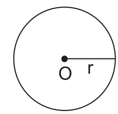
16 Area of Circle, Sector and Segment

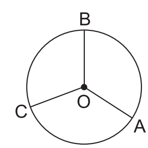


CIRCLE The set of points which are at a constant distance of r units from a fixed point O is called a circle with *centre* O and *radius* $= r$ units. The circle is denoted by $C(O, r)$.

In other words, a circle is the locus of a point which moves in such a way that its distance from a fixed point O remains constant at r units.

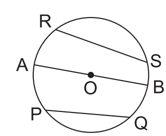
The fixed point O is called the *centre* and the constant distance r units is called its *radius.*

CIRCUMFERENCE The perimeter (or length of boundary) of a circle is called its circumference.



RADIUS A line segment joining the centre of a circle and a point on the circle is called a radius of the circle.

Plural of radius is *radii*. In the given figure, OA, OB, OC are three radii of the circle.



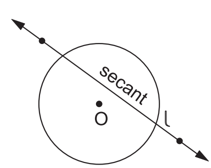
CHORD A line segment joining any two points on a circle is called a chord of the circle.

In the given figure, PQ, RS and AOB are three chords of a circle with centre O.

DIAMETER A chord of a circle passing through its centre is called a diameter of the circle.

Diameter is the longest chord of a circle. In the above figure, AOB is a diameter.

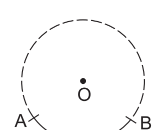
Diameter = 2 × radius.



SECANT A line which intersects a circle at two points is called a secant of the circle.

In the given figure, line l is a secant of the circle with centre O.

ARC A continuous piece of a circle is called an arc of the circle.

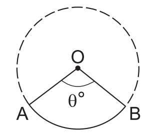


In the given figure, AB is an arc of a circle, with centre O, denoted by $\overset{\frown}{AB}$. The remaining part of the circle, shown by the dotted lines, represents $\overset{\frown}{BA}$.

CENTRAL ANGLE An angle subtended by an arc at the centre of a circle is called its central angle.

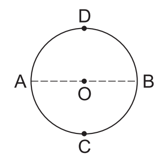
In the given figure of a circle with centre O,

$$\text{central angle of } \overset{\frown}{AB} = \angle AOB = \theta°.$$



If $\theta° < 180°$ then $\overset{\frown}{AB}$ is called the *minor arc* and $\overset{\frown}{BA}$ is called the *major arc*.

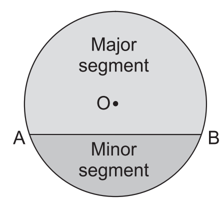
SEMICIRCLE A diameter divides a circle into two equal arcs. Each of these two arcs is called a semicircle.



In the given figure of a circle with centre O, $\overset{\frown}{ACB}$ and $\overset{\frown}{BDA}$ are semicircles.

An arc whose length is less than the arc of a semicircle is called a *minor arc*. An arc whose length is more than the arc of a semicircle is called a *major arc*.

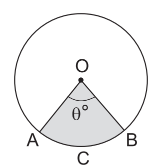
SEGMENT A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.



The segment containing the minor arc is called a *minor segment*, while the segment containing the major arc is the *major segment*.

The centre of the circle lies in the major segment.

SECTOR OF A CIRCLE The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.



In the given figure, $OACBO$ is a sector of the circle with centre O.

If arc AB is a minor arc then $OACBO$ is called the *minor sector* of the circle.

The remaining part of the circle is called the *major sector* of the circle.

QUADRANT One-fourth of a circular disc is called a quadrant. The central angle of a quadrant is 90°.

FORMULAE ON AREA OF CIRCLE, SECTOR AND SEGMENT

1. *For a circle of radius r, we have*

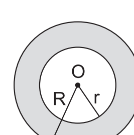
   (i) Circumference $= 2\pi r$ (ii) Area $= \pi r^2$.

2. *For a semicircle of radius r, we have*

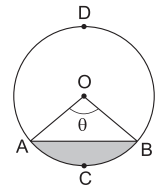
   (i) Perimeter $= (\pi r + 2r)$ (ii) Area $= \frac{1}{2}\pi r^2$.

3. *For a ring having outer radius = R and inner radius = r, we have*

   $$\textit{Area of the ring} = \pi(R^2 - r^2).$$



4. *Let an arc ACB make an angle θ° at the centre of a circle of radius r. Then, we have*



(i) Length of minor arc $ACB = \frac{2\pi r\theta}{360}$.

Length of major arc $BDA = \left(2\pi r - \frac{2\pi r\theta}{360}\right)$.

(ii) Area of minor sector $OACBO = \frac{\pi r^2\theta}{360} = \left(\frac{1}{2} \times \text{radius} \times \text{arc length}\right)$.

Area of major sector $OADBO = \left(\pi r^2 - \frac{\pi r^2\theta}{360}\right)$.

(iii) Area of minor segment $ACBA = \left(\frac{\pi r^2\theta}{360} - \frac{1}{2} r^2 \sin\theta\right)$.

Area of major segment $BDAB = [\pi r^2 - (\text{area of minor segment})]$.

(iv) Perimeter of sector $OACBO = \left(2r + \frac{2\pi r\theta}{360}\right)$.

5. *For Rotation of the Hands of a Clock:*

(i) Angle described by minute hand in 60 minutes = 360°.

(ii) Angle described by hour hand in 12 hours = 360°.

6. *For Rotating Wheels:*

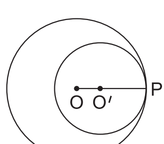
(i) Distance moved by a wheel in 1 rotation = its cicumference.

(ii) Number of rotations in 1 minute $= \frac{\text{distance moved in 1 minute}}{\text{circumference}}$.

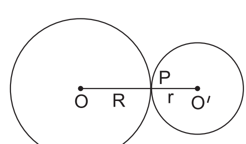
7. *Touching Circles:*

(i) When two circles touch internally [see fig. (i)], then distance between their centres = difference of their radii.

(ii) When two circles touch externally [see fig. (ii)], then distance between their centres = sum of their radii.



(i)



(ii)

SOLVED EXAMPLES

EXAMPLE 1 *Find the circumference and area of a circle of diameter* 28 cm.

SOLUTION Diameter = 28 cm $\Rightarrow$ radius r = 14 cm.

$\therefore$ circumference of the circle = $2\pi r$

$$= \left(2 \times \frac{22}{7} \times 14\right) \text{cm} = 88 \text{ cm}.$$

Area of the circle = $\pi r^2 = \left(\frac{22}{7} \times 14 \times 14\right) \text{cm}^2$

$$= 616 \text{ cm}^2.$$

EXAMPLE 2 *Find the area of a circle whose circumference is* 66 cm.

SOLUTION Let the radius of the circle be r cm.

Then, its circumference = $(2\pi r)$ cm.

$\therefore$ $2\pi r = 66 \Rightarrow 2 \times \frac{22}{7} \times r = 66$

$$\Rightarrow r = \left(66 \times \frac{7}{44}\right) = \frac{21}{2}.$$

$\therefore$ area of the circle = $\pi r^2 \text{cm}^2 = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2$

$$= 346.5 \text{ cm}^2.$$

EXAMPLE 3 *A steel wire, when bent in the form of a square, encloses an area of* 121 cm^2. *The same wire is bent in the form of a circle. Find the area of the circle.*

SOLUTION Area of the square = 121 cm^2.

Side of the square = $\sqrt{121}$ cm = 11 cm.

Perimeter of the square = 4 × 11 cm = 44 cm.

$\therefore$ length of the wire = 44 cm.

$\therefore$ circumference of the circle = length of the wire = 44 cm.

Let the radius of the circle be r cm.

Then, $2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7$.

$\therefore$ area of the circle = πr^2

$$= \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 154 \text{ cm}^2.$$

EXAMPLE 4 *A wire is looped in the form of a circle of radius* 28 cm. *It is rebent into a square form. Determine the length of the side of the square.*

SOLUTION Length of the wire = circumference of the circle

$$= \left(2 \times \frac{22}{7} \times 28\right) \text{ cm} = 176 \text{ cm}.$$

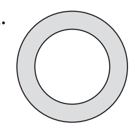
$\therefore$ perimeter of the square = length of the wire = 176 cm.

Hence, the side of the square $= \left(\frac{176}{4}\right)$ cm = 44 cm.

EXAMPLE 5 *A circular park,* 42 m *in diameter, has a path* 3.5 m *wide running round it on the outside. Find the cost of gravelling the path at* ₹ 20 *per* m^2.

SOLUTION Radius of the circular park = 21 m.

Radius of the outer circle = 21 m + 3.5 m = 24.5 m.



Area of the path $= \pi \times [(24.5)^2 - (21)^2]\ m^2$

$$= \frac{22}{7} \times (24.5 + 21)(24.5 - 21)\ m^2$$

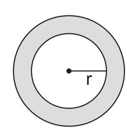
$$= \left(\frac{22}{7} \times 45.5 \times 3.5\right) m^2 = 500.5\ m^2.$$

$\therefore$ cost of gravelling the path = ₹ (500.5 × 20) = ₹ 10010.

EXAMPLE 6 *A road which is* 7 m *wide surrounds a circular park whose circumference is* 352 m. *Find the area of the road.*

SOLUTION Let the radius of the park be r m.

Then, its circumference = $2\pi r$ m.



$\therefore \quad 2\pi r = 352 \Rightarrow 2 \times \frac{22}{7} \times r = 352$

$$\Rightarrow r = 352 \times \frac{7}{44} = 56.$$

Thus, inner radius = 56 m,

outer radius = (56 + 7) m = 63 m.

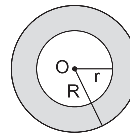
Area of the road $= \pi[(63)^2 - (56)^2]\ m^2$

$$= \frac{22}{7} \times (63 + 56)(63 - 56)\ m^2$$

$$= \left(\frac{22}{7} \times 119 \times 7\right) m^2 = 2618\ m^2.$$

EXAMPLE 7 *A racetrack is in the form of a ring whose inner and outer circumferences are* 437 m *and* 503 m *respectively. Find the width of the track and also its area.*

SOLUTION Let r m and R m be the radii of inner and outer boundaries.



Then, $2\pi r = 437$ and $2\pi R = 503$

$\Rightarrow \quad r = \frac{437}{2\pi}$ and $R = \frac{503}{2\pi}\cdot$

Width of the track $= (R - r)$ m

$$= \left(\frac{503}{2\pi} - \frac{437}{2\pi}\right) \text{ m}$$

$$= \frac{1}{2\pi} \times (503 - 437) \text{ m}$$

$$= \left(\frac{1}{2} \times \frac{7}{22} \times 66\right) \text{ m} = 10.5 \text{ m}.$$

Area of the track $= \pi(R^2 - r^2) \text{ m}^2 = \pi(R + r)(R - r) \text{ m}^2$

$$= \left[\pi\left(\frac{503}{2\pi} + \frac{437}{2\pi}\right) \times 10.5\right] \text{m}^2$$

$[\because \; (R - r) = 10.5]$

$$= \left[\left(\pi \times \frac{940}{2\pi}\right) \times 10.5\right] \text{m}^2 = \left(470 \times \frac{21}{2}\right) \text{m}^2$$

$$= (235 \times 21) \text{ m}^2 = 4935 \text{ m}^2.$$

EXAMPLE 8 *If the perimeter of a semicircular protractor is* 36 cm, *find its diameter.*

SOLUTION Let the radius of the protractor be r cm.

Then, perimeter $= (\pi r + 2r)$ cm $= (\pi + 2)r$ cm

$$= \left(\frac{22}{7} + 2\right) r \text{ cm} = \frac{36}{7} r \text{ cm}.$$

$$\therefore \quad \frac{36}{7} r = 36 \;\Rightarrow\; r = \left(36 \times \frac{7}{36}\right) = 7.$$

Hence, diameter $= 2r$ cm $= 14$ cm.

EXAMPLE 9 *A bicycle wheel makes* 5000 *revolutions in moving* 11 km. *Find the diameter of the wheel.*

SOLUTION Distance covered by the wheel in 1 revolution

$$= \left(\frac{11 \times 1000 \times 100}{5000}\right) \text{ cm} = 220 \text{ cm}.$$

$\therefore$ the circumference of the wheel = 220 cm.

Let the diameter of the wheel be d cm.

Then, $\pi d = 220 \Rightarrow \frac{22}{7} \times d = 220 \Rightarrow d = 220 \times \frac{7}{22} = 70.$

Hence, the diameter of the wheel is 70 cm.

EXAMPLE 10 *The diameter of each wheel of a bus is* 140 cm. *How many revolutions per minute must a wheel make in order to move at a speed of* 66 km *per hour?*

SOLUTION Distance covered by a wheel in 1 minute

$$= \left(\frac{66 \times 1000 \times 100}{60}\right) \text{cm} = 110000 \text{ cm}.$$

Circumference of a wheel $= \left(2 \times \frac{22}{7} \times 70\right)$ cm = 440 cm.

