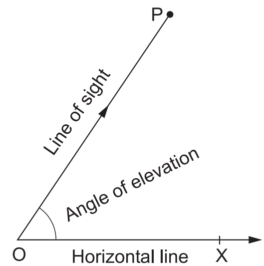
14 Heights and Distances

LINE OF SIGHT *When an observer looks from a point O at an object P then the line OP is called the line of sight.*

ANGLE OF ELEVATION

Suppose that from a point O, you look up at an object P, *placed above the level of your eye*. Then, the angle which the line of sight makes with the horizontal line through O is called the *angle of elevation* of P, as seen from O.



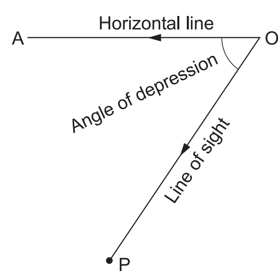
Example Let OX be a horizontal line on the level ground and let a person at O be looking up towards an object P, say an aeroplane or the top of a tree or the top of a tower or a flag at the top of a house.

Then, $\angle XOP$ is the angle of elevation of P from O.

ANGLE OF DEPRESSION

Now, suppose that from a point O, you look down at an object P, *placed below the level of your eye.*

Then, the angle which the line of sight makes with the horizontal line through O is called the *angle of depression* of P, as seen from O.

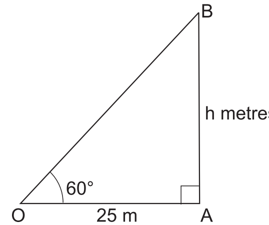


SOLVED EXAMPLES

EXAMPLE 1 *A vertical pole stands on the level ground. From a point on the ground,* 25 m *away from the foot of the pole, the angle of elevation of its top is found to be* 60°. *Find the height of the pole.* [*Take* $\sqrt{3} = 1.732$.]

SOLUTION Let AB be the pole standing on a level ground and let O be the position of the observer. Then, $OA = 25$ m, $\angle OAB = 90°$ and $\angle AOB = 60°$.

Let $AB = h$ metres.



From the right $\triangle OAB$, we have

$$\frac{AB}{OA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{h}{25} = \sqrt{3}$$

$$\Rightarrow \quad h = 25 \times \sqrt{3} = 25 \times 1.732 = 43.3.$$

Hence, the height of the pole is 43.3 m.

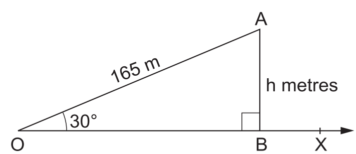
EXAMPLE 2 *A kite is flying, attached to a thread which is* 165 m *long. The thread makes an angle of* 30° *with the ground. Find the height of the kite from the ground, assuming that there is no slack in the thread.*

SOLUTION Let OX be the horizontal ground and let A be the position of the kite. Let O be the position of the observer and OA be the thread. Draw $AB \perp OX$.

Then, $\angle BOA = 30°$, $OA = 165$ m and $\angle OBA = 90°$.

Height of the kite from the ground $= AB$.

Let $AB = h$ metres.



From right $\triangle OBA$, we have

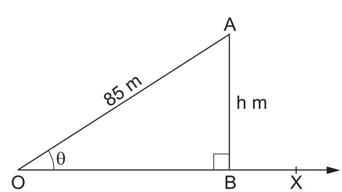
$$\frac{AB}{OA} = \sin 30° = \frac{1}{2}$$

$$\Rightarrow \quad \frac{h}{165} = \frac{1}{2} \quad \Rightarrow \quad h = \frac{165}{2} = 82.5.$$

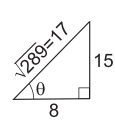
Hence, the height of the kite from the ground = 82.5 m.

EXAMPLE 3 *The length of a string between a kite and a point on the ground is* 85 m. *If the string makes an angle* θ *with the ground level such that* $\tan\theta = 15/8$ *then find the height of the kite from the ground. Assume that there is no slack in the string.* [CBSE 2014]

SOLUTION. Let OX be the horizontal ground and let A be the position of the kite. Let O be the position of the observer and OA be the string. Draw $AB \perp OX$.



Then, $\angle BOA = \theta$ such that $\tan\theta = \frac{15}{8}$, $OA = 85$ m and $\angle OBA = 90°$.



Let $AB = h$ metres.

From right $\triangle OBA$, we have

$$\frac{AB}{OA} = \sin\theta = \frac{15}{17} \qquad \left[\because\ \tan\theta = \frac{15}{8} \Rightarrow \sin\theta = \frac{15}{17}\right]$$

$$\Rightarrow \frac{h}{85} = \frac{15}{17} \Rightarrow h = \frac{15}{17} \times 85 = 75.$$

Hence, the height of the kite from the ground is 75 m.

EXAMPLE 4 *A ladder* 15 m *long just reaches the top of a vertical wall. If the ladder makes an angle of* 60° *with the wall, find the height of the wall.* [CBSE 2013]

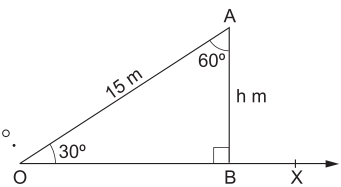
SOLUTION Let OX be the horizontal ground and let OA be the ladder leaning against the wall AB. Then,

$OA = 15$ m, $\angle OAB = 60°$ and

$\angle OBA = 90°$.

Let $AB = h$ metres.

Now, $\angle AOB = (90° - 60°) = 30°$.



From right $\triangle OBA$, we have

$$\frac{AB}{OA} = \sin 30° = \frac{1}{2} \Rightarrow \frac{h}{15} = \frac{1}{2}$$

$$\Rightarrow h = 15 \times \frac{1}{2} = 7.5.$$

Hence, the height of the wall is 7.5 metres.

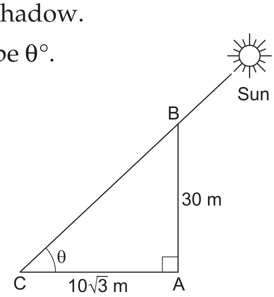
EXAMPLE 5 *If a tower* 30 m *high, casts a shadow* $10\sqrt{3}$ m *long on the ground, then what is the angle of elevation of the sun?* [CBSE 2017]

SOLUTION Let AB be the pole and let AC be its shadow.

Let the angle of elevation of the sun be $\theta°$.

Then, $\angle ACB = \theta$, $\angle CAB = 90°$,

$AB = 30$ m and $AC = 10\sqrt{3}$ m.



From right $\triangle CAB$, we have

$$\tan\theta = \frac{AB}{AC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \theta = 60°.$$

Hence, the angular elevation of the sun is 60°.

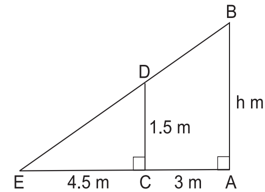
EXAMPLE 6 *If a* 1.5-m-*tall girl stands at a distance of* 3 m *from a lamp-post and casts a shadow of length* 4.5 m *on the ground then find the height of the lamp-post.*

SOLUTION Let AB be the lamp-post and CD be the girl.

Let CE be the shadow of CD. Then,

$CD = 1.5\,\text{m}, CE = 4.5\text{ m and } AC = 3\text{ m}.$

Let $AB = h$ metres.



Now, ΔAEB and ΔCED are similar.

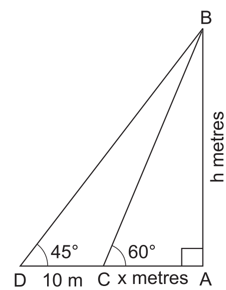
$$\therefore \quad \frac{AB}{AE} = \frac{CD}{CE} \Rightarrow \frac{h}{(3+4.5)} = \frac{1.5}{4.5} = \frac{1}{3}$$

$$\Rightarrow h = \frac{1}{3} \times 7.5 = 2.5.$$

Hence, the height of the lamp-post is 2.5 metres.

EXAMPLE 7 *The shadow of a tower, when the angle of elevation of the sun is* $45°$, *is found to be* 10 *metres longer than when the angle of elevation is* $60°$. *Find the height of the tower.* [*Given* $\sqrt{3} = 1.732$.]

SOLUTION Let AB be the tower and let AC and AD be its shadows when the angles of elevation of the sun are $60°$ and $45°$ respectively.



$$\therefore \quad \angle ACB = 60°, \angle ADB = 45°,$$

$$\angle DAB = 90° \text{ and } CD = 10\text{ m}.$$

Let $AB = h$ metres and $AC = x$ metres.

From right ΔCAB, we get

$$\frac{AC}{AB} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}. \qquad \ldots \text{(i)}$$

From right ΔDAB, we get

$$\frac{AD}{AB} = \cot 45° = 1 \Rightarrow \frac{x+10}{h} = 1$$

$$\Rightarrow \quad x + 10 = h \Rightarrow x = h - 10. \qquad \ldots \text{(ii)}$$

Equating the values of x from (i) and (ii), we get

$$\frac{h}{\sqrt{3}} = h - 10 \Rightarrow h = \sqrt{3}h - 10\sqrt{3} \Rightarrow (\sqrt{3}-1)h = 10\sqrt{3}$$

$$\Rightarrow \quad h = \frac{10\sqrt{3}}{(\sqrt{3}-1)} = \left\{\frac{10\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}\right\}$$

$$= 5\sqrt{3}(\sqrt{3}+1) = 15 + 5\sqrt{3}$$

$$\Rightarrow \quad h = (5 + 5 \times 1.732) = (15 + 8.66) = 23.66.$$

Hence, the height of the tower is 23.66 m.

SUN'S ALTITUDE *The angle of elevation of the sun from the earth is called the sun's altitude.*

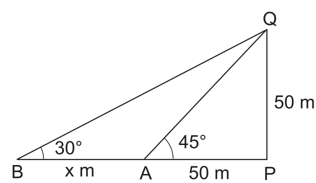
The sun's altitudes are different at different times of the day.

EXAMPLE 8 *A tower is* 50 m *high. Its shadow is* x *metres shorter when the sun's altitude is* 45° *than when it is* 30°. *Find the value of* x. [*Given* $\sqrt{3} = 1.732$.]

SOLUTION Let PQ be the tower and let PA and PB be its shadows when the altitudes of the sun are 45° and 30° respectively. Then,

$$\angle PAQ = 45°, \angle PBQ = 30°, \angle BPQ = 90°, PQ = 50 \text{ m}.$$

Let $AB = x$ metres.



From right $\triangle APQ$, we have

$$\frac{AP}{PQ} = \cot 45° = 1$$

$$\Rightarrow \quad \frac{AP}{50 \text{ m}} = 1 \quad \Rightarrow \quad AP = 50 \text{ m}.$$

From right $\triangle BPQ$, we have

$$\frac{BP}{PQ} = \cot 30° = \sqrt{3} \Rightarrow \frac{x+50}{50} = \sqrt{3} \Rightarrow \quad x = 50(\sqrt{3} - 1).$$

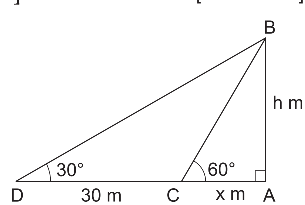
$$\Rightarrow \quad x = 50(1.732 - 1) = (50 \times 0.732) = 36.6$$

Hence, $x = 36.6$.

EXAMPLE 9 *The shadow of a tower standing on a level ground is found to be* 30 m *longer when the sun's altitude is* 30°, *than when it was* 60°. *Find the height of the tower.* [*Take* $\sqrt{3} = 1.732$.] [CBSE 2012]

SOLUTION Let AB be the tower and let AC and AD be the lengths of its shadows when $\angle ACB = 60°$ and $\angle ADB = 30°$.

Let $AB = h$ metres, and $AC = x$ metres.



From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \left(h \times \frac{1}{\sqrt{3}}\right) = \frac{h}{\sqrt{3}}. \qquad \text{... (i)}$$

From right $\triangle DAB$, we have

$$\frac{AD}{AB} = \cot 30° = \sqrt{3} \Rightarrow \frac{x+30}{h} = \sqrt{3}$$

$\Rightarrow \quad x = (\sqrt{3}h - 30).$ ... (ii)

Equating the values of x from (i) and (ii), we get

$$\frac{h}{\sqrt{3}} = (\sqrt{3}h - 30) \Rightarrow h = 3h - 30\sqrt{3}$$

$\Rightarrow \quad 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = (15 \times 1.732) = 25.98.$

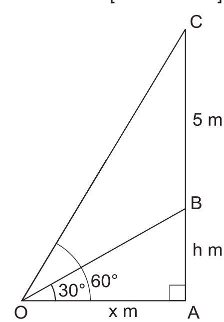
Hence, the height of the tower is 25.98 m.

<u>EXAMPLE 10</u> *From a point O on the ground, the angle of elevation of the top of a tower is* 30° *and that of the top of the flagstaff on the top of the tower is* 60°. *If the length of the flagstaff is* 5 *metres, find the height of the tower.* [CBSE 2015]

SOLUTION Let AB be the tower and BC be the flagstaff.

Let O be a point on the ground such that $\angle AOB = 30°$ and $\angle AOC = 60°$ $BC = 5$ m (given).

