15 Perimeter and Area of Plane Figures

PERIMETER *The perimeter of a plane figure is the length of its boundary.*

In case of a triangle or a polygon, the perimeter is the sum of the lengths of its sides.

The unit of perimeter is same as the unit of length.

AREA *The area of a plane figure is the measure of the surface enclosed by its boundary.*

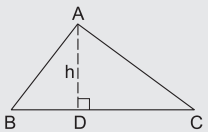
The area of a triangle or a polygon is the measure of the surface enclosed by its sides.

Area is measured in square units such as square centimetres and square metres, written as cm^2 and m^2 respectively.

PERIMETER AND AREA OF TRIANGLES [FORMULAE]

1. (i) Area of a triangle $= \frac{1}{2} \times$ base $\times$ corresponding height.

$$\text{ar}(\triangle ABC) = \left(\frac{1}{2} \times BC \times AD\right) \text{ sq units.}$$



(ii) Heron's formula:

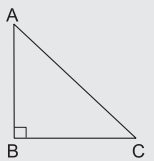
Let a, b, c be the sides of a $\triangle ABC$. Then,

$s = \frac{1}{2}(a + b + c)$ is called its semiperimeter.

$$\text{ar}(\triangle ABC) = \sqrt{s(s-a)(s-b)(s-c)}.$$

2. In a right $\triangle ABC$, let $\angle B = 90°$. Then,

$$\text{ar}(\triangle ABC) = \left(\frac{1}{2} \times BC \times AB\right) \text{ sq units.}$$

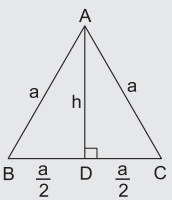


3. In an equilateral triangle of side a, we have

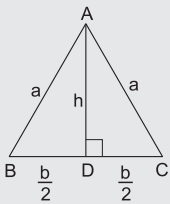
(i) Height $= \left(\frac{\sqrt{3}}{2} a\right)$ units.

(ii) Area $= \left(\frac{\sqrt{3}}{4} a^2\right)$ sq units.

(iii) Perimeter $= 3a$ units.



4. For an isosceles $\triangle ABC$ in which $AB = AC = a$ and $BC = b$, we have



(i) Height $= \dfrac{\sqrt{4a^2 - b^2}}{2}$ units.

(ii) Area $= \left(\dfrac{1}{4} b \sqrt{4a^2 - b^2}\right)$ sq units.

(iii) Perimeter $= (2a + b)$ units.

SOLVED EXAMPLES

EXAMPLE 1 *Find the area of a triangle having base* 25 cm *and height* 10.8 cm.

SOLUTION Area of the given triangle $= \left(\dfrac{1}{2} \times \text{base} \times \text{height}\right)$

$= \left(\dfrac{1}{2} \times 25 \times 10.8\right) \text{cm}^2 = 135 \text{ cm}^2.$

EXAMPLE 2 *The base of a triangular field is three times its altitude. If the cost of sowing the field at* ₹ 960 *per hectare is* ₹ 12960, *find its base and height.*

SOLUTION Total cost of sowing the field = ₹ 12960.

Rate per hectare = ₹ 960.

$\text{Area} = \dfrac{\text{total cost}}{\text{rate}} = \left(\dfrac{12960}{960}\right) \text{hectares} = 13.5 \text{ hectares}$

$= (13.5 \times 10000)\,\text{m}^2 = 135000\,\text{m}^2.$

Let the altitude of the field be x metres.

Then, its base $= 3x$ metres.

$\text{Area} = \left(\dfrac{1}{2} \times \text{base} \times \text{height}\right) = \left(\dfrac{1}{2} \times 3x \times x\right) \text{m}^2 = \left(\dfrac{3x^2}{2}\right) \text{m}^2.$

$\therefore \quad \dfrac{3x^2}{2} = 135000 \Rightarrow x^2 = \left(135000 \times \dfrac{2}{3}\right) = 90000$

$\Rightarrow x = \sqrt{90000} = 300.$

$\therefore$ base $= (3 \times 300)\,\text{m} = 900\,\text{m}$ and altitude $= 300$ m.

EXAMPLE 3 *The lengths of the sides of a triangle are in the ratio* 3 : 4 : 5, *and its perimeter is* 144 cm. *Find (i) the area of the triangle, and (ii) the height corresponding to the longest side.*

SOLUTION On dividing 144 cm in the ratio 3 : 4 : 5, we get

$$a = \left(144 \times \frac{3}{12}\right) \text{cm} = 36 \text{ cm}, b = \left(144 \times \frac{4}{12}\right) \text{cm} = 48 \text{ cm}$$

and $c = \left(144 \times \frac{5}{12}\right) \text{cm} = 60 \text{ cm}.$

$\therefore \quad s = \frac{1}{2}(36 + 48 + 60) \text{ cm} = 72 \text{ cm}.$

$(s - a) = (72 - 36) \text{ cm} = 36 \text{ cm},$

$(s - b) = (72 - 48) \text{ cm} = 24 \text{ cm}$

and $(s - c) = (72 - 60) \text{ cm} = 12 \text{ cm}.$

(i) Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72 \times 36 \times 24 \times 12} \text{ cm}^2$

$= 72 \times 12 \text{ cm}^2 = 864 \text{ cm}^2.$

(ii) Let base = 60 cm and the corresponding height = h cm.

Then, area of the triangle $= \left(\frac{1}{2} \times 60 \times h\right) \text{cm}^2 = (30h) \text{ cm}^2.$

$\therefore \quad 30h = 864 \Rightarrow h = \frac{864}{30} = 28.8.$

Longest side = 60 cm, corresponding height = 28.8 cm.

EXAMPLE 4 *Each side of an equilateral triangle measures* 10 cm. *Calculate (i) the area of the triangle, and (ii) the height of the triangle.* [*Given,* $\sqrt{3} = 1.732.$]

SOLUTION Here, $a = 10$ cm.

(i) Area of the triangle $= \left(\frac{\sqrt{3}}{4} \times a^2\right)$ sq units

$= \left(\frac{\sqrt{3}}{4} \times 10 \times 10\right) \text{cm}^2 = (25 \times \sqrt{3}) \text{ cm}^2$

$= (25 \times 1.732) \text{ cm}^2 = 43.3 \text{ cm}^2.$

(ii) Height of the triangle $= \left(\frac{\sqrt{3}}{2} \times a\right)$ units

$= \left(\frac{\sqrt{3}}{2} \times 10\right) \text{cm} = (5 \times \sqrt{3}) \text{ cm}$

$= (5 \times 1.732) \text{ cm} = 8.66 \text{ cm}.$

EXAMPLE 5 *The height of an equilateral triangle is* 15 cm. *Find its area.* [*Given,* $\sqrt{3} = 1.73.$]

SOLUTION Let each side of the triangle be a cm. Then,

its height $= \left(\frac{\sqrt{3}}{2} \times a\right) \text{cm}.$

$$\therefore \quad \frac{\sqrt{3}}{2} \times a = 15 \;\Rightarrow\; a = \left(\frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 10\sqrt{3}.$$

Thus, each side of the triangle = a cm = $10\sqrt{3}$ cm.

$$\text{Area of the triangle} = \left(\frac{\sqrt{3}}{4} \times a^2\right) = \left(\frac{\sqrt{3}}{4} \times 10\sqrt{3} \times 10\sqrt{3}\right) \text{cm}^2$$

$$= (75\sqrt{3}) \text{ cm}^2 = \left(\frac{75 \times 173}{100}\right) \text{cm}^2$$

$$= \frac{519}{4} \text{ cm}^2 = 129.75 \text{ cm}^2.$$

EXAMPLE 6 *Find the area of an isosceles triangle, each of whose equal sides is* 13 cm *and whose base is* 24 cm.