Number of revolutions in 1 min $= \left(\frac{110000}{440}\right) = 250.$

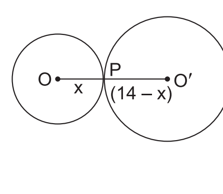
EXAMPLE 11 *Two circles touch externally. The sum of their areas is* 130π sq cm *and the distance between their centres is* 14 cm. *Find the radii of the circles.*

SOLUTION Since the given circles touch externally, we have

sum of their radii = distance between their centres = 14 cm.

Let the radii of the given circles be x cm and $(14 - x)$ cm.

Sum of their areas $= [\pi x^2 + \pi(14 - x)^2]$ cm^2.



$\therefore \quad \pi x^2 + \pi(14 - x)^2 = 130\pi$

$\Rightarrow \quad x^2 + (14 - x)^2 = 130$

$\Rightarrow \quad 2x^2 - 28x + 66 = 0$

$\Rightarrow \quad x^2 - 14x + 33 = 0$

$\Rightarrow \quad (x - 11)(x - 3) = 0$

$\Rightarrow \quad x - 11 = 0$ or $x - 3 = 0$

$\Rightarrow \quad x = 11$ or $x = 3$.

Now, $x = 11 \Rightarrow (14 - x) = (14 - 11) = 3$.

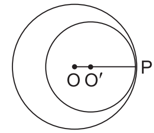
And, $x = 3 \Rightarrow (14 - x) = (14 - 3) = 11$.

Hence, the radii of the given circles are 11 cm and 3 cm.

EXAMPLE 12 *Two circles touch internally. The sum of their areas is* (116π) cm^2 *and the distance between their centres is* 6 cm. *Find the radii of the circles.*

[CBSE 2017]

SOLUTION The circles touch internally.



$\therefore$ difference of their radii

= distance between their centres = 6 cm.

Let the radii of given circles be r cm and $(r + 6)$ cm.

Sum of their areas = $[\pi r^2 + \pi(r+6)^2]$ cm^2 = $\pi[r^2 + (r+6)^2]$ cm^2.

$\therefore \quad \pi[r^2 + (r+6)^2] = 116\pi$

$\Rightarrow \quad r^2 + (r+6)^2 = 116$

$\Rightarrow \quad 2r^2 + 12r - 80 = 0$

$\Rightarrow \quad r^2 + 6r - 40 = 0$

$\Rightarrow \quad (r+10)(r-4) = 0$

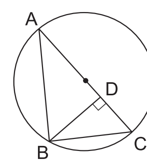
$\Rightarrow \quad r + 10 = 0$ or $r - 4 = 0$

$\Rightarrow \quad r = 4$ [neglecting $r = -10$, as radius cannot be negative]

$\therefore$ the radii of the given circles are 4 cm and 10 cm.

EXAMPLE 13 *Find the area of a right-angled triangle, if the radius of its circum-circle is* 7.5 cm *and the altitude drawn to the hypotenuse is* 6 cm *long.*

SOLUTION Let ΔABC be right-angled at B.



Hypotenuse AC

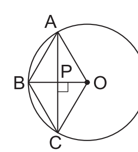
= diameter of its circumcircle

= (2×7.5) cm = 15 cm.

Let $BD \perp AC$. Then, $BD = 6$ cm.

$\therefore \quad ar(\Delta ABC) = \left(\frac{1}{2} AC \times BD\right) = \left(\frac{1}{2} \times 15 \times 6\right)$ cm^2 = 45 cm^2.

EXAMPLE 14 *In the given figure, OABC is a rhombus whose three vertices A, B, C lie on a circle of radius* 10 cm *and centre O. Find the area of the rhombus.* [*Take* $\sqrt{3} = 1.732$.]



SOLUTION Clearly, $OA = OB = OC = 10$ cm. Let OB and AC intersect at P.

Since the diagonals of a rhombus bisect each other at right angles, we have $OP = 5$ cm and $\angle OPC = 90°$.

Now, $AC = 2CP$ and

$$CP = \sqrt{OC^2 - OP^2} \text{ cm} = \sqrt{(10)^2 - 5^2} \text{ cm}$$

$$= \sqrt{75} \text{ cm} = 5\sqrt{3} \text{ cm}.$$

$\therefore \quad AC = (2 \times 5\sqrt{3})$ cm $= 10\sqrt{3}$ cm

$= (10 \times 1.732)$ cm $= 17.32$ cm.

$$\therefore \quad \text{ar(rhombus } OABC) = \left(\frac{1}{2} \times OB \times AC\right)$$

$$= \left(\frac{1}{2} \times 10 \times 17.32\right) \text{cm}^2$$

$$= 86.6 \text{ cm}^2.$$

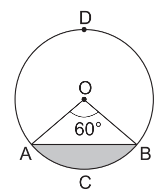
EXAMPLE 15 *In a circle of radius* 21 cm, *an arc subtends an angle of* 60° *at the centre. Find (i) the length of the arc, (ii) the area of the sector, (iii) the area of the minor segment, and (iv) the area of the major segment.* [*Given*, $\sqrt{3} = 1.72$.]

SOLUTION Let ACB be the given arc subtending an angle of 60° at the centre. Then, $r = 21$ cm and $\theta = 60°$.

(i) Length of the arc $ACB = \dfrac{2\pi r\theta}{360}$

$$= \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right) \text{cm}$$

$$= 22 \text{ cm}.$$



(ii) Area of the sector $OACBO = \dfrac{\pi r^2\theta}{360}$

$$= \left(\frac{22}{7} \times 21 \times 21 \times \frac{60}{360}\right) \text{cm}^2$$

$$= 231 \text{ cm}^2.$$

(iii) Area of the minor segment $ACBA$

= (area of the sector $OACB$) – (area of the $\triangle OAB$)

$$= \left(231 - \frac{1}{2}r^2 \sin\theta\right) \text{cm}^2 = \left[231 - \left(\frac{1}{2} \times 21 \times 21 \times \sin 60°\right)\right] \text{cm}^2$$

$$= \left(231 - \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2}\right) \text{cm}^2$$

$$= \left(231 - \frac{1}{4} \times 441 \times 1.72\right) \text{cm}^2$$

$$= (231 - 189.63) \text{ cm}^2 = 41.37 \text{ cm}^2.$$

(iv) Area of the major segment $BDAB$

= (area of the circle) – (area of the minor segment)

$$= \left\{\left(\frac{22}{7} \times 21 \times 21\right) - 41.37\right\} \text{cm}^2$$

$$= (1386 - 41.37) \text{ cm}^2 = 1344.63 \text{ cm}^2.$$

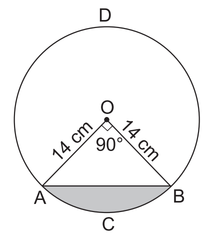
EXAMPLE 16 *A chord of a circle of radius* 14 cm *makes a right angle at the centre. Find the areas of the minor and the major segments of the circle.*

SOLUTION Let AB be the chord of a circle of centre O and radius = 14 cm such that $\angle AOB = 90°$.

$\therefore$ area of the sector $OACBO = \frac{\pi r^2 \theta}{360}$ cm^2

$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{90}{360}\right)$ cm^2

$= 154$ cm^2.



Area of $\triangle OAB = \frac{1}{2} r^2 \sin\theta$

$= \left(\frac{1}{2} \times 14 \times 14 \times \sin 90°\right)$ cm^2

$= 98$ cm^2.

Area of the minor segment $ACBA$

= (area of the sector $OACBO$) – (area of the $\triangle OAB$)

$= (154 - 98)$ cm^2 $= 56$ cm^2.

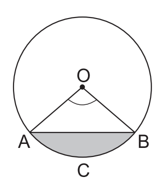
Area of the major segment $BDAB$

= (area of the circle) – (area of the minor segment)

$= \left[\left(\frac{22}{7} \times 14 \times 14\right) - 56\right]$ cm^2 $= (616 - 56)$ cm^2 $= 560$ cm^2.

EXAMPLE 17 *The perimeter of a sector of a circle of radius* 14 cm *is* 68 cm. *Find the area of the sector.*

SOLUTION Let O be the centre of a circle of radius 14 cm.



Let $OACBO$ be the sector whose perimeter is 68 cm.

Then, $OA + OB +$ arc $ACB = 68$ cm

$\Rightarrow$ 14 cm + 14 cm + arc $ACB = 68$ cm

$\Rightarrow$ arc $ACB = (68 - 28)$ cm $= 40$ cm.

$\therefore$ ar(sector $OACBO$) $= \left(\frac{1}{2} \times \text{radius} \times \text{arc length}\right)$

$= \left(\frac{1}{2} \times 14 \times 40\right)$ cm^2

$= 280$ cm^2.

EXAMPLE 18 *The minute hand of a clock is* 12 cm *long. Find the area of the face of the clock described by the minute hand in* 35 *minutes.*

SOLUTION Angle described by the minute hand in 60 minutes = 360°.

Angle described by the minute hand in 35 minutes

$$= \left(\frac{360}{60} \times 35\right)^\circ = 210^\circ.$$

$\therefore \quad \theta = 210^\circ$ and $r = 12$ cm.

Area swept by the minute hand in 35 minutes

$$= \left(\frac{\pi r^2 \theta}{360}\right) = \left(\frac{22}{7} \times 12 \times 12 \times \frac{210}{360}\right) \text{cm}^2 = 264 \text{ cm}^2.$$

EXAMPLE 19 *A car has wheels which are* 80 cm *in diameter. How many complete revolutions does each wheel make in* 10 *minutes, when the car is travelling at a speed of* 66 km *an hour?*

SOLUTION Distance covered by wheel in 1 minute

$$= \left(\frac{66 \times 1000 \times 100}{60}\right) \text{cm} = 110000 \text{ cm}.$$

Circumference of a wheel $= 2\pi R = \left(2 \times \frac{22}{7} \times 40\right)$ cm

$$= \frac{1760}{7} \text{ cm}.$$

Number of revolutions made by a wheel in 1 minute

$$= \left(110000 \times \frac{7}{1760}\right) \text{cm}.$$

Number of revolutions made by a wheel in 10 minutes

$$= \left(110000 \times \frac{7}{1760} \times 10\right) = 4375.$$

EXAMPLE 20 *A car has two wipers which do not overlap. Each wiper has a blade of length* 21 cm, *sweeping through an angle of* 120°. *Find the total area cleaned at each sweep of the blades.*

SOLUTION Clearly, each wiper sweeps a sector of a circle of radius 21 cm, making an angle of 120° at the centre of the circle.

So, the required area swept by two wipers

$$= \left(2 \times \frac{\pi r^2 \theta}{360}\right) = \left(2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120}{360}\right) \text{cm}^2$$

$$= 924 \text{ cm}^2.$$

EXAMPLE 21 *To warn ships for underwater rocks, a lighthouse spreads a red-coloured light over a sector of angle* 72° *to a distance of* 15 km. *Find the area of the sea over which the ships are warned.* [*Use* $\pi = 3.14$.]

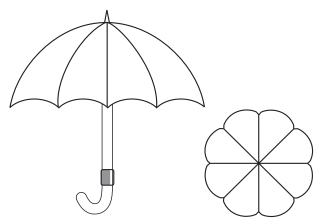
SOLUTION Required area = area of the sector in which, $r = 15$ km and $\theta = 72°$

$$= \left(\frac{\pi r^2 \theta}{360}\right) = \left(3.14 \times 15 \times 15 \times \frac{72}{360}\right) \text{km}^2$$

$$= \left(\frac{314 \times 45}{100}\right) \text{km}^2 = \frac{1413}{10} \text{km}^2$$

$$= 141.3 \text{ km}^2.$$

EXAMPLE 22 *An umbrella has 8 ribs which are equally spaced (as shown in the figure). Assuming the umbrella to be a flat circle of radius* 42 cm, *find the area between the two consecutive ribs of the umbrella.*



SOLUTION Since the ribs are equally spaced, so the angle made by two consecutive ribs at the centre $= \left(\frac{360}{8}\right)^\circ = 45°$.

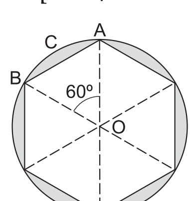
Area between two consecutive ribs

$= $ area of a sector of a circle with $r = 42$ cm and $\theta = 45°$

$$= \left(\frac{45}{360} \times \frac{22}{7} \times 42 \times 42\right) \text{cm}^2$$

$$= 693 \text{ cm}^2.$$

EXAMPLE 23 *A round table cover has six equal designs, as shown in the figure. If the radius of the cover is* 28 cm, *find the cost of making the designs at the rate of* ₹ 0.35 *per* cm^2. [*Use* $\sqrt{3} = 1.7$.] [CBSE 2009]



SOLUTION Let O be the centre of the table cover and let it be divided into six equal designs, each being a segment. Let one of these segments be $ACBA$. Clearly, $\angle AOB = \left(\frac{360}{6}\right)^\circ = 60^\circ$.

ar(segment $ACBA$) = ar(sector $OACBO$) – ar(equilateral ΔOAB)

$$= \left(\frac{\pi r^2\theta}{360} - \frac{\sqrt{3}}{4}a^2\right)$$

$$= \left[\left(\frac{22}{7}\times 28\times 28\times\frac{60}{360}\right) - \left(\frac{\sqrt{3}}{4}\times 28\times 28\right)\right]\text{cm}^2$$

$[\because\ a = OA = 28 \text{ cm}]$

$$= \left(\frac{1232}{3} - \frac{17}{10}\times 7\times 28\right)\text{cm}^2$$

$$= \left(\frac{1232}{3} - \frac{1666}{5}\right)\text{cm}^2$$

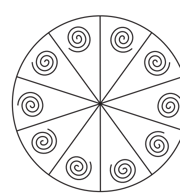
$$= \frac{(6160-4998)}{15}\ \text{cm}^2$$

$$= \frac{1162}{15}\ \text{cm}^2.$$

ar(all the six segments) $= \left(\frac{1162}{15}\times 6\right)\text{cm}^2 = \frac{2324}{5}\ \text{cm}^2.$

Cost of designs = ₹$\left(\frac{2324}{5}\times\frac{35}{100}\right)$ = ₹162.68.