Let $AB = h$ metres and $OA = x$ metres.



From right $\triangle OAB$, we have

$$\frac{OA}{AB} = \cot 30° = \sqrt{3} \Rightarrow \frac{x}{h} = \sqrt{3}$$

$$\Rightarrow x = h\sqrt{3}.$$ ... (i)

From right $\triangle OAC$, we have

$$\frac{OA}{AC} = \cot 60° = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \frac{x}{h+5} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h+5}{\sqrt{3}}.$ ... (ii)

Equating the values of x from (i) and (ii), we get

$$h\sqrt{3} = \frac{h+5}{\sqrt{3}} \Rightarrow 3h = h+5 \Rightarrow 2h = 5 \Rightarrow h = 2.5.$$

Hence, the height of the tower is 2.5 metres.

<u>EXAMPLE 11</u> *Two pillars of equal heights stand on either side of a road which is* 100 m *wide. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are* 60° *and* 30°. *Find the height of each pillar and position of the point on the road.* [*Take* $\sqrt{3} = 1.732$.]

[CBSE 2011, '13]

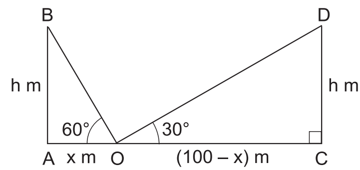
SOLUTION Let AB and CD be two pillars, each of height h metres and let AC be the road such that $AC = 100$ m.

Let O be the point of observation on AC.

Let $OA = x$ metres and $OC = (100 - x)$ m.

Also, $\angle AOB = 60°$ and $\angle COD = 30°$.

$AB \perp AC$ and $CD \perp AC$.



From right $\triangle OAB$, we have

$$\frac{AB}{OA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3} \Rightarrow h = \sqrt{3}x. \qquad \text{... (i)}$$

From right $\triangle OCD$, we have

$$\frac{CD}{OC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{(100 - x)} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad h = \frac{(100 - x)}{\sqrt{3}}. \qquad \text{... (ii)}$$

Equating the values of h from (i) and (ii), we get

$$\sqrt{3}x = \frac{(100 - x)}{\sqrt{3}} \Rightarrow 3x = (100 - x) \Rightarrow 4x = 100 \Rightarrow x = 25.$$

Putting $x = 25$ m in (i), we get

$$h = (25 \times \sqrt{3}) = (25 \times 1.732) = 43.3.$$

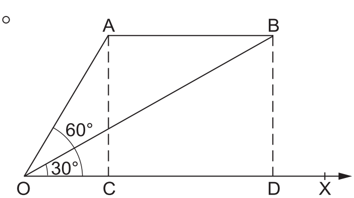
Hence, the height of each pillar is 43.3 m and the point of observation is 25 m away from the first pillar.

EXAMPLE 12 *The angle of elevation of an aeroplane from a point on the ground is* $60°$. *After a flight of* 15 *seconds, the angle of elevation changes to* $30°$. *If the aeroplane is flying at a constant height of* $1500\sqrt{3}$ m, *find the speed of the plane in* km/hr. [CBSE 2015]

SOLUTION Let OX be the horizontal ground and let O be the point of observation. Let A and B be the two positions of the aeroplane. Let $AC \perp OX$ and $BD \perp OX$. Then,

$$\angle COA = 60°, \angle DOB = 30°$$

and $AC = BD = 1500\sqrt{3}$ m.



From right $\triangle OCA$, we have

$$\frac{OC}{AC} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{OC}{1500\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow OC = 1500 \text{ m}.$$

From right $\triangle ODB$, we have

$$\frac{OD}{BD} = \cot 30° = \sqrt{3} \Rightarrow \frac{OD}{1500\sqrt{3}\text{ m}} = \sqrt{3}$$

$$\Rightarrow \quad OD = (1500 \times 3) \text{ m} = 4500 \text{ m}.$$

$$\therefore \quad CD = (OD - OC) = (4500 - 1500) \text{ m} = 3000 \text{ m}.$$

Thus, the aeroplane covers 300 m in 15 seconds.

$$\therefore \quad \text{speed of the aeroplane} = \left(\frac{3000}{15} \times \frac{60 \times 60}{1000}\right) \text{km/hr}$$

$$= 720 \text{ km/hr}.$$

Hence, the speed of the aeroplane is 720 km/hr.

EXAMPLE 13 *The angles of elevation and depression of the top and bottom of a tower from the top of a building* 60 m *high are* 30° *and* 60° *respectively. Find the difference between the heights of the building and the tower and also the distance between them.* [CBSE 2013C, '14]

SOLUTION Let AB be the tower and CD be the building. Then, $CD = 60$ m.

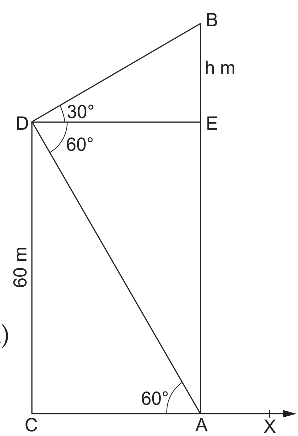
Let CAX be the horizontal ground.

Draw $DE \perp AB$. Now, $\angle EDB = 30°$

and $\angle CAD = \angle ADE = 60°$.

Also, $AE = CD = 60$ m.

Let $BE = h$ metres.



From right $\triangle ACD$, we have

$$\frac{CA}{CD} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{CA}{60 \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow CA = \frac{60}{\sqrt{3}}. \quad \text{... (i)}$$

From right $\triangle EDB$, we have

$$\frac{DE}{BE} = \cot 30° = \sqrt{3} \Rightarrow \frac{DE}{h \text{ m}} = \sqrt{3}$$

$$\Rightarrow DE = h\sqrt{3} \text{ m}. \quad \text{... (ii)}$$

But, $CA = DE$.

$\therefore$ from (i) and (ii), we get

$$\frac{60}{\sqrt{3}} = h\sqrt{3} \Rightarrow 3h = 60 \Rightarrow h = 20.$$

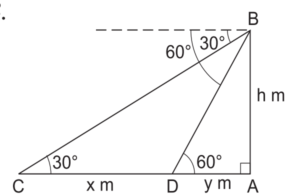
Hence, the difference between the heights of the tower and the building is 20 metres.

From (i), we get $CA = \left(\frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right)$ m $= 20\sqrt{3}$ metres.

Hence, the distance between the tower and the building is $20\sqrt{3}$ m.

EXAMPLE 14 *A man on a cliff observes a boat at an angle of depression of* $30°$ *which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be* $60°$. *Find the total time taken by the boat to reach the shore.* [CBSE 2014]

SOLUTION Let AB be the cliff with the man at B.



Let C and D be the two positions of the boat approaching the shore at A. Then,

$$\angle ACB = 30° \text{ and } \angle ADB = 60°.$$

Let $AB = h$ metres, $CD = x$ metres and $DA = y$ metres.

From right $\triangle DAB$, we have

$$\frac{AD}{AB} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow \frac{y}{h} = \frac{1}{\sqrt{3}} \Rightarrow y = \frac{h}{\sqrt{3}}. \quad \text{... (i)}$$

From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \frac{x+y}{h} = \sqrt{3} \Rightarrow x + y = h\sqrt{3}. \quad \text{... (ii)}$$

On subtracting (i) from (ii), we get

$$x = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right) \Rightarrow x = \frac{2h}{\sqrt{3}}. \quad \text{... (iii)}$$

From (iii) and (i), we get

$$x : y = \frac{2h}{\sqrt{3}} : \frac{h}{\sqrt{3}} = 2:1.$$

Let the time taken to cover y units be t minutes. Then,

ratio of distances = ratio of times taken to cover them.

So, $2:1 = 6:t \Rightarrow \frac{2}{1} = \frac{6}{t} \Rightarrow 2t = 6 \Rightarrow t = 3.$

Thus, the boat takes 3 minutes to cover the distance DA.

Hence, the total time taken by the boat to reach the shore is $(6+3)$ minutes = 9 minutes.

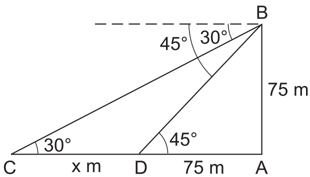
<u>EXAMPLE 15</u> *Observed from the top of a* 75-m-*high lighthouse (from sea level), the angles of depression of two ships are* 30° *and* 45°. *If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the ships.* (*Use* $\sqrt{3} = 1.73$.) [CBSE 2014]

SOLUTION Let AB be the lighthouse and C and D be the positions of two ships. Then,

$AB = 75$ m, $\angle ACB = 30°$

and $\angle ADB = 45°$.

Let $CD = x$ metres.



From right $\triangle DAB$, we have

$$\frac{AD}{AB} = \cot 45° = 1$$

$$\Rightarrow \quad \frac{AD}{75 \text{ m}} = 1 \Rightarrow AD = 75 \text{ m}.$$

From right $\triangle CAB$, we have

$$\frac{CA}{AB} = \cot 30° = \sqrt{3} \Rightarrow \frac{x+75}{75} = \sqrt{3}$$

$$\Rightarrow \quad x + 75 = 75\sqrt{3} \Rightarrow x = 75(\sqrt{3} - 1) = 75(1.73 - 1)$$

$$\Rightarrow \quad x = (75 \times 0.73) = 54.75.$$

Hence, the distance between the ships is 54.75 metres.

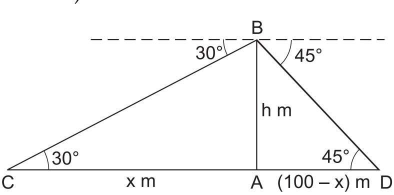
<u>EXAMPLE 16</u> *Two ships are approaching a lighthouse from opposite directions. The angles of depression of the two ships from the top of a lighthouse are* 30° *and* 45°. *If the distance between the two ships is* 100 *metres, find the height of the lighthouse.* (*Use* $\sqrt{3} = 1.732$.) [CBSE 2014]

SOLUTION Let AB be the lighthouse and C and D be the positions of the two ships.

Then, $\angle ACB = 30°$ and $\angle ADB = 45°$.

Let $AB = h$ metres, $CA = x$ metres.

Then, $AD = (100 - x)$ m.



From right $\triangle BAD$, we have

$$\frac{AD}{AB} = \cot 45° = 1$$

$$\Rightarrow \quad \frac{(100-x)}{h} = 1 \quad \Rightarrow \quad x = (100-h). \qquad \text{... (i)}$$

From right ΔBAC, we have

$$\frac{AC}{AB} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{x}{h} = \sqrt{3} \Rightarrow x = h\sqrt{3}. \qquad \text{... (ii)}$$

Equating the values of x from (i) and (ii), we get

$$100 - h = h\sqrt{3} \Rightarrow h(\sqrt{3}+1) = 100$$

$$\Rightarrow \quad h = \left\{\frac{100}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}\right\} = 50(\sqrt{3}-1)$$

$$\Rightarrow \quad h = 50(1.732-1) = (50 \times 0.732) = 36.6.$$

Hence, the height of the lighthouse is 36.6 m.

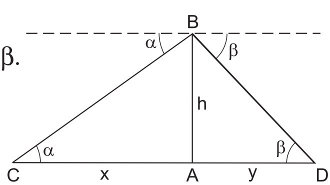
EXAMPLE 17 *From the top of a lighthouse, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the lighthouse be h metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is* $\dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \tan\beta}$ *metres.* [HOTS]

SOLUTION Let AB be the lighthouse and C and D be the positions of the two ships. Then, $AB = h$ metres.

Clearly, $\angle ACB = \alpha$ and $\angle ADB = \beta$.

Let $AC = x$ metres and

$AD = y$ metres.



From right ΔCAB, we have

$$\frac{AC}{AB} = \cot\alpha \Rightarrow \frac{x}{h} = \cot\alpha \Rightarrow x = h\cot\alpha. \qquad \text{... (i)}$$

From right ΔDAB, we have

$$\frac{AD}{AB} = \cot\beta \Rightarrow \frac{y}{h} = \cot\beta \Rightarrow y = h\cot\beta. \qquad \text{... (ii)}$$

Adding the corresponding sides of (i) and (ii), we get

$$x + y = h(\cot\alpha + \cot\beta) = h\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

$$\Rightarrow \quad x + y = \frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \tan\beta}.$$

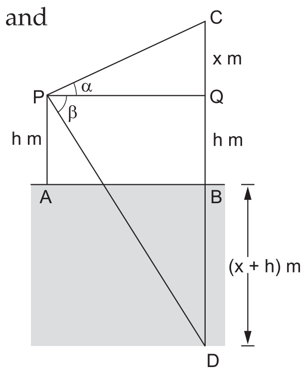
Hence, the distance between the ships is $\dfrac{h(\tan\alpha + \tan\beta)}{\tan\alpha \tan\beta}$ m.

<u>EXAMPLE 18</u> *The angle of elevation of a cloud from a point h metres above a lake is* α *and the angle of depression of its reflection in the lake is* β*. Prove that the height of the cloud is* $\frac{h(\tan\beta+\tan\alpha)}{(\tan\beta-\tan\alpha)}$ *metres.* [HOTS]

[CBSE 2008]

SOLUTION Let AB be the surface of the lake and let P be a point vertically above A such that $AP = h$ metres.

