is* 161, *find the other.*

SOLUTION For two numbers a and b, we know that

$$(a \times b) = \{\text{HCF}(a, b)\} \times \{\text{LCM}(a, b)\}.$$

Here $a = 161$, HCF = 23 and LCM = 1449.

And, we have to find b.

$$\therefore \quad (161 \times b) = (23 \times 1449) \Rightarrow b = \frac{(23 \times 1449)}{161} = 207.$$

Hence, the other number is 207.

EXAMPLE 7 *Given that HCF*(252, 594) = 18, *find LCM*(252, 594).

SOLUTION We have

$$\text{LCM} = \frac{\text{product of two given numbers}}{\text{their HCF}}$$

$$= \frac{(252 \times 594)}{18} = 8316.$$

Hence, LCM(252, 594) = 8316.

EXAMPLE 8 *Find the simplest form of* $\frac{148}{185}$.

SOLUTION We find HCF(148, 185), which is 37.

So, we divide the numerator and denominator of the given fraction by 37.

$$148\overline{)185}(1$$
$$148$$
$$37\overline{)148}(4$$
$$148$$
$$0$$

$$\therefore \quad \frac{148}{185} = \frac{148 \div 37}{185 \div 37} = \frac{4}{5}.$$

Hence, the simplest form of the given fraction is $\frac{4}{5}$.

EXAMPLE 9 *Find the HCF and LCM of* 108, 120 *and* 252 *using prime factorisation method.*

SOLUTION By prime factorisation, we get

2	108	2	120	2	252
2	54	2	60	2	126
3	27	2	30	3	63
3	9	3	15	3	21
	3		5		7

$$108 = (2^2 \times 3^3)$$

$$120 = (2^3 \times 3 \times 5)$$

$$252 = (2^2 \times 3^2 \times 7)$$

HCF(108, 120, 252) = product of common terms with lowest power

$$= (2^2 \times 3) = (4 \times 3) = 12.$$

LCM(108, 120, 252) = product of prime factors with highest power

$= (2^3 \times 3^3 \times 5 \times 7) = 7560.$

$\therefore$ HCF = 12 and LCM = 7560.

EXAMPLE 10 *Find the largest number which divides* 245 *and* 1037, *leaving remainder* 5 *in each case.*

SOLUTION Clearly, the required number divides $(245-5) = 240$ and $(1037-5) = 1032$ exactly.

So, the required number is HCF(240, 1032).

2	240
2	120
2	60
2	30
3	15
	5

2	1032
2	516
2	258
3	129
	43

Now, $240 = (2^4 \times 3 \times 5)$

and $1032 = (2^3 \times 3 \times 43)$.

$\therefore$ HCF(240, 1032) $= (2^3 \times 3) = 24$.

Hence, the required number is 24.

EXAMPLE 11 *Find the largest number which divides* 129 *and* 545, *leaving remainders* 3 *and* 5 *respectively.*

SOLUTION Clearly, the required number divides $(129-3) = 126$ and $(545 - 5) = 540$ exactly.

2	126
3	63
3	21
	7

2	540
2	270
3	135
3	45
3	15
	5

$\therefore$ required number = HCF(126, 540).

Now, $126 = (2 \times 3 \times 3 \times 7) = (2 \times 3^2 \times 7)$

and $540 = (2 \times 2 \times 3 \times 3 \times 3 \times 5)$

$= (2^2 \times 3^3 \times 5)$.

$\therefore$ HCF(126, 540) = product of common terms with lowest power

$= (2 \times 3^2) = (2 \times 9) = 18.$

Hence, the required number is 18.

EXAMPLE 12 *Find the largest number that will divide* 398, 436 *and* 542, *leaving remainders* 7, 11 *and* 15 *respectively.*

SOLUTION Clearly, the required number divides $(398-7) = 391$, $(436-11) = 425$ and $(542-15) = 527$ exactly.

$\therefore$ required number = HCF(391, 425, 527).

17	391
	23

5	425
5	85
	17

Now, $391 = (17 \times 23)$,

$425 = (5^2 \times 17)$,

$527 = (17 \times 31)$.

$\therefore$ HCF(391, 425, 527) = 17.

Hence, the required number is 17.

EXAMPLE 13 *Two tanks contain 504 and 735 litres of milk respectively. Find the maximum capacity of a container which can measure the milk of either tank an exact number of times.*

SOLUTION Resolving 504 and 735 into prime factors, we get

$$504 = (2^3 \times 3^2 \times 7)$$

and $735 = (5 \times 3 \times 7^2)$.

$$\begin{array}{r|r} 2 & 504 \\ \hline 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 5 & 735 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$\therefore$ HCF(504, 735) = (3×7) = 21.

$\therefore$ capacity of the required container = 21 litres.

EXAMPLE 14 *An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?*

SOLUTION Clearly, the maximum number of columns = HCF(612, 48).

Now, $612 = (2^2 \times 3^2 \times 17)$

and $48 = (2^4 \times 3)$.

$$\begin{array}{r|r} 2 & 612 \\ \hline 2 & 306 \\ \hline 3 & 153 \\ \hline 3 & 51 \\ \hline & 17 \end{array} \qquad \begin{array}{r|r} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$\therefore$ HCF(612, 48) = $(2^2 \times 3) = (4 \times 3) = 12$.

$\therefore$ required number of columns = 12.

EXAMPLE 15 *A sweetseller has 420 kaju burfis and 150 badam burfis. He wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. How many of these can be placed in each stack? How many stacks are formed?*

SOLUTION Maximum number of *burfis* in each stack = HCF(420, 150).

Now, $420 = (2^2 \times 3 \times 5 \times 7)$

and $150 = (2 \times 3 \times 5^2)$.

$$\begin{array}{r|r} 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 2 & 150 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\therefore$ HCF(420, 150) = $(2 \times 3 \times 5) = 30$.

$\therefore$ maximum number of *burfis* in each stack = 30.

$\therefore$ number of stacks $= \left(\frac{420}{30} + \frac{150}{30}\right) = (14 + 5) = 19$.

EXAMPLE 16 *Ravi and Sikha drive around a circular sports field. Ravi takes 16 minutes to take one round, while Sikha completes the round in 20 minutes. If both start at the same point, at the same time and go in the same direction, after how much time will they meet at the starting point?*

SOLUTION Required number of minutes = LCM(16, 20).

Now, $16 = 2^4$ and $20 = (2^2 \times 5)$.

$\therefore$ LCM(16, 20) $= (2^4 \times 5) = (16 \times 5) = 80$.

Hence, both will meet at the starting point after 80 minutes.

EXAMPLE 17 *In a school there are two sections, namely A and B, of class X. There are 30 students in section A and 28 students in section B. Find the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.*

SOLUTION Clearly, the required number of books are to be distributed equally among the students of section A or B.

So, the number of these books must be a multiple of 30 as well as that of 28.

Consequently, the required number is LCM(30, 28).

Now, $30 = 2 \times 3 \times 5$

and $28 = 2^2 \times 7$.

$\therefore$ LCM(30, 28) = product of prime factors with highest power

$= (2^2 \times 3 \times 5 \times 7) = (4 \times 3 \times 5 \times 7) = 420$.

Hence, the required number of books = 420.