EXAMPLE 24 *A broach is made with silver wire in the form of a circle with diameter* 35 mm. *The wire is also used in making* 5 *diameters which divide the circle into ten equal sectors, as shown in the figure.*



Find (i) the total length of the silver wire required,

(ii) the area of each sector of the broach.

SOLUTION (i) Total length of the wire required

= (circumference of given circle) + (length of five diameters)

$$= \left(2\times\frac{22}{7}\times\frac{35}{2}\right)\text{mm} + (5\times 35)\ \text{mm} = (110+175)\ \text{mm}$$

= 285 mm = 28.5 cm.

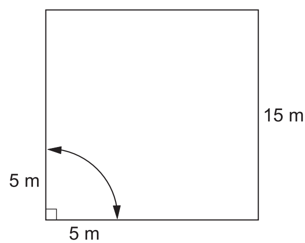
(ii) The given circle has been divided into 10 equal sectors.

$$\text{Area of each sector} = \frac{1}{10} \times \text{area of the given circle}$$
$$= \left(\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) \text{mm}^2$$
$$= \frac{385}{4} \text{ mm}^2 = 96.25 \text{ mm}^2.$$

EXAMPLE 25 *A horse is tied to a peg at one corner of a square-shaped grass field of side* 15 m *by means of a* 5-m-*long rope (as shown in the figure).*

Find (i) the area of that part of the field in which the horse can graze,

(ii) the increase in the grazing area if the rope were 10 m *long instead of* 5 m. [*Use* $\pi = 3.14$.]



SOLUTION (i) Clearly, the required area is the area of a quadrant of a circle of radius 5 m.

$$\therefore \text{ required area} = \frac{1}{4} \times \pi r^2 = \left(\frac{1}{4} \times 3.14 \times 5^2\right) \text{m}^2 = 19.625 \text{ m}^2.$$

(ii) Let the length of the rope be 10 m. Then,

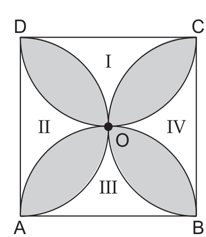
$$\text{area grazed} = \frac{1}{4} \times \pi R^2 = \left(\frac{1}{4} \times 3.14 \times 10^2\right) \text{m}^2 = 78.50 \text{ m}^2.$$

$$\text{Increase in grazing area} = (78.50 - 19.625) \text{ m}^2$$
$$= 58.875 \text{ m}^2.$$

AREAS OF COMBINATIONS OF PLANE FIGURES

SOLVED EXAMPLES

EXAMPLE 1 *In the given figure, ABCD is a square of side* 10 cm *and semicircles are drawn with each side of the square as diameter. Find the area of the shaded region.* [*Use* $\pi = 3.14$.]



SOLUTION Let us mark the unshaded regions as I, II, III and IV as shown in the given figure.

Let these regions meet at a common point O. Then,

(area of I) + (area of III)

$= \text{ar(sq } ABCD) - \{\text{ar(semicircle } AOD) + \text{ar(semicircle } BOC)\}$

$$= \left\{(10 \times 10) - \left(\frac{1}{2} \times 3.14 \times 5^2 + \frac{1}{2} \times 3.14 \times 5^2\right)\right\} \text{ cm}^2$$

$= (100 - 78.5) \text{ cm}^2 = 21.5 \text{ cm}^2.$

Similarly, (area of II) + (area of IV) = 21.5 cm^2.

Area of shaded region = ar(sq $ABCD$) – ar(I + II + III + IV)

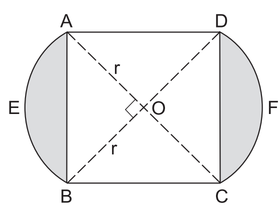
$= \{(10 \times 10) - (2 \times 21.5)\} \text{ cm}^2$

$= (100 - 43) \text{ cm}^2 = 57 \text{ cm}^2.$

EXAMPLE 2 *In the given figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side AB = 42 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find*

(i) the sum of the areas of the lawn and the flower beds,

(ii) the sum of the areas of two flower beds. [CBSE 2015]



SOLUTION Area of the square lawn $ABCD = (42 \times 42) \text{ m}^2$.

Let $OA = OB = r$ metres. Then, $\angle AOB = 90°$.

$\therefore \quad OA^2 + OB^2 = AB^2 \Rightarrow r^2 + r^2 = (42)^2$

$\Rightarrow 2r^2 = 1764 \Rightarrow r^2 = 882.$

(i) Sum of the areas of the lawn and the flower beds

$= \text{ar(sector } OAEBO) + \text{ar(sector } OCFDO) + \text{ar}(\triangle OAD) + \text{ar}(\triangle OBC)$

$$= \left(\frac{22}{7} \times 882 \times \frac{90}{360}\right) \text{m}^2 + \left(\frac{22}{7} \times 882 \times \frac{90}{360}\right) \text{m}^2 + \left(\frac{1}{2} \times r \times r\right) \text{m}^2 + \left(\frac{1}{2} \times r \times r\right) \text{m}^2$$

$$= 2\left\{\left(\frac{22}{7} \times 882 \times \frac{90}{360}\right) + \frac{1}{2}r^2\right\} \text{ m}^2$$

$$= 2\left(693 + \frac{1}{2} \times 882\right)\text{m}^2 = 2(693 + 441) \text{ m}^2$$

$$= 2268 \text{ m}^2.$$

Hence, the sum of the areas of the lawn and the two flower beds is 2268 m^2.

(ii) Sum of the areas of the two flower beds

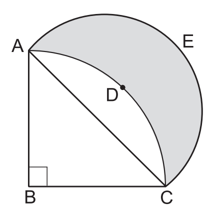
$$= 2\{\text{ar(sector } OAEBO) - \text{ar}(\triangle OAB)\}$$

$$= 2\left\{\left(\frac{22}{7} \times 882 \times \frac{90}{360}\right) - \frac{1}{2}r^2\right\} \text{ m}^2$$

$$= 2\left(693 - \frac{1}{2} \times 882\right) \text{m}^2 = 2(693 - 441) \text{ m}^2$$

$$= 504 \text{ m}^2.$$

EXAMPLE 3 *In the given figure, ABCD is a quadrant of a circle of radius* 28 cm *and a semicircle AECA is drawn with AC as diameter. Find the area of the shaded region.* [CBSE 2007, '08, '12, '14]



SOLUTION Since $ABCD$ is a quadrant of a circle, we have $\angle ABC = 90°$.

$\therefore \quad AC^2 = AB^2 + BC^2 = [(28)^2 + (28)^2] \text{ cm}^2$

$= (784 + 784) \text{ cm}^2 = 1568 \text{ cm}^2$

$\Rightarrow \quad AC = \sqrt{1568} \text{ cm} = 28\sqrt{2} \text{ cm}.$

$\therefore \quad$ radius of the semicircle $= \frac{1}{2}AC = 14\sqrt{2}$ cm.

Area of the shaded region

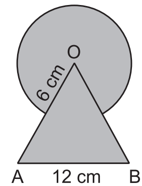
$= \text{ar}(\triangle ABC) + \text{ar(semicircle } ACEA) - \text{ar(quadrant } BCDA)$

$$= \left[\left(\frac{1}{2} \times 28 \times 28\right) + \left\{\frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2\right\} - \left\{\frac{22}{7} \times 28 \times 28 \times \frac{90}{360}\right\}\right]\text{cm}^2$$

$$= (392 + 616 - 616) \text{ cm}^2 = 392 \text{ cm}^2.$$

Hence, the area of the shaded region is 392 cm^2.

EXAMPLE 4 *Find the area of the shaded region in the given figure, where a circular arc of radius* 6 cm *has been drawn with vertex O of an equilateral triangle OAB of side* 12 cm *as centre.*
[*Take* $\sqrt{3} = 1.73$ *and* $\pi = 3.14$.]



SOLUTION Since $\triangle OAB$ is equilateral, we have $\angle AOB = 60°$.

Required area

= {(area of equilateral $\triangle OAB$) + (area of circle with r = 6 cm)}
– (area of sector of a circle with r = 6 cm and $\theta = 60°$)

$$= \left\{\left(\frac{\sqrt{3}}{4}\times 12\times 12\right)+(\pi\times 6\times 6)-\left(\pi\times 6\times 6\times\frac{60}{360}\right)\right\}\text{ cm}^2$$

$$= \{(1.73\times 36)+36\pi-6\pi\}\text{ cm}^2 = \{(1.73\times 36)+(30\pi)\}\text{ cm}^2$$

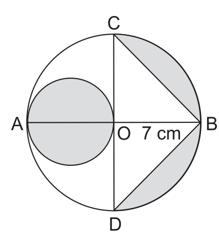
$$= \left\{\left(\frac{173}{100}\times 36\right)+\left(30\times\frac{314}{100}\right)\right\}\text{ cm}^2 = \left(\frac{1557}{25}+\frac{471}{5}\right)\text{ cm}^2$$

$$= \left(\frac{1557+2355}{25}\right)\text{ cm}^2 = \frac{3912}{25}\text{ cm}^2 = 156.48\text{ cm}^2.$$

Hence, the area of the shaded region is 156.48 cm^2.

EXAMPLE 5 *In the given figure, AB and CD are the diameters of a circle with centre O, perpendicular to each other. OA is the diameter of the smaller circle. If* $OB = 7$ cm, *find the area of the shaded region.*

[CBSE 2010, '13]



SOLUTION Clearly, the diameter of the larger circle is 14 cm and the diameter of the smaller circle is 7 cm.

So, the radius of the larger circle is 7 cm and that of the smaller circle is 3.5 cm.

Area of the shaded region

= {(area of smaller circle) + (area of larger semicircle)}
– (area of $\triangle CBD$)

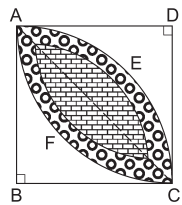
$$= \left[\left\{ \pi \times \left(\frac{7}{2} \right)^2 \right\} + \left\{ \frac{1}{2} \times \pi \times 7 \times 7 \right\} - \left\{ \frac{1}{2} \times CD \times OB \right\} \right] \text{cm}^2$$

$$= \left\{ \left(\frac{22}{7} \times \frac{49}{4} \right) + \left(\frac{1}{2} \times \frac{22}{7} \times 49 \right) - \left(\frac{1}{2} \times 14 \times 7 \right) \right\} \text{cm}^2$$

$$= \left(\frac{77}{2} + 77 - 49 \right) \text{cm}^2 = (38.5 + 28)\ \text{cm}^2 = 66.5\ \text{cm}^2.$$

Hence, the area of the shaded region is $66.5\ \text{cm}^2$.

EXAMPLE 6 *Calculate the area of the designed region in the given figure, common between two quadrants of circles of radius* 7 cm *each.*



SOLUTION Area of the designed region

$$= 2[\text{ar(quadrant } ABCE) - \text{ar}(\Delta ABC)]$$

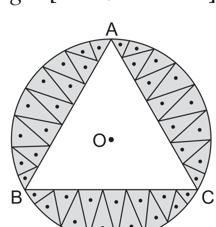
$$= 2\left[\frac{1}{4} \pi \times (7)^2 - \frac{1}{2} \times 7 \times 7 \right] \text{cm}^2$$

$$= 2\left[\left(\frac{1}{4} \times \frac{22}{7} \times 49 \right) - \left(\frac{49}{2} \right) \right] \text{cm}^2$$

$$= 2\left(\frac{77}{2} - \frac{49}{2} \right) \text{cm}^2 = \left(2 \times \frac{28}{2} \right) \text{cm}^2 = 28\ \text{cm}^2.$$

Hence, the area of the designed region is $28\ \text{cm}^2$.

EXAMPLE 7 *In a circular table cover of radius* 42 cm, *a design is formed, leaving an equilateral triangle ABC in the middle, as shown in the figure. Find the area of the design.* [*Use* $\sqrt{3} = 1.73$.] [CBSE 2013C]

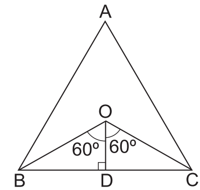


SOLUTION Let O be the centre of the circle. Join BO and CO.