SOLUTION Here, each equal side, $a = 13$ cm and base, $b = 24$ cm.

$$\therefore \quad \text{area of the triangle} = \left(\frac{1}{4} b \cdot \sqrt{4a^2 - b^2}\right) \text{cm}^2$$

$$= \left[\frac{1}{4} \times 24 \times \sqrt{4 \times 169 - 24 \times 24}\right] \text{cm}^2$$

$$= 60 \text{ cm}^2.$$

EXAMPLE 7 *The base of an isosceles triangle measures* 24 cm *and its area is* 192 cm^2. *Find its perimeter.*

SOLUTION Here, base, $b = 24$ cm, and let each equal side be a cm. Then,

$$\text{area} = \frac{1}{4} b \sqrt{4a^2 - b^2} \text{ sq units} = \frac{1}{4} \times 24 \times \sqrt{4a^2 - 576} \text{ cm}^2$$

$$= 12 \times \sqrt{a^2 - 144} \text{ cm}^2.$$

But, area = 192 cm^2 [given].

$$\therefore \quad 12 \times \sqrt{a^2 - 144} = 192 \;\Rightarrow\; \sqrt{a^2 - 144} = 16$$

$$\Rightarrow \quad a^2 - 144 = 256 \;\Rightarrow\; a^2 = 400 \;\Rightarrow\; a = 20.$$

$$\therefore \quad \text{perimeter of the triangle} = (2a + b) \text{ cm}$$

$$= (2 \times 20 + 24) \text{ cm} = 64 \text{ cm}.$$

EXAMPLE 8 *The difference between the sides at right angles in a right-angled triangle is* 14 cm. *The area of the triangle is* 120 cm^2. *Calculate the perimeter of the triangle.*

SOLUTION Let the sides containing the right angle be x cm and $(x - 14)$ cm.

$$\text{Then, its area} = \left[\frac{1}{2} \times x \times (x - 14)\right] \text{cm}^2.$$

But, area = 120 cm^2 [given].

$$\therefore \quad \frac{1}{2} x(x - 14) = 120 \;\Rightarrow x^2 - 14x - 240 = 0$$

$\Rightarrow \quad x^2 - 24x + 10x - 240 = 0 \Rightarrow x(x - 24) + 10(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x + 10) = 0 \Rightarrow x = 24 \quad$ [neglecting $x = -10$].

$\therefore$ one side = 24 cm, and other side = (24 – 14) cm = 10 cm.

Hypotenuse = $\sqrt{(24)^2 + (10)^2}$ cm = $\sqrt{576 + 100}$ cm

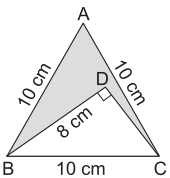
= $\sqrt{676}$ cm = 26 cm.

$\therefore$ perimeter of the triangle = (24 + 10 + 26) cm = 60 cm.

EXERCISE 15A

1. Find the area of the triangle whose base measures 24 cm and the corresponding height measures 14.5 cm.
2. Find the area of the triangle whose sides are 42 cm, 34 cm and 20 cm. Also, find the height corresponding to the longest side.
3. Find the area of the triangle whose sides are 18 cm, 24 cm and 30 cm. Also, find the height corresponding to the smallest side.
4. The sides of a triangle are in the ratio 5 : 12 : 13, and its perimeter is 150 m. Find the area of the triangle.
5. The perimeter of a triangular field is 540 m, and its sides are in the ratio 25 : 17 : 12. Find the area of the field. Also, find the cost of ploughing the field at ₹ 40 per 100 m^2.
6. The perimeter of a right triangle is 40 cm and its hypotenuse measures 17 cm. Find the area of the triangle.
7. The difference between the sides at right angles in a right-angled triangle is 7 cm. The area of the triangle is 60 cm^2. Find its perimeter.
8. The lengths of the two sides of a right triangle containing the right angle differ by 2 cm. If the area of the triangle is 24 cm^2, find the perimeter of the triangle.
9. Each side of an equilateral triangle is 10 cm. Find (i) the area of the triangle and (ii) the height of the triangle.
10. The height of an equilateral triangle is 6 cm. Find its area. [Take $\sqrt{3} = 1.73$.]
11. If the area of an equilateral triangle is $36\sqrt{3}$ cm^2, find its perimeter.
12. If the area of an equilateral triangle is $81\sqrt{3}$ cm^2, find its height.
13. The base of a right-angled triangle measures 48 cm and its hypotenuse measures 50 cm. Find the area of the triangle.
14. The hypotenuse of a right-angled triangle is 65 cm and its base is 60 cm. Find the length of perpendicular and the area of the triangle.

15. Find the area of a right-angled triangle, the radius of whose circumcircle measures 8 cm and the altitude drawn to the hypotenuse measures 6 cm.
16. Find the length of the hypotenuse of an isosceles right-angled triangle whose area is 200 cm^2. Also, find its perimeter. [Given, $\sqrt{2} = 1.41$.]
17. The base of an isosceles triangle measures 80 cm and its area is 360 cm^2. Find the perimeter of the triangle.
18. Each of the equal sides of an isosceles triangle measures 2 cm more than its height, and the base of the triangle measures 12 cm. Find the area of the triangle.
19. Find the area and perimeter of an isosceles right triangle, each of whose equal sides measures 10 cm. [Take $\sqrt{2} = 1.41$.]
20. In the given figure, $\triangle ABC$ is an equilateral triangle the length of whose side is equal to 10 cm, and $\triangle DBC$ is right-angled at D and $BD = 8$ cm. Find the area of the shaded region. [Take $\sqrt{3} = 1.732$.]



ANSWERS (EXERCISE 15A)

1. 174 cm^2 **2.** 336 cm^2, 16 cm **3.** 216 cm^2, 24 cm **4.** 750 m^2
5. 9000 m^2, ₹ 3600 **6.** 60 cm^2 **7.** 40 cm **8.** 24 cm
9. (i) 43.3 cm^2 (ii) 8.66 cm **10.** 20.76 cm^2 **11.** 36 cm **12.** $9\sqrt{3}$ cm
13. 336 cm^2 **14.** 25 cm, 750 cm^2 **15.** 48 cm^2 **16.** 28.2 cm, 68.2 cm
17. 162 cm **18.** 48 cm^2 **19.** 50 cm^2, 34.1 cm **20.** 19.3 cm^2

HINTS TO SOME SELECTED QUESTIONS

6. Sum of two sides = $(40 - 17)$ cm = 23 cm.
Let these sides be x cm and $(23 - x)$ cm.
$\therefore \quad x^2 + (23 - x)^2 = (17)^2 \Rightarrow x^2 + x^2 - 46x + 529 = 289$
$\Rightarrow 2x^2 - 46x + 240 = 0 \Rightarrow x^2 - 23x + 120 = 0$
$\Rightarrow (x - 15)(x - 8) = 0 \Rightarrow x = 15$ or $x = 8$.
$\therefore$ base = 15 cm and height = 8 cm. Find the area of the triangle.

7. Let these sides be x cm and $(x - 7)$ cm.
$\therefore \quad \frac{1}{2} \times x \times (x - 7) = 60 \Rightarrow x^2 - 7x - 120 = 0 \Rightarrow (x - 15)(x + 8) = 0$
$\Rightarrow x = 15 \quad [\because x \neq -8]$.
$\therefore$ these sides are 15 cm and 8 cm.
Hypotenuse = $\sqrt{(15)^2 + 8^2}$ cm = $\sqrt{289}$ cm = 17 cm.

9. (i) Area of the triangle $= \left(\frac{\sqrt{3}}{4}\times a^2\right) = \left(\frac{1.732}{4}\times 10\times 10\right)\text{ cm}^2 = 43.3\text{ cm}^2.$

(ii) $\frac{1}{2}\times a\times \text{height} = 43.3\text{ cm}^2 \Rightarrow \frac{1}{2}\times 10\text{ cm}\times h = 43.3\text{ cm}^2 \Rightarrow h = \frac{43.3\text{ cm}^2}{5\text{ cm}} = 8.66\text{ cm}.$

10. Height $= \frac{\sqrt{3}a}{2} \Rightarrow \frac{\sqrt{3}a}{2} = 6\text{ cm} \Rightarrow a = \frac{12}{\sqrt{3}}\text{ cm} = \frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\text{ cm} = 4\sqrt{3}\text{ cm}.$

$\therefore$ area $= \frac{\sqrt{3}}{4}\times a^2 = \frac{\sqrt{3}}{4}\times(4\sqrt{3})^2\text{ cm}^2 = \left(\frac{\sqrt{3}}{4}\times 48\right)\text{cm}^2 = (12\times 1.73)\text{ cm}^2.$

11. $\frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}\text{ cm}^2 \Rightarrow a^2 = 144\text{ cm}^2 \Rightarrow a = \sqrt{144}\text{ cm} = 12\text{ cm}.$

12. $\frac{\sqrt{3}}{4}a^2 = 81\sqrt{3}\text{ cm}^2 \Rightarrow a^2 = (81\times 4)\text{ cm}^2 \Rightarrow a = \sqrt{196}\text{ cm} = 18\text{ cm}.$

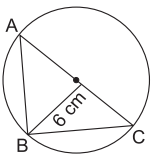
Height $= \frac{\sqrt{3}a}{2} = \left(\frac{\sqrt{3}\times 18}{2}\right)\text{cm} = 9\sqrt{3}\text{ cm}.$

13. Height $= \sqrt{(50)^2 - (48)^2}\text{ cm} = \sqrt{98\times 2}\text{ cm} = (7\times 2)\text{ cm} = 14\text{ cm}.$

15. The circumcentre of a right triangle is the midpoint of its hypotenuse.

$\therefore$ hypotenuse $= 2\times$ (radius of the circumcircle)
$= (2\times 8)\text{ cm} = 16\text{ cm}.$

$\therefore$ base $= 16$ cm, height $= 6$ cm.