Let C be the position of the cloud and let D be its reflection in the lake.



Draw $PQ \perp CD$. Then,

$$\angle QPC = \alpha, \angle QPD = \beta,$$

$$BQ = AP = h \text{ metres.}$$

Let $CQ = x$ metres. Then,

$$BD = BC = (x+h) \text{ metres.}$$

From right $\triangle PQC$, we have

$$\frac{PQ}{CQ} = \cot\alpha \Rightarrow \frac{PQ}{x\text{ m}} = \cot\alpha$$

$$\Rightarrow \quad PQ = x\cot\alpha \text{ metres.} \qquad \ldots\text{(i)}$$

From right $\triangle PQD$, we have

$$\frac{PQ}{QD} = \cot\beta \quad \Rightarrow \quad \frac{PQ}{(x+2h)\text{ m}} = \cot\beta$$

$$\Rightarrow \quad PQ = (x+2h)\cot\beta \text{ metres.} \qquad \ldots\text{(ii)}$$

From (i) and (ii), we get

$$x\cot\alpha = (x+2h)\cot\beta$$

$$\Rightarrow \quad x(\cot\alpha - \cot\beta) = 2h\cot\beta \Rightarrow x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x\left(\frac{\tan\beta-\tan\alpha}{\tan\alpha+\tan\beta}\right) = \frac{2h}{\tan\beta} \Rightarrow x = \frac{2h\tan\alpha}{(\tan\beta-\tan\alpha)}.$$

$\therefore$ height of the cloud from the surface of the lake

$$= (x+h) = \left\{\frac{2h\tan\alpha}{(\tan\beta-\tan\alpha)} + h\right\}\text{ m}$$

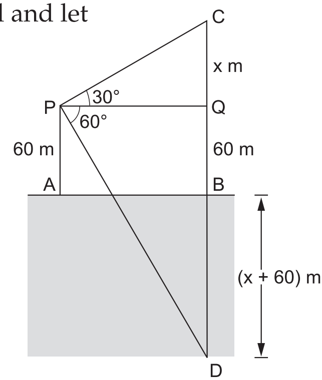
$$= \frac{h(\tan\alpha+\tan\beta)}{(\tan\beta-\tan\alpha)} \text{ metres.}$$

<u>EXAMPLE 19</u> *The angle of elevation of a cloud from a point* 60 m *above the surface of the water of a lake is* 30° *and the angle of depression of its shadow*

in water of lake is 60°. Find the height of the cloud from the surface of water. [CBSE 2010, '15, '17]

SOLUTION Let AB be the surface of the lake and let P be a point vertically above A such that $AP = 60$ m.

Let C be the position of the cloud and let D be its reflection in the lake.



Draw $PQ \perp CD$. Then,

$$\angle QPC = 30°, \angle QPD = 60°,$$

$$BQ = AP = 60 \text{ m}.$$

Let $CQ = x$ metres. Then,

$$BD = BC = (x + 60) \text{ m}.$$

From right $\triangle PQC$, we have

$$\frac{PQ}{CQ} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \frac{PQ}{x \text{ m}} = \sqrt{3} \Rightarrow PQ = x\sqrt{3} \text{ m}. \quad \ldots(i)$$

From right $\triangle PQD$, we have

$$\frac{PQ}{QD} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{PQ}{(x + 60 + 60) \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{(x + 120)}{\sqrt{3}} \text{ m}. \quad \ldots(ii)$$

Equating the values of PQ from (i) and (ii), we get

$$x\sqrt{3} = \frac{(x + 120)}{\sqrt{3}}$$

$$\Rightarrow 3x = x + 120 \Rightarrow 2x = 120 \Rightarrow x = 60.$$

$\therefore$ height of the cloud from the surface of the lake

$$= BC = (60 + x) \text{ m} = (60 + 60) \text{ m} = 120 \text{ m}.$$

Hence, the height of the cloud from the surface of the lake is 120 metres.

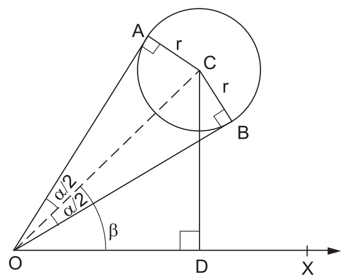
EXAMPLE 20 *A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is*

$$\left(r \sin \beta \operatorname{cosec} \frac{\alpha}{2}\right).$$ [HOTS]

SOLUTION Let us represent the balloon by a circle with centre C and radius r. Let OX be the horizontal ground and let O be the point of observation. From O, draw tangents OA and OB to the circle. Join CA, CB and CO. Draw $CD \perp OX$.

$\therefore \quad \angle AOB = \alpha, \angle DOC = \beta$ and

$\angle AOC = \angle BOC = \frac{\alpha}{2}\cdot$



From right ΔOAC, we have

$$\frac{OC}{AC} = \operatorname{cosec}\frac{\alpha}{2}$$

$$\Rightarrow \quad \frac{OC}{r} = \operatorname{cosec}\frac{\alpha}{2}$$

$$\Rightarrow \quad OC = r\operatorname{cosec}\frac{\alpha}{2}\cdot \qquad \text{... (i)}$$

From right ΔODC, we have

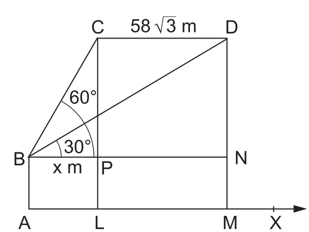
$$\frac{CD}{OC} = \sin\beta \;\Rightarrow\; CD = (OC)\times \sin\beta$$

$$\Rightarrow \quad CD = r\sin\beta\operatorname{cosec}\frac{\alpha}{2} \qquad \text{[using (i)].}$$

Hence, the height of the centre of the balloon from the ground is $r\sin\beta\operatorname{cosec}\frac{\alpha}{2}\cdot$

EXAMPLE 21 *A boy whose eye level is* 1.3 m *from the ground, spots a balloon moving with the wind in a horizontal line at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at an instant is* $60°$. *After* 2 *seconds, the angle of elevation reduces to* $30°$. *If the speed of the wind is* $29\sqrt{3}$ m/s *then find the height of the balloon from the ground.* [HOTS] [CBSE 2009C]

SOLUTION Let AB be the position of the boy and AX be the horizontal ground. Let C and D be the two positions of the balloon. Draw $CL \perp AX$, $DM \perp AX$ and $BN \perp DM$, intersecting CL at P.



Then, $\angle CBN = 60°$ and $\angle DBN = 30°$.

Distance covered by the balloon in 2 seconds

$$= (29\sqrt{3}\times 2)\text{ m} = 58\sqrt{3}\text{ m}.$$

$\therefore \quad CD = 58\sqrt{3}$ m.

Let $BP = x$ metres. Then,

$$BN = (BP + PN) = (BP + CD) = (x + 58\sqrt{3})\text{ m and } DN = CP.$$

From right $\triangle BND$, we have

$$\frac{DN}{BN} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{DN}{(x+58\sqrt{3})\text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow DN = \frac{(x+58\sqrt{3})}{\sqrt{3}}\text{ m.} \quad \ldots \text{(i)}$$

From right $\triangle BPC$, we have

$$\frac{CP}{BP} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{CP}{x\text{ m}} = \sqrt{3} \Rightarrow CP = (x\sqrt{3})\text{ m.} \quad \ldots \text{(ii)}$$

$$\text{Now, } DN = CP \Rightarrow \frac{(x+58\sqrt{3})}{\sqrt{3}} = (x\sqrt{3})$$

$$\Rightarrow (3x - x) = 58\sqrt{3} \Rightarrow 2x = 58\sqrt{3} \Rightarrow x = 29\sqrt{3}.$$

From (ii), we get

$$CP = (29\sqrt{3} \times \sqrt{3})\text{ m} = (29 \times 3)\text{ m} = 87\text{ m.}$$

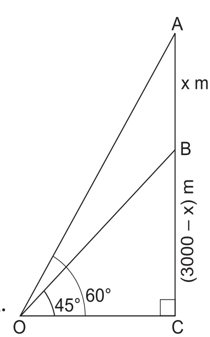
$\therefore$ height of the balloon from the ground

$$= CL = CP + PL = CP + AB = 87\text{ m} + 1.3\text{ m} = 88.3\text{ m.}$$

Hence, the height of the balloon from the ground is 88.3 metres.

EXAMPLE 22 *An aeroplane when flying at a height of* 3000 *metres from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are* 60° *and* 45° *respectively. Find the vertical distance between the aeroplanes at that instant.* [*Take* $\sqrt{3} = 1.73$.] [CBSE 2008]

SOLUTION Let O be the point of observation. Let A and B be the positions of the two planes at the given instant when A is vertically above B.



Let AB when produced meet the ground at C.

Then, $\angle COA = 60°$, $\angle COB = 45°$, $\angle OCB = \angle OCA = 90°$ and $AC = 3000$ m.

Let $AB = x$ metres. Then, $BC = (3000 - x)$m.

From right $\triangle OCB$, we have

$$\frac{OC}{BC} = \cot 45^\circ = 1 \Rightarrow \frac{OC}{(3000 - x)\text{ m}} = 1$$

$\Rightarrow \quad OC = (3000 - x)$ m. ... (i)

From right $\triangle OCA$, we have

$$\frac{OC}{AC} = \cot 60^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{OC}{3000\text{ m}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad OC = \left(\frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right)\text{ m} = 1000\sqrt{3}$ m. ... (ii)

From (i) and (ii), we get

$$(3000 - x) = 1000\sqrt{3}$$

$\Rightarrow \quad x = (3000 - 1000\sqrt{3}) = (3000 - 1000 \times 1.73)$

$\qquad = (3000 - 1730) = 1270.$

Hence, the required distance between the two aeroplanes is 1270 metres.

EXAMPLE 23 *A man standing on the deck of a ship, which is* 10 m *above the water level, observes the angle of elevation of the top of a hill as* 60°, *and the angle of depression of the base of the hill as* 30 . *Find the distance of the hill from the ship and the height of the hill.* [CBSE 2006, '10]

SOLUTION Let AB be the deck and CD be the hill.

Let the man be at B.

Then, $AB = 10$ m.

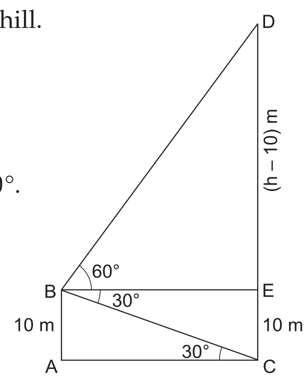
Let $BE \perp CD$ and $AC \perp CD$.

Then, $\angle EBD = 60^\circ$ and $\angle EBC = 30^\circ$.

$\therefore \quad \angle ACB = \angle EBC = 30^\circ$.

Let $CD = h$ metres.

Then, $CE = AB = 10$ m and $ED = (h - 10)$ m.



From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{AC}{10\text{ m}} = \sqrt{3}$$

$\Rightarrow \quad AC = 10\sqrt{3}$ m. ... (i)

$\therefore \quad BE = AC = 10\sqrt{3}$ m.

From right $\triangle BED$, we have

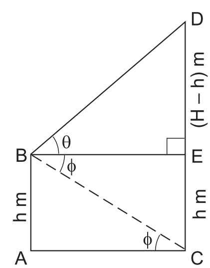
$$\frac{DE}{BE} = \tan 60° = \sqrt{3} \Rightarrow \frac{h-10}{10\sqrt{3}} = \sqrt{3} \quad \text{[using (i)]}$$

$\Rightarrow \quad h - 10 = 30 \Rightarrow h = 40.$

Hence, the distance of the ship from the hill is $10\sqrt{3}$ metres and the height of the hill is 40 metres.

EXAMPLE 24 *From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are* θ *and* ϕ *respectively. Show that the height of the opposite house is* $h(1 + \tan\theta \cot\phi)$ *metres.* [CBSE 2006]

SOLUTION Let AB be the house with window at B and let CD be the another house. Then, $AB = h$ metres.



Draw $BE \parallel AC$, meeting CD at E. Then,

$\angle EBD = \theta$ and $\angle ACB = \angle EBC = \phi$.

Let $CD = H$ metres. Then,

$CE = AB = h$ metres and

$ED = (H - h)$ m.

From right $\triangle ACB$, we have

$$\frac{AC}{AB} = \cot\phi \Rightarrow \frac{AC}{h} = \cot\phi \Rightarrow AC = h\cot\phi \text{ metres.}$$

From right $\triangle BED$, we have

$$\frac{DE}{BE} = \tan\theta \Rightarrow \frac{(H-h)}{h\cot\phi} = \tan\theta \qquad [\because BE = AC = h\cot\phi \text{ m}]$$

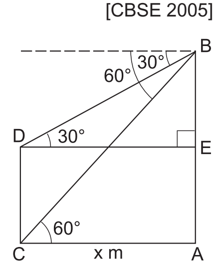
$\Rightarrow \quad (H - h) = h\tan\theta \cot\phi$

$\Rightarrow \quad H = h(1 + \tan\theta \cot\phi).$

Hence, the height of the opposite house is $h(1 + \tan\theta \cot\phi)$ metres.