EXERCISE 1B

1. Using prime factorisation, find the HCF and LCM of:

(i) 36, 84 (ii) 23, 31 (iii) 96, 404

(iv) 144, 198 (v) 396, 1080 (vi) 1152, 1664

In each case, verify that:

HCF × LCM = product of given numbers.

2. Using prime factorisation, find the HCF and LCM of:

(i) 8, 9, 25 (ii) 12, 15, 21 (iii) 17, 23, 29

(iv) 24, 36, 40 (v) 30, 72, 432 (vi) 21, 28, 36, 45

3. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.
4. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, find the other.
5. The HCF of two numbers is 18 and their product is 12960. Find their LCM.
6. Is it possible to have two numbers whose HCF is 18 and LCM is 760? Give reason.

   HINT HCF always divides LCM completely.
7. Find the simplest form of:

   (i) $\frac{69}{92}$ (ii) $\frac{473}{645}$ (iii) $\frac{1095}{1168}$ (iv) $\frac{368}{496}$
8. Find the largest number which divides 438 and 606, leaving remainder 6 in each case.
9. Find the largest number which divides 320 and 457, leaving remainders 5 and 7 respectively.
10. Find the least number which when divided by 35, 56 and 91 leaves the same remainder 7 in each case.
11. Find the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 respectively.
12. Find the smallest number which when increased by 17 is exactly divisible by both 468 and 520.
13. Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.
14. Find the largest four-digits number which when divided by 4, 7 and 13 leaves a remainder of 3 in each case.
15. Find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3.
16. Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.
17. Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.
18. In a seminar, the number of participants in Hindi, English and mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required, if in each room, the same number of participants are to be seated and all of them being in the same subject.

19. Three sets of English, mathematics and science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subjectwise and the height of each stack is the same. How many stacks will be there?
20. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank? How many planks are formed?
21. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.
22. Find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils.
23. Find the least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad.
24. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.
25. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. At what time will they beep together at the earliest?
26. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 a.m. then at what time will they again change simultaneously?
27. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 minutes respectively. In 30 hours, how many times do they toll together?

ANSWERS (EXERCISE 1B)

1. (i) HCF = 12, LCM = 252 (ii) HCF = 1, LCM = 713
(iii) HCF = 4, LCM = 9696 (iv) HCF = 18, LCM = 1584
(v) HCF = 36, LCM = 11880 (vi) HCF = 128, LCM = 14976

2. (i) HCF = 1, LCM = 1800 (ii) HCF = 3, LCM = 420
(iii) HCF = 1, LCM = 11339 (iv) HCF = 4, LCM = 360
(v) HCF = 6, LCM = 2160 (vi) HCF = 1, LCM = 1260

3. 207 **4.** 435 **5.** 720

6. No, since HCF does not divide LCM exactly

7. (i) $\frac{3}{4}$ (ii) $\frac{11}{15}$ (iii) $\frac{15}{16}$ (iv) $\frac{23}{31}$

8. 24	**9.** 45	**10.** 3647	**11.** 204	**12.** 4663	**13.** 9720
14. 9831	**15.** 23	**16.** 4	**17.** 1394	**18.** 21	**19.** 14
20. 7 m, 22 planks		**21.** 35 cm	**22.** 91	**23.** 814	**24.** 9.6 m
25. 10 : 31 hrs		**26.** 8 : 7 : 12 hrs		**27.** 16 times	

HINTS TO SOME SELECTED QUESTIONS

8. Required number = HCF(432, 600) = 24.

9. Required number = HCF(315, 450) = 45.

10. Required number = {LCM(35, 56, 91) + 7} = (3640 + 7) = 3647.

11. Here, $(28-8) = 20$ and $(32-12) = 20$.

    ∴ required number = {LCM(28, 32) − 20} = (224 − 20) = 204.

12. Required number = {LCM(468, 520) − 17} = (4680 − 17) = 4663.

13. Greatest number of four digits = 9999.

    LCM(15, 24, 36) = 360.

    On dividing 9999 by 360, remainder = 279.

    ∴ required number = (9999 − 279) = 9720.

14. Required number = (greatest number of 4-digits divisible by 4, 7 and 13) + 3.

15. LCM of 5, 6, 4 and 3 = 60.

    On dividing 2497 by 60, remainder = 37.

    ∴ number to be added = (60 − 37) = 23.

16. Required number = HCF of (91 − 43), (183 − 91) and (183 − 43), i.e., HCF of 48, 92 and 140.

17. Here, (20 − 14) = 6, (25 − 19) = 6, (35 − 29) = 6, (40 − 34) = 6.

    ∴ required number = (LCM of 20, 25, 35, 40) – 6.

18. Maximum number of participants in each room = HCF(60, 84, 108) = 12.

    Minimum number of rooms required $= \left(\frac{60}{12} + \frac{84}{12} + \frac{108}{12}\right) = 21.$

19. HCF(336, 240, 96) = 48. So, we make stacks of 48 books each.

    Number of stacks $= \left(\frac{336}{48} + \frac{240}{48} + \frac{96}{48}\right) = (7 + 5 + 2) = 14.$

20. Required length of each plank in metres = HCF(42 m, 49 m, 63 m) = 7 m.

    Number of planks $= \left(\frac{42}{7} + \frac{49}{7} + \frac{63}{7}\right) = (6 + 7 + 9) = 22.$

21. Required length = HCF(700 cm, 385 cm, 1295 cm) = 35 cm.

22. Maximum number of students = HCF(1001, 910) = 91.
23. Side of each square tile = HCF(1517 cm, 902 cm) = 41 cm.

    Required number of tiles $= \left(\frac{1517 \times 902}{41 \times 41}\right) = 814.$
24. Required length = LCM(64 cm, 80 cm, 96 cm) = 960 cm = 9.6 m.
25. Interval of beeping together = LCM (60 seconds, 62 seconds) = 1860 seconds

    $= \frac{1860}{60}$ min = 31 min.

    So, they will beep together again at 10 : 31 a.m.
26. Interval of change = LCM (48 seconds, 72 seconds, 108 seconds) = 432 seconds

    = 7 min 12 seconds.

    Required time of simultaneous change = 8 : 7 : 12 hours.
27. LCM of 2, 4, 6, 8, 10, 12 = 120.

    After every 2 hours they toll together.

    Required number of times $= \left(\frac{30}{2} + 1\right)$ times = 16 times.

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RATIONAL NUMBERS

NATURAL NUMBERS *Counting numbers* 1, 2, 3, 4, …, *etc., are known as natural numbers.*

WHOLE NUMBERS *All counting numbers together with* 0 *form the collection of whole numbers.*

Thus, 0, 1, 2, 3, 4, 5, …, etc., are whole numbers.

INTEGERS *All counting numbers, negatives of counting numbers and* 0 *form the collection of all integers.*

Thus, …, –4, –3, –2, –1, 0, 1, 2, 3, …, etc., are integers.