16. Let base = height = a cm. Then,

$\frac{1}{2}\times a\times a = 200\text{ cm}^2 \Rightarrow a^2 = 400\text{ cm}^2 \Rightarrow a = \sqrt{400}\text{ cm}^2 = 20\text{ cm}.$

$\therefore$ hypotenuse $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2\times 400}\text{ cm}$
$= 20\sqrt{2}\text{ cm} = (20\times 1.41)\text{ cm}.$

17. $\frac{1}{2}\times 80\times\sqrt{4a^2 - 6400} = 360 \Rightarrow 80\times\sqrt{a^2 - 1600} = 360 \Rightarrow \sqrt{a^2 - 1600} = 9$
$\Rightarrow a^2 - 1600 = 81 \Rightarrow a^2 = 1681 \Rightarrow a = \sqrt{1681} = 41\text{ cm}.$

Perimeter $= (41 + 41 + 80)\text{ cm} = 162\text{ cm}.$

18. Let the height be h cm. Then, $a = (h + 2)$ cm and $b = 12$ cm.

$\therefore$ $\frac{1}{2}\times 12\times h = \frac{1}{4}\times 12\times\sqrt{4(h+2)^2 - 144}.$

20. $CD = \sqrt{(10)^2 - 8^2}\text{ cm} = \sqrt{36}\text{ cm} = 6\text{ cm}.$

Required area $= \text{ar}(\triangle ABC) - \text{ar}(\triangle DBC)$
$= \left\{\frac{\sqrt{3}}{4}\times(10)^2 - \frac{1}{2}\times 8\times 6\right\}\text{cm}^2.$

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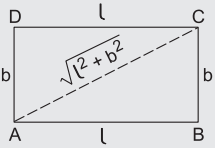
PERIMETER AND AREA OF QUADRILATERALS [FORMULAE]

1. Area and perimeter of a rectangle:



   (i) Area = (length × breadth) sq units.

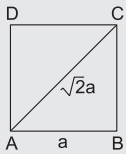
   (ii) Length = $\frac{\text{area}}{\text{breadth}}$ and Breadth = $\frac{\text{area}}{\text{length}}$.

   (iii) Perimeter = $2(l + b)$ units.

   (iv) Diagonal = $\sqrt{l^2 + b^2}$ units.

2. Area of 4 walls of a room = $[2(l + b) \times h)]$ sq units.

3. Area and perimeter of a square:



   (i) Area = a^2 sq units.

   (ii) Area = $\left\{\frac{1}{2} \times (\text{diagonal})^2\right\}$ sq units.

   (iii) Perimeter = $4a$ units.

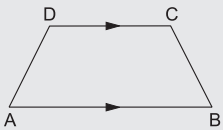
   (iv) Diagonal = $\sqrt{2}a$ units.

4. (i) Area of a parallelogram = {(base) × (height)} sq units.

   (ii) Area of a rhombus = $\left(\frac{1}{2} \times \text{product of diagonals}\right)$ sq units.

5. Area of a trapezium

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times (\text{distance between them}).$$

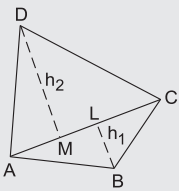


6. Area of a quadrilateral:

   (i) Let $ABCD$ be a quadrilateral with diagonal AC.

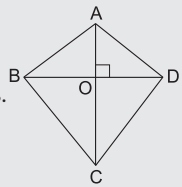
   Let $BL \perp AC$ and $DM \perp AC$. Let $BL = h_1$ and $DM = h_2$.

   ar (quad. $ABCD$) = $\left\{\frac{1}{2}(h_1 + h_2) \times AC\right\}$ sq units.



   (ii) When diagonals of a quadrilateral are perpendicular to each other:

   ar(quad. $ABCD$) = $\left(\frac{1}{2} \times \text{product of diagonals}\right)$ sq units.



SOLVED EXAMPLES

EXAMPLE 1 *The length of a rectangular field exceeds its breadth by* 8 m *and the area of the field is* 240 m^2. *Find the dimensions of the field.*

[CBSE 2014]

SOLUTION Let the breadth of the field be x metres.

Then, its length = $(x+8)$ m.

$\therefore$ area = $(x+8)\cdot x$ m^2.

$\therefore$ $(x+8)x = 240 \Rightarrow x^2+8x-240=0$

$\Rightarrow$ $x^2+20x-12x-240=0 \Rightarrow x(x+20)-12(x+20)=0$

$\Rightarrow$ $(x+20)(x-12)=0 \Rightarrow x=12$ [$\because$ $x \neq -20$].

$\therefore$ breadth = 12 m and length = 20 m.

EXAMPLE 2 *Sum of the areas of two squares is* 400 cm^2. *If the difference of their perimeters is* 16 cm, *find the sides of the two squares.* [CBSE 2013]

SOLUTION Let the sides of the given squares be a cm and b cm respectively. Then,

$a^2+b^2 = 400$ ... (i)

and $4a-4b=16 \Rightarrow a-b=4$. ... (ii)

Squaring (ii) on both sides, we get

$(a-b)^2 = 16 \Rightarrow (a^2+b^2)-2ab=16$

$\Rightarrow$ $400-2ab=16$ [using (i)]

$\Rightarrow$ $2ab = 384 \Rightarrow ab = 192$. ... (iii)

Now, $(a+b)^2-(a-b)^2 = 4ab$

$\Rightarrow$ $(a+b)^2-16 = 4\times 192$.

$\Rightarrow$ $(a+b)^2 = 768+16 = 784 \Rightarrow a+b = \sqrt{784} = 28$.

On solving $a+b=28$ and $a-b=4$, we get $a=16$ and $b=12$.

Hence, the sides of the given squares are 16 cm and 12 cm.

EXAMPLE 3 *The length of the diagonal of a square is* 24 cm. *Find (i) the area of the square and (ii) its perimeter.* [*Given*, $\sqrt{2}=1.41$.]

SOLUTION (i) Area of the square = $\left\{\frac{1}{2}\times(\text{diagonal})^2\right\}$ sq units.

$= \left(\frac{1}{2}\times 24\times 24\right)$ cm^2 = 288 cm^2.

(ii) Side of the square = $\sqrt{288}$ cm = $\sqrt{144\times 2}$ cm

$= 12\sqrt{2}$ cm = (12×1.41) cm = 16.92 cm.

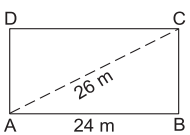
$\therefore$ perimeter of the square = (4×16.92) cm = 67.68 cm.

EXAMPLE 4 *The longer side of a rectangular hall is* 24 m, *and the length of its diagonal is* 26 m. *Find the area of the hall.*

SOLUTION Let $ABCD$ be the hall in which AB = 24 m and AC = 26 m.

By Pythagoras' theorem, we have

$BC = \sqrt{AC^2-AB^2}$ units = $\sqrt{(26)^2-(24)^2}$ m



$= \sqrt{(26-24)(26+24)}$ m $= \sqrt{50 \times 2}$ m

$= \sqrt{100}$ m $= 10$ m.

Thus, length = 24 m, breadth = 10 m.

$\therefore$ area of the hall $= (24 \times 10)$ m^2 $= 240$ m^2.

EXAMPLE 5 *The length and the breadth of a rectangular park are in the ratio* 8 : 5. *A path,* 1.5 m *wide, running all around the outside of the park has an area of* 594 m^2. *Find the dimensions of the park.*

SOLUTION Let the length and the breadth of the park be $8x$ metres and $5x$ metres respectively.