EXAMPLE 25 *From the top of a building* 60 m *high, the angles of depression of the top and bottom of a tower are observed to be* 30° *and* 60°*. Find the height of the tower.* [CBSE 2005]

SOLUTION Let AB be the building and CD be the tower such that $\angle BDE = 30°$, $\angle BCA = 60°$ and $AB = 60$ m.



Let $CA = DE = x$ metres.

From right $\triangle CAB$, we have

$$\frac{CA}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{60} = \frac{1}{\sqrt{3}} \Rightarrow x = 60 \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 60 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$$

$$\Rightarrow CA = DE = 20\sqrt{3} \text{ m.} \quad \ldots \text{(i)}$$

From right $\triangle BED$, we have

$$\frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3} \text{ m}} \Rightarrow \frac{BE}{20\sqrt{3} \text{ m}} = \frac{1}{\sqrt{3}} \quad \text{[using (i)]}$$

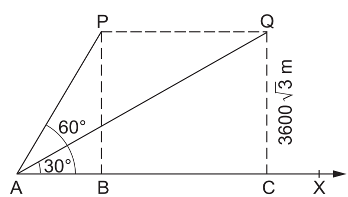
$$\Rightarrow BE = 20\sqrt{3} \times \frac{1}{\sqrt{3}} \text{ m} = 20 \text{ m.}$$

$$\therefore CD = AE = AB - BE = 60 \text{ m} - 20 \text{ m} - 40 \text{ m.}$$

Hence, the height of the tower is 40 m.

EXAMPLE 26 *The angle of elevation of a jet plane from a point A on the ground is* 60°. *After a flight of 30 seconds, the angle of elevation changes to* 30°. *If the jet plane is flying at a constant height of* $3600\sqrt{3}$ *metres, find the speed of the jet plane.* [CBSE 2008]

SOLUTION Let A be the point of observation and let AX be a horizontal line through A and $QC \perp AX$. Let P and Q be the two positions of the plane. Let $PB \perp AX$.



Then, $PB = QC = 3600\sqrt{3}$ m, $\angle BAP = 60^\circ$ and $\angle BAQ = 30^\circ$.

From right $\triangle ABP$, we have

$$\frac{AB}{BP} = \cot 60^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{AB}{3600\sqrt{3} \text{ m}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB = 3600\sqrt{3} \times \frac{1}{\sqrt{3}} \text{ m} = 3600 \text{ m.} \quad \ldots \text{(i)}$$

Let $BC = PQ = x$ metres.

Then, $AC = AB + BC = (x + 3600)$ m [using (i)].

From right $\triangle ACQ$, we have

$$\frac{AC}{CQ} = \cot 30° = \sqrt{3} \Rightarrow \frac{x+3600}{3600\sqrt{3}} = \sqrt{3}$$

$\Rightarrow \quad x + 3600 = (3600 \times 3) = 10800$

$\Rightarrow \quad x = 10800 - 3600 = 7200.$

Thus, $PQ = 7200$ metres.

Now, 7200 m is covered in 30 seconds.

$\therefore$ speed of the jet plane $= \left(\frac{7200}{30} \times \frac{60 \times 60}{1000}\right)$ km/hr

$= 864$ km/hr.

Hence, the speed of the jet plane is 864 km/hr.

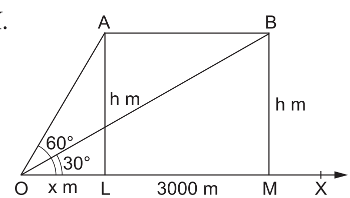
EXAMPLE 27 *The angle of elevation of a jet fighter from a point A on the ground is* 60°. *After a flight of* 15 *seconds, the angle of elevation changes to* 30°. *If the jet is flying at a speed of* 720 km/hour, *find the constant height at which the jet is flying.* [*Use* $\sqrt{3} = 1.732$.] [CBSE 2008]

SOLUTION Let O be the point of observation on the ground OX.

Let A and B be the two positions of the jet.

Then, $\angle XOA = 60°$ and $\angle XOB = 30°$.

Draw $AL \perp OX$ and $BM \perp OX$.



Let $AL = BM = h$ metres.

Speed of the jet

$= \left(720 \times \frac{5}{18}\right)$ m/s

$= 200$ m/s.

Time taken to cover the distance $AB = 15$ s.

Distance covered = speed × time

= 200 m/s × 15 s

= 200 × 15 m = 3000 m.

$\therefore \quad LM = AB = 3000$ m.

Let $OL = x$ metres.

From right $\triangle OLA$, we have

$$\frac{OL}{AL} = \cot 60° = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}$. ... (i)

From right $\triangle OMB$, we have

$$\frac{OM}{BM} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \quad \frac{(x+3000)}{h} = \sqrt{3} \Rightarrow x = (h\sqrt{3} - 3000). \quad \text{... (ii)}$$

Equating the values of x from (i) and (ii), we get

$$\frac{h}{\sqrt{3}} = h\sqrt{3} - 3000 \Rightarrow h = 3h - 3000\sqrt{3}$$

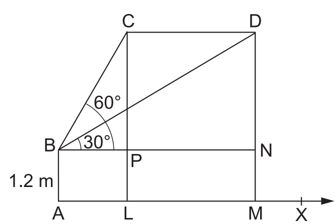
$$\Rightarrow \quad 2h = 3000\sqrt{3} \Rightarrow h = 1500\sqrt{3} = 1500 \times 1.732 = 2598.$$

Hence, the required height is 2598 m.

EXAMPLE 28 *A* 1.2-m-*tall girl spots a balloon moving with the wind in a horizontal line at a height of* 88.2 *metres from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is* $60°$. *After some time, the angle of elevation reduces to* $30°$. *Find the distance travelled by the balloon during the interval.*

SOLUTION Let AB be the position of the girl and AX be the horizontal ground. Let C and D be the two positions of the balloon.

Draw $CL \perp AX$, $DM \perp AX$ and $BN \perp DM$, intersecting CL at P.



Then, $\angle CBP = 60°$, $\angle DBN = 30°$, $AB = PL = NM = 1.2$ m and $CL = DM = 88.2$ m.

$\therefore \quad CP = 88.2 \text{ m} - 1.2 \text{ m} = 87 \text{ m}.$

From right $\triangle BPC$, we have

$$\frac{BP}{CP} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow \frac{BP}{87 \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow BP = \frac{87 \text{ m}}{\sqrt{3}}$$

$$\Rightarrow \quad BP = \frac{87 \text{ m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 29\sqrt{3} \text{ m}.$$

From right $\triangle BND$, we have

$$\frac{BN}{DN} = \cot 30° = \sqrt{3} \Rightarrow \frac{BP + PN}{CP} = \sqrt{3}$$

$$\Rightarrow \quad \frac{29\sqrt{3} \text{ m} + CD}{87 \text{ m}} = \sqrt{3} \quad [\because PN = CD \text{ and } DN = CP]$$

$\Rightarrow \quad 29\sqrt{3}\text{ m} + CD = 87\sqrt{3}\text{ m}$

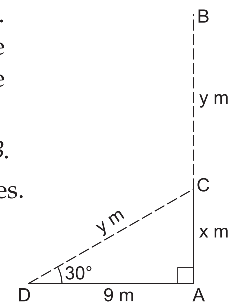
$\Rightarrow \quad CD = 87\sqrt{3}\text{ m} - 29\sqrt{3}\text{ m} = 58\sqrt{3}\text{ m}.$

Hence, the required distance travelled by the balloon is $58\sqrt{3}$ m.

EXAMPLE 29 *A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle* 30° *with it. The distance between the foot of the tree to the point where the top touches the ground is* 9 m. *Find the height of the tree.*

SOLUTION Let AB be the original height of the tree. Suppose it got bent at a point C and let the part CB take the position CD, meeting the ground at D. Then,

$$AD = 9\text{ m}, \angle ADC = 30° \text{ and } CD = CB.$$



Let $AC = x$ metres and $CD = CB = y$ metres.

From right ΔDAC, we have

$$\frac{AC}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{9}{\sqrt{3}} \Rightarrow x = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3}.$$

Also, from right ΔDAC, we have

$$\frac{CD}{AD} = \sec 30° = \frac{2}{\sqrt{3}} \Rightarrow \frac{y}{9} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad y = \frac{18}{\sqrt{3}} \Rightarrow y = \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 6\sqrt{3}.$$

$\therefore \quad AC = 3\sqrt{3}\text{ m and } CB = 6\sqrt{3}\text{ m}.$

Total height of the tree $= 3\sqrt{3}\text{ m} + 6\sqrt{3}\text{ m} = 9\sqrt{3}$ metres.

EXAMPLE 30 *A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is* 60°. *When he moves* 30 *metres away from the bank, he finds the angle of elevation to be* 30°. *Find the height of the tree and the width of the river.* [*Take* $\sqrt{3} = 1.732$.] [CBSE 2008C]

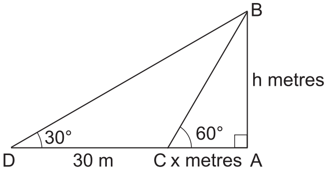
SOLUTION Let AB be the tree and AC be the river.

Let C and D be the two positions of the person.

Then, $\angle ACB = 60°$, $\angle ADB = 30°$, $\angle DAB = 90°$ and $CD = 30$ m.

Let $AB = h$ metres and $AC = x$ metres.

From right $\triangle CAB$, we have



$$\frac{AC}{AB} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}. \quad \ldots \text{(i)}$$

From right $\triangle DAB$, we have

$$\frac{AD}{AB} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \frac{x+30}{h} = \sqrt{3} \Rightarrow x = \sqrt{3}h - 30. \quad \ldots \text{(ii)}$$

Equating the values of x from (i) and (ii), we get

$$\frac{h}{\sqrt{3}} = \sqrt{3}h - 30 \Rightarrow h = 3h - 30\sqrt{3}$$

$$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98.$$

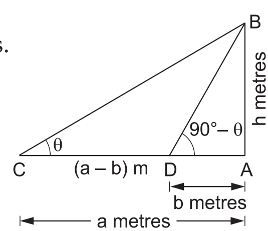
Putting $h = 15\sqrt{3}$ in (i), we get $x = \frac{15\sqrt{3}}{\sqrt{3}} = 15$.

Hence, the height of the tree is 25.98 m and the width of the river is 15 metres.

EXAMPLE 31 *The angles of elevation of the top of a tower from two points on the ground at distances a metres and b metres from the base of the tower and in the same straight line are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.* [HOTS] [CBSE 2000C, '05C]

SOLUTION Let AB be the tower and let C and D be the two positions of the observer. Then,

$AC = a$ metres and $AD = b$ metres.



Let $\angle ACB = \theta$. Then, $\angle ADB = (90° - \theta)$.

Let $AB = h$ metres.

From right $\triangle DAB$, we have

$$\frac{AB}{AD} = \tan(90° - \theta) \Rightarrow \frac{h}{b} = \cot\theta$$

$$\Rightarrow h = b\tan\theta. \quad \cdots \text{(i)}$$

From right $\triangle CAB$, we have

$$\frac{AB}{AC} = \tan\theta \Rightarrow \frac{h}{a} = \tan\theta \Rightarrow h = a\tan\theta. \quad \cdots \text{(ii)}$$

From (i) and (ii), we get $h^2 = ab \Rightarrow h = \sqrt{ab}$.

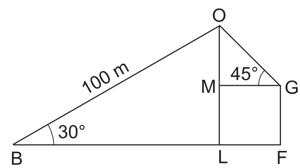
Hence, the height of the tower is $\sqrt{ab}$ metres.

EXAMPLE 32 *A boy standing on a horizontal plane finds a bird flying at a distance of* 100 m *from him at an elevation of* 30°. *A girl standing on the roof of a* 20-m-*high building, finds the angle of elevation of the same bird to be* 45°. *The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl.* [*Given* $\sqrt{2} = 1.41$.] [HOTS]

[CBSE 2007]

SOLUTION Let O be the position of the bird, B be the position of the boy and FG be the building at which G is the position of the girl.

Let OL BF and GM OL. Then,



$BO = 100$ m, $\angle OBL = 30°$,

$FG = 20$ m and $\angle OGM = 45°$.

From right $\triangle OLB$, we have

$$\frac{OL}{BO} = \sin 30° \Rightarrow \frac{OL}{100 \text{ m}} = \frac{1}{2}$$

$$\Rightarrow OL = 100 \text{ m} \times \frac{1}{2} = 50 \text{ m}.$$

$$OM = OL - ML = OL - FG = 50 \text{ m} - 20 \text{ m} = 30 \text{ m}.$$

From right $\triangle OMG$, we have

$$\frac{OM}{OG} = \sin 45° = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad OG = \sqrt{2} \times OM = \sqrt{2} \times 30 \text{ m}$$

$$\Rightarrow \quad OG = 30 \times 1.41 \text{ m} = 42.3 \text{ m}.$$

Distance of the bird from the girl = 42.3 m.