RATIONAL NUMBERS *The numbers of the form* $\frac{p}{q}$, *where p and q are integers, and* $q \neq 0$ *are called rational numbers.*

RATIONAL NUMBERS IN DECIMAL FORM *Every rational number when expressed in decimal form is expressible either in terminating or in nonterminating repeating decimal form.*

AN IMPORTANT OBSERVATION

To Test Whether a Given Rational Number is a Terminating or Repeating Decimal

Let x be a rational number whose simplest form is $\frac{p}{q}$, *where p and q are integers and* $q \neq 0$. *Then,* *(i) x is a terminating decimal only when q is of the form* $(2^m \times 5^n)$ *for some non-negative integers m and n.*

(ii) x is a nonterminating repeating decimal, if $q \neq (2^m \times 5^n)$.

AN IMPORTANT TEST

Let $\frac{p}{q}$ be the simplest form of a given rational number.

(i) If $q = (2^m \times 5^n)$ for some non-negative integers m and n then $\frac{p}{q}$ is a ***terminating decimal.***

(ii) If $q \neq (2^m \times 5^n)$ then $\frac{p}{q}$ is a ***nonterminating repeating decimal.***

SOLVED EXAMPLES

EXAMPLE 1 *Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form.*

(i) $\frac{31}{(2^2 \times 5^3)}$ *(ii)* $\frac{33}{50}$ *(iii)* $\frac{41}{1000}$ *(iv)* $\frac{17}{625}$

SOLUTION (i) The given number is $\frac{31}{(2^2 \times 5^3)}$.

Clearly, none of 2 and 5 is a factor of 31.

So, the given rational is in its simplest form.

Clearly, $(2^2 \times 5^3)$ is of the form $(2^m \times 5^n)$.

So, the given number is a terminating decimal.

Now, $\frac{31}{(2^2 \times 5^3)} = \frac{31 \times 2}{(2^3 \times 5^3)} = \frac{62}{(2 \times 5)^3} = \frac{62}{(10)^3} = \frac{62}{1000}$

$= 0.062.$

(ii) The given number is $\frac{33}{50}$.

Now, $50 = (2 \times 5^2)$ and none of 2 and 5 is a factor of 33.

So, the given rational number is in its simplest form.

Clearly, $50 = (2 \times 5^2) = (2^m \times 5^n)$, where $m = 1$ and $n = 2$.

So, the given number is a terminating decimal.

Now, $\frac{33}{50} = \frac{33}{(2 \times 5^2)} = \frac{33 \times 2}{(2^2 \times 5^2)} = \frac{66}{(2 \times 5)^2}$

$= \frac{66}{(10)^2} = \frac{66}{100} = 0.66.$

(iii) The given number is $\frac{41}{1000}$.

Now, $1000 = (8 \times 125) = (2^3 \times 5^3)$.

Clearly, none of 2 and 5 is a factor of 41.

So, the given number is in its simplest form.

Now, $1000 = (2^3 \times 5^3)$ which is of the form $(2^m \times 5^n)$, where $m = 3$ and $n = 3$.

So, the given number is a terminating decimal.

And, $\frac{41}{1000} = 0.041$.

(iv) The given number is $\frac{17}{625}$.

And, $625 = 5^4$ and 5 is not a factor of 17.

So, the given number is in its simplest form.

Now, $625 = 5^4$ is of the form $(2^m \times 5^n)$, where $m = 0, n = 4$.

So, the given number is a terminating decimal.

$$\text{Now, } \frac{17}{625} = \frac{17}{5^4} = \frac{17 \times 2^4}{5^4 \times 2^4} = \frac{17 \times 16}{(5 \times 2)^4} = \frac{272}{(10)^4}$$

$$= \frac{272}{10000} = 0.0272.$$

EXAMPLE 2 *Without actual division, show that each of the following rational numbers is a nonterminating repeating decimal.*

(i) $\frac{121}{(2^3 \times 3^2 \times 7^5)}$ *(ii)* $\frac{17}{90}$ *(iii)* $\frac{53}{343}$ *(iv)* $\frac{66}{180}$

SOLUTION (i) Given number is $\frac{121}{(2^3 \times 3^2 \times 7^5)}$.

Clearly, none of 2, 3 and 7 is a factor of 121.

So, the given rational number is in its simplest form.

And, $(2^3 \times 3^2 \times 7^5) \neq (2^m \times 5^n)$.

$\therefore$ $\frac{121}{(2^3 \times 3^2 \times 7^5)}$ is a nonterminating repeating decimal.

(ii) Given number is $\frac{17}{90}$.

And, $90 = (2 \times 3^2 \times 5)$.

Clearly, none of 2, 3 and 5 is a factor of 17.

$\therefore \quad \frac{17}{90}$ is in its simplest form.

Also, $90 = (2 \times 3^2 \times 5) \neq (2^m \times 5^n)$.

$\therefore \quad \frac{17}{90}$ is a nonterminating repeating decimal.

(iii) Given number is $\frac{53}{343}$.

Now, $343 = 7^3$ and 7 is not a factor of 53.

$\therefore \quad \frac{53}{343}$ is in its simplest form.

Also, $343 = 7^3 \neq (2^m \times 5^n)$.

$\therefore \quad \frac{53}{343}$ is a nonterminating repeating decimal.

(iv) Given number is $\frac{66}{180}$ and HCF(66, 180) = 6.

$\therefore \quad \frac{66}{180} = \frac{66 \div 6}{180 \div 6} = \frac{11}{30}$.

Now, $30 = (2 \times 3 \times 5)$ and none of 2, 3, 5 is a factor of 11.

$\therefore \quad \frac{11}{30}$ is in its simplest form.

Also, $30 = (2 \times 3 \times 5) \neq (2^m \times 5^n)$.

$\therefore \quad \frac{11}{30}$ and hence $\frac{66}{180}$ is a nonterminating repeating decimal.

EXAMPLE 3 *The decimal expansion of the rational number* $\frac{43}{2^4 \cdot 5^3}$, *will terminate after how many places of decimals?* [CBSE 2009]

SOLUTION We have

$$\frac{43}{2^4 \cdot 5^3} = \frac{43 \times 5}{2^4 \times 5^4} = \frac{215}{(2 \times 5)^4} = \frac{215}{10^4} = \frac{215}{10000} = 0.0215.$$

So, it will terminate after 4 places of decimals.

EXAMPLE 4 *Express each of the following as a rational number in simplest form.*

(a) $0.\overline{6}$ *(b)* $1.\overline{8}$ *(c)* $0.1\overline{6}$

SOLUTION (a) Let $x = 0.\overline{6}$. Then,

$x = 0.666\ldots$ … (i)

$\therefore \quad 10x = 6.666\ldots$ … (ii)

On subtracting (i) from (ii), we get

$$9x = 6 \Rightarrow x = \frac{6}{9} = \frac{2}{3}.$$

Hence, $0.\overline{6} = \frac{2}{3}$.

(b) Let $x = 1.\overline{8}$. Then,

$x = 1.888\ldots$ ... (i)

$\therefore\ 10x = 18.888\ldots$ ... (ii)

On subtracting (i) from (ii), we get

$$9x = 17 \Rightarrow x = \frac{17}{9} = 1\frac{8}{9}.$$

Hence, $1.\overline{8} = 1\frac{8}{9}$.

(c) Let $x = 0.1\overline{6}$. Then,

$x = 0.1666\ldots$ ... (i)

$\therefore\ 10x = 1.6666\ldots$ ... (ii)

And, $100x = 16.6666\ldots$ ... (iii)

On subtracting (ii) from (iii), we get

$$90x = 15 \Rightarrow x = \frac{15}{90} = \frac{1}{6}.$$

$\therefore\ 0.1\overline{6} = \frac{1}{6}$.