Then, area of the park $= (8x \times 5x)$ m^2

$= (40x^2)$ m^2.

Length of the park including the path $= (8x + 3)$ m.

Breadth of the park including the path $= (5x + 3)$ m.

Area of the park including the path $= (8x + 3)(5x + 3)$ m^2.

Area of the path $= [(8x + 3)(5x + 3) - 40x^2]$ m^2

$= (39x + 9)$ m^2.

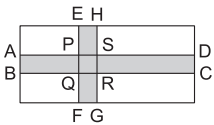
$\therefore \quad 39x + 9 = 594 \Rightarrow 39x = 585$

$\Rightarrow x = \dfrac{585}{39} = 15.$

$\therefore$ length $= (8 \times 15)$ m $= 120$ m,

breadth $= (5 \times 15)$ m $= 75$ m.

EXAMPLE 6 *A rectangular lawn,* 75 m *by* 60 m, *has two roads, each* 4 m *wide, running through the middle of the lawn, one parallel to length and the other parallel to breadth, as shown in the figure. Find the cost of gravelling the roads at* ₹ 50 *per* m^2.



SOLUTION Area of the road $ABCD = (75 \times 4)$ m^2 $= 300$ m^2.

Area of the road $EFGH = (60 \times 4)$ m^2 $= 240$ m^2.

Clearly, area $PQRS$ is common to both the roads.

Area $PQRS = (4 \times 4)$ m^2 $= 16$ m^2.

Area of the roads to be gravelled

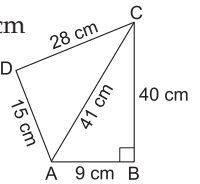
$= (300 + 240 - 16)$ m^2 $= 524$ m^2.

Cost of gravelling the roads

$=$ ₹ $(524 \times 50) =$ ₹ 26200.

EXAMPLE 7 *Find the area of the quadrilateral whose sides measure* 9 cm, 40 cm, 28 cm *and* 15 cm, *and in which the angle between the first two sides is a right angle.*

SOLUTION Let $ABCD$ be the given quadrilateral in which $AB = 9$ cm, $BC = 40$ cm, $CD = 28$ cm, $DA = 15$ cm and $\angle ABC = 90°$.



By Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2} \text{ units} = \sqrt{(9)^2 + (40)^2} \text{ cm}$$

$$= \sqrt{1681} \text{ cm} = 41 \text{ cm}.$$

$$\text{Area of } \triangle ABC = \left(\frac{1}{2} \times AB \times BC\right)$$

$$= \left(\frac{1}{2} \times 9 \times 40\right) \text{cm}^2 = 180 \text{ cm}^2.$$

In $\triangle ACD$,

let $a = AC = 41$ cm, $b = CD = 28$ cm and $c = DA = 15$ cm.

$$\therefore \quad s = \frac{1}{2}(41 + 28 + 15) \text{ cm} = 42 \text{ cm}.$$

$$\therefore \quad (s - a) = 1 \text{ cm}, (s - b) = 14 \text{ cm and } (s - c) = 27 \text{ cm}.$$

$$\text{Area of } \triangle ACD = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq units}$$

$$= \sqrt{42 \times 1 \times 14 \times 27} \text{ cm}^2$$

$$= (14 \times 3 \times 3) \text{ cm}^2 = 126 \text{ cm}^2.$$

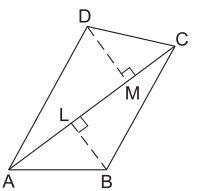
Area of the quadrilateral $ABCD$

$$= \text{area of } \triangle ABC + \text{area of } \triangle ACD$$

$$= (180 + 126) \text{ cm}^2 = 306 \text{ cm}^2.$$

EXAMPLE 8 *In a four-sided field, the length of the longer diagonal is* 120 m. *The lengths of perpendiculars from the opposite vertices on this diagonal are* 20 m *and* 15 m. *Find the area of the field.*

SOLUTION Let $ABCD$ be the field and AC be its longer diagonal.



Let $BL \perp AC$ and $DM \perp AC$. Then,

$AC = 120$ m, $BL = 20$ m

and $DM = 15$ m.

$$\text{Area of the field} = \left\{\frac{1}{2} \times (BL + DM) \times AC\right\} \text{ sq units}$$

$$= \left\{\frac{1}{2} \times (20 + 15) \times 120\right\} \text{ m}^2 = 2100 \text{ m}^2.$$

EXAMPLE 9 *Find the area of a parallelogram one of whose sides measures* 40 cm *and the corresponding height measures* 19.5 cm.

SOLUTION Area of the given parallelogram

$$= (\text{base} \times \text{height}) \text{ sq units}$$

$$= (40 \times 19.5) \text{ cm}^2 = \left(40 \times \frac{195}{10}\right) \text{cm}^2$$

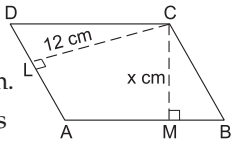
$$= 780 \text{ cm}^2.$$

EXAMPLE 10 *The adjacent sides of a parallelogram are* 36 cm *and* 27 cm *in length. If the distance between the shorter sides is* 12 cm, *find the distance between the longer sides.*

SOLUTION Longer side = 36 cm, shorter side = 27 cm.

Distance between the shorter sides = 12 cm.

Let the distance between the longer sides be x cm.



Area of the ||gm

= (longer side × distance between the longer sides)

= (shorter side × distance between the shorter sides)

$\therefore \quad 36 \times x = 27 \times 12 \Rightarrow x = \dfrac{27 \times 12}{36} = 9.$

$\therefore$ distance between the longer sides = 9 cm.

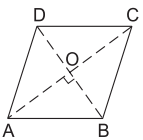
EXAMPLE 11 *The diagonals of a rhombus are* 48 cm *and* 20 cm *long. Find (i) the area of the rhombus, and (ii) the perimeter of the rhombus.*

SOLUTION (i) Area of the rhombus

$$= \frac{1}{2} \times (\text{product of diagonals}) \text{ sq units}$$

$$= \left(\frac{1}{2} \times 48 \times 20\right) \text{cm}^2$$

$$= 480 \text{ cm}^2.$$



(ii) We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore \quad OA = OC = 24$ cm, $OB = OD = 10$ cm and $\angle AOB = 90°$.

$\therefore$ by Pythagoras' theorem, we have

$$AB = \sqrt{OA^2 + OB^2} \text{ units} = \sqrt{(24)^2 + (10)^2} \text{ cm}$$

$$= \sqrt{676} \text{ cm} = 26 \text{ cm}.$$

$\therefore$ perimeter of the rhombus = (4 × 26) cm = 104 cm.

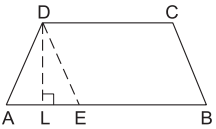
EXAMPLE 12 *Find the area of a trapezium whose parallel sides are* 35 cm *and* 23 cm *long and the distance between them is* 15 cm.

SOLUTION Area of the trapezium

$$= \frac{1}{2} \times (\text{sum of parallel sides}) \times (\text{distance between them})$$

$$= \left\{ \frac{1}{2} \times (35 + 23) \times 15 \right\} \text{cm}^2 = 435 \text{ cm}^2.$$

EXAMPLE 13 *Find the area of a trapezium ABCD in which* $AB \parallel DC$, $AB = 77$ cm, $BC = 25$ cm, $CD = 60$ cm *and* $DA = 26$ cm. [HOTS]



SOLUTION Draw $DE \parallel BC$ and $DL \perp AB$.

Then, $DE = BC = 25$ cm.

$AE = (AB - EB) = (AB - DC) = (77 - 60)$ cm $= 17$ cm.

In $\triangle DAE$, we have

$a = AE = 17$ cm, $b = DE = 25$ cm and $c = DA = 26$ cm.

$\therefore \quad s = \frac{1}{2}(17 + 25 + 26)$ cm $= 34$ cm,

$(s - a) = 17$ cm, $(s - b) = 9$ cm and $(s - c) = 8$ cm.