Now, $\angle BAC = 60° \Rightarrow \angle BOC = 2(\angle BAC) = 120°$.

Draw $OD \perp BC$. Then, $\angle BOD = \angle COD = 60°$.

From right ΔBDO, we have



$$\frac{OD}{OB} = \cos 60° = \frac{1}{2} \Rightarrow \frac{OD}{42 \text{ cm}} = \frac{1}{2}$$

$$\Rightarrow \quad OD = \left(42 \times \frac{1}{2}\right) \text{cm} = 21 \text{ cm},$$

and $\dfrac{BD}{OB} = \sin 60° = \dfrac{\sqrt{3}}{2} \Rightarrow \dfrac{BD}{42 \text{ cm}} = \dfrac{\sqrt{3}}{2}$

$$\Rightarrow \quad BD = \left(42 \times \frac{\sqrt{3}}{2}\right) \text{cm} = 21\sqrt{3} \text{ cm}.$$

$$\therefore \quad BC = 2 \times BD = (2 \times 21\sqrt{3}) \text{ cm} = (42\sqrt{3}) \text{ cm}.$$

Area of the designed region

= (area of the circle with $r = 42$ cm)

− (area of equilateral ΔABC with $a = 42\sqrt{3}$ cm)

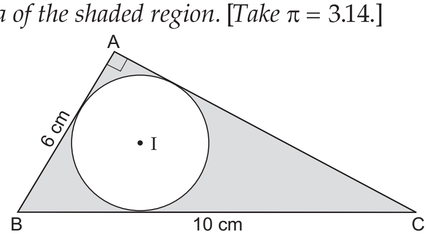
$$= \left(\pi r^2 - \frac{\sqrt{3}}{4}a^2\right) = \left\{\left(\frac{22}{7} \times 42 \times 42\right) - \left(\frac{1.73}{4} \times (42\sqrt{3})^2\right)\right\} \text{cm}^2$$

$$= \left\{5544 - \left(\frac{1.73}{4} \times 1764 \times 3\right)\right\} \text{cm}^2 = (5544 - 2288.79) \text{ cm}^2$$

$$= 3255.21 \text{ cm}^2.$$

Hence, the required area is 3255.21 cm^2.

EXAMPLE 8 *In the given figure, ABC is a right-angled triangle, right-angled at A, in which $AB = 6$ cm, $BC = 10$ cm and I is the incentre of ΔABC. Find the area of the shaded region.* [*Take* $\pi = 3.14$.] [CBSE 2009]



SOLUTION From right ΔBAC, we have

$$AC^2 = BC^2 - AB^2 = \{(10)^2 - 6^2\} \text{ cm}^2$$
$$= (100 - 36) \text{ cm}^2 = 64 \text{ cm}^2$$

$$\Rightarrow \quad AC = \sqrt{64} \text{ cm} = 8 \text{ cm}.$$

$$\text{ar}(\Delta ABC) = \left(\frac{1}{2} \times \text{base} \times \text{height}\right) = \left(\frac{1}{2} \times AB \times AC\right)$$

$$= \left(\frac{1}{2} \times 6 \times 8\right) \text{cm}^2 = 24 \text{ cm}^2.$$

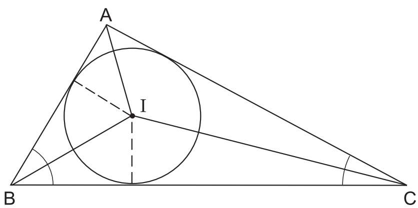
Let the radius of the incircle be r. Then,

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle IAB) + \text{ar}(\triangle IBC) + (\triangle ICA)$$

$$\Rightarrow \quad 24 \text{ cm}^2 = \left(\frac{1}{2} \times AB \times r\right) + \left(\frac{1}{2} \times BC \times r\right) + \left(\frac{1}{2} \times CA \times r\right)$$

$$= \frac{1}{2} r(AB + BC + CA) = \frac{1}{2} r(6 \text{ cm} + 10 \text{ cm} + 8 \text{ cm}) = (12 \text{ cm})r$$

$$\Rightarrow \quad r = 2 \text{ cm}.$$



$\therefore$ area of the shaded region

$= \text{ar}(\triangle ABC) - (\text{area of incircle with } r = 2 \text{ cm})$

$= (24 - \pi r^2) \text{ cm}^2 = (24 - 3.14 \times 2 \times 2) \text{ cm}^2$

$= (24 - 12.56) \text{ cm}^2 = 11.44 \text{ cm}^2.$

Hence, the required area is 11.44 cm^2.

EXAMPLE 9 *In an equilateral triangle of side* 12 cm, *a circle is inscribed touching its sides. Find the area of the portion of the triangle not included in the circle.* [*Take* $\sqrt{3} = 1.73$ *and* $\pi = 3.14$.]

SOLUTION Let ABC be an equilateral triangle of side 12 cm.

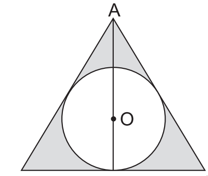
Let $AD \perp BC$. Then, D is the midpoint of BC.

$\therefore \quad BD = DC = 6$ cm and $AB = 12$ cm.

$\therefore \quad AD = \sqrt{AB^2 - BD^2}$ [by Pythagoras' theorem]

$= \sqrt{(12)^2 - 6^2}$ cm $= \sqrt{(144 - 36)}$ cm

$= \sqrt{108}$ cm $= 6\sqrt{3}$ cm.



Let O be the centre of the inscribed circle.

Then, O is the centroid of $\triangle ABC$.

$\therefore \quad AO : OD = 2 : 1$ and $OD = \frac{1}{3} AD = \frac{1}{3} \times 6\sqrt{3}$ cm $= 2\sqrt{3}$ cm.

$\therefore \quad r = OD = 2\sqrt{3}$ cm.

Required area = area of the shaded region

$= \text{ar}(\triangle ABC) - (\text{area of the incircle})$

$$= \left[\left(\frac{\sqrt{3}}{4} \times 12 \times 12\right) - \{3.14 \times (2\sqrt{3})^2\}\right] \text{cm}^2$$

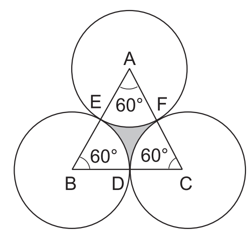
$= [36\sqrt{3} - (3.14 \times 12)]\ \text{cm}^2$

$= (36 \times 1.73 - 37.68)\ \text{cm}^2$

$= (62.28 - 37.68)\ \text{cm}^2 = 24.6\ \text{cm}^2.$

Hence, the required area is $24.6\ \text{cm}^2$.

EXAMPLE 10 *The area of an equilateral triangle is* $100\sqrt{3}\ \text{cm}^2$. *Taking each vertex as centre, a circle is described with radius equal to half the length of the side of the triangle, as shown in the figure. Find the area of that part of the triangle which is not included in the circles.* [*Take* $\pi = 3.14$ *and* $\sqrt{3} = 1.732$.]



SOLUTION Let each side of the triangle be a cm. Then,

area of the triangle $= \left(\frac{\sqrt{3}}{4}a^2\right) \text{cm}^2.$

$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 100\sqrt{3} \Rightarrow a^2 = 400 \Rightarrow a = 20.$

Thus, the length of each side of $\triangle ABC$ is 20 cm.

$\therefore$ radius of each circle = 10 cm.

Required area

$= (\text{area of } \triangle ABC) - 3(\text{area of a sector with } r = 10 \text{ cm}, \theta = 60°)$

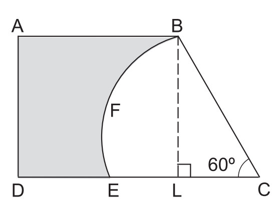
$$= \left(100\sqrt{3} - 3 \times 3.14 \times 10 \times 10 \times \frac{60}{360}\right) \text{cm}^2$$

$= \{(100 \times 1.732) - 157\}\ \text{cm}^2 = (173.2 - 157)\ \text{cm}^2 = 16.2\ \text{cm}^2.$

Hence, the area of the required part is $16.2\ \text{cm}^2$.

EXAMPLE 11 *In the given figure, ABCD is a trapezium with* $AB \parallel CD$ *and* $\angle BCD = 60°$. *If BFEC is a sector of a circle with centre C and* $AB = BC = 7$ cm *and* $DE = 4$ cm, *then find the area of the shaded region.* [*Use* $\pi = 22/7$ *and* $\sqrt{3} = 1.73$.]

[CBSE 2010]



SOLUTION Draw $BL \perp CD$.

Clearly, $CE = CB = 7$ cm.

And, $CD = CE + DE = 7 \text{ cm} + 4 \text{ cm} = 11$ cm.

From right $\triangle CLB$, we have

$$\frac{BL}{BC} = \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow \frac{BL}{7 \text{ cm}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad BL = \frac{7\sqrt{3}}{2} \text{ cm} = \frac{7 \times 1.73}{2} \text{ cm} = \frac{12.11}{2} \text{ cm} = 6.05 \text{ cm}.$$

Required shaded area

$= \text{ar(trapezium } ABCD) - \text{ar(sector } BCEFB)$

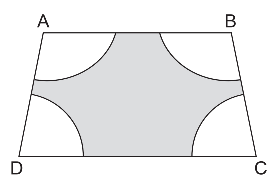
$= \left\{\frac{1}{2} \times (AB + CD) \times BL\right\} - \left\{\frac{\pi r^2 \theta}{360}\right\}$, where $r = CB = 7$ cm

$$= \left[\left\{\frac{1}{2}(7 + 11) \times 6.05\right\} - \left\{\frac{22}{7} \times 7 \times 7 \times \frac{60}{360}\right\}\right] \text{cm}^2$$

$= (54.45 - 25.66) \text{ cm}^2 = 28.79 \text{ cm}^2$.

Hence, the area of the shaded region is 28.79 cm^2.

EXAMPLE 12 *In the given figure, ABCD is a trapezium in which* $AB \parallel DC$, $AB = 18$ cm, $DC = 32$ cm *and the distance between AB and DC is* 14 cm. *If arcs of equal radii* 7 cm *have been drawn with centres A, B, C and D then find the area of the shaded region.* [CBSE 2015, '17]



SOLUTION $\text{ar(trap. } ABCD) = \frac{1}{2}$ (sum of parallel sides)

× (distance between them)

$$= \left\{\frac{1}{2}(18 + 32) \times 14\right\} \text{cm}^2 = 350 \text{ cm}^2.$$

Sum of the areas of the 4 sectors

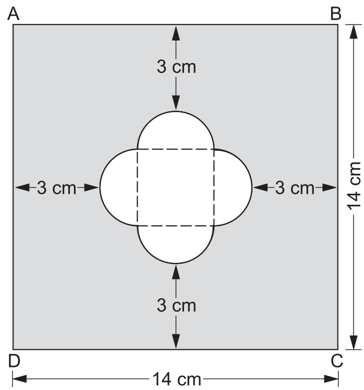
= area of a circle of radius 7 cm

$$= \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 154 \text{ cm}^2.$$

Area of the shaded region $= (350 - 154) \text{ cm}^2 = 196 \text{ cm}^2$.

Hence, the area of the shaded region is 196 cm^2.

EXAMPLE 13 *Find the area of the shaded region in the figure, given below.* [*Take* $\pi = 3.14$.] [CBSE 2015]



SOLUTION We have

$$\text{ar(sq } ABCD) = (14 \times 14) \text{ cm}^2 = 196 \text{ cm}^2.$$

Let the radius of each semicircle be a cm.

Then, $(a + 2a + a) = \{14 - (3 + 3)\}$ cm = 8 cm.

$\therefore \quad 4a = 8 \Rightarrow a = 2.$

So, the radius of each semicircle = 2 cm.

Area of 4 semicircles $= \left(4 \times \frac{1}{2} \times 3.14 \times 2 \times 2\right) \text{ cm}^2 = 25.12 \text{ cm}^2.$

Length of each side of the smaller square

$= 2a$ cm $= (2 \times 2)$ cm = 4 cm.

Area of smaller square $= (4 \times 4) \text{ cm}^2 = 16 \text{ cm}^2.$

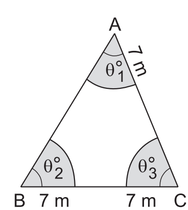
Area of unshaded region $= (25.12 + 16) \text{ cm}^2 = 41.12 \text{ cm}^2$

Area of shaded region $= (196 - 41.12) \text{ cm}^2 = 154.88 \text{ cm}^2$

Hence, the required area is 154.88 cm^2.

EXAMPLE 14 *Three horses are tethered with 7-metre-long ropes at the three corners of a triangular field having sides* 20 m, 34 m *and* 42 m. *Find the area of the plot which can be grazed by the horses. Also, find the area of the plot which remains ungrazed.* [CBSE 2009]

SOLUTION Let $\angle A = \theta_1^\circ$, $\angle B = \theta_2^\circ$ and $\angle C = \theta_3^\circ$.