EXAMPLE 5 *Express* $0.\overline{32}$ *as a fraction in simplest form.*

SOLUTION Let $x = 0.\overline{32}$. Then,

$x = 0.3232\ldots$ ... (i)

$\therefore\ 100x = 32.3232\ldots$ ... (ii)

On subtracting (i) from (ii), we get

$$99x = 32 \Rightarrow x = \frac{32}{99}.$$

Hence, $0.\overline{32} = \frac{32}{99}$.

EXAMPLE 6 *Express* $0.2\overline{54}$ *as a fraction in simplest form.*

SOLUTION Let $x = 0.2\overline{54}$. Then,

$x = 0.2545454\ldots$ ... (i)

$\therefore\ 10x = 2.545454\ldots$ ... (ii)

and $1000x = 254.545454\ldots$ ... (iii)

On subtracting (ii) from (iii), we get

$$990x = 252 \Rightarrow x = \frac{252}{990} = \frac{126}{495} = \frac{42}{165} = \frac{14}{55}.$$

$\therefore\ 0.2\overline{54} = \frac{14}{55}$.

EXAMPLE 7 *Show that each of the following numbers is rational. What can you say about the prime factors of their denominators?*
(i) 23.123456789 *(ii)* $32.\overline{123456789}$

SOLUTION (i) Clearly, the given number 23.123456789 is a terminating decimal. So, it is rational and the prime factors of its denominator are 2 or 5 or both.

(ii) Clearly, the given number $32.\overline{123456789}$ is a nonterminating repeating decimal. So, it is rational and the prime factors of its denominator are other than 2 or 5 also.

EXAMPLE 8 *Decide whether the number* 0.12012001200012 … *is rational or not. Give reason to support your answer.*

SOLUTION Clearly, the given number 0.12012001200012… is a nonterminating and nonrepeating decimal. So, it is not rational.

EXERCISE 1C

1. Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form.

(i) $\frac{23}{(2^3 \times 5^2)}$ (ii) $\frac{24}{125}$ (iii) $\frac{171}{800}$ (iv) $\frac{15}{1600}$

(v) $\frac{17}{320}$ (vi) $\frac{19}{3125}$

2. Without actual division, show that each of the following rational numbers is a nonterminating repeating decimal.

(i) $\frac{11}{(2^3 \times 3)}$ (ii) $\frac{73}{(2^2 \times 3^3 \times 5)}$ (iii) $\frac{129}{(2^2 \times 5^3 \times 7^2)}$ (iv) $\frac{9}{35}$

(v) $\frac{77}{210}$ (vi) $\frac{32}{147}$ (vii) $\frac{29}{343}$ (viii) $\frac{64}{455}$

3. Express each of the following as a fraction in simplest form.

(i) $0.\overline{8}$ (ii) $2.\overline{4}$ (iii) $0.\overline{24}$ (iv) $0.1\overline{2}$

(v) $2.2\overline{4}$ (vi) $0.\overline{365}$

ANSWERS (EXERCISE 1C)

1. (i) 0.115 (ii) 0.192 (iii) 0.21375 (iv) 0.009375 (v) 0.053125
(vi) 0.00608

3. (i) $\frac{8}{9}$ (ii) $\frac{22}{9}$ (iii) $\frac{8}{33}$ (iv) $\frac{11}{90}$ (v) $\frac{101}{45}$ (vi) $\frac{181}{495}$

HINTS TO SOME SELECTED QUESTIONS

1. (i) $\frac{23}{(2^3 \times 5^2)} = \frac{23 \times 5}{(2^3 \times 5^3)} = \frac{115}{(10)^3} = \frac{115}{1000} = 0.115.$

(ii) $\frac{24}{125} = \frac{24}{5^3} \times \frac{2^3}{2^3} = \frac{24 \times 8}{(5 \times 2)^3} = \frac{192}{(10)^3} = \frac{192}{1000} = 0.192.$

(iii) $\frac{171}{800} = \frac{171}{8} \times \frac{1}{100} = \frac{21.375}{100} = 0.21375.$

(iv) $\frac{15}{1600} = \frac{15}{16} \times \frac{1}{100} = \frac{0.9375}{100} = 0.009375.$

(v) $\frac{17}{320} = \frac{17 \times 5}{320 \times 5} = \frac{85}{1600} = \frac{85}{16} \times \frac{1}{100} = \frac{5.3125}{100} = 0.053125.$

(vi) $\frac{19}{3125} = \frac{19 \times 8}{3125 \times 8} = \frac{152}{25000} = \frac{152}{25} \times \frac{1}{1000} = \frac{6.08}{1000} = 0.00608.$

2. (iv) $\frac{9}{35} = \frac{9}{(5 \times 7)}$ (v) $\frac{77}{210} = \frac{11}{30} = \frac{11}{(2 \times 3 \times 5)}$ (vi) $\frac{32}{147} = \frac{32}{(3 \times 7^2)}$

(vii) $\frac{29}{343} = \frac{29}{7^3}$ (viii) $\frac{64}{455} = \frac{64}{(5 \times 7 \times 13)}$

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IRRATIONAL NUMBERS

IRRATIONAL NUMBERS *The numbers which when expressed in decimal form are expressible as nonterminating and nonrepeating decimals are known as irrational numbers.*

Examples Note that every nonterminating and nonrepeating decimal is irrational.

Type 1. (i) Clearly, 0.1010010001… is a nonterminating and nonrepeating decimal. So, it is irrational.

(ii) 0.2020020002 , 0.3030030003…, etc., are all irrational.

(iii) 0.12112111211112… is irrational,

0.13113111311113… is irrational, and so on.

(iv) 0.232232223… is irrational,

0.343343334… is irrational, and so on.

Type 2. If m is a positive integer which is not a perfect square then $\sqrt{m}$ is irrational.

Thus, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \sqrt{11}$, etc., are all irrational.

Type 3. If m is a positive integer which is not a perfect cube then $\sqrt[3]{m}$ is irrational.

Thus, $\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5}, \sqrt[3]{6},$ etc., are all irrational.

Type 4. π is irrational, while $\frac{22}{7}$ is rational.

SOME RESULTS ON IRRATIONALS

THEOREM 1 *Let p be a prime number and a be a positive integer. If p divides a^2 then show that p divides a.*

PROOF Let p be a prime number and a be a positive integer such that p divides a^2.

We know that every positive integer can be expressed as the product of primes.

Let $a = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes, not necessarily all distinct. Then,

$$a^2 = (p_1 p_2 \ldots p_n)(p_1 p_2 \ldots p_n)$$

$$\therefore \quad a^2 = (p_1^2 p_2^2 \ldots p_n^2).$$

Now, p divides a^2

$\Rightarrow$ p is a prime factor of a^2

$\Rightarrow$ p is one of $p_1, p_2, \ldots, p_n$

[$\because$ prime factors of a^2 are $p_1, p_2, \ldots, p_n$]

$\Rightarrow$ p divides a [$\because$ $a = p_1 p_2 \ldots p_n$].

Thus, (p divides a^2) $\Rightarrow$ (p divides a).

Using the above result, we can prove the following.