$\therefore$ area of $\triangle DAE = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{34 \times 17 \times 9 \times 8} \text{ cm}^2$$

$$= (17 \times 3 \times 4) \text{ cm}^2 = 204 \text{ cm}^2.$$

Also, area of $\triangle DAE = \frac{1}{2} \times AE \times DL$ sq units

$$= \frac{1}{2} \times 17 \text{ cm} \times DL.$$

$$\therefore \quad \frac{1}{2} \times 17 \text{ cm} \times DL = 204 \text{ cm}^2 \Rightarrow DL = \left(\frac{204 \times 2}{17} \right) \text{cm} = 24 \text{ cm}.$$

$$\text{ar(trap. } ABCD) = \left\{ \frac{1}{2}(AB + DC) \times DL \right\} \text{ sq units}$$

$$= \left\{ \frac{1}{2}(77 + 60) \times 24 \right\} \text{cm}^2 = 1644 \text{ cm}^2.$$

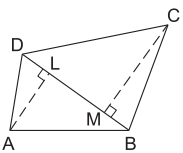
EXERCISE 15B

1. The perimeter of a rectangular plot of land is 80 m and its breadth is 16 m. Find the length and area of the plot.
2. The length of a rectangular park is twice its breadth and its perimeter is 840 m. Find the area of the park.

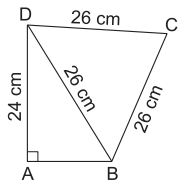
3. One side of a rectangle is 12 cm long and its diagonal measures 37 cm. Find the other side and the area of the rectangle.
4. The area of a rectangular plot is 462 m^2 and its length is 28 m. Find its perimeter.
5. A lawn is in the form of a rectangle whose sides are in the ratio 5 : 3. The area of the lawn is 3375 m^2. Find the cost of fencing the lawn at ₹ 65 per metre.
6. A room is 16 m long and 13.5 m broad. Find the cost of covering its floor with 75-m-wide carpet at ₹ 60 per metre.
7. The floor of a rectangular hall is 24 m long and 18 m wide. How many carpets, each of length 2.5 m and breadth 80 cm, will be required to cover the floor of the hall?
8. A 36-m-long, 15-m-broad verandah is to be paved with stones, each measuring 6 dm by 5 dm. How many stones will be required?
9. The area of a rectangle is 192 cm^2 and its perimeter is 56 cm. Find the dimensions of the rectangle.
10. A rectangular park 35 m long and 18 m wide is to be covered with grass, leaving 2.5 m uncovered all around it. Find the area to be laid with grass.
11. A rectangular plot measures 125 m by 78 m. It has a gravel path 3 m wide all around on the outside. Find the area of the path and the cost of gravelling it at ₹ 75 per m^2.
12. (i) A footpath of uniform width runs all around the inside of a rectangular field 54 m long and 35 m wide. If the area of the path is 420 m^2, find the width of the path.

    (ii) A carpet is laid on the floor of a room 8 m by 5 m. There is a border of constant width all around the carpet. If the area of the border is 12 m^2, find its width.
13. The length and the breadth of a rectangular garden are in the ratio 9 : 5. A path 3.5 m wide, running all around inside it has an area of 1911 m^2. Find the dimensions of the garden.
14. A room 4.9 m long and 3.5 m broad is covered with carpet, leaving an uncovered margin of 25 cm all around the room. If the breadth of the carpet is 80 cm, find its cost at ₹ 80 per metre.
15. In a rectangular park of dimensions 50 m × 40 m, a rectangular pond is constructed so that the area of grass strip of uniform width surrounding the pond would be 1184 m^2. Find the length and breadth of the pond. [CBSE 2017]
16. A 80 m by 64 m rectangular lawn has two roads, each 5 m wide, running through its middle, one parallel to its length and the other parallel to its breadth. Find the cost of gravelling the roads at ₹ 40 per m^2.

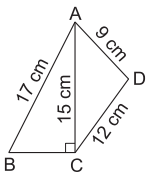
17. The dimensions of a room are 14 m × 10 m × 6.5 m. There are two doors and 4 windows in the room. Each door measures 2.5 m × 1.2 m and each window measures 1.5 m × 1 m. Find the cost of painting the four walls of the room at ₹ 35 per m^2.
18. The cost of painting the four walls of a room 12 m long at ₹ 30 per m^2 is ₹ 7560 and the cost of covering the floor with mat at ₹ 25 per m^2 is ₹ 2700. Find the dimensions of the room.
19. Find the area and perimeter of a square plot of land whose diagonal is 24 m long. [Take $\sqrt{2} = 1.41$.]
20. Find the length of the diagonal of a square whose area is 128 cm^2. Also, find its perimeter.
21. The area of a square field is 8 hectares. How long would a man take to cross it diagonally by walking at the rate of 4 km per hour?
22. The cost of harvesting a square field at ₹ 900 per hectare is ₹ 8100. Find the cost of putting a fence around it at ₹ 18 per metre.
23. The cost of fencing a square lawn at ₹ 14 per metre is ₹ 28000. Find the cost of mowing the lawn at ₹ 54 per 100 m^2.
24. In the given figure, $ABCD$ is a quadrilateral in which diagonal $BD = 24$ cm, $AL \perp BD$ and $CM \perp BD$ such that $AL = 9$ cm and $CM = 12$ cm. Calculate the area of the quadrilateral.



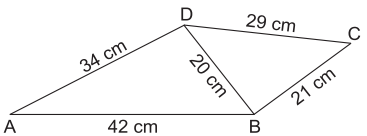
25. Find the area of the quadrilateral $ABCD$ in which $AD = 24$ cm, $\angle BAD = 90°$ and $\triangle BCD$ is an equilateral triangle having each side equal to 26 cm. Also, find the perimeter of the quadrilateral. [Given, $\sqrt{3} = 1.73$.]



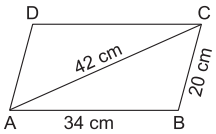
26. Find the perimeter and area of the quadrilateral $ABCD$ in which $AB = 17$ cm, $AD = 9$ cm, $CD = 12$ cm, $\angle ACB = 90°$ and $AC = 15$ cm.



27. Find the area of the quadrilateral $ABCD$ in which $AB = 42$ cm, $BC = 21$ cm, $CD = 29$ cm, $DA = 34$ cm and diagonal $BD = 20$ cm.



28. Find the area of a parallelogram with base equal to 25 cm and the corresponding height measuring 16.8 cm.
29. The adjacent sides of a parallelogram are 32 cm and 24 cm. If the distance between the longer sides is 17.4 cm, find the distance between the shorter sides.
30. The area of a parallelogram is 392 m^2. If its altitude is twice the corresponding base, determine the base and the altitude.
31. The adjacent sides of a parallelogram *ABCD* measure 34 cm and 20 cm, and the diagonal *AC* measures 42 cm. Find the area of the parallelogram.



32. Find the area of the rhombus, the lengths of whose diagonals are 30 cm and 16 cm. Also, find the perimeter of the rhombus.
33. The perimeter of a rhombus is 60 cm. If one of its diagonals is 18 cm long, find (i) the length of the other diagonal, and (ii) the area of the rhombus.
34. The area of a rhombus is 480 cm^2, and one of its diagonals measures 48 cm. Find (i) the length of the other diagonal, (ii) the length of each of its sides, and (iii) its perimeter.
35. The parallel sides of a trapezium are 12 cm and 9 cm and the distance between them is 8 cm. Find the area of the trapezium.
36. The shape of the cross section of a canal is a trapezium. If the canal is 10 m wide at the top, 6 m wide at the bottom and the area of its cross section is 640 m^2, find the depth of the canal.
37. Find the area of a trapezium whose parallel sides are 11 m and 25 m long, and the nonparallel sides are 15 m and 13 m long. [HOTS]

ANSWERS (EXERCISE 15B)

1. Length = 24 m, area = 384 m^2 **2.** 39200 m^2 **3.** 35 cm, 420 cm^2
4. 89 m **5.** ₹ 15600 **6.** ₹ 17280 **7.** 216 **8.** 1800
9. Length = 16 cm, breadth = 12 cm **10.** 240 m^2 **11.** 1254 m^2, ₹ 94050
12. (i) 2.5 m (ii) 50 cm **13.** Length = 180 m, breadth = 100 m
14. ₹ 1320 **15.** length = 34 m, breadth = 24 m **16.** ₹ 27800 **17.** ₹ 10500
18. Length = 12 m, breadth = 9 m and height = 6 m **19.** 288 m^2, 67.68 m
20. 16 cm, 45.12 cm **21.** 6 minutes **22.** ₹ 21600 **23.** ₹ 135000 **24.** 252 cm^2
25. 412.37 cm^2, 86 cm **26.** 46 cm, 114 cm^2 **27.** 546 cm^2
28. 420 cm^2 **29.** 23.2 cm **30.** Base = 14 m, altitude = 28 m
31. 672 cm^2 **32.** 240 cm^2, 68 cm **33.** (i) 24 cm (ii) 216 cm^2
34. (i) 20 cm (ii) 26 cm (iii) 104 cm **35.** 84 cm^2 **36.** 80 m **37.** 216 m^2

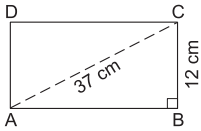
HINTS TO SOME SELECTED QUESTIONS

3. Other side $= \sqrt{(37)^2 - (12)^2}$ cm

$= \sqrt{(37 + 12)(37 - 12)}$ cm

$= \sqrt{49 \times 25}$ cm $= (7 \times 5)$ cm

$= 35$ cm.