Area which can be grazed by the horses

= sum of the areas of three sectors with central angles θ_1°, θ_2° and θ_3°, and each with $r = 7$ m

$$= \left(\frac{\pi r^2 \theta_1}{360} + \frac{\pi r^2 \theta_2}{360} + \frac{\pi r^2 \theta_3}{360} \right) \text{m}^2, \text{ where } r = 7 \text{ m}$$

$$= \frac{\pi r^2}{360}(\theta_1 + \theta_2 + \theta_3) \text{ m}^2 = \left(\frac{\pi r^2 \times 180}{360} \right) \text{m}^2$$

$$[\because \theta_1 + \theta_2 + \theta_3 = \angle A + \angle B + \angle C = 180°]$$

$$= \left(\frac{22}{7} \times 7 \times 7 \times \frac{1}{2} \right) \text{m}^2 = 77 \text{ m}^2.$$

Sides of the plot are $a = 20$ m, $b = 34$ m, $c = 42$ m.

$\therefore \quad s = \frac{1}{2}(a + b + c) = 48$ m, $(s - a) = 28$ m, $(s - b) = 14$ m and

$(s - c) = 6$ m.

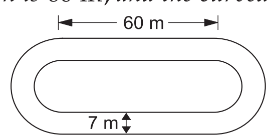
$\therefore \quad$ area of the whole plot = area of $\triangle ABC$

$$= \sqrt{s(s-a)(s-b)(s-c)} \text{ sq units}$$

$$= \sqrt{48 \times 28 \times 14 \times 6} \text{ m}^2 = 336 \text{ m}^2.$$

Area ungrazed $= (336 - 77)$ m^2 $= 259$ m^2.

EXAMPLE 15 *The inside perimeter of a running track, as shown in the figure, is* 340 m. *The length of each straight portion is* 60 m, *and the curved portions are semicircles. If the track is* 7 m *wide, find the area of the track. Also, find the outer perimeter of the track.* [CBSE 2008C]



SOLUTION Length of inner curved portion $= (340 - 2 \times 60)$ m $= 220$ m.

$\therefore \quad$ the length of each inner curved part = 110 m.

Let the radius of each inner curved part be r.

Then, $\pi r = 110 \text{ m} \Rightarrow \frac{22}{7} \times r = 110$ m

$$\Rightarrow r = \left(110 \times \frac{7}{22} \right) \text{m} = 35 \text{ m}.$$

$\therefore \quad$ inner radius = 35 m, outer radius $= (35 + 7)$ m $= 42$ m.

$\therefore \quad$ area of the track

= (area of 2 rectangles, each 60 m × 7 m)
+ (area of the circular ring with $R = 42$ m, $r = 35$ m)

$$= \left[(2 \times 60 \times 7) + \frac{22}{7} \times \{(42)^2 - (35)^2\} \right] \text{m}^2$$

$$= \left[840 + \frac{22}{7} \times (42 + 35)(42 - 35) \right] \text{m}^2$$

$$= (840 + 1694)\ \text{m}^2 = 2534\ \text{m}^2.$$

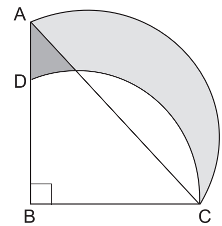
Length of the outer boundary of the track

$$= \left(2 \times 60 + 2 \times \frac{22}{7} \times 42\right) \text{m} = 384\ \text{m}.$$

Hence, the outer perimeter of the track is 384 m.

EXAMPLE 16 *In the given figure, $\triangle ABC$ is a right-angled triangle with $\angle B = 90°$, $AB = 48$ cm and $BC = 14$ cm. With AC as diameter a semicircle is drawn and with BC as radius, a quadrant of a circle is drawn. Find the area of the shaded region.*

[*Use* $\pi = 22/7$.] [CBSE 2009C]



SOLUTION In right $\triangle ABC$, we have

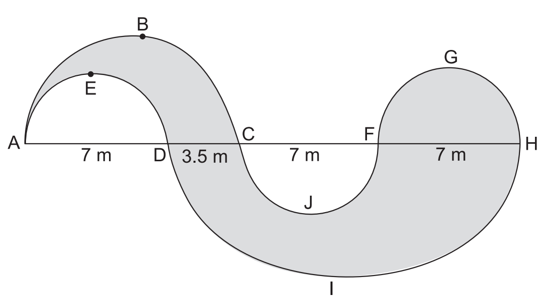
$$AC^2 = AB^2 + BC^2 = [(48)^2 + (14)^2]\ \text{cm}^2$$
$$= (2304 + 196)\ \text{cm}^2 = 2500\ \text{cm}^2$$
$$\Rightarrow \quad AC = \sqrt{2500}\ \text{cm} = 50\ \text{cm} \Rightarrow R = \frac{50}{2}\ \text{cm} = 25\ \text{cm}.$$

Area of the shaded region

$$= \text{ar}(\triangle ABC) + (\text{area of semicircle on } AC) - \text{ar}(\text{quadrant } BCD)$$
$$= \left(\frac{1}{2} \times 14 \times 48\right) + \frac{1}{2}\pi \times 25 \times 25 - \frac{1}{4}\pi \times BC^2$$
$$= \left[336 + \frac{1}{2} \times \frac{22}{7} \times 625 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right] \text{cm}^2$$
$$= \left(336 + \frac{6875}{7} - 154\right) \text{cm}^2 = (182 + 982.14)\ \text{cm}^2$$
$$= 1164.14\ \text{cm}^2.$$

Hence, the area of the shaded region is $1164.14\ \text{cm}^2$.

EXAMPLE 17 *Find the area of the shaded region of the figure given below.* [CBSE 2011]



SOLUTION Area of the shaded region

= ar($ABCDEA$) + ar($FGHF$) + ar($HIDCJFH$)

= ar(semicircle ABC) – ar(semicircle AED) + ar(semicircle FGH) – ar(semicircle HID) – ar(semicircle FJC)

$$=\left[\left\{\frac{1}{2}\pi\times\frac{21}{4}\times\frac{21}{4}\right\}-\left\{\frac{1}{2}\pi\times\frac{7}{2}\times\frac{7}{2}\right\}+\left\{\frac{1}{2}\pi\times\frac{7}{2}\times\frac{7}{2}\right\}+\left\{\frac{1}{2}\pi\times\frac{35}{4}\times\frac{35}{4}\right\}-\left\{\frac{1}{2}\pi\times\frac{7}{2}\times\frac{7}{2}\right\}\right]\text{m}^2$$

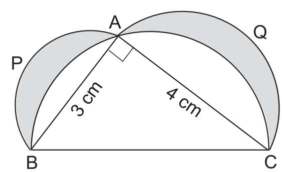
$$=\left\{\left(\frac{22}{7}\times\frac{441}{32}\right)+\left(\frac{1}{2}\times\frac{22}{7}\times\frac{35}{4}\times\frac{35}{4}\right)-\left(\frac{22}{7}\times\frac{49}{8}\right)\right\}\text{ m}^2$$

$$=\left\{\left(\frac{11\times 63}{16}\right)+\left(\frac{55\times 35}{16}\right)-\frac{77}{4}\right\}\text{ m}^2=\left(\frac{693}{16}+\frac{1925}{16}-\frac{77}{4}\right)\text{m}^2$$

$$=\left(\frac{693+1925-308}{16}\right)\text{m}^2=\frac{1155}{8}\text{ m}^2=144.38\text{ m}^2.$$

Hence, the required area is 144.38 m^2.

EXAMPLE 18 *In the given figure, $\triangle ABC$ is right angled at A. Semicircles are drawn on AB, AC and BC as diameters. It is given that AB* = 3 cm *and AC* = 4 cm. *Find the area of the shaded region.* [CBSE 2017]



SOLUTION From right $\triangle BAC$, we get

$$BC=\sqrt{AB^2+AC^2}=\sqrt{3^2+4^2}\text{ cm}$$

$$=\sqrt{9+16}\text{ cm}=\sqrt{25}\text{ cm}=5\text{ cm}.$$

Area of the shaded region

= {ar($\triangle ABC$) + ar(semicircle APB) + ar(semicircle AQC)} – ar(semicircle BAC)

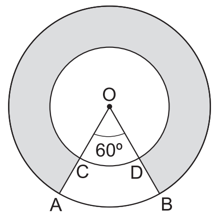
$$=\left[\left(\frac{1}{2}\times 3\times 4\right)+\left(\frac{1}{2}\pi\times\frac{3}{2}\times\frac{3}{2}\right)+\left(\frac{1}{2}\pi\times 2\times 2\right)-\left(\frac{1}{2}\pi\times\frac{5}{2}\times\frac{5}{2}\right)\right]\text{ cm}^2$$

$$=\left\{6+\frac{1}{2}\pi\left(\frac{9}{4}+4-\frac{25}{4}\right)\right\}\text{ cm}^2=(6+0)\text{ cm}^2=6\text{ cm}^2.$$

Hence, the area of the shaded region is 6 cm^2.

EXAMPLE 19 *In the given figure, two concentric circles with centre O, have radii* 21 cm *and* 42 cm. *If* $\angle AOB = 60°$ *find the area of the shaded region.*

[CBSE 2014]



SOLUTION ar(region $ACDB$) = ar(sector AOB) – ar(sector COD)

$$= \left\{\left(\frac{22}{7} \times 42 \times 42 \times \frac{60}{360}\right) - \left(\frac{22}{7} \times 21 \times 21 \times \frac{60}{360}\right)\right\} \text{cm}^2$$

$$= (924 - 231)\ \text{cm}^2 = 693\ \text{cm}^2.$$

$$\text{ar(circular ring)} = \left\{\left(\frac{22}{7} \times 42 \times 42\right) - \left(\frac{22}{7} \times 21 \times 21\right)\right\} \text{cm}^2$$

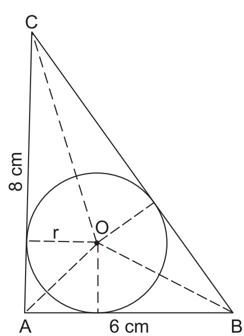
$$= (5544 - 1386)\ \text{cm}^2 = 4158\ \text{cm}^2.$$

Required area = ar(circular ring) – ar(region $ACDB$)

$$= (4158 - 693)\ \text{cm}^2 = 3465\ \text{cm}^2.$$

Hence, the area of the shaded region is $3465\ \text{cm}^2$.

EXAMPLE 20 *In the given figure,* $\triangle ABC$ *is right-angled at A with* $AB = 6$ cm *and* $AC = 8$ cm. *A circle with centre O has been inscribed inside the triangle. Find the value of r, the radius of the inscribed circle.*



SOLUTION $BC = \sqrt{AB^2 + AC^2} = \sqrt{6^2 + 8^2}$

$$= \sqrt{36 + 64} = \sqrt{100} = 10\ \text{cm}.$$

Let O be the centre of the inscribed circle and r be its radius. Join OA, OB and OC.

$$\text{ar}(\triangle OAC) + \text{ar}(\triangle OAB) + \text{ar}(\triangle BOC) = \text{ar}(\triangle ABC)$$

$$\Rightarrow \left(\frac{1}{2}\times 8\times r\right)+\left(\frac{1}{2}\times 6\times r\right)+\left(\frac{1}{2}\times 10\times r\right)=\frac{1}{2}\times 6\times 8.$$

$$\Rightarrow (4r + 3r + 5r) = 24 \Rightarrow 12r = 24 \Rightarrow r = 2 \text{ cm}.$$

Hence, $r = 2$ cm.

EXERCISE 16A

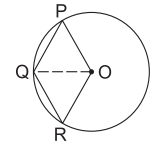
Use $\pi = \frac{22}{7}$, unless stated otherwise.

1. The circumference of a circle is 39.6 cm. Find its area.
2. The area of a circle is 98.56 cm^2. Find its circumference.
3. The circumference of a circle exceeds its diameter by 45 cm. Find the circumference of the circle.
4. A copper wire when bent in the form of a square encloses an area of 484 cm^2. The same wire is now bent in the form of a circle. Find the area enclosed by the circle.
5. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm^2. The same wire is bent to form a circle. Find the area enclosed by the circle.
6. The length of a chain used as the boundary of a semicircular park is 108 m. Find the area of the park.
7. The sum of the radii of two circles is 7 cm, and the difference of their circumferences is 8 cm. Find the circumferences of the circles.
8. Find the area of a ring whose outer and inner radii are respectively 23 cm and 12 cm.
9. (i) A path of 8 m width runs around the outside of a circular park whose radius is 17 m. Find the area of the path.

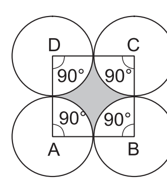
   (ii) A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path, if its cost is ₹ 110 per sq m. [CBSE 2017]
10. A racetrack is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width and the area of the track.
11. A sector is cut from a circle of radius 21 cm. The angle of the sector is 150°. Find the length of the arc and the area of the sector.

12. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of the circle. Find the area of major and minor segments of the circle. [CBSE 2017]
13. The length of an arc of a circle, subtending an angle of $54°$ at the centre is 16.5 cm. Calculate the radius, circumference and area of the circle.
14. The radius of a circle with centre O is 7 cm. Two radii OA and OB are drawn at right angles to each other. Find the areas of minor and major segments.
15. Find the lengths of the arcs cut off from a circle of radius 12 cm by a chord 12 cm long. Also, find the area of the minor segment. [Take $\pi = 3.14$ and $\sqrt{3} = 1.73$.]
16. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the areas of both the segments. [Take $\pi = 3.14$.]
17. Find the area of both the segments of a circle of radius 42 cm with central angle 120°. [Given, $\sin 120° = \frac{\sqrt{3}}{2}$ and $\sqrt{3} = 1.73$.]
18. A chord of a circle of radius 30 cm makes an angle of 60° at the centre of the circle. Find the areas of the minor and major segments. [Take $\pi = 3.14$ and $\sqrt{3} = 1.732$.]
19. In a circle of radius 10.5 cm, the minor arc is one-fifth of the major arc. Find the area of the sector corresponding to the major arc.
20. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. [Take $\pi = 3.14$.]
21. Find the area of a quadrant of a circle whose circumference is 88 cm.
22. A rope by which a cow is tethered is increased from 16 m to 23 m. How much additional ground does it have now to graze?
23. A horse is placed for grazing inside a rectangular field 70 m by 52 m. It is tethered to one corner by a rope 21 m long. On how much area can it graze? How much area is left ungrazed?
24. A horse is tethered to one corner of a field which is in the shape of an equilateral triangle of side 12 m. If the length of the rope is 7 m, find the area of the field which the horse cannot graze. Take $\sqrt{3} = 1.732$. Write the answer correct to 2 places of decimal.
25. Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed? [Take $\pi = 3.14$.]

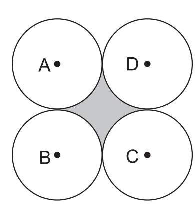
26. In the given figure, *OPQR* is a rhombus, three of whose vertices lie on a circle with centre *O*. If the area of the rhombus is $32\sqrt{3}\ \text{cm}^2$, find the radius of the circle.



27. The side of a square is 10 cm. Find (i) the area of the inscribed circle, and (ii) the area of the circumscribed circle. [Take π = 3.14.]
28. If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.
29. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle. [Take $\sqrt{3} = 1.73$.]
30. The radius of the wheel of a vehicle is 42 cm. How many revolutions will it complete in a 19.8-km-long journey?
31. The wheels of the locomotive of a train are 2.1 m in radius. They make 75 revolutions in one minute. Find the speed of the train in km per hour.
32. The wheels of a car make 2500 revolutions in covering a distance of 4.95 km. Find the diameter of a wheel.
33. A boy is cycling in such a way that the wheels of his bicycle are making 140 revolutions per minute. If the diameter of a wheel is 60 cm, calculate the speed (in km/hr) at which the boy is cycling.
34. The wheel of a motorcycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/hr?
35. The diameters of the front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that a rear wheel makes to cover the distance which the front wheel covers in 800 revolutions.
36. Four equal circles are described about the four corners of a square so that each touches two of the others, as shown in the figure. Find the area of the shaded region, if each side of the square measures 14 cm. [CBSE 2007]

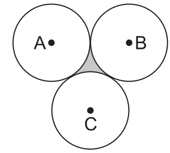


37. Four equal circles, each of radius 5 cm, touch each other, as shown in the figure. Find the area included between them. [Take π = 3.14.]



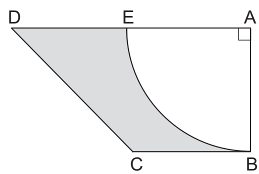
38. Four equal circles, each of radius *a* units, touch each other. Show that the area between them is $\left(\frac{6}{7}a^2\right)$ sq units.

39. Three equal circles, each of radius 6 cm, touch one another as shown in the figure. Find the area enclosed between them. [Take $\pi = 3.14$ and $\sqrt{3} = 1.732$.]

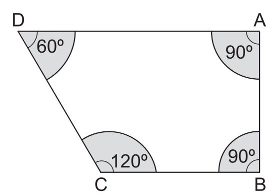


40. If three circles of radius a each, are drawn such that each touches the other two, prove that the area included between them is equal to $\frac{4}{25}a^2$.
[Take $\sqrt{3} = 1.73$ and $\pi = 3.14$.]

41. In the given figure, $ABCD$ is a trapezium of area 24.5 cm^2. If $AD \parallel BC$, $\angle DAB = 90°$, $AD = 10$ cm, $BC = 4$ cm and ABE is quadrant of a circle then find the area of the shaded region. [CBSE 2014]



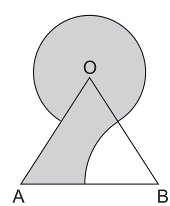
42. $ABCD$ is a field in the shape of a trapezium, $AD \parallel BC$, $\angle ABC = 90°$ and $\angle ADC = 60°$. Four sectors are formed with centres A, B, C and D, as shown in the figure. The radius of each sector is 14 m. Find the following:



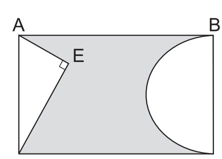
(i) total area of the four sectors,

(ii) area of the remaining portion, given that $AD = 55$ m, $BC = 45$ m and $AB = 30$ m. [CBSE 2013C]

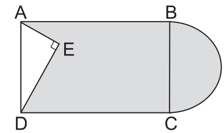
43. Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex of an equilateral triangle of side 12 cm as centre and a sector of circle of radius 6 cm with centre B is made. [Use $\sqrt{3} = 1.73$ and $\pi = 3.14$.] [CBSE 2014]



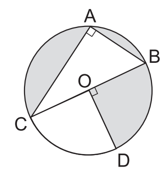
44. In the given figure, $ABCD$ is a rectangle with $AB = 80$ cm and $BC = 70$ cm, $\angle AED = 90°$ and $DE = 42$ cm. A semicircle is drawn, taking BC as diameter. Find the area of the shaded region. [CBSE 2014]



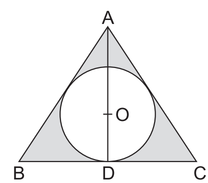
45. In the given figure, from a rectangular region $ABCD$ with $AB = 20$ cm, a right triangle AED with $AE = 9$ cm and $DE = 12$ cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. [Use $\pi = 3.14$.] [CBSE 2014]



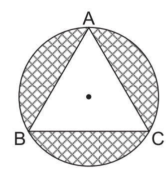
46. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of shaded region. [Use $\pi = 3.14$.] [CBSE 2012, '17]



47. In the given figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. [Use $\sqrt{3} = 1.73$ and $\pi = 3.14$.] [CBSE 2014]



48. On a circular table cover of radius 42 cm, a design is formed by a girl leaving an equilateral triangle ABC in the middle, as shown in the figure. Find the covered area of the design. [Use $\sqrt{3} = 1.73$ and $\pi = \frac{22}{7}$.] [CBSE 2013C]



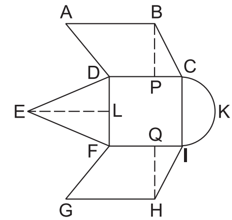
49. The perimeter of the quadrant of a circle is 25 cm. Find its area. [CBSE 2012]

50. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the minor segment. [Use $\pi = 3.14$.] [CBSE 2012]

51. The radius of a circular garden is 100 m. There is a road 10 m wide, running all around it. Find the area of the road and the cost of levelling it at ₹ 20 per m^2. [Use $\pi = 3.14$.] [CBSE 2011]

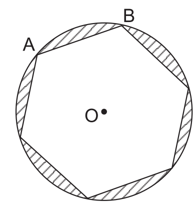
52. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles. [Take $\sqrt{3} = 1.73$.] [CBSE 2009]

53. A child draws the figure of an aeroplane as shown. Here, the wings $ABCD$ and $FGHI$ are parallelograms, the tail DEF is an isosceles triangle, the cockpit CKI is a semicircle and $CDFI$ is a square. In the given figure, $BP \perp CD$, $HQ \perp FI$ and $EL \perp DF$. If $CD = 8$ cm, $BP = HQ = 4$ cm and $DE = EF = 5$ cm, find the area of the whole figure. [Take π = 3.14.]

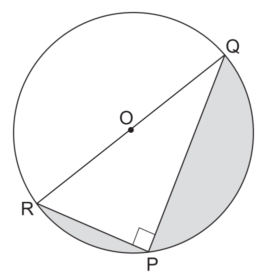


54. A circular disc of radius 6 cm is divided into three sectors with central angles $90°$, $120°$ and $150°$. What part of the whole circle is the sector with central angle $150°$? Also, calculate the ratio of the areas of the three sectors.

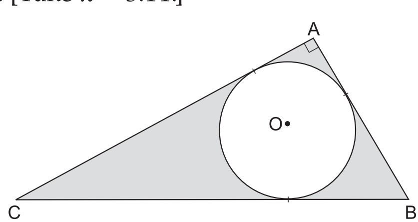
55. A round table cover has six equal designs as shown in the given figure. If the radius of the cover is 35 cm then find the total area of the design. [Use $\sqrt{3} = 1.732$ and π = 3.14.] [CBSE 2009, '14]



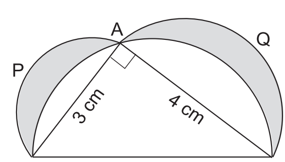
56. In the given figure, $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle. Find the area of the shaded region. [Take π = 3.14.] [CBSE 2009]



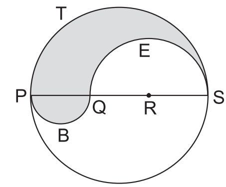
57. In the given figure, $\triangle ABC$ is right-angled at A. Find the area of the shaded region if $AB = 6$ cm, $BC = 10$ cm and O is the centre of the incircle of $\triangle ABC$. [Take π = 3.14.] [CBSE 2009]



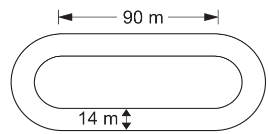
58. In the given figure, $\triangle ABC$ is right-angled at A. Semicircles are drawn on AB, AC and BC as diameters. It is given that $AB = 3$ cm and $AC = 4$ cm. Find the area of the shaded region. [CBSE 2017]



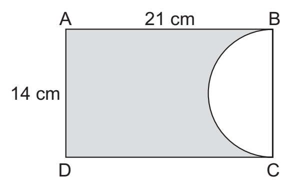
59. *PQRS* is a diameter of a circle of radius 6 cm. The lengths *PQ*, *QR* and *RS* are equal. Semicircles are drawn with *PQ* and *QS* as diameters, as shown in the given figure. If *PS* = 12 cm, find the perimeter and area of the shaded region. [Take π = 3.14.]



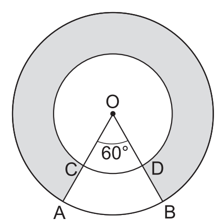
60. The inside perimeter of a running track shown in the figure is 400 m. The length of each of the straight portions is 90 m, and the ends are semicircles. If the track is 14 m wide everywhere, find the area of the track. Also, find the length of the outer boundary of the track.



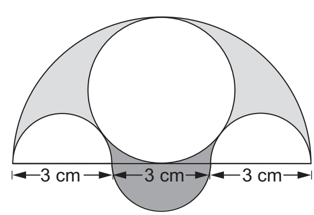
61. In the given figure, *ABCD* is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with *BC* as diameter. Find the area and the perimeter of the shaded region in the figure. [CBSE 2017]



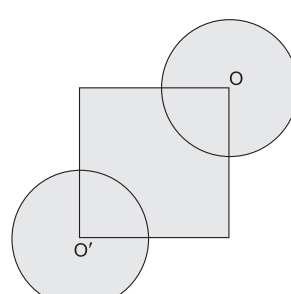
62. In the given figure, two concentric circles with centre *O* have radii 21 cm and 42 cm. If ∠*AOB* = 60°, find the area of the shaded region. [Use $\pi = \frac{22}{7}$·] [CBSE 2017]



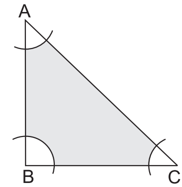
63. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region. [CBSE 2017]



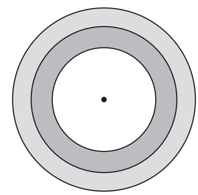
64. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square, where *O* and *O′* are centres of the circles. Find the area of shaded region. [CBSE 2017]



65. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in the given figure. If $AB = 14$ cm, $BC = 48$ cm and $CA = 50$ cm then find the area of the shaded region. [Use $\pi = 3.14$.]



66. If the diameters of the concentric circles shown in the figure below are in the ratio 1 : 2 : 3 then find the ratio of the areas of three regions.