THEOREM 2 *Prove that $\sqrt{2}$ is irrational.* [CBSE 2008, '09]

PROOF If possible, let $\sqrt{2}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{2} = \frac{a}{b} \Rightarrow 2 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 2b^2 = a^2$ ... (i)

$\Rightarrow$ 2 divides a^2 [$\because$ 2 divides $2b^2$]

$\Rightarrow$ 2 divides a

[$\because$ 2 is prime and divides $b^2 \Rightarrow$ 2 divides b].

Let $a = 2c$ for some integer c.

Putting $a = 2c$ in (i), we get

$2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$

$\Rightarrow$ 2 divides b^2 [$\because$ 2 divides $2c^2$]

$\Rightarrow$ 2 divides b

[$\because$ 2 is prime and 2 divides $b^2 \Rightarrow$ 2 divides b].

Thus, 2 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational.

THEOREM 3 *Prove that $\sqrt{3}$ is irrational.* [CBSE 2008, '09C]

PROOF If possible, let $\sqrt{3}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{3} = \frac{a}{b} \Rightarrow 3 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 3b^2 = a^2$ … (i)

$\Rightarrow$ 3 divides a^2 [$\because$ 3 divides $3b^2$]

$\Rightarrow$ 3 divides a

[$\because$ 3 is prime and 3 divides $a^2 \Rightarrow$ 3 divides a].

Let $a = 3c$ for some integer c.

Putting $a = 3c$ in (i), we get

$3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$

$\Rightarrow$ 3 divides b^2 [$\because$ 3 divides $3c^2$]

$\Rightarrow$ 3 divides b

[$\because$ 3 is prime and 3 divides $b^2 \Rightarrow$ 3 divides b].

Thus, 3 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{3}$ is rational.

Hence, $\sqrt{3}$ is irrational.

THEOREM 4 *Prove that* $\sqrt{5}$ *is irrational.* [CBSE 2008, '09]

PROOF If possible, let $\sqrt{5}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{5} = \frac{a}{b} \Rightarrow 5 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 5b^2 = a^2$ ... (i)

$\Rightarrow$ 5 divides a^2 [$\because$ 5 divides $5b^2$]

$\Rightarrow$ 5 divides a

[$\because$ 5 is prime and 5 divides $a^2 \Rightarrow$ 5 divides a].

Let $a = 5c$ for some integer c.

Putting $a = 5c$ in (i), we get

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

$\Rightarrow$ 5 divides b^2 [$\because$ 5 divides $5c^2$]

$\Rightarrow$ 5 divides b

[$\because$ 5 is prime and 5 divides $b^2 \Rightarrow$ 5 divides b].

Thus, 5 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{5}$ is rational.

Hence, $\sqrt{5}$ is irrational.

THEOREM 5 *Prove that* $\sqrt{11}$ *is irrational.*

PROOF If possible, let $\sqrt{11}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{11} = \frac{a}{b} \Rightarrow 11 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 11b^2 = a^2$ ... (i)

$\Rightarrow$ 11 divides a^2 [$\because$ 11 divides $11b^2$]

$\Rightarrow$ 11 divides a

[$\because$ 11 is prime and 11 divides $a^2 \Rightarrow$ 11 divides a].

Let $a = 11c$ for some positive integer c.

Putting $a = 11c$ in (i), we get

$$11b^2 = 121c^2 \Rightarrow b^2 = 11c^2$$

$\Rightarrow$ 11 divides b^2 [$\because$ 11 divides $11c^2$]

$\Rightarrow$ 11 divides b

[$\because$ 11 is prime and 11 divides $b^2 \Rightarrow$ 11 divides b].

Thus, 11 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ is irrational.

THEOREM 6 *If p is a prime number then prove that $\sqrt{p}$ is irrational.*

PROOF Let p be a prime number and if possible, let $\sqrt{p}$ be rational.

Let its simplest form be $\sqrt{p} = \frac{m}{n}$, where m and n are integers having no common factor other than 1, and $n \neq 0$.

Then, $\sqrt{p} = \frac{m}{n} \Rightarrow p = \frac{m^2}{n^2}$ [on squaring both sides]

$\Rightarrow pn^2 = m^2$ … (i)

$\Rightarrow p$ divides m^2 [$\because$ p divides pn^2]

$\Rightarrow p$ divides m

[$\because$ p is prime and p divides $m^2 \Rightarrow p$ divides m].

Let $m = pq$ for some integer q.

Putting $m = pq$ in (i), we get

$$pn^2 = p^2q^2 \Rightarrow n^2 = pq^2$$

$\Rightarrow p$ divides n^2 [$\because$ p divides pq^2]

$\Rightarrow p$ divides n

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n].

Thus, p is a common factor of m and n.

But, this contradicts the fact that m and n have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{p}$ is rational.

Hence, $\sqrt{p}$ is irrational.

THEOREM 7 *If a is rational and* $\sqrt{b}$ *is irrational then prove that* $(a+\sqrt{b})$ *is irrational.*

PROOF Let a be rational and $\sqrt{b}$ be irrational. Then, we have to prove that $(a+\sqrt{b})$ is irrational.

If possible, let $(a+\sqrt{b})$ be rational. Then,

$(a+\sqrt{b})$ is rational and a is rational

$\Rightarrow$ $\{(a+\sqrt{b})-a\}$ is rational [$\because$ difference of rationals is rational]

$\Rightarrow$ $\sqrt{b}$ is rational.

This contradicts the fact that $\sqrt{b}$ is irrational.

The contradiction arises by assuming that $(a+\sqrt{b})$ is rational.

Hence, $(a+\sqrt{b})$ is irrational.

THEOREM 8 *If a is a nonzero rational and* $\sqrt{b}$ *is irrational then show that* $a\sqrt{b}$ *is irrational.*

PROOF Let a be a nonzero rational and let $\sqrt{b}$ be irrational.

Then, we have to show that $a\sqrt{b}$ is irrational.

If possible, let $a\sqrt{b}$ be rational.

Then, $a\sqrt{b}=\frac{p}{q}$, where p and q are nonzero integers, having no common factor other than 1.

Now, $a\sqrt{b}=\frac{p}{q} \Rightarrow \sqrt{b}=\frac{p}{aq}$. ... (i)

But, p and aq are both rational and $aq \neq 0$.

$\therefore$ $\frac{p}{aq}$ is rational.

Thus, from (i), it follows that $\sqrt{b}$ is rational.

This contradicts the fact that $\sqrt{b}$ is irrational.

The contradiction arises by assuming that $a\sqrt{b}$ is rational.

Hence, $a\sqrt{b}$ is irrational.

SOLVED EXAMPLES

EXAMPLE 1 *Show that* $(2+\sqrt{3})$ *is an irrational number.*

SOLUTION Let us assume, to the contrary, that $(2+\sqrt{3})$ is rational.

Then, there exist co-primes a and b $(b \neq 0)$ such that

$$(2+\sqrt{3}) = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a}{b} - 2 \;\Rightarrow\; \sqrt{3} = \frac{a-2b}{b}.$$

Since a and b are integers, so $\frac{a-2b}{b}$ is rational.

Thus, $\sqrt{3}$ is also rational.

But, this contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $(2+\sqrt{3})$ is irrational.