EXAMPLE 33 *A* 1.5-m-*tall boy is standing at some distance from a* 30-m-*tall building. The angle of elevation from his eyes to the top of the building increases from* 30° *to* 60° *as he walks towards the building. Find the distance he walked towards the building.* [HOTS]

SOLUTION Let AB be the building and let CD and EF be the two positions of the boy. Draw $DFG \parallel CEA$. Then,

$CD = EF = 1.5$ m, $\angle GDB = 30°$ and $\angle GFB = 60°$,

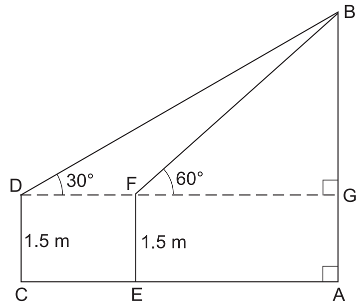
$AB = 30$ m, $GB = 30 \text{ m} - 1.5 \text{ m} = 28.5$ m.

From right $\triangle DGB$, we have

$$\frac{DG}{GB} = \cot 30° \Rightarrow \frac{DG}{28.5 \text{ m}} = \sqrt{3} \Rightarrow DG = \frac{57\sqrt{3}}{2} \text{ m}.$$

From right $\triangle FGB$, we have

$$\frac{FG}{GB} = \cot 60° \Rightarrow \frac{FG}{28.5 \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow FG = \frac{57}{2\sqrt{3}} \text{ m}.$$



$$\therefore \quad DF = DG - FG = \left(\frac{57\sqrt{3}}{2} - \frac{57}{2\sqrt{3}}\right) \text{m} = \left(\frac{171-57}{2\sqrt{3}}\right) \text{m}$$

$$= \frac{114}{2\sqrt{3}} \text{ m} = \frac{57}{\sqrt{3}} \text{ m} = \frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ m} = 19\sqrt{3} \text{ m}.$$

Hence, the required distance is $19\sqrt{3}$ m.

EXERCISE 14

1. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of its top is found to be 60°. Find the height of the tower. [Take $\sqrt{3} = 1.732$.]
2. A kite is flying at a height of 75 m from the level ground, attached to a string inclined at 60° to the horizontal. Find the length of the string, assuming that there is no slack in it. [Take $\sqrt{3} = 1.732$.]
3. An observer 1.5 m tall is 30 m away from a chimney. The angle of elevation of the top of the chimney from his eye is 60°. Find the height of the chimney. [CBSE 2013C]
4. The angles of elevation of the top of a tower from two points at distances of 5 metres and 20 metres from the base of the tower and in the same straight line with it, are complementary. Find the height of the tower. [CBSE 2014]
5. The angle of elevation of the top of a tower at a distance of 120 m from a point *A* on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at *A* is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.732$.] [CBSE 2014]
6. From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The angle of elevation of

the top of a water tank (on the top of the tower) is 45°. Find (i) the height of the tower, (ii) the depth of the tank.

7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 6 m. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 30° and that of the top of the flagstaff is 60°. Find the height of the tower. [Use $\sqrt{3} = 1.732$.] [CBSE 2011]

8. A statue 1.46 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. [Use $\sqrt{3} = 1.73$.] [CBSE 2008]

9. The angle of elevation of the top of an unfinished tower at a distance of 75 m from its base is 30°. How much higher must the tower be raised so that the angle of elevation of its top at the same point may be 60°? [Take $\sqrt{3} = 1.732$.]

10. On a horizontal plane there is a vertical tower with a flagpole on the top of the tower. At a point, 9 metres away from the foot of the tower, the angle of elevation of the top and bottom of the flagpole are 60° and 30° respectively. Find the height of the tower and the flagpole mounted on it. [Take $\sqrt{3} = 1.73$.] [CBSE 2005]

11. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point *P* between them on the road, the angle of elevation of the top of one pole is 60° and the angle of depression from the top of another pole at *P* is 30°. Find the height of each pole and distances of the point *P* from the poles. [CBSE 2015]

12. Two men are on opposite sides of a tower. They measure the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 50 metres, find the distance between the two men. [Take $\sqrt{3} = 1.732$.]

13. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45° respectively. Find the distance between the cars. [Take $\sqrt{3} = 1.732$.] [CBSE 2011, '17]

14. A straight highway leads to the foot of a tower. A man standing on the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower form this point.

[**HINT** See Solved Example 14.]

15. A TV tower stands vertically on a bank of canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

16. The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building. [CBSE 2013]

17. The horizontal distance between two towers is 60 metres. The angle of depression of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 90 metres, find the height of the first tower. [Use $\sqrt{3} = 1.732$.]

18. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 metres, find the height of the chimney.

    According to pollution control norms, the minimum height of a smoke-emitting chimney should be 100 metres. State if the height of the above-mentioned chimney meets the pollution norms. What value is discussed in this question? [CBSE 2014]

19. From the top of a 7-metre-high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. [Use $\sqrt{3} = 1.732$.] [CBSE 2017]

20. The angle of depression from the top of a tower of a point A on the ground is 30°. On moving a distance of 20 metres from the point A towards the foot of the tower to a point B, the angle of elevation of the top of the tower from the point B is 60°. Find the height of the tower and its distance from the point A. [CBSE 2012]

21. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 30°. Find the height of the tower. [CBSE 2011]

22. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}$-m-high cliff are 45° and 60° respectively. Find the height of the tower. [CBSE 2012]

23. A man on the deck of a ship, 16 m above water level, observes that the angles of elevation and depression respectively of the top and bottom

of a cliff are 60° and 30°. Calculate the distance of the cliff from the ship and height of the cliff. [Take $\sqrt{3} = 1.732$.] [CBSE 2007C]

24. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°. Find the height of tower PQ. [Take $\sqrt{3} = 1.73$.] [CBSE 2003C]

25. The angle of elevation of an aeroplane from a point on the ground is 45°. After flying for 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 2500 metres, find the speed of the aeroplane.

26. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 m towards the foot of the tower, the angle of elevation becomes 60°. Show that the height of the tower is 129.9 metres. [Given $\sqrt{3} = 1.732$.] [CBSE 2006C]

27. As observed from the top of a lighthouse, 100 m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation. [Use $\sqrt{3} = 1.732$.] [CBSE 2004, '08C]

28. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 2.5 m from the banks, find the width of the river. [Take $\sqrt{3} = 1.732$.]

29. The angles of elevation of the top of a tower from two points at distances of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Show that the height of the tower is 6 metres.

30. A ladder of length 6 metres makes an angle of 45° with the floor while leaning against one wall of a room. If the foot of the ladder is kept fixed on the floor and it is made to lean against the opposite wall of the room, it makes an angle of 60° with the floor. Find the distance between two walls of the room. [CBSE 2011]

31. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [CBSE 2011]

32. An electrician has to repair an electric fault on a pole of height 4 metres. He needs to reach a point 1 metre below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, which when inclined at an angle of 60° to the horizontal would enable him to reach the required position? [Use $\sqrt{3} = 1.73$.]

33. From the top of a building *AB*, 60 m high, the angles of depression of the top and bottom of a vertical lamp-post *CD* are observed to be 30° and 60° respectively. Find
    (i) the horizontal distance between *AB* and *CD*,
    (ii) the height of the lamp-post,
    (iii) the difference between the heights of the building and the lamp-post. [CBSE 2009]

34. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower. [CBSE 2017]

35. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$.] [CBSE 2017]

36. From a point on the ground the angles of elevation of the bottom and top of a communication tower fixed on the top of a 20-m-high building are 45° and 60° respectively. Find the height of the tower. [Take $\sqrt{3} = 1.732$.] [CBSE 2017]

37. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. [CBSE 2017]

38. If at some time of the day the ratio of the height of a vertically standing pole to the length of its shadow on the ground is $\sqrt{3}:1$ then find the angle of elevation of the sun at that time. [CBSE 2017]

ANSWERS (EXERCISE 14)

1. 34.64 m **2.** 86.6 m **3.** 53.46 m **4.** 10 m **5.** 87.84 m
6. (i) 23.1 m (ii) 16.9 m **7.** 3 m **8.** 2 m **9.** 86.6 m
10. 5.19 m, 10.38 m
11. $20\sqrt{3}$ m; 20 m from left pole and 60 m from right pole
12. 136.6 m **13.** 273.2 m **14.** 3 seconds **15.** $10\sqrt{3}$ m, 10 m **16.** 20 m
17. 55.36 m **18.** 120 m **19.** 19.12 m **20.** 17.32 m, 30 m **21.** 15 m
22. 43.92 m **23.** 27.71 m, 64 m **24.** 94.6 m **25.** 439.2 km/hr
27. 115.46 m **28.** 6.83 m **30.** 7.24 m **31.** 136.6 m **32.** 3.46 m

33. (i) 34.64 m (ii) 40 m (iii) 20 m **34.** $6(\sqrt{3}+1)$ minutes

35. 473.2 m **36.** 14.64 m **37.** $\frac{\sqrt{3}+1}{2}$ km **38.** 60°

HINTS TO SOME SELECTED QUESTIONS

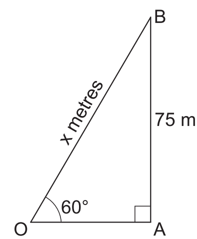
2. Let the length of the string be x metres.

Then, $\frac{x}{75} = \text{cosec } 60° = \frac{2}{\sqrt{3}}$

$\Rightarrow \quad x = 75 \times \frac{2}{\sqrt{3}}$

$\therefore \quad x = 75 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 50\sqrt{3}.$



3. Let AB be the chimney and CD be the observer.

Draw $DE \perp AB$.

Then, $AE = CD = 1.5$ m.

Let $AB = x$ metres. Then, $BE = (x - 1.5)$ m.

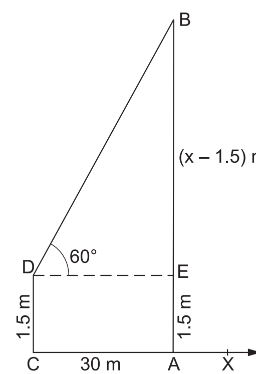
$\angle EDB = 60°$.

Also, $DE = CA = 30$ m.

From right $\triangle BED$, we have

$$\frac{BE}{DE} = \tan 60°$$

$$\Rightarrow \quad \frac{x - 1.5}{30} = \sqrt{3}.$$



4. $\frac{AB}{AC} = \tan\theta \Rightarrow \frac{h}{5} = \tan\theta$

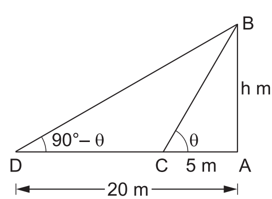
$\Rightarrow \quad h = 5\tan\theta.$ ... (i)

$\frac{AB}{AD} = \tan(90° - \theta) = \cot\theta.$

$\Rightarrow \quad \frac{h}{20} = \cot\theta \Rightarrow h = 20\cot\theta$ ... (ii)

Multiplying (i) and (ii), we get

$\therefore \quad h^2 = 100\tan\theta \times \cot\theta = 100 \Rightarrow h = 10.$

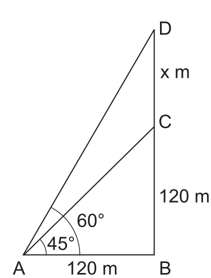


5. Let BC be the tower and CD be the flagstaff and let AB be the horizontal ground such that $AB = 120$ m, $\angle BAC = 45°$ and $\angle BAD = 60°$. Then,

$$\frac{BC}{AB} = \tan 45° = 1 \Rightarrow \frac{BC}{120 \text{ m}} = 1 \Rightarrow BC = 120 \text{ m}.$$

Let $CD = x$ metres. Then,

$$\frac{BD}{AB} = \tan 60° = \sqrt{3} \Rightarrow \frac{120 + x}{120} = \sqrt{3}. \text{ Find } x.$$



6. Let BC be the tower and CD be the water tank.

Let A be the point of observation. Then, $\angle BAC = 30°$, $\angle BAD = 45°$ and $AB = 40$ m.

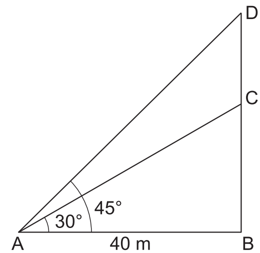
From right $\triangle ABD$, we have

$$\frac{BD}{AB} = \tan 45° = 1 \Rightarrow \frac{BD}{40 \text{ m}} = 1 \Rightarrow BD = 40 \text{ m}.$$

From right $\triangle ABC$, we have

$$\frac{BC}{AB} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{BC}{40 \text{ m}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BC = \frac{40 \text{ m}}{\sqrt{3}} \Rightarrow BC = \frac{40 \text{ m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ m}.$$



(i) Height of the tower $= BC = \frac{40\sqrt{3}}{3}$ m $= 23.1$ m.

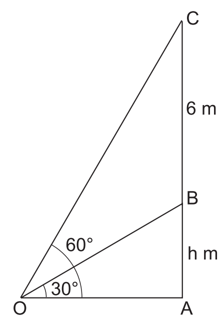
(ii) Depth of the tank $= CD = (BD - BC) = (40 - 23.1)$ m $= 16.9$ m.