6. Length of the carpet $= \dfrac{\text{area of the floor}}{\text{width of the carpet}} = \left(\dfrac{16 \times 13.5}{0.75}\right)$ m $= 288$ m.

7. Area of the floor $= (24 \times 18)$ m^2.

Area of each carpet $= \left(\dfrac{25}{10} \times \dfrac{80}{100}\right)$ m^2 $= 2$ m^2.

Number of carpets $= \dfrac{\text{area of the floor}}{\text{area of each carpet}}$.

8. Number of stones $= \dfrac{\text{area of the verandah}}{\text{area of each stone}} = \left(\dfrac{36 \times 15}{0.6 \times 0.5}\right)$.

9. $l + b = 28$ and $lb = 192$.

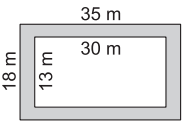
$(l - b)^2 = (l + b)^2 - 4lb = (28)^2 - 4 \times 192 = 16 \Rightarrow l - b = 4$.

On solving $l + b = 28$ and $l - b = 4$, we get $l = 16$, $b = 12$.

10. Outer area $= (35 \times 18)$ m^2 $= 630$ m^2.

Inner area $= (30 \times 13)$ m^2 $= 390$ m^2.

Grass area $= (630 - 390)$ m^2 $= 240$ m^2.



11. Area of the path $= [(125 + 6) \times (78 + 6) - (125 \times 78)]$ m^2.

12. (i) Let the width of the path be x metres. Then,

$(54 \times 35) - \{(54 - 2x) \times (35 - 2x)\} = 420$

$\Rightarrow \quad 2x^2 - 89x + 210 = 0 \Rightarrow 2x^2 - 84x - 5x + 210 = 0$

$\Rightarrow \quad (x - 42)(2x - 5) = 0 \Rightarrow x = 42$ or $x = 2.5$ [$\because$ width of the path $\neq 42$ m].

(ii) Let the width of the border be x metres. Then,

$(8 \times 5) - (8 - 2x)(5 - 2x) = 12 \Rightarrow 2x^2 - 13x + 6 = 0 \Rightarrow (x - 6)(2x - 1) = 0$.

$\therefore \quad x = \dfrac{1}{2}$ m $= 50$ cm.

13. Let the length and breadth be $(9x)$ m and $(5x)$ m respectively.

Then, $(9x \times 5x) - (9x - 7)(5x - 7) = 1911$.

14. Area to be carpeted $= (4.9 - 0.5)(3.5 - 0.5)$ m^2 $= 13.2$ m^2.

Length of the carpet $= \left(\dfrac{13.2}{0.80}\right)$ m $= 16.5$ m.

16. Area of the roads $= \{(80 \times 5) + (64 \times 5) - (5 \times 5)\}$ m^2 $= 695$ m^2.

17. Area of 4 walls $= [2(l + b) \times h] = \left(2 \times 24 \times \dfrac{13}{2}\right)$ m^2 $= 312$ m^2.

Area not to be painted $= \left\{\left(2 \times \dfrac{5}{2} \times \dfrac{12}{10}\right) + \left(4 \times \dfrac{3}{2} \times 1\right)\right\}$ m^2 $= 12$ m^2.

18. Area of 4 walls $= \dfrac{\text{total cost of painting}}{\text{rate per m}^2} = \left(\dfrac{7560}{30}\right)\text{m}^2 = 252\ \text{m}^2.$

Area of the floor $= \dfrac{\text{total cost of matting}}{\text{rate per m}^2} = \left(\dfrac{2700}{25}\right)\text{m}^2 = 108\ \text{m}^2.$

Let breadth = b metres and height = h metres. Then,

$12 \times b = 108 \Rightarrow$ breadth $= \dfrac{108}{12}\ \text{m} = 9\ \text{m}.$

Also, $2(l + b) \times h = 252 \Rightarrow 2(12 + 9) \times h = 252 \Rightarrow h = 6\ \text{m}.$

19. Area $= \dfrac{1}{2}d^2 = \left(\dfrac{1}{2} \times 24 \times 24\right)\text{m}^2 = 288\ \text{m}^2.$

Now, $a^2 = 288 \Rightarrow a = \sqrt{288} = 12\sqrt{2} \Rightarrow 4a = 48\sqrt{2}\ \text{m}.$

20. $\dfrac{1}{2}d^2 = 128 \Rightarrow d^2 = 256 = (16)^2 \Rightarrow d = 16.$

$a^2 = 128 \Rightarrow a = \sqrt{128} = 8\sqrt{2} \Rightarrow 4a = 32\sqrt{2}.$

21. Area $= (8 \times 10000)\,\text{m}^2 \Rightarrow \dfrac{1}{2} \times (\text{diagonal})^2 = 8 \times 10000$

$\Rightarrow (\text{diagonal})^2 = 160000 \Rightarrow$ diagonal $= 400\ \text{m}.$

24. ar (quad. $ABCD$) = ar($\triangle ABD$) + ar($\triangle CBD$) $= \left(\dfrac{1}{2} \times BD \times AL\right) + \left(\dfrac{1}{2} \times BD \times CM\right)$

$= \dfrac{1}{2} \times BD \times (AL + CM)\ \text{cm}^2.$

25. $AB = \sqrt{BD^2 - AD^2} = \sqrt{(26)^2 - (24)^2}\ \text{cm} = \sqrt{(26 + 24)(26 - 24)}\ \text{cm}$

$= \sqrt{50 \times 2}\ \text{cm} = \sqrt{100}\ \text{cm} = 10\ \text{cm}.$

ar(quad. $ABCD$) = ar(right $\triangle BAD$) + ar(equilateral $\triangle BCD$).

29. Area of the parallelogram $= (32 \times 17.4)\ \text{cm}^2.$

$24 \times h = 32 \times 17.4$. Find h.

31. ar(llgm $ABCD$) $= 2 \times$ ar($\triangle ABC$).

32. We know that the diagonals of a rhombus bisect each other at right angles.

So, $OA = 15$ cm, $OB = 8$ cm and $\angle AOB = 90°$.

$\therefore \quad AB^2 = (OA^2 + OB^2) = (15)^2 + (8)^2 = 225 + 64 = 289$

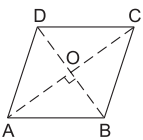
$\Rightarrow \quad AB = \sqrt{289} = 17$ cm.

So, perimeter $= (4 \times 17)$ cm $= 68$ cm.

33. Each side = 15 cm.

$(15)^2 = 9^2 + x^2 \Rightarrow x^2 = (225 - 81) = 144 \Rightarrow x = \sqrt{144} = 12$ cm.

$\therefore$ other diagonal $= 2x = 24$ cm. Area $= \left(\dfrac{1}{2} \times d_1 \times d_2\right)\text{cm}^2.$



36. $\dfrac{1}{2}(10 + 6) \times d = 640 \Rightarrow d = 80$ cm.