ANSWERS (EXERCISE 16A)

1. 124.74 cm^2 **2.** 35.2 cm **3.** 66 cm **4.** 616 cm^2
5. 346.5 cm^2 **6.** 693 m^2 **7.** 26 cm, 18 cm **8.** 1210 cm^2
9. (i) 1056 m^2 (ii) ₹ 1863.40 **10.** 7 m, 2618 m^2
11. 55 cm, 577.5 cm^2 **12.** 9.08 cm^2, 305.2 cm^2
13. 17.5 cm, 110 cm, 962.5 cm^2 **14.** 14 cm^2, 140 cm^2
15. (12.56 cm, 62.8 cm), 13.08 cm **16.** 14.25 cm^2, 142.75 cm^2
17. 1085.07 cm^2, 4458.93 cm^2 **18.** 81.3 cm^2, 2744.7 cm^2
19. 288.75 cm^2 **20.** 1909.12 cm^2 **21.** 154 cm^2 **22.** 858 m^2
23. 346.5 m^2, 3293.5 m^2 **24.** 36.69 m^2 **25.** 537.5 m^2
26. 8 cm **27.** (i) 78.5 cm^2 (ii) 157 cm^2 **28.** $\pi : 2$ **29.** 72.66 cm
30. 7500 **31.** 59.4 km/hr **32.** 63 cm **33.** 15.84 km/hr **34.** 500
35. 320 **36.** 42 cm^2 **37.** 21.5 cm^2 **39.** 5.76 cm^2 **41.** 14.875 cm^2
42. (i) 616 m^2 (ii) 884 m^2 **43.** 137.64 cm^2 **44.** 2499 cm^2 **45.** 334.31 cm^2
46. 283.96 cm^2 **47.** $r = 2\sqrt{3}$ cm, ar(shaded region) = 24.6 cm^2
48. 3255.21 cm^2 **49.** 38.5 cm^2 **50.** 28.5 cm^2
51. 6594 m^2, ₹ 131880 **52.** 7.77 cm^2 **53.** 65.12 cm^2 **54.** $\frac{5}{12}$, 3 : 4 : 5
55. 663.95 cm^2 **56.** 161.31 cm^2 **57.** 11.44 cm^2 **58.** 6 cm^2
59. 37.68 cm, 37.68 cm^2 **60.** 6216 m^2, 488 m **61.** 217 cm^2, 78 cm
62. 3465 cm^2 **63.** $12\frac{3}{8}$ cm^2 **64.** 1708 cm^2 **65.** 296.75 cm^2 **66.** 1 : 3 : 5

HINTS TO SOME SELECTED QUESTIONS

3. $2\pi R - 2R = 45 \Rightarrow 2R(\pi - 1) = 45 \Rightarrow 2R\left(\frac{22}{7} - 1\right) = 45 \Rightarrow R = 10.5$ cm.

4. $a^2 = 484 \Rightarrow a = \sqrt{484} = 22$ cm.

   Length of the wire $= 4a = (4 \times 22)$ cm $= 88$ cm.

   $2\pi R = 88$.

5. $\frac{\sqrt{3}}{4}a^2 = 121\sqrt{3} \Rightarrow a^2 = 484 \Rightarrow a = \sqrt{484} = 22$ cm.

   Length of the wire $= 3a = (3 \times 22)$ cm $= 66$ cm.

   $2\pi R = 66$.

6. $\pi R + 2R = 168 \Rightarrow \left(\frac{22}{7} + 2\right)R = 168 \Rightarrow R = 21$ cm.

   Area of the park $= \frac{1}{2}\pi R^2$.

7. Let the radii of the circles be x cm and $(7 - x)$ cm.

   Then, $2\pi x - 2\pi(7 - x) = 8 \Rightarrow 2\pi x = 26$.

   Substitute this value of $2\pi x$ and find $2\pi(7 - x)$.

9. (i) $R_1 = 17$ m and $R_2 = (17 + 8)$ m $= 25$ m.

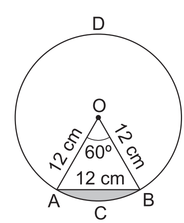
   Area of the path $= \pi\{(25)^2 - (17)^2\}$ m^2.

13. $2 \times \frac{22}{7} \times r \times \frac{54}{360} = 16.5$. Find r.

14. Area of the minor segment $= \left\{\frac{22}{7} \times 7 \times 7 \times \frac{90}{360} - \frac{1}{2} \times 7 \times 7 \times \sin 90°\right\}$ cm^2 $= 14$ cm^2.

    Area of the major segment $= \left\{\left(\frac{22}{7} \times 7 \times 7\right) - 14\right\}$ cm^2 $= 140$ cm^2.

15. ΔOAB is equilateral. So, $\angle AOB = 60°$.



    arc $ACB = \left(\frac{2\pi R\theta}{360}\right) = \left(\frac{2\pi \times 12 \times 60}{360}\right)$ cm

    $= 4\pi$ cm $= (4 \times 3.14)$ cm $= 12.56$ cm.

    arc $BDA = (2\pi r - \text{arc } ACB)$

    $= (2 \times 3.14 \times 12 - 12.56)$ cm $= 62.8$ cm.

    ar(minor segment $ACBA$) = ar(sector $OACBO$) – ar(ΔOAB)

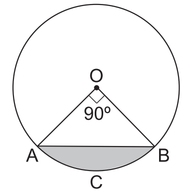
    $= \left(\frac{\pi r^2\theta}{360} - \frac{\sqrt{3}}{4}a^2\right)$

    $= \left\{\frac{3.14 \times 12 \times 12 \times 60}{360} - \frac{\sqrt{3} \times 12 \times 12}{4}\right\}$ cm^2

    $= (75.36 - 62.28)$ cm^2 $= 13.08$ cm^2.

16. Let $OA = 5\sqrt{2}$ cm, $OB = 5\sqrt{2}$ cm and $AB = 10$ cm.

    Then, $OA^2 + OB^2 = AB^2 \Rightarrow \angle AOB = 90°$.



ar(minor segment $ACBA$)

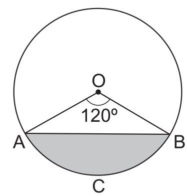
$= \text{ar(sector } OACBO) - \text{ar}(\Delta OAB)$

$= \left\{\frac{3.14 \times (5\sqrt{2})^2 \times 90}{360} - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}\right\} \text{cm}^2$

$= \frac{57}{4} \text{cm}^2 = 14.25 \text{cm}^2.$

ar(major segment) $= \{3.14 \times (5\sqrt{2})^2 - 14.25\} \text{cm}^2 = (157 - 14.25) \text{cm}^2$

$= 142.75 \text{cm}^2.$



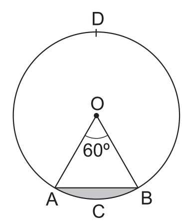
17. ar(minor segment $ACBA$)

$= \text{ar(sector } OACBO) - \text{ar}(\Delta OAB)$

$= \left(\frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta\right) = (1848 - 762.93) \text{cm}^2$

$= 1085.07 \text{ cm}^2.$

ar(major segment) $= \left[\left(\frac{22}{7} \times 42 \times 42\right) - 1085.07\right] \text{cm}^2.$



18. ar(minor segment $ACBA$)

$= \text{ar(sector } OACBO) - \text{ar}(\Delta OAB) = \left(\frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin\theta\right)$

$= \left(\frac{3.14 \times 30 \times 30 \times 60}{360} - \frac{1}{2} \times 30 \times 30 \times \sin 60°\right)$

$= (471 - 389.7) = 81.3 \text{cm}^2.$

ar(major segment $BDAB$) $= (\pi R^2 - 81.3) \text{ cm}^2 = (2826 - 81.3) = 2744.7 \text{ cm}^2.$

19. Let the length of the major arc be x cm. Then,

the length of the minor arc $= \frac{x}{5}$ cm.

Circumference of the circle $= \left(x + \frac{x}{5}\right) \text{cm} = \frac{6x}{5}$ cm.

$\therefore \quad \frac{6x}{5} = \left(2 \times \frac{22}{7} \times \frac{21}{2}\right) \Rightarrow x = 55.$

Length of the major arc = 55 cm.

Area of the sector corresponding to major arc $= \left(\frac{1}{2} rl\right) = \left(\frac{1}{2} \times \frac{21}{2} \times 55\right) \text{cm}^2.$

20. In 2 days, the short hand will complete 4 rounds.

$\therefore$ length moved by it = 4(circumference of the circle with $r = 4$ cm)

$= (4 \times 2\pi \times 4) \text{ cm} = 32\pi \text{ cm}.$

In 2 days, the long hand will complete 48 rounds.

$\therefore$ length moved by it = 48(circumference of the circle with $r = 6$ cm)

$= (48 \times 2\pi \times 6) \text{ cm} = 576\pi \text{ cm}.$

$\therefore$ sum of the lengths moved $= (32\pi + 576\pi) \text{ cm} = (608\pi) \text{ cm}.$

21. $2\pi R = 88 \Rightarrow 2\times\frac{22}{7}\times R = 88 \Rightarrow R = 14\,\text{cm}.$

Required area $= \left(\frac{1}{4}\pi R^2\right)\text{cm}^2 = 154\,\text{cm}^2.$

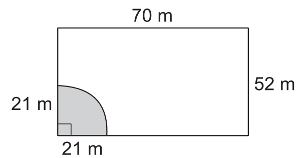
22. Additional ground $= \pi\times[(23)^2 - (16)^2]\,\text{m}^2.$

23. Area which can be grazed

= area of the quadrant of radius 21 m

$= \left(\frac{1}{4}\times\frac{22}{7}\times 21\times 21\right)\text{m}^2 = 346.5\ \text{m}^2.$

Area ungrazed $= [(70\times 52) - 346.5]\ \text{m}^2.$



24. Each angle of an equilateral triangle is 60°.

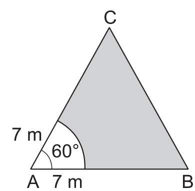
Area which cannot be grazed

= (area of the equilateral $\triangle ABC$)

− (area of the sector with $r = 7$ m, $\theta = 60°$)

$= \left[\frac{\sqrt{3}}{4}\times(12)^2 - \frac{22}{7}\times(7)^2\times\frac{60}{360}\right]\text{m}^2 = \frac{(108\sqrt{3}-77)}{3}\ \text{m}^2$

$= 36.69\ \text{m}^2.$

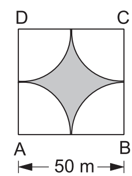


25. Ungrazed area = shaded area

$= \left[(50\times 50) - 4\times\frac{\pi\times(25)^2\times 90}{360}\right]\text{m}^2.$

$= [2500 - (625\times 3.14)]\,\text{m}^2$

$= (2500 - 1962.5)\,\text{m}^2 = 537.5\,\text{m}^2.$



26. $OP = OR = OQ = r.$

Let OQ and PR intersect at S.

Since diagonals of a rhombus bisect each other at right angles, we have

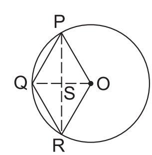
$OS = \frac{1}{2}r$ and $\angle OSR = 90°.$

$\therefore\quad SR = \sqrt{OR^2 - OS^2} = \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}r}{2}.$

$\therefore\quad PR = 2\times SR = \sqrt{3}r.$

Area of the rhombus $= \frac{1}{2}\times OQ\times PR = \frac{1}{2}\times r\times\sqrt{3}r = \frac{\sqrt{3}}{2}r^2.$

$\therefore\quad \frac{\sqrt{3}}{2}r^2 = 32\sqrt{3} \Rightarrow r = 8\ \text{cm}.$



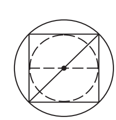
27. Diameter of the inscribed circle = side of the square = 10 cm.

$\therefore$ radius of the inscribed circle = 5 cm.

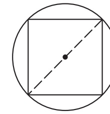
Diameter of the circumscribed circle = diagonal of the square

$= (\sqrt{2}\times 10)$ cm.

$\therefore$ radius of the circumscribed circle $= 5\sqrt{2}$ cm.



28. Let the radius of the circle be r cm.

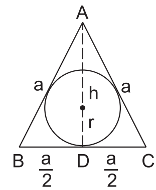


Then, diagonal of the square = diameter of the circle = $2r$ cm.

Area of the circle = (πr^2) cm^2.

Area of the square = $\frac{1}{2}\times(\text{diagonal})^2 = \frac{1}{2}\times 4r^2 = (2r^2)$ cm^2.

29. Let the radius of the incircle be r cm.



Then, $\pi r^2 = 154 \Rightarrow r^2 = \left(154\times\frac{7}{22}\right) \Rightarrow r = 7$ cm.

Let each side of the triangle be a cm and its height be h cm.

Then, $r = \frac{h}{3} \Rightarrow h = 3r = 21$ cm.

$$h = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}\,a}{2} \Rightarrow \frac{\sqrt{3}\,a}{2} = 21.$$

$$\therefore \quad a = \frac{42}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = 14\sqrt{3}.$$

30. Circumference = $\left(2\times\frac{22}{7}\times 42\right)$ cm = 264 cm.

Required number of revolutions = $\left(\frac{19.8\times 1000\times 100}{264}\right) = 7500.$

31. Circumference = $\left(2\times\frac{22}{7}\times 2.1\right)$ m = $\frac{66}{5}$ m.

Distance covered in 60 minutes = $\left(\frac{66}{5}\times 75\times 60\right)$ m,

speed in km/hr = $\left(\frac{66}{5}\times 75\times 60\times\frac{1}{1000}\right)$ km/hr.