7. $\frac{OA}{AB} = \cot 30° \Rightarrow \frac{OA}{h \text{ m}} = \sqrt{3} \Rightarrow OA = h\sqrt{3}$ m. ... (i)

$\frac{OA}{AC} = \cot 60° \Rightarrow \frac{OA}{(h+6) \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow OA = \frac{(h+6)}{\sqrt{3}}$ m. ... (ii)

$$\therefore \quad h\sqrt{3} = \frac{(h+6)}{\sqrt{3}} \Rightarrow 3h = h + 6.$$

$$\Rightarrow \quad 2h = 6 \Rightarrow h = 3.$$



9. Let AB be the unfinished tower and let AC be the complete tower. Let O be the point of observation.

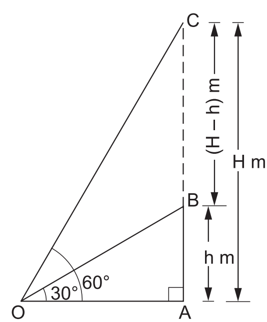
Then, $OA = 75$ m, $\angle AOB = 30°$ and $\angle AOC = 60°$.

Let $AB = h$ metres and $AC = H$ metres.

$$\frac{AB}{OA} = \tan 30° \Rightarrow \frac{h}{75 \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow h = 25\sqrt{3} \text{ m}.$$

And, $\frac{AC}{OA} = \tan 60° \Rightarrow \frac{H}{75 \text{ m}} = \sqrt{3} \Rightarrow H = 75\sqrt{3}$ m.

Hence, the required height is $(H - h)$ m $= (50\sqrt{3})$ m.



11. From right $\triangle PAB$, we have

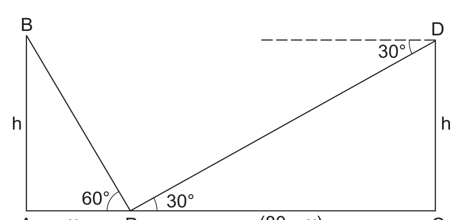
$$\frac{AB}{AP} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad h = x\sqrt{3}. \quad \text{... (i)}$$

From right $\triangle PCD$, we have

$$\frac{CD}{PC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{(80 - x)}{\sqrt{3}}. \quad \text{.... (ii)}$$



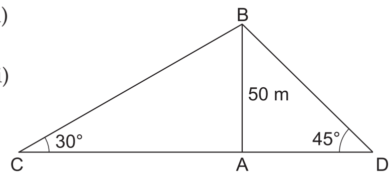
$\therefore \quad x\sqrt{3} = \dfrac{(80 - x)}{\sqrt{3}} \Rightarrow 3x + x = 80$

$\Rightarrow \quad 4x = 80 \Rightarrow x = 20$

$\therefore \quad$ height $= 20\sqrt{3}$ m.

P is 20 m from left pole and 60 m from right pole.

12. $\dfrac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow AC = 50\sqrt{3}$ m. ... (i)



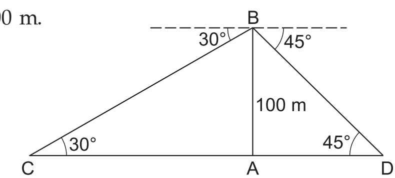
$\dfrac{AD}{AB} = \cot 45° = 1 \Rightarrow AD = 50$ m. ... (ii)

Required distance

$= AC + AD = 50(\sqrt{3} + 1)$ m

$= (50 \times 2.732)$ m $= 136.6$ m.

13. From right ΔBAD, we have



$\dfrac{AD}{AB} = \cot 45° = 1 \Rightarrow \dfrac{AD}{100 \text{ m}} \Rightarrow AD = 100$ m.

From right ΔBAC, we have

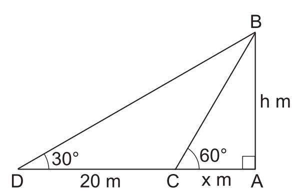
$\dfrac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \dfrac{AC}{100 \text{ m}} = \sqrt{3}$

$\Rightarrow \quad AC = (100 \times 1.732)$ m $= 173.2$ m.

$\therefore \quad$ distance between the cars

$= (173.2 + 100)$ m $= 273.2$ m.

15. Let AB be the tower and AC be the canal.



$\dfrac{x}{h} = \cot 60° = \dfrac{1}{\sqrt{3}} \Rightarrow x = \dfrac{h}{\sqrt{3}}$. ... (i)

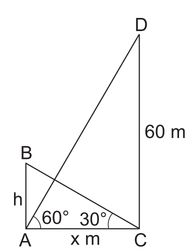
$\dfrac{(x + 20)}{h} = \cot 30° = \sqrt{3} \Rightarrow x = (h\sqrt{3} - 20)$. ... (ii)

From (i) and (ii), we get $\dfrac{h}{\sqrt{3}} = (h\sqrt{3} - 20)$.

$\therefore \quad h = (3h - 20\sqrt{3}) \Rightarrow 2h = 20\sqrt{3} \Rightarrow h = 10\sqrt{3}$.

Putting $h = 10\sqrt{3}$ in (i), we get x m $= 10$ m (width of the canal).

16. Let AB be the building and CD be the tower.



From right ΔACD, we have

$\dfrac{AC}{CD} = \cot 60° = \dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{x}{60} = \dfrac{1}{\sqrt{3}} \Rightarrow x = \dfrac{60}{\sqrt{3}}$

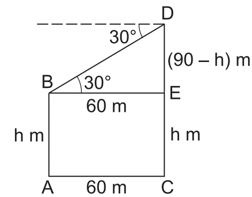
$\Rightarrow \quad x = \dfrac{60}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$.

From right ΔBAC, we have

$\dfrac{AB}{AC} = \tan 30° = \dfrac{1}{\sqrt{3}} \Rightarrow \dfrac{h}{x} = \dfrac{1}{\sqrt{3}} \Rightarrow h = \dfrac{x}{\sqrt{3}}$

$\Rightarrow \quad h = \dfrac{20\sqrt{3}}{\sqrt{3}} = 20$.

17. From right $\triangle BED$, we have



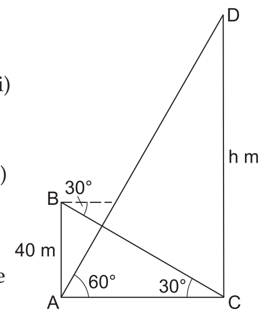
$$\frac{90-h}{60} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow 90-h = \frac{60}{\sqrt{3}}$$

$$\Rightarrow \quad 90-h = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$$

$$\Rightarrow \quad h = 90 - 20\sqrt{3} = 90 - 20 \times 1.732$$

$$\Rightarrow \quad h = 90 - 34.64 = 55.36.$$

18. Let AB be the tower and CD be the chimney.



From right $\triangle ACD$, we have

$$\frac{AC}{CD} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow \frac{AC}{h} = \frac{1}{\sqrt{3}} \Rightarrow AC = \frac{h}{\sqrt{3}}. \quad \text{... (i)}$$

From right $\triangle CAB$, we have

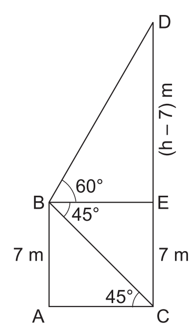
$$\frac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow AC = 40\sqrt{3}. \quad \text{... (ii)}$$

From (i) and (ii), we get $\frac{h}{\sqrt{3}} = 40\sqrt{3} \Rightarrow h = 120.$

Clearly, the height of the given chimney meets the pollution norms.

We should comply with the prescribed rules and contribute to the cleanliness of the environment.

19. Let AB be the building and CD be the cable tower.



Draw $BE \perp CD$. Let $CD = h$ metres.

Then, $CE = AB = 7$ m and $DE = (h-7)$ m.

From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 45° = 1 \Rightarrow \frac{AC}{7 \text{ m}} = 1$$

$$\Rightarrow \quad AC = 7.$$

$$\therefore \quad BE = AC = 7 \text{ m}.$$

From right $\triangle BED$, we have

$$\frac{DE}{BE} = \tan 60° = \sqrt{3} \Rightarrow \frac{h-7}{7} = \sqrt{3}.$$

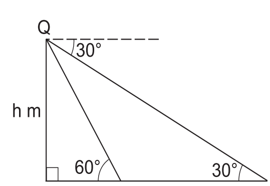
$$\therefore \quad h = 7\sqrt{3} + 7 = 7(\sqrt{3}+1) = 7(1.732+1)$$

$$\Rightarrow \quad h = (7 \times 2.732) = 19.12.$$

20. From right $\triangle APQ$, we have

$$\frac{PQ}{PA} = \tan 30° \Rightarrow \frac{h}{x+20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{x+20}{\sqrt{3}}. \quad \text{... (i)}$$



From right $\triangle BPQ$, we have

$$\frac{PQ}{PB} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow h = x\sqrt{3}. \quad \text{... (ii)}$$

From (i) and (ii), we get

$$\frac{(x+20)}{\sqrt{3}} = x\sqrt{3} \Rightarrow x = 10.$$

$\therefore \quad h = 10\sqrt{3} = 10 \times 1.732 = 17.32.$

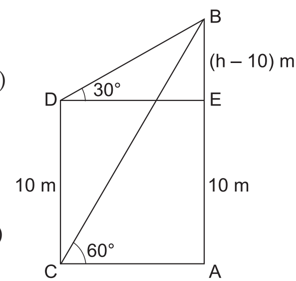
Distance $AP = (20 + x)$ m $= (20 + 10)$ m $= 30$ m.

21. Let the height of the tower be h metres.

From right $\triangle CAB$, we have

$$\frac{CA}{AB} = \cot 60° \Rightarrow \frac{CA}{h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow CA = \frac{h}{\sqrt{3}}. \quad \text{... (i)}$$



From right $\triangle BED$, we have

$$\frac{DE}{BE} = \cot 30° \Rightarrow \frac{DE}{(h-10)} = \sqrt{3}$$

$$\Rightarrow DE = \sqrt{3}(h-10). \quad \text{... (ii)}$$

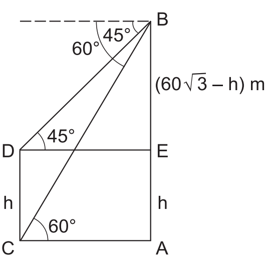
But, $CA = DE$.

$$\therefore \quad \frac{h}{\sqrt{3}} = \sqrt{3}(h-10) \Rightarrow 3h - 30 = h$$

$$\Rightarrow 2h = 30 \Rightarrow h = 15.$$

22. Let AB be the cliff and CD be the tower.

Let $AE = CD = h$ metres. Then, $BE = (60\sqrt{3} - h)$ m.



From right $\triangle BED$, we have

$$\frac{DE}{BE} = \cot 45° \Rightarrow \frac{DE}{60\sqrt{3} - h} = 1$$

$$\Rightarrow DE = 60\sqrt{3} - h. \quad \text{... (i)}$$

From right $\triangle CAB$, we have

$$\frac{CA}{AB} = \cot 60° \Rightarrow \frac{CA}{60\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow CA = \frac{60\sqrt{3}}{\sqrt{3}} = 60. \quad \text{... (ii)}$$

But, $CA = DE$.