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MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. The length of a rectangular hall is 5 m more than its breadth. If the area of the hall is 750 m^2 then its length is
   (a) 15 m (b) 20 m (c) 25 m (d) 30 m
2. The length of a rectangular field is 23 m more than its breadth. If the perimeter of the field is 206 m, then its area is
   (a) 2420 m^2 (b) 2520 m^2 (c) 2480 m^2 (d) 2620 m^2
3. The length of a rectangular field is 12 m and the length of its diagonal is 15 m. The area of the field is
   (a) 108 m^2 (b) 180 m^2 (c) $30\sqrt{3}$ m^2 (d) $12\sqrt{15}$ m^2
4. The cost of carpeting a room 15 m long with a carpet 75 cm wide, at ₹ 70 per metre, is ₹ 8400. The width of the room is
   (a) 9 m (b) 8 m (c) 6 m (d) 12 m
5. The length of a rectangle is thrice its breadth and the length of its diagonal is $8\sqrt{10}$ cm. The perimeter of the rectangle is
   (a) $15\sqrt{10}$ cm (b) $16\sqrt{10}$ cm (c) $24\sqrt{10}$ cm (d) 64 cm
6. On increasing the length of a rectangle by 20% and decreasing its breadth by 20%, what is the change in its area?
   (a) 20% increase (b) 20% decrease
   (c) No change (d) 4% decrease
7. A rectangular ground 80 m × 50 m has a path 1 m wide outside around it. The area of the path is
   (a) 264 m^2 (b) 284 m^2 (c) 400 m^2 (d) 464 m^2
8. The length of the diagonal of a square is $10\sqrt{2}$ cm. Its area is
   (a) 200 cm^2 (b) 100 cm^2 (c) 150 cm^2 (d) $100\sqrt{2}$ cm^2
9. The area of a square field is 6050 m^2. The length of its diagonal is
   (a) 135 m (b) 120 m (c) 112 m (d) 110 m
10. The area of a square field is 0.5 hectare. The length of its diagonal is
    (a) 150 m (b) $100\sqrt{2}$ m (c) 100 m (d) $50\sqrt{2}$ m
11. The area of an equilateral triangle is $4\sqrt{3}$ cm^2. Its perimeter is
    (a) 9 cm (b) 12 cm (c) $12\sqrt{3}$ cm (d) $6\sqrt{3}$ cm
12. Each side of an equilateral triangle is 8 cm. Its area is
    (a) 24 cm^2 (b) $24\sqrt{3}$ cm^2 (c) $16\sqrt{3}$ cm^2 (d) $8\sqrt{3}$ cm^2

13. Each side of an equilateral triangle is $6\sqrt{3}$ cm. The altitude of the triangle is
    (a) 8 cm (b) 9 cm (c) $3\sqrt{3}$ cm (d) 6 cm
14. The height of an equilateral triangle is $3\sqrt{3}$ cm. Its area is
    (a) $6\sqrt{3}$ cm^2 (b) 27 cm^2 (c) $9\sqrt{3}$ cm^2 (d) $27\sqrt{3}$ cm^2
15. The base and height of a triangle are in the ratio 3 : 4 and its area is 216 cm^2. The height of the triangle is
    (a) 18 cm (b) 24 cm (c) 21 cm (d) 28 cm
16. The lengths of the sides of a triangular field are 20 m, 21 m and 29 m. The cost of cultivating the field at ₹ 9 per m^2 is
    (a) ₹ 2610 (b) ₹ 3780 (c) ₹ 1890 (d) ₹ 1800
17. The side of a square is equal to the side of an equilateral triangle. The ratio of their areas is
    (a) 4 : 3 (b) 2 : $\sqrt{3}$ (c) 4 : $\sqrt{3}$ (d) none of these
18. The side of an equilateral triangle is equal to the radius of a circle whose area is 154 cm^2. The area of the triangle is
    (a) 49 cm^2 (b) $\frac{49\sqrt{3}}{4}$ cm^2 (c) $\frac{7\sqrt{3}}{4}$ cm^2 (d) 77 cm^2
19. The area of a rhombus is 480 cm^2 and the length of one of its diagonals is 20 cm. The length of each side of the rhombus is
    (a) 24 cm (b) 30 cm (c) 26 cm (d) 28 cm
20. One side of a rhombus is 20 cm long and one of its diagonals measures 24 cm. The area of the rhombus is
    (a) 192 cm^2 (b) 480 cm^2 (c) 240 cm^2 (d) 384 cm^2

ANSWERS (MCQ)

1. (d) **2.** (b) **3.** (a) **4.** (c) **5.** (d) **6.** (d) **7.** (a) **8.** (b) **9.** (d) **10.** (c)
11. (b) **12.** (c) **13.** (b) **14.** (c) **15.** (b) **16.** (c) **17.** (c) **18.** (b) **19.** (c) **20.** (d)

HINTS TO SOME SELECTED QUESTIONS

1. Let the length be x metres. Then, breadth = $(x - 5)$ m.
   $\therefore\ x(x-5) = 750 \Rightarrow x^2 - 5x - 750 = 0 \Rightarrow (x - 30)(x + 25) = 0 \Rightarrow x = 30$ [$\because x \neq -25$].
   Hence, length = 30 m.
2. Let the breadth be x metres. Then, length = $(x + 23)$ m.
   $\therefore\ 2[(x + 23) + x] = 206 \Rightarrow 4x = 160 \Rightarrow x = 40$.
   $\therefore\ b = 40$ m and $l = 63$ m.
   Hence, area = (63×40) m^2 = 2520 m^2.

3. $l = 12\,\text{m}$ and diagonal $(d) = 15\,\text{m}$.
$b^2 = (d^2 - l^2) = (15)^2 - (12)^2 = (15 + 12)(15 - 12) = 27 \times 3 = 81$
$\Rightarrow \quad b = \sqrt{81}\,\text{m} = 9\,\text{m}.$
$\therefore \quad \text{area} = (12 \times 9)\,\text{m}^2 = 108\,\text{m}^2.$

4. Length of the carpet $= \dfrac{\text{total cost}}{\text{rate per m}} = \left(\dfrac{8400}{70}\right)\text{m} = 120\,\text{m}.$
Area of the room = area of the carpet $= \left(120 \times \dfrac{3}{4}\right)\text{m}^2 = 90\,\text{m}^2.$
Width of the room $= \dfrac{\text{area}}{\text{length}} = \left(\dfrac{90}{15}\right)\text{m} = 6\,\text{m}.$

5. Let the breadth be x cm. Then, length $= 3x$ cm.
$\therefore \quad \text{diagonal} = \sqrt{(3x)^2 + x^2}\ \text{cm} = x\sqrt{10}\ \text{cm}.$
Thus, $x\sqrt{10} = 8\sqrt{10} \Rightarrow x = 8.$
Perimeter $= 2(3x + x)\ \text{cm} = 8x\ \text{cm} = (8 \times 8)\ \text{cm} = 64\ \text{cm}.$

6. Let the length be l m and breadth be b m.
Then, original area $= (l \times b)\,\text{m}^2 = lb\,\text{m}^2.$
New length $= \left(l \times \dfrac{120}{100}\right)\text{m} = \dfrac{6l}{5}\,\text{m}$, new breadth $= \left(b \times \dfrac{80}{100}\right)\text{m} = \dfrac{4b}{5}\,\text{m}.$
New area $= \left(\dfrac{24lb}{25}\right)\text{m}^2 = 96\,\text{m}^2.$ Change in area $= \left(ab - \dfrac{24lb}{25}\right)\text{m}^2 = \dfrac{lb}{25}\,\text{m}^2.$
$\therefore \quad \%\ \text{change} = \dfrac{lb \times 100}{25 \times lb}\% = 4\%.$

7. Area of the ground without path $= (80 \times 50)\,\text{m}^2 = 4000\,\text{m}^2.$
Area of the ground with path $= (82 \times 52)\,\text{m}^2 = 4264\,\text{m}^2.$
Area of the path $= (4264 - 4000)\,\text{m}^2 = 264\,\text{m}^2.$

8. Area $= \dfrac{1}{2} \times (\text{diagonal})^2 = \dfrac{1}{2} \times (10\sqrt{2})^2\ \text{cm}^2 = 100\ \text{cm}^2.$

9. $\dfrac{1}{2} \times (\text{diagonal})^2 = 6050 \Rightarrow (\text{diagonal})^2 = 12100 = (110)^2 \Rightarrow \text{diagonal} = 110\,\text{m}.$

10. Area $= (0.5 \times 10000)\,\text{m}^2 = 5000\,\text{m}^2.$
$\dfrac{1}{2} \times d^2 = 5000 \Rightarrow d^2 = 10000 = (100)^2 \Rightarrow d = 100\,\text{m}.$

11. $\dfrac{\sqrt{3}}{4} \times a^2 = 4\sqrt{3} \ \Rightarrow\ a^2 = 16 \ \Rightarrow\ a = 4.$
$\therefore \quad \text{perimeter} = 3a = (3 \times 4)\ \text{cm} = 12\ \text{cm}.$