EXAMPLE 2 *Show that* $2\sqrt{3}$ *is irrational.*

SOLUTION Let us assume, to the contrary, that $2\sqrt{3}$ is rational.

Then, there exist co-primes a and b $(b \neq 0)$ such that

$$2\sqrt{3} = \frac{a}{b} \;\Rightarrow\; \sqrt{3} = \frac{a}{2b}.$$

Since a and b are integers, so $\frac{a}{2b}$ is rational.

Thus, $\sqrt{3}$ is also rational.

But, this contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $2\sqrt{3}$ is irrational.

EXAMPLE 3 *Show that* $\frac{1}{\sqrt{2}}$ *is irrational.*

SOLUTION Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational.

Then, there exist co-primes a and b $(b \neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b} \;\Rightarrow\; \sqrt{2} = \frac{b}{a}.$$

Since a and b are integers, so $\frac{b}{a}$ is rational.

Thus, $\sqrt{2}$ is also rational.

But, this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $\frac{1}{\sqrt{2}}$ is irrational.

EXAMPLE 4 *Prove that* $(3+5\sqrt{2})$ *is irrational.*

SOLUTION Let us assume, to the contrary, that $(3+5\sqrt{2})$ is rational.

Then, there exist co-primes a and b $(b \neq 0)$ such that

$$3+5\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 5\sqrt{2} = \frac{a}{b} - 3 = \frac{a-3b}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a-3b}{5b}.$$

Since a and b are integers, so $\frac{a-3b}{5b}$ is rational.

Thus, $\sqrt{2}$ is also rational.

But, this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $(3+5\sqrt{2})$ is irrational.

EXAMPLE 5 *Prove that* $(\sqrt{2}+\sqrt{3})$ *is irrational.*

SOLUTION Let us assume that $(\sqrt{2}+\sqrt{3})$ is irrational.

Then, there exist co-primes a and b such that

$$\sqrt{2}+\sqrt{3} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a}{b} - \sqrt{2}$$

$$\Rightarrow \quad (\sqrt{3})^2 = \left(\frac{a}{b} - \sqrt{2}\right)^2 \qquad \text{[squaring both sides]}$$

$$\Rightarrow \quad 3 = \frac{a^2}{b^2} - \frac{2a}{b}\sqrt{2} + 2$$

$$\Rightarrow \quad \frac{2a}{b}\sqrt{2} = \frac{a^2}{b^2} - 1$$

$$\Rightarrow \quad \sqrt{2} = \frac{a^2-b^2}{2ab}.$$

Since a and b are integers, so $\frac{a^2-b^2}{2ab}$ is rational.

Thus, $\sqrt{2}$ is also rational.

But, this contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is incorrect.

Hence, $(\sqrt{2}+\sqrt{3})$ is irrational.

EXAMPLE 6 *Prove that* $\sqrt{p}+\sqrt{q}$ *is irrational, where* p *and* q *are primes.*

SOLUTION Let us assume that $\sqrt{p}+\sqrt{q}$ is rational.

Then, there exist co-primes a and b such that

$$\sqrt{p}+\sqrt{q}=\frac{a}{b}$$

$$\Rightarrow \quad \sqrt{p}=\frac{a}{b}-\sqrt{q}$$

$$\Rightarrow \quad (\sqrt{p})^2=\left(\frac{a}{b}-\sqrt{q}\right)^2 \qquad \text{[squaring both sides]}$$

$$\Rightarrow \quad p=\frac{a^2}{b^2}-\frac{2a}{b}\sqrt{q}+q$$

$$\Rightarrow \quad \frac{2a}{b}\sqrt{q}=\frac{a^2}{b^2}+q-p$$

$$\Rightarrow \quad \sqrt{q}=(a^2+b^2q-b^2p)\times\frac{b}{2a}=\frac{a^2b+b^3(q-p)}{2a}.$$

Since a, b, p, q are integers so $\dfrac{a^2b+b^3(q-p)}{2a}$ is rational.

Thus, $\sqrt{q}$ is also rational.

But, q being prime, $\sqrt{q}$ is irrational.

Since, a contradiction arises so our assumption is incorrect.

Hence, $(\sqrt{p}+\sqrt{q})$ is irrational.

EXERCISE 1D

1. Define (i) rational numbers (ii) irrational numbers (iii) real numbers.
2. Classify the following numbers as rational or irrational:

   (i) $\frac{22}{7}$ (ii) 3.1416 (iii) π (iv) $3.\overline{142857}$

   (v) 5.636363 … (vi) 2.040040004 … (vii) 1.535335333 …

   (viii) 3.121221222 … (ix) $\sqrt{21}$ (x) $\sqrt[3]{3}$

3. Prove that each of the following numbers is irrational.

   (i) $\sqrt{6}$ (ii) $(2-\sqrt{3})$ [CBSE 2008]

   (iiii) $(3+\sqrt{2})$ [CBSE 2009] (iv) $(2+\sqrt{5})$ [CBSE 2008C]

   (v) $(5+3\sqrt{2})$ [CBSE 2008] (vi) $3\sqrt{7}$

   (vii) $\frac{3}{\sqrt{5}}$ (viii) $(2-3\sqrt{5})$ [CBSE 2010]

   (ix) $(\sqrt{3}+\sqrt{5})$

4. Prove that $\frac{1}{\sqrt{3}}$ is irrational.

HINT $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \cdot \sqrt{3}$.

5. (i) Give an example of two irrationals whose sum is rational.
   (ii) Give an example of two irrationals whose product is rational.

HINT (i) Take $(2+\sqrt{3})$ and $(2-\sqrt{3})$.
(ii) Take $(3+\sqrt{2})$ and $(3-\sqrt{2})$.

6. State whether the given statement is true or false.
   (i) The sum of two rationals is always rational.
   (ii) The product of two rationals is always rational.
   (iii) The sum of two irrationals is always an irrational.
   (iv) The product of two irrationals is always an irrational.
   (v) The sum of a rational and an irrational is irrational.
   (vi) The product of a rational and an irrational is irrational.

7. Prove that $(2\sqrt{3}-1)$ is an irrational number. [CBSE 2010]

8. Prove that $(4-5\sqrt{2})$ is an irrational number. [CBSE 2010]

9. Prove that $(5-2\sqrt{3})$ is an irrational number. [CBSE 2010]

10. Prove that $5\sqrt{2}$ is irrational.

11. Prove that $\frac{2}{\sqrt{7}}$ is irrational.

HINT $\frac{2}{\sqrt{7}} = \left(\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}\right) = \frac{2}{7} \cdot \sqrt{7}$.