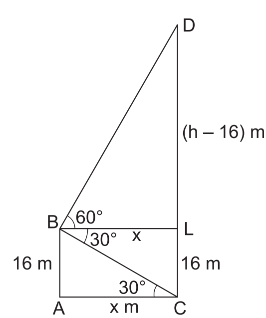
$$\therefore \quad 60\sqrt{3} - h = 60 \Rightarrow h = 60(\sqrt{3} - 1) = 60 \times (1.732 - 1)$$

$$= (60 \times 0.732) = 43.92.$$

23. Let the height of the cliff be h metres and the distance of the cliff from the ship be x metres.

From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 30° \Rightarrow \frac{x}{16} = \sqrt{3} \Rightarrow x = 16\sqrt{3}.$$



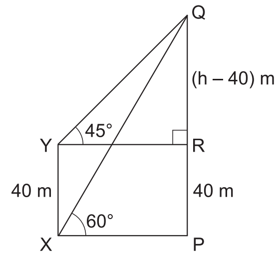
From right $\triangle BLD$, we have

$$\frac{DL}{BL} = \tan 60° = \sqrt{3} \Rightarrow \frac{h-16}{16\sqrt{3}} = \sqrt{3}$$

$$[\because BL = x = 16\sqrt{3}]$$

$$\Rightarrow h - 16 = 48 \Rightarrow h = 64.$$

24. Let the height of tower PQ be h metres.



From right $\triangle XPQ$, we have

$$\frac{XP}{PQ} = \cot 60° \Rightarrow \frac{XP}{h} = \frac{1}{\sqrt{3}} \Rightarrow XP = \frac{h}{\sqrt{3}}. \qquad \text{... (i)}$$

From right $\triangle YRQ$, we have

$$\frac{YR}{QR} = \cot 45° \Rightarrow \frac{YR}{h-40} = 1 \Rightarrow YR = (h-40). \qquad \text{... (ii)}$$

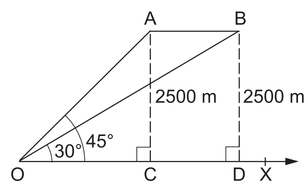
But, $YR = XP$.

$$\therefore \quad h - 40 = \frac{h}{\sqrt{3}} \Rightarrow h(\sqrt{3}-1) = 40\sqrt{3}$$

$$\Rightarrow h = \left\{\frac{40\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}\right\} = 20(3+\sqrt{3})$$

$$\Rightarrow h = 20(3 + 1.73) = (20 \times 4.73) = 94.6.$$

25. Let A and B be the two positions of the aeroplane and let O be the point of observation.



From right $\triangle OCA$, we have

$$\frac{OC}{AC} = \cot 45° \Rightarrow \frac{OC}{2500 \text{ m}} = 1 \Rightarrow OC = 2500 \text{ m}.$$

From right $\triangle ODB$, we have

$$\frac{OD}{BD} = \cot 30° \Rightarrow \frac{OD}{2500 \text{ m}} = \sqrt{3} \Rightarrow OD = 2500\sqrt{3} \text{ m}.$$

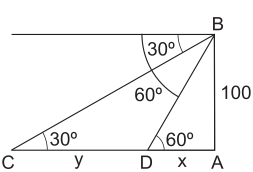
$$\therefore \quad AB = CD = OD - OC = 2500(\sqrt{3}-1) \text{ m} = (25000 \times 0.732) \text{ m} = 1830 \text{ m}.$$

Thus, the aeroplane covers 1830 m in 15 seconds.

$$\therefore \quad \text{its speed} = \left(\frac{1830}{15} \times \frac{60 \times 60}{1000}\right) \text{km/hr} = 439.2 \text{ km/hr}.$$

27. Let AB be the lighthouse and C and D be the two positions of the ship.

Then, $\dfrac{x}{100 \text{ m}} = \cot 60° \Rightarrow x = \dfrac{100}{\sqrt{3}}$ m.



$$\frac{x+y}{100 \text{ m}} = \cot 30° = \sqrt{3} \Rightarrow x + y = (100\sqrt{3}) \text{ m}.$$

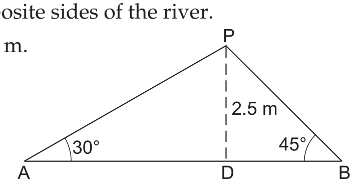
$$\therefore \quad y = x + y - x = 100\sqrt{3} \text{ m} - \frac{100}{\sqrt{3}} \text{ m} = \frac{200}{\sqrt{3}} \text{ m}$$

$$= \frac{200 \text{ m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{200 \text{ m}}{3} \times \sqrt{3}$$

$$\Rightarrow \quad y = \frac{200 \text{ m}}{3} \times 1.732 = \frac{346.4 \text{ m}}{3} = 115.46 \text{ m}.$$

28. Let A and B be two points on the banks on opposite sides of the river.

Let P be a point on the bridge at a height of 2.5 m.



Then, $DP = 2.5$ m.

From right $\triangle PDB$, we have

$$\frac{DB}{PD} = \cot 45° \Rightarrow \frac{DB}{2.5 \text{ m}} = 1 \Rightarrow DB = 2.50 \text{ m}.$$

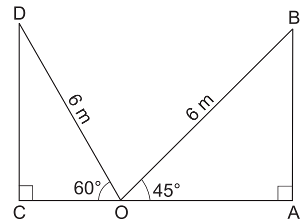
From right $\triangle PDA$, we have

$$\frac{AD}{PD} = \cot 30° \Rightarrow \frac{AD}{2.5 \text{ m}} = \sqrt{3} \Rightarrow AD = 2.5\sqrt{3} \text{ m}.$$

$\therefore$ Width of the river $= AB = AD + DB = 2.5\sqrt{3}$ m $+ 2.5$ m

$$= 2.5(\sqrt{3} + 1) \text{ m} = \frac{5}{2} \times (1.732 + 1) \text{ m} = 6.83 \text{ m}.$$

30. Let AB and CD be the two opposite walls and let the foot of the ladder be fixed at the point O on the ground. Let OB and OD be the two positions of the ladder.



From right $\triangle OAB$, we have

$$\frac{OA}{OB} = \cos 45° \Rightarrow \frac{OA}{6 \text{ m}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad OA = \frac{6 \text{ m}}{\sqrt{2}} \Rightarrow \quad OA = \frac{6 \text{ m}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2} \text{ m}.$$

From right $\triangle OCD$, we have

$$\frac{OC}{OD} = \cos 60° \Rightarrow \frac{OC}{6 \text{ m}} = \frac{1}{2} \Rightarrow OC = 3 \text{ m}.$$

Distance between two walls $= AC = OA + OC = 3(\sqrt{2} + 1)$ m $= 3(1.414 + 1)$ m

$$= (3 \times 2.414) \text{ m} = 7.242 \text{ m} \approx 7.24 \text{ m}.$$

32. Let AB be the electric pole such that $AB = 4$ m. Let C be a point 1 m below B.

Then, $AC = 4$ m $- 1$ m $= 3$ m.

Let OC be the ladder. Then, $\angle AOC = 60°$.

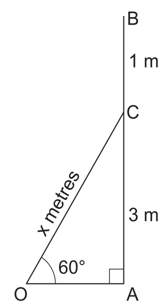
Let $OC = x$ metres.

From right $\triangle OAC$, we have

$$\frac{OC}{AC} = \text{cosec } 60° \Rightarrow \frac{x}{3} = \frac{2}{\sqrt{3}} \Rightarrow x = \frac{6}{\sqrt{3}} \Rightarrow x = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}.$$

$$\Rightarrow x = (2 \times 1.73) = 3.46.$$

Hence, the length of the ladder is 3.46 m.



. . . .

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. If the height of a vertical pole is equal to the length of its shadow on the ground, the angle of elevation of the sun is [CBSE 2014]

   (a) 0° (b) 30° (c) 45° (d) 60°

2. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground then the angle of elevation of the sun at that time is [CBSE 2012, '14]

   (a) 30° (b) 45° (c) 60° (d) 75°

3. If the length of the shadow of a tower is $\sqrt{3}$ times its height then the angle of elevation of the sun is [CBSE 2012]

   (a) 45° (b) 30° (c) 60° (d) 90°

4. If a pole 12 m high casts a shadow $4\sqrt{3}$ m long on the ground then the sun's elevation is [CBSE 2013C]

   (a) 60° (b) 45° (c) 30° (d) 90°

5. The shadow of a 5-m-long stick is 2 m long. At the same time, the length of the shadow of a 12.5-m-high tree is [CBSE 2011]

   (a) 3 m (b) 3.5 m (c) 4.5 m (d) 5 m

6. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, the length of the ladder is [CBSE 2014]

   (a) $\frac{4}{\sqrt{3}}$ m (b) $4\sqrt{3}$ m (c) $2\sqrt{2}$ m (d) 4 m

7. A ladder 15 m long makes an angle of 60° with the wall. Find the height of the point, where the ladder touches the wall. [CBSE 2017]

   (a) $15\sqrt{3}$ m (b) $\frac{15\sqrt{3}}{2}$ m (c) $\frac{15}{2}$ m (d) 15 m

8. From a point on the ground, 30 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The height of the tower is [CBSE 2014]

   (a) 30 m (b) $10\sqrt{3}$ m (c) 10 m (d) $30\sqrt{3}$ m

9. The angle of depression of a car parked on the road from the top of a 150-m-high tower is 30°. The distance of the car from the tower is [CBSE 2014]

   (a) $50\sqrt{3}$ m (b) $150\sqrt{3}$ m (c) $150\sqrt{2}$ m (d) 75 m

10. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is [CBSE 2012]

    (a) 45° (b) 30° (c) 60° (d) 90°

11. From the top of a cliff 20 m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is [CBSE 2013C]

    (a) 20 m (b) 40 m (c) 60 m (d) 80 m

12. If a 1.5-m-tall girl stands at a distance of 3 m from a lamp-post and casts a shadow of length 4.5 m on the ground, then the height of the lamp-post is

    (a) 1.5 m (b) 2 m (c) 2.5 m (d) 2.8 m

13. The length of the shadow of a tower standing on level ground is found to be $2x$ metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower is

(a) $(2\sqrt{3}x)$ m (b) $(3\sqrt{2}x)$ m (c) $(\sqrt{3}-1)x$ m (d) $(\sqrt{3}+1)x$ m

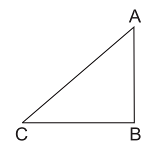
14. The lengths of a vertical rod and its shadow are in the ratio $1:\sqrt{3}$. The angle of elevation of the sun is

(a) 30° (b) 45° (c) 60° (d) 90°

15. A pole casts a shadow of length $2\sqrt{3}$ m on the ground when the sun's elevation is 60°. The height of the pole is [CBSE 2015]

(a) $4\sqrt{3}$ m (b) 6 m (c) 12 m (d) 3 m

16. In the given figure, a tower AB is 20 m high and BC, its shadow on the ground is $20\sqrt{3}$ m long. The sun's altitude is [CBSE 2015]



(a) 30° (b) 45°

(c) 60° (d) none of these

17. The tops of two towers of heights x and y, standing on a level ground subtend angles of 30° and 60° respectively at the centre of the line joining their feet. Then, $x:y$ is [CBSE 2015]

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

18. The angle of elevation of the top of a tower from a point on the ground 30 m away from the foot of the tower is 30°. The height of the tower is

(a) 30 m (b) $10\sqrt{3}$ m (c) 20 m (d) $10\sqrt{2}$ m

19. The string of a kite is 100 m long and it makes an angle of 60° with the horizontal. If there is no slack in the string, the height of the kite from the ground is

(a) $50\sqrt{3}$ m (b) $100\sqrt{3}$ m (c) $50\sqrt{2}$ m (d) 100 m

20. If the angles of elevation of the top of a tower from two points at distances a and b from the base and in the same straight line with it are complementary then the height of the tower is

(a) $\sqrt{\frac{a}{b}}$ (b) $\sqrt{ab}$ (c) $\sqrt{a+b}$ (d) $\sqrt{a-b}$

21. On the level ground, the angle of elevation of a tower is 30°. On moving 20 m nearer, the angle of elevation is 60°. The height of the tower is

(a) 10 m (b) $10\sqrt{3}$ m (c) 15 m (d) 20 m

22. In a rectangle, the angle between a diagonal and a side is 30° and the length of this diagonal is 8 cm. The area of the rectangle is

(a) 16 cm^2 (b) $\frac{16}{\sqrt{3}}$ cm^2 (c) $16\sqrt{3}$ cm^2 (d) $8\sqrt{3}$ cm^2

23. From the top of a hill, the angles of depression of two consecutive km stones due east are found to be 30° and 45°. The height of the hill is

(a) $\frac{1}{2}(\sqrt{3}-1)$ km (b) $\frac{1}{2}(\sqrt{3}+1)$ km

(c) $(\sqrt{3}-1)$ km (d) $(\sqrt{3}+1)$ km

24. If the elevation of the sun changes from 30° to 60° then the difference between the lengths of shadows of a pole 15 m high, is

(a) 7.5 m (b) 15 m (c) $10\sqrt{3}$ m (d) $5\sqrt{3}$ m

25. An observer 1.5 m tall is 28.5 m away from a tower and the angle of elevation of the top of the tower from the eye of the observer is 45°. The height of the tower is

(a) 27 m (b) 30 m (c) 28.5 m (d) none of these

ANSWERS (MCQ)

1. (c) **2.** (c) **3.** (b) **4.** (a) **5.** (d) **6.** (d) **7.** (c) **8.** (b) **9.** (b) **10.** (b)

11. (a) **12.** (c) **13.** (d) **14.** (a) **15.** (b) **16.** (a) **17.** (c) **18.** (b) **19.** (a) **20.** (b)

21. (b) **22.** (c) **23.** (b) **24.** (c) **25.** (b)

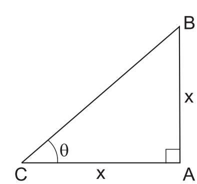
HINTS TO SOME SELECTED QUESTIONS

1. Let AB be the pole and AC be its shadow such that $AB = AC$.

Let $\angle ACB = \theta$. Then,

$$\tan\theta = \frac{AB}{AC} = 1 \Rightarrow \tan\theta = \tan 45°$$

$\Rightarrow \quad \theta = 45°$.

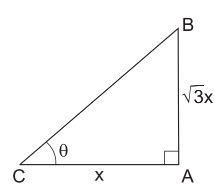


2. Let AB be the pole and AC be its shadow.

Let $AC = x$ m. Then, $AB = \sqrt{3}x$ m.

Let $\angle ACB = \theta$. Then,

$$\tan\theta = \frac{AB}{AC} = \frac{\sqrt{3}x}{x} = \sqrt{3} \Rightarrow \theta = 60°.$$



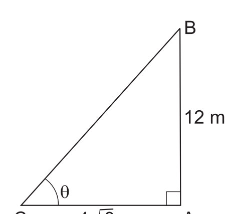
4. Let AB be the pole and AC be its shadow.