13. Area of the triangle $= \left(\dfrac{\sqrt{3}}{4} \times a^2\right) = \left\{\dfrac{\sqrt{3}}{4} \times (6\sqrt{3})^2\right\}\text{cm}^2$
$= \left(\dfrac{\sqrt{3}}{4} \times 108\right)\text{cm}^2 = (27\sqrt{3})\,\text{cm}^2.$
$\dfrac{1}{2} \times 6\sqrt{3} \times h = 27\sqrt{3} \Rightarrow h = \dfrac{27\sqrt{3}}{3\sqrt{3}} = 9\,\text{cm}.$

14. $\frac{1}{2}\times a\times h=\frac{\sqrt{3}}{4}a^2\Rightarrow a=\frac{2h}{\sqrt{3}}=\left(\frac{2}{\sqrt{3}}\times 3\sqrt{3}\right)\text{cm}=6\text{ cm}.$

$\therefore$ area $=\left(\frac{\sqrt{3}}{4}\times 6\times 6\right)\text{cm}^2=9\sqrt{3}\text{ cm}^2.$

15. Let the base be $3x$ cm and height be $4x$ cm. Then,

$\frac{1}{2}\times 3x\times 4x=216\Rightarrow x^2=36\Rightarrow x=6.$

$\therefore$ height $=(4\times 6)\text{ cm}=24\text{ cm}.$

16. $s=\frac{1}{2}(20+21+29)=35,\ (s-a)=15,\ (s-b)=14$ and $(s-c)=6.$

$\therefore$ area $=\sqrt{35\times 15\times 14\times 6}\text{ m}^2=(7\times 5\times 3\times 2)\text{ m}^2=210\text{ m}^2.$

Required cost $=₹(210\times 9)=₹\,1890.$

17. Let the side of the square = side of the triangle $=a$. Then,

$\frac{\text{area of the square}}{\text{area of the triangle}}=\frac{a^2}{\frac{\sqrt{3}}{4}a^2}=\frac{4}{\sqrt{3}}=4:\sqrt{3}.$

18. Area of the circle $=154\text{ cm}^2.$

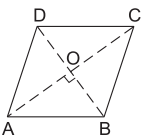
$\therefore\quad \pi R^2=154\ \Rightarrow\ \frac{22}{7}\times R^2=154$

$\Rightarrow\quad R^2=\left(154\times\frac{7}{22}\right)=7^2\ \Rightarrow\ R=7\text{ cm}.$

$\therefore$ side of the triangle = radius of the circle $=7$ cm.

Area of the triangle $=\left(\frac{\sqrt{3}}{4}\times 7\times 7\right)\text{cm}^2=\frac{49\sqrt{3}}{4}\text{ cm}^2.$

19. $\frac{1}{2}\times d_1\times d_2=480\text{ cm}^2\Rightarrow\frac{1}{2}\times 20\times d_2=480\text{ cm}^2\Rightarrow d_2=48\text{ cm}.$



$\therefore$ $OB=10$ cm, $OA=24$ cm and $\angle AOB=90°.$

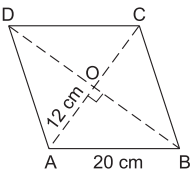
$\therefore$ $AB^2=OA^2+OB^2=(24)^2+(10)^2=576+100=676$

$\Rightarrow$ $AB=\sqrt{676}\text{ cm}=26\text{ cm}.$

Hence, each side of the rhombus is 26 cm.

20. Let $ABCD$ be the rhombus in which $AB=20$ cm and

$OA=\left(\frac{1}{2}\times 24\right)\text{cm}=12\text{ cm}.$



$\therefore$ $OB^2=AB^2-OA^2=(20)^2-(12)^2=256=(16)^2$

$\Rightarrow$ $OB=16$ cm.

$\therefore$ $BD=(2\times 16)\text{cm}=32\text{ cm}.$

$\therefore$ area $=\left(\frac{1}{2}\times 24\times 32\right)\text{cm}^2=384\text{ cm}^2.$

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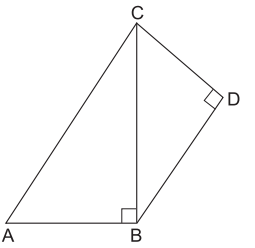
TEST YOURSELF

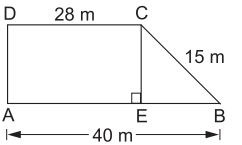
MCQ

1. In the given figure $ABDC$ is a quadrilateral in which $\angle ABC = 90°$, $\angle BDC = 90°$, $AC = 17$ cm, $BC = 15$ cm, $BD = 12$ cm and $CD = 9$ cm. The area of quad. $ABDC$ is



   (a) 102 cm^2 (b) 114 cm^2
   (c) 95 cm^2 (d) 57 cm^2

2. In the given figure $ABCD$ is a trapezium in which $AB = 40$ m, $BC = 15$ m, $CD = 28$ m, $AD = 9$ m and $CE \perp AB$. Area of trap. $ABCD$ is



   (a) 306 m^2 (b) 316 m^2
   (c) 296 m^2 (d) 284 m^2

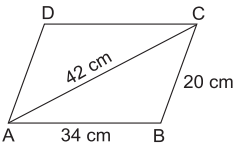
3. The sides of a triangle are in the ratio 12 : 14 : 25 and its perimeter is 25.5 cm. The largest side of the triangle is
   (a) 7 cm (b) 14 cm (c) 12.5 cm (d) 18 cm

4. The parallel sides of a trapezium are 9.7 cm and 6.3 cm, and the distance between them is 6.5 cm. The area of the trapezium is
   (a) 104 cm^2 (b) 78 cm^2 (c) 52 cm^2 (d) 65 cm^2

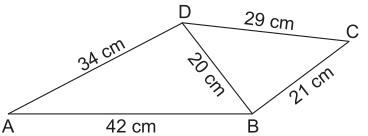
Short-Answer Questions

5. Find the area of an equilateral triangle having each side of length 10 cm. [Take $\sqrt{3} = 1.732$.]
6. Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.
7. The longer side of a rectangular hall is 24 m and the length of its diagonal is 26 m. Find the area of the hall.
8. The length of the diagonal of a square is 24 cm. Find its area.
9. Find the area of a rhombus whose diagonals are 48 cm and 20 cm long.
10. Find the area of a triangle whose sides are 42 cm, 34 cm and 20 cm.
11. A lawn is in the form of a rectangle whose sides are in the ratio 5 : 3 and its area is 3375 m^2. Find the cost of fencing the lawn at ₹ 20 per metre.
12. Find the area of a rhombus each side of which measures 20 cm and one of whose diagonals is 24 cm.

13. Find the area of a trapezium whose parallel sides are 11 cm and 25 cm long and nonparallel sides are 15 cm and 13 cm.
14. The adjacent sides of a ||gm $ABCD$ measure 34 cm and 20 cm and the diagonal AC is 42 cm long. Find the area of the ||gm.



15. The cost of fencing a square lawn at ₹ 14 per metre is ₹ 2800. Find the cost of mowing the lawn at ₹ 54 per 100 m^2.
16. Find the area of quad. $ABCD$ in which $AB = 42$ cm, $BC = 21$ cm, $CD = 29$ cm, $DA = 34$ cm and diag. $BD = 20$ cm.



Long-Answer Questions

17. A parallelogram and a rhombus are equal in area. The diagonals of the rhombus measure 120 m and 44 m. If one of the sides of the ||gm is 66 m long, find its corresponding altitude.
18. The diagonals of a rhombus are 48 cm and 20 cm long. Find the perimeter of the rhombus.
19. The adjacent sides of a parallelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12 cm, find the distance between the longer sides.
20. In a four-sided field, the length of the longer diagonal is 128 m. The lengths of perpendiculars from the opposite vertices upon this diagonal are 22.7 m and 17.3 m. Find the area of the field.

ANSWERS (TEST YOURSELF)

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** 43.3 cm^2
6. 60 cm^2	**7.** 240 m^2	**8.** 288 cm^2	**9.** 480 cm^2	**10.** 336 cm^2
11. ₹ 4800	**12.** 384 cm^2	**13.** 216 cm^2	**14.** 672 cm^2	**15.** ₹ 1350
16. 546 cm^2	**17.** 40 m	**18.** 104 cm	**19.** 9 cm	**20.** 2560 m^2

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