ANSWERS (EXERCISE 1D)

2. (i) rational (ii) rational (iii) irrational (iv) rational (v) rational (vi) irrational (vii) irrational (viii) irrational (ix) irrational (x) irrational

6. (i) True (ii) True (iii) False (iv) False (v) True (vi) True

EXERCISE 1E

Very-Short-Answer Questions

1. State Euclid's division lemma.

2. State fundamental theorem of arithmetic.
3. Express 360 as product of its prime factors.
4. If a and b are two prime numbers then find HCF(a, b).
5. If a and b are two prime numbers then find LCM(a, b).
6. If the product of two numbers is 1050 and their HCF is 25, find their LCM.
7. What is a composite number?
8. If a and b are relatively prime then what is their HCF?
9. If the rational number $\frac{a}{b}$ has a terminating decimal expansion, what is the condition to be satisfied by b? [CBSE 2008]
10. Simplify: $\frac{(2\sqrt{45}+3\sqrt{20})}{2\sqrt{5}}$. [CBSE 2010]
11. Write the decimal expansion of $\frac{73}{(2^4 \times 5^3)}$. [CBSE 2009]
12. Show that there is no value of n for which $(2^n \times 5^n)$ ends in 5.
13. Is it possible to have two numbers whose HCF is 25 and LCM is 520?
14. Give an example of two irrationals whose sum is rational.
15. Give an example of two irrationals whose product is rational.
16. If a and b are relatively prime, what is their LCM?
17. The LCM of two numbers is 1200. Show that the HCF of these numbers cannot be 500. Why?

Short-Answer Questions

18. Express $0.\overline{4}$ as a rational number in simplest form.
19. Express $0.\overline{23}$ as a rational number in simplest form.
20. Explain why 0.15015001500015 … is an irrational number.
21. Show that $\frac{\sqrt{2}}{3}$ is irrational.
22. Write a rational number between $\sqrt{3}$ and 2.
23. Explain why $3.\overline{1416}$ is a rational number.

ANSWERS (EXERCISE 1E)

3. $(2^3 \times 3^2 \times 5)$ **4.** 1 **5.** ab **6.** 42 **8.** 1
9. $b = (2^m \times 5^n)$, where m and n are some non-negative integers
10. 6 **11.** 0.0365 **13.** No **16.** ab **17.** since 500 is not a factor of 1200
18. $\frac{4}{9}$ **19.** $\frac{23}{99}$ **22.** 1.8

HINTS TO SOME SELECTED QUESTIONS

7. A number having at least 3 factors is called a composite number.

11. $\frac{73}{(2^4 \times 5^3)} = \frac{73 \times 5}{(2^4 \times 5^4)} = \frac{365}{(2 \times 5)^4} = \frac{365}{(10)^4} = \frac{365}{10000} = 0.0365.$

12. $(2^n \times 5^n) = (2 \times 5)^n = 10^n$, which always ends in a zero.

13. HCF always divides the LCM completely.

14. $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

15. $(3+\sqrt{2})$ and $(3-\sqrt{2})$.

20. Given number is nonterminating and nonrepeating decimal.

22. Clearly, $\sqrt{3} = 1.732\ldots$. So, we may take 1.8 as the required rational number between $\sqrt{3}$ and 2.

23. Clearly, it is a nonterminating repeating decimal.

....

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. Which of the following is a pair of co-primes?

   (a) (14, 35) (b) (18, 25) (c) (31, 93) (d) (32, 62)

2. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$ then HCF(a, b) = ?

   (a) 90 (b) 180 (c) 360 (d) 540

3. HCF of $(2^3 \times 3^2 \times 5)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3 \times 5^3 \times 7)$ is

   (a) 30 (b) 48 (c) 60 (d) 105

4. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is

   (a) 40 (b) 560 (c) 1120 (d) 1680

5. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number?

   (a) 36 (b) 45 (c) 9 (d) 81

6. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is

   (a) 8000 (b) 1600 (c) 320 (d) 1605

7. What is the largest number that divides each one of 1152 and 1664 exactly?

   (a) 32 (b) 64 (c) 128 (d) 256

8. What is the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively?

   (a) 13 (b) 9 (c) 3 (d) 585

9. What is the largest number that divides 245 and 1029, leaving remainder 5 in each case?

   (a) 15 (b) 16 (c) 9 (d) 5

10. The simplest form of $\frac{1095}{1168}$ is

    (a) $\frac{17}{26}$ (b) $\frac{25}{26}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$

11. Euclid's division lemma states that for any positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy

    (a) $1 < r < b$ (b) $0 < r \le b$ (c) $0 \le r < b$ (d) $0 < r < b$

12. A number when divided by 143 leaves 31 as remainder. What will be the remainder when the same number is divided by 13?

    (a) 0 (b) 1 (c) 3 (d) 5

13. Which of the following is an irrational number?

    (a) $\frac{22}{7}$ (b) 3.1416

    (c) $3.\overline{1416}$ (d) 3.141141114 …

14. π is

    (a) an integer (b) a rational number

    (c) an irrational number (d) none of these

15. $2.\overline{35}$ is

    (a) an integer (b) a rational number

    (c) an irrational number (d) none of these

16. 2.13113111311113 ... is

    (a) an integer (b) a rational number

    (c) an irrational number (d) none of these

17. The number 3.24636363 … is

    (a) an integer (b) a rational number

    (c) an irrational number (d) none of these

18. Which of the following rational numbers is expressible as a terminating decimal?

    (a) $\frac{124}{165}$ (b) $\frac{131}{30}$ (c) $\frac{2027}{625}$ (d) $\frac{1625}{462}$

19. The decimal expansion of the rational number $\frac{37}{2^2 \times 5}$ will terminate after
    (a) one decimal place (b) two decimal places
    (c) three decimal places (d) four decimal places
20. The decimal expansion of the number $\frac{14753}{1250}$ will terminate after
    (a) one decimal place (b) two decimal places
    (c) three decimal places (d) four decimal places
21. The number 1.732 is
    (a) an irrational number (b) a rational number
    (c) an integer (d) a whole number
22. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, the least prime factor of $(a+b)$ is
    (a) 2 (b) 3 (c) 5 (d) 8
23. $\sqrt{2}$ is
    (a) a rational number
    (b) an irrational number
    (c) a terminating decimal
    (d) a nonterminating repeating decimal
24. $\frac{1}{\sqrt{2}}$ is
    (a) a fraction (b) a rational number
    (c) an irrational number (d) none of these
25. $(2+\sqrt{2})$ is
    (a) an integer (b) a rational number
    (c) an irrational number (d) none of these
26. What is the least number that is divisible by all the natural numbers from 1 to 10 (both inclusive)?
    (a) 100 (b) 1260 (c) 2520 (d) 5040

ANSWERS (MCQ)

1. (b) **2.** (b) **3.** (c) **4.** (d) **5.** (d) **6.** (c) **7.** (c) **8.** (a) **9.** (b)
10. (d) **11.** (c) **12.** (d) **13.** (d) **14.** (c) **15.** (b) **16.** (c) **17.** (b) **18.** (c)
19. (b) **20.** (d) **21.** (b) **22.** (a) **23.** (b) **24.** (c) **25.** (c) **26.** (c)

HINTS TO SOME SELECTED QUESTIONS

2. HCF(a, b) = product of common terms with lowest power

$$= (2^2 \times 3^2 \times 5) = (4 \times 9 \times 5) = 180.$$

3. HCF = product of common terms with lowest power

$$= (2^2 \times 3 \times 5) = (4 \times 3 \times 5) = 60.$$

4. LCM = product of prime factors with highest power

$$= (2^4 \times 3 \times 5 \times 7) = (16 \times 3 \times 5 \times 7) = 1680.$$

5. Other number $= \dfrac{\text{HCF} \times \text{LCM}}{\text{given number}} = \dfrac{27 \times 162}{54} = 81.$

6. LCM $= \dfrac{\text{product of two numbers}}{\text{their HCF}} = \dfrac{1600}{5} = 320.$

7. Required number = HCF(1152, 1664) = 128.

8. Required number = HCF{(70 – 5), (125 – 8)} = HCF(65, 117) = 13.

9. Required number = HCF{(245 – 5), (1029 – 5)} = HCF(240, 1024) = 16.

10. HCF{1095, 1168} = 73.

$$\therefore \quad \frac{1095}{1168} = \frac{1095 \div 73}{1168 \div 73} = \frac{15}{16}.$$

11. On dividing a by b, let q be the quotient and r be the remainder.

Then, we have

$$a = bq + r, \text{ where } 0 \le r < b.$$

12. Let the given number when divided by 143 give q as quotient and 31 as remainder.

Then, number $= 143q + 31 = \{13 \times 11q + 13 \times 2 + 5\} = 13 \times (11q + 2) + 5.$

So, the same number when divided by 13 gives 5 as remainder.