$AB = 12$ m and $AC = 4\sqrt{3}$ m.

Let $\angle ACB = \theta$. Then, $\tan\theta = \frac{AB}{AC} = \frac{12}{4\sqrt{3}}$

$$\Rightarrow \quad \tan\theta = \frac{12}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

$\Rightarrow \quad \theta = 60°$.



5. Ratio of lengths of objects = ratio of lengths of their shadows.

Let the length of shadow of the tree be x m. Then,

$$\frac{5}{12.5} = \frac{2}{x} \Rightarrow 5x = 2 \times 12.5 = 25 \Rightarrow x = 5$$

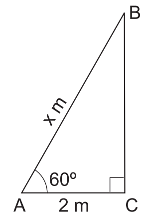
6. Let AB be the ladder and BC be the wall.

Then, $\angle CAB = 60°$ and $CA = 2$ m.

Let $AB = x$ metres. Then,

$$\frac{AC}{AB} = \cos 60° = \frac{1}{2} \Rightarrow \frac{2}{x} = \frac{1}{2}$$

$\Rightarrow \quad x = 4.$



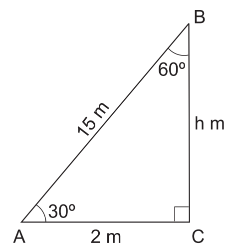
7. Let AB be the ladder and BC be the wall.

Then, $\angle ABC = 60° \Rightarrow \angle CAB = (90° - 60°) = 30°$.

Let $BC = h$ m. Then,

$$\frac{BC}{AB} = \sin 30° \Rightarrow \frac{h}{15} = \frac{1}{2}$$

$\Rightarrow \quad h = \frac{15}{2}.$

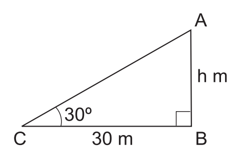


8. Let AB be the tower and $BC = 30$ m be the ground such that $\angle BCA = 30°$.

Let $AB = h$ metres. Then,

$$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{30}{\sqrt{3}}$$

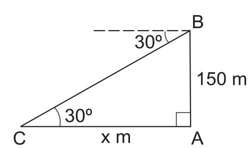
$$\Rightarrow \quad h = \left(\frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 10\sqrt{3}.$$



9. Let AB be the tower and C be the position of the car on the ground such that $\angle ACB = 30°$.

Let $AC = x$ metres. Then, $\frac{AC}{AB} = \cot 30° \Rightarrow \frac{x}{150} = \sqrt{3}$

$\Rightarrow \quad x = 150\sqrt{3}.$



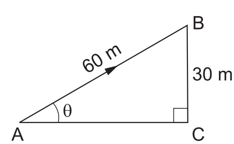
10. Let AB be the string and B be the kite.

Let AC be the horizontal and let $BC \perp AC$.

Let $\angle CAB = \theta$.

$BC = 30$ m and $AB = 60$ m. Then,

$$\frac{BC}{AB} = \sin\theta \Rightarrow \sin\theta = \frac{30}{60} = \frac{1}{2} \Rightarrow \sin\theta = \sin 30° \Rightarrow \theta = 30°.$$



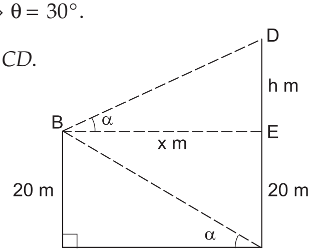
11. Let AB be the cliff and CD be the tower. Draw $BE \perp CD$.

Let $\angle ACB = \angle EBD = \alpha$ and let $DE = h$ metres.

Also, $AB = 20$ m. Let $AC = BE = x$ m. Then

$$\frac{x}{h} = \cot\alpha \text{ and } \frac{x}{20} = \cot\alpha$$

Thus, $\frac{x}{h} = \frac{x}{20} \Rightarrow h = 20$ m.

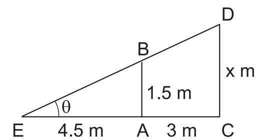


12. Let AB be the position of the girl and let CD be the lamp-post. Let AE be the shadow of AB. Then, $AB = 1.5$ m, $AC = 3$ m and $AE = 4.5$ m.

Let $CD = x$ metres. Now, ΔAEB and ΔCED are similar.

$$\therefore \quad \frac{CD}{EC} = \frac{AB}{AE} \Rightarrow \frac{x}{7.5} = \frac{1.5}{4.5} = \frac{1}{3} \Rightarrow x = \left(7.5 \times \frac{1}{3}\right) = 2.5.$$

Hence, the height of the lamp-post is 2.5 m.



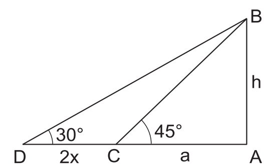
13. Let h m be the height of the tower and let a m and $(a + 2x)$ m be the lengths of the shadows of the tower when sun's elevation is 45° and 30° respectively. Then,

$$\frac{h}{a} = \tan 45° = 1 \Rightarrow a = h.$$

$$\frac{h}{a + 2x} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{h + 2x} = \frac{1}{\sqrt{3}}.$$

$$\therefore \quad \sqrt{3}h = h + 2x \Rightarrow 2x = (\sqrt{3} - 1)h \Rightarrow h = \frac{2x}{(\sqrt{3} - 1)}$$

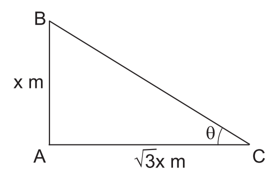
$$\therefore \quad h = \left\{\frac{2x}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}\right\} = x(\sqrt{3} + 1).$$



14. Let AB be the rod of length x metres and let AC be its shadow of length $(\sqrt{3}x)$ m. Let $\angle ACB = \theta$.

$$\text{Then, } \frac{AB}{AC} = \tan\theta \Rightarrow \tan\theta = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \tan 30°$$

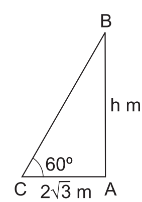
$$\Rightarrow \theta = 30°.$$



15. Let the height of the pole be h metres.

$$\text{Then, } \frac{h}{2\sqrt{3}} = \tan 60° = \sqrt{3}$$

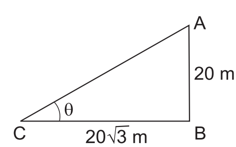
$$\Rightarrow h = (2\sqrt{3} \times \sqrt{3}) = 6.$$



16. Let $AB = 20$ m be the tower and $BC = 20\sqrt{3}$ m be the length of its shadow. Let $\angle ACB = \theta$.

$$\text{Then, } \tan\theta = \frac{AB}{AC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30°.$$

$$\therefore \quad \theta = 30°.$$



17. Let AB and CD be the given pillars and O be the midpoint of AC.

Then, $AB = x$, $CD = y$, $\angle AOB = 30°$ and $\angle COD = 60°$.

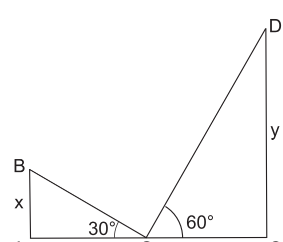
From right ΔOAB, we have

$$\frac{OA}{AB} = \cot 30° \Rightarrow \frac{OA}{x} = \sqrt{3}$$

$$\Rightarrow OA = x\sqrt{3}. \qquad \text{... (i)}$$

From right ΔOCD, we have

$$\frac{OC}{OD} = \cot 60° \Rightarrow \frac{OC}{y} = \frac{1}{\sqrt{3}} \Rightarrow OC = \frac{y}{\sqrt{3}}. \qquad \text{... (ii)}$$

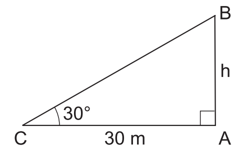


But, $OA = OC$.

$\therefore \quad x\sqrt{3} = \frac{y}{\sqrt{3}} \Rightarrow 3x = y \Rightarrow \frac{x}{y} = \frac{1}{3} \Rightarrow x : y = 1 : 3.$

18. $\frac{h}{30} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{30 \text{ m}}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{30 \text{ m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}.$



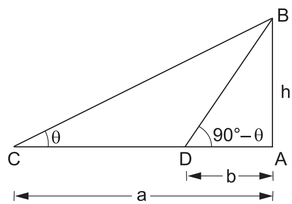
20. $\frac{h}{a} = \tan\theta \Rightarrow h = a\tan\theta.$ ... (i)

$\frac{h}{b} = \tan(90° - \theta) = \cot\theta$

$\Rightarrow \quad h = b\cot\theta$ ... (ii)

From (i) and (ii), we get

$h^2 = ab$ and hence $h = \sqrt{ab}$.

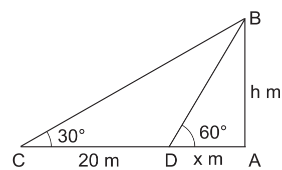


21. $\frac{x}{h} = \cot 60° = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}$ ... (i)

$\frac{x + 20}{h} = \cot 30° = \sqrt{3} \Rightarrow x = (h\sqrt{3} - 20)$ ... (ii)

$\therefore \quad \frac{h}{\sqrt{3}} = (h\sqrt{3} - 20) \Rightarrow h = 3h - 20\sqrt{3}$

$\Rightarrow \quad 2h = 20\sqrt{3} \Rightarrow h = 10\sqrt{3}.$



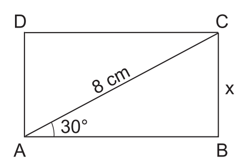
22. Let $ABCD$ be the rectangle.

$\frac{BC}{AC} = \sin 30° = \frac{1}{2} \Rightarrow \frac{x}{8 \text{ cm}} = \frac{1}{2} \Rightarrow x = 4 \text{ cm}.$

$\therefore \quad AB^2 = (8)^2 \text{ cm}^2 - (4)^2 = (64 - 16) \text{ cm}^2 = 48 \text{ cm}^2$

$\Rightarrow \quad AB = \sqrt{48} \text{ cm}^2 = 4\sqrt{3} \text{ cm}.$

$\therefore \quad \text{ar}(ABCD) = (4\sqrt{3} \times 4) \text{ cm}^2 = 16\sqrt{3} \text{ cm}^2.$



23. Let AB be the hill.

From right $\triangle BAD$, we have

$\frac{AD}{AB} = \cot 45° \Rightarrow \frac{x}{h} = 1 \Rightarrow x = h.$ ... (i)

From right $\triangle BAC$, we have

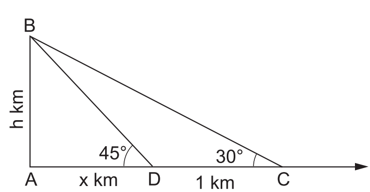
$\frac{AC}{AD} = \cot 30° \Rightarrow \frac{x+1}{h} = \sqrt{3} \Rightarrow x = (h\sqrt{3} - 1)$ ... (ii)

From (i) and (ii),we have

$h = (h\sqrt{3} - 1) \Rightarrow h(\sqrt{3} - 1) = 1$

$\Rightarrow \quad h = \frac{1}{(\sqrt{3} - 1)}$

$\Rightarrow \quad h = \left\{\frac{1}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}\right\} = \frac{1}{2}(\sqrt{3} + 1).$

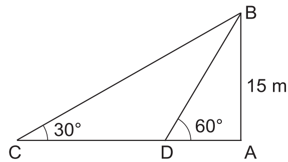


24. Let AB be the pole and AC and AD be its shadows when $\angle ACB = 30°$ and $\angle ADB = 60°$.

From right $\triangle CAB$, we have

$$\frac{AC}{AB} = \cot 30° \Rightarrow \frac{AC}{15 \text{ m}} = \sqrt{3}$$

$$\Rightarrow \quad AC = 15\sqrt{3} \text{ m.} \qquad \text{... (i)}$$



From right $\triangle DAB$, we have

$$\frac{AD}{AB} = \cot 60° \Rightarrow \frac{AD}{15 \text{ m}} = \frac{1}{\sqrt{3}} \Rightarrow AD = \frac{15 \text{ m}}{\sqrt{3}}$$

$$\Rightarrow \quad AD = \frac{15 \text{ m}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3} \text{ m} \qquad \text{... (ii)}$$

Required difference $= (15\sqrt{3} - 5\sqrt{3})$ m $= 10\sqrt{3}$ m.

25. Let AB be the observer and $CD = h$ metres be the tower.

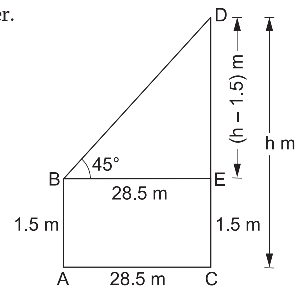
$BE = AC = 28.5$ m.

From right $\triangle BED$, we have

$$\frac{DE}{BE} = \tan 45° \Rightarrow \frac{DE}{28.5 \text{ m}} = 1$$

$$\Rightarrow DE = 28.5 \text{ m}$$

$\therefore \quad h - 1.5 = 28.5 \Rightarrow h = 30.$



✿