13. 3.141141114 … is a nonterminating, nonrepeating decimal. So, it is irrational.

14. π is an irrational number.

15. $2.\overline{35} = 2.353535\ldots$, which is a repeating decimal.

$\therefore \quad 2.\overline{35}$ is rational.

16. 2.13113111311113 … is a nonterminating, nonrepeating decimal. So, it is irrational.

17. The number 3.24636363 … is a nonterminating repeating decimal.

So, it is a rational number.

18. $\dfrac{2027}{625} = \dfrac{2027}{(5^4 \times 2^0)}.$

So, it is expressible as a terminating decimal.

19. $\dfrac{37}{2^2 \times 5} = \dfrac{37 \times 5}{2^2 \times 5^2} = \dfrac{185}{(2 \times 5)^2} = \dfrac{185}{(10)^2} = \dfrac{185}{100} = 1.85.$

So, it will terminate after 2 decimal places.

20. $\frac{14753}{1250} \times \frac{8}{8} = \frac{14753 \times 8}{10000} = \frac{14753 \times 8}{(10)^4}$.

So, it will terminate after 4 decimal places.

21. $1.732 = \frac{1732}{1000}$, which is a rational number.

22. Clearly, 2 is neither a factor of a nor that of b.

$\therefore$ a and b are both odd.

Hence, $(a + b)$ is even.

$\therefore$ least prime factor of $(a + b)$ is 2.

24. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{2} \cdot \sqrt{2}$.

Here, $\frac{1}{2}$ is rational and $\sqrt{2}$ is irrational.

And, the product of a rational and an irrational is irrational.

$\therefore$ $\frac{1}{2} \cdot \sqrt{2}$ and hence $\frac{1}{\sqrt{2}}$ is irrational.

25. The sum of a rational and an irrational is irrational.

Here, 2 is rational and $\sqrt{2}$ is irrational.

$\therefore$ $(2 + \sqrt{2})$ is irrational.

26. Required number = LCM{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$= \text{LCM}\{1, 2, 3, 2^2, 5, 2 \times 3, 7, 2^3, 3^2, 2 \times 5\}$

$= (1 \times 2^3 \times 3^2 \times 5 \times 7) = (8 \times 9 \times 5 \times 7) = 2520.$

SUMMARY OF RESULTS

1. Euclid's Division Lemma

Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r$, *where* $0 \le r < b$.

2. Euclid's Division Algorithm According to this, we find the HCF of two positive integer a and b with $a > b$ in following steps.

Step 1. Apply the division lemma to find q and r such that $a = bq + r$, where $0 \le r < b$.

Step 2. If $r = 0$ then HCF $= b$. If $r \neq 0$, apply Euclid's division lemma to b and r.

Step 3. Continue the process till the remainder is zero. The divisor at this stage is HCF(a, b).

3. The Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

4. *If p is prime and p divides* a^2 *then p divides a, where a is a positive integer.*

5. *To prove that* $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$, *etc., are irrationals.*

6. *Let x be a rational number which can be expressed as a terminating decimal. If we put it in the simplest form* $\frac{p}{q}$ *then* $q = (2^m \times 5^n)$ *for some non-negative integers m and n.*

7. *Let* $x = \frac{p}{q}$ *be a rational number such that* $q \neq (2^m \times 5^n)$ *then x has a nonterminating repeating decimal expansion.*

8. *A number which can be expressed as a nonterminating and nonrepeating decimal is an irrational number.*

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \ldots, \pi, e,$ etc., are all irrational numbers.

....

TEST YOURSELF

MCQ

1. The decimal representation of $\frac{71}{150}$ is
   (a) a terminating decimal
   (b) a nonterminating, repeating decimal
   (c) a nonterminating and nonrepeating decimal
   (d) none of these
2. Which of the following has a terminating decimal expansion?
   (a) $\frac{32}{91}$ (b) $\frac{19}{80}$ (c) $\frac{23}{45}$ (d) $\frac{25}{42}$
3. On dividing a positive integer n by 9, we get 7 as remainder. What will be the remainder if $(3n - 1)$ is divided by 9?
   (a) 1 (b) 2 (c) 3 (d) 4
4. $0.\overline{68} + 0.\overline{73} = ?$
   (a) $1.\overline{41}$ (b) $1.\overline{42}$ (c) $0.\overline{141}$ (d) None of these

Short-Answer Questions

5. Show that any number of the form $4^n, n \in N$ can never end with the digit 0.

6. The HCF of two numbers is 27 and their LCM is 162. If one of the number is 81, find the other.
7. Examine whether $\frac{17}{30}$ is a terminating decimal.
8. Find the simplest form of $\frac{148}{185}$.
9. Which of the following numbers are irrational?

   (a) $\sqrt{2}$ (b) $\sqrt[3]{6}$ (c) 3.142857 (d) $2.\overline{3}$

   (e) π (f) $\frac{22}{7}$ (g) 0.232332333… (h) $5.27\overline{41}$

10. Prove that $(2+\sqrt{3})$ is irrational.
11. Find the HCF and LCM of 12, 15, 18, 27.
12. Give an example of two irrationals whose sum is rational.
13. Give prime factorisation of 4620.
14. Find the HCF of 1008 and 1080 by prime factorisation method.
15. Find the HCF and LCM of $\frac{8}{9}, \frac{10}{27}$ and $\frac{16}{81}$.
16. Find the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively.

Long-Answer Questions

17. Prove that $\sqrt{3}$ is an irrational number.
18. Show that every positive odd integer is of the form $(4q + 1)$ or $(4q + 3)$ for some integer q.
19. Show that one and only one out of n, $(n + 2)$ and $(n + 4)$ is divisible by 3, where n is any positive integer.
20. Show that $(4+3\sqrt{2})$ is irrational.

ANSWERS (TEST YOURSELF)

1. (b) **2.** (b) **3.** (b) **4.** (b) **6.** 54 **7.** No **8.** $\frac{4}{5}$

9. $\sqrt{2}, \sqrt[3]{6}, \pi$, 0.232332333… **11.** HCF = 3, LCM = 540

12. $(2+\sqrt{3})$ and $(2-\sqrt{3})$ **13.** $(2^2 \times 3 \times 5 \times 7 \times 11)$

14. 72 **15.** HCF $= \frac{2}{81}$, LCM $= \frac{80}{9}$ **16.** 108

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