32. Distance covered in 1 revolution = $\left(\frac{4.95\times 1000\times 100}{2500}\right)$ cm = 198 cm.

$2\pi R = 198 \Rightarrow 2R\times\frac{22}{7} = 198 \Rightarrow 2R = \left(198\times\frac{7}{22}\right) = 63$ cm.

33. Number of revolutions per hour = $(140\times 60) = 8400$.

Distance covered in 1 revolution = $2\pi R = \left(\frac{22}{7}\times 60\right)$ cm.

Distance covered in 1 hour = distance covered in 8400 revolutions

$$= \left(\frac{22}{7}\times 60\times 8400\right)\text{cm} = \left(22\times 72000\times\frac{1}{100}\times\frac{1}{1000}\right)\text{km}$$

$$= \frac{1584}{100}\text{ km} = 15.84\text{ km} = \text{speed per hr.}$$

35. Radius of the front wheel = 40 cm = $\frac{2}{5}$ m.

Circumference of the front wheel = $\left(2\pi\times\frac{2}{5}\right)$ m = $\frac{4\pi}{5}$ m.

Distance moved by it in 800 revolutions $= \left(\frac{4\pi}{5} \times 800\right)$ m $= (640\pi)$ m.

Circumference of the rear wheel $= (2\pi \times 1)$ m $= (2\pi)$ m.

Required number of revolutions $= \left(\frac{640\pi}{2\pi}\right) = 320.$

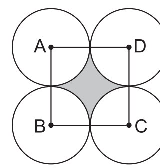
36. Required area = ar(square $ABCD$) – 4 ar(a segment with $r = 14$ cm and $\theta = 90°$)

$$= \left\{(14 \times 14) - 4\left(\frac{22}{7} \times 14 \times 14 \times \frac{90}{360}\right)\right\} \text{cm}^2$$

$$= (196 - 154)\,\text{cm}^2 = 42\,\text{cm}^2.$$

37. Let A, B, C, D be the centres of these circles.

Join AB, BC, CD and DA.



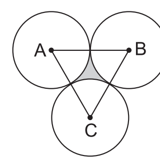
Required area = (area of the square $ABCD$ with $a = 10$ cm)

– 4(area of a sector with $r = 5$ cm, $\theta = 90°$).

38. Required area $= \left\{(2a)^2 - \frac{22}{7}a^2\right\}$ sq units $= \left(\frac{6}{7}a^2\right)$ sq units.

39. Let A, B, C be the centres of these circles.

Join AB, BC and CA.



Required area = (area of the $\triangle ABC$ with each side $a = 12$ cm)

– 3(area of a sector with $r = 6$ cm, $\theta = 60°$).

40. As discussed in the previous question, we have

required area $= \left[\frac{\sqrt{3}}{4} \times (2a)^2 - \frac{3\pi a^2 \times 60}{360}\right] = (1.73a^2 - 1.57a^2) = \left(\frac{4}{25}a^2\right)$ sq units.

41. Area of the trapezium $ABCD = \frac{1}{2}(AD + BC) \times AB = \frac{1}{2}AB \times (10\text{ cm} + 4\text{ cm})$

$= AB \times 7$ cm.

$\therefore\ AB \times 7\text{ cm} = 24.5\text{ cm}^2 \Rightarrow AB = 3.5$ cm.

Area of the shaded region = ar(trapezium $ABCD$) – ar(quadrant ABE)

$$= \left\{24.5 - \frac{1}{4}\pi \times (AB)^2\right\} \text{cm}^2$$

$$= \left(24.5 - \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$$

$$= \left(24.5 - \frac{77}{8}\right) \text{cm}^2 = (24.5 - 9.625)\,\text{cm}^2$$

$$= 14.875\text{ cm}^2.$$

42. Since $AD \parallel BC$ and $\angle ABC = 90°$, so $\angle BAD = 90°$.

Also, $\angle ADC = 60°$ (given).

$\therefore\ \angle BCD = 360° - (90° + 90° + 60°) = 120°.$

(i) Total area of 4 sectors $= \left\{\pi \times (14)^2 \times \left(\frac{90}{360} + \frac{90}{360} + \frac{120}{360} + \frac{60}{360}\right)\right\}$ m^2

$$= \left\{ \frac{22}{7} \times 14 \times 14 \times \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3} + \frac{1}{6} \right) \right\} \text{m}^2$$

$$= (22 \times 28 \times 1)\,\text{m}^2 = 616\,\text{m}^2.$$

(ii) Area of remaining portion = ar(trapezium $ABCD$) – total area of 4 sectors

$$= \left[\left\{ \frac{1}{2}(55 + 45) \times 30 \right\} - 616 \right] \text{m}^2$$

$$= (1500 - 616)\,\text{m}^2 = 884\,\text{m}^2.$$

43. Since $\triangle OAB$ is equilateral, we have $\angle O = \angle A = \angle B = 60°$ and $OA = OB = AB = 12$ cm.

Radius of the circle = 6 cm.

ar(shaded circular part) $= \left(\pi R^2 - \dfrac{\pi r^2 \times 60}{360} \right)$, where $r = 6$ cm

$$= \pi R^2 \left(1 - \frac{1}{6} \right) \text{cm}^2$$

$$= \left(3.14 \times 6 \times 6 \times \frac{5}{6} \right) \text{cm}^2 = 94.2\,\text{cm}^2.$$

ar(shaded triangular region) $= \left\{ \left(\dfrac{\sqrt{3}}{4} \times 12 \times 12 \right) - \dfrac{\pi \times 6 \times 6 \times 60}{360} \right\} \text{cm}^2$

$$= \{(36 \times 1.73) - (3.14 \times 6)\}\,\text{cm}^2 = \left(\frac{1557}{25} - \frac{471}{25} \right) \text{cm}^2$$

$$= \frac{1086}{25}\,\text{cm}^2 = 43.44\,\text{cm}^2.$$

Area of the shaded region $= (94.2 + 43.44)\,\text{cm}^2 = 137.64\,\text{cm}^2$.

44. $AE^2 = (AD^2 - DE^2) = \{(70)^2 - (42)^2\} = (70 + 42)(70 - 42) = (112 \times 28) = (4 \times 28 \times 28)$

$\therefore \quad AE = (2 \times 28) = 56$ cm.

$$\text{ar}(\triangle AED) = \frac{1}{2} \times DE \times AE = \left(\frac{1}{2} \times 42 \times 56 \right) \text{cm}^2 = 1176\,\text{cm}^2.$$

$$\text{ar (semicircle)} = \frac{1}{2} \pi \times (35)^2 = \left\{ \frac{1}{2} \times \frac{22}{7} \times 35 \times 35 \right\} \text{cm}^2 = 1925\,\text{cm}^2.$$

Area of the shaded region $= \{(80 \times 70) - (1176 + 1925)\}\,\text{cm}^2$

$$= (5600 - 3101)\,\text{cm}^2 = 2499\,\text{cm}^2.$$

45. $AD^2 = (DE^2 + AE^2) = (144 + 81)\,\text{cm}^2 = 225\,\text{cm}^2$

$\Rightarrow \quad AD = \sqrt{225}$ cm = 15 cm $\Rightarrow r = \dfrac{15}{2}$ cm = 7.5 cm.

ar(shaded region) = ar(rectangle $ABCD$) + ar(semicircle on BC) – ar($\triangle ADE$)

$$= \left\{ (20 \times 15) + \left(\frac{1}{2} \times 3.14 \times \frac{15}{2} \times \frac{15}{2} \right) - \left(\frac{1}{2} \times 9 \times 12 \right) \right\} \text{cm}^2$$

$$= (300 + 88.3125 - 54)\,\text{cm}^2 = (388.31 - 54)\,\text{cm}^2 = 334.31\,\text{cm}^2.$$

46. $\angle BAC = 90°$ (angle in a semicircle).

$$CB^2 = AC^2 + AB^2 = [(24)^2 + 7^2]\,\text{cm}^2 = (576 + 49)\,\text{cm}^2 = 625\,\text{cm}^2$$

$\Rightarrow \quad CB = \sqrt{625}$ cm = 25 cm.

$\therefore \quad OC = \dfrac{1}{2} CB = \dfrac{25}{2}$ cm = radius of the circle.

Required area = (area of the circle) – {ar(ΔBAC) + ar(quadrant COD)}

$$= \left(3.14 \times \frac{25}{2} \times \frac{25}{2}\right) - \left\{\left(\frac{1}{2} \times 24 \times 7\right) + \left(\frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\right)\right\}$$

$$= \left\{\left(\frac{3}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\right) - 84\right\} \text{cm}^2 = (367.96 - 84)\,\text{cm}^2 = 283.96\,\text{cm}^2.$$

47. Draw $AOD \perp BC$. Then, D is the midpoint of BC.

$\therefore$ $AC = 12$ cm and $DC = 6$ cm.

$AD^2 = AC^2 - DC^2 = (12)^2 - 6^2 = 108 \Rightarrow AD = \sqrt{108} = 6\sqrt{3}$ cm.

$\therefore$ $h = 6\sqrt{3}$ cm. Now, $3r = h = 6\sqrt{3} \Rightarrow r = 2\sqrt{3}$ cm.

So, the radius of inscribed circle is $2\sqrt{3}$ cm.

$$\text{ar(shaded region)} = \left[\left(\frac{\sqrt{3}}{4} \times 12 \times 12\right) - \{3.14 \times (2\sqrt{3})^2\}\right] \text{cm}^2$$

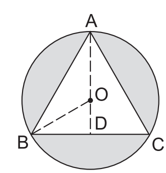
$$= [(36 \times 1.73) - (12 \times 3.14)]\ \text{cm}^2$$

$$= (62.28 - 37.68)\ \text{cm}^2$$

$$= 24.6\ \text{cm}^2.$$

Note $h = 3r$ in case of incircle.

48. Let O be the centre of the circumcircle. Join OB and draw $OD \perp BC$. Then, $OB = 42$ cm and $\angle OBD = 30°$.



$$\frac{OD}{OB} = \sin 30° = \frac{1}{2} \Rightarrow \frac{OD}{42} = \frac{1}{2} \Rightarrow OD = 21\,\text{cm}.$$

$$BD^2 = OB^2 - OD^2 = (42)^2 - (21)^2$$

$$= (42 + 21)(42 - 21) = 63 \times 21.$$

$\therefore$ $BD = \sqrt{63 \times 21} = 21\sqrt{3}$ cm $\Rightarrow BC = 2 \times 21\sqrt{3} = 42\sqrt{3}$ cm.

Area of the shaded region

= (area of a circle with $r = 42$ cm) – ar(equilateral ΔABC with $a = 42\sqrt{3}$ cm)

$$= \left\{\left(\frac{22}{7} \times 42 \times 42\right) - \frac{1.73}{4} \times 42\sqrt{3} \times 42\sqrt{3}\right\} \text{cm}^2$$

$$= (5544 \times 2288.79)\,\text{cm}^2 = 3255.21\ \text{cm}^2.$$

Note : $a = r\sqrt{3}$ in case of circumcircle.

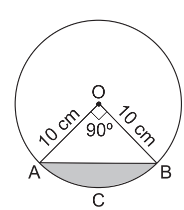
49. Perimeter of the quadrant $= \left(2r + \dfrac{2\pi r \times 90}{360}\right)$ cm $= \dfrac{25r}{7}$ cm.

$\therefore$ $\dfrac{25r}{7} = 25 \Rightarrow r = 7.$

Area of the quadrant $= \dfrac{1}{4}\pi r^2 = \left(\dfrac{1}{4} \times \dfrac{22}{7} \times 7 \times 7\right) \text{cm}^2 = \dfrac{77}{2}\ \text{cm}^2 = 38.5\ \text{cm}^2.$

50. ar(minor segment)

= ar(sector $OACBO$) – ar(ΔOAB)



$$= \left\{\left(3.14 \times 10 \times 10 \times \frac{90}{360}\right) - \left(\frac{1}{2} \times 10 \times 10\right)\right\} \text{cm}^2$$

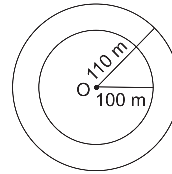
$$= (78.5 - 50)\ \text{cm}^2 = 28.5\ \text{cm}^2.$$

51. Area of the road $= \pi(R^2 - r^2)$

$= 3.14 \times \{(110)^2 - (100)^2\}\ m^2$

$= (3.14 \times 210 \times 10)\ m^2 = 6954\ m^2.$

Cost of levelling = ₹(6594×20) = ₹131880.



52. $\frac{\sqrt{3}}{4} a^2 = 49\sqrt{3}\ cm^2 \Rightarrow a^2 = 49 \times 4\ cm^2 \Rightarrow a = 7 \times 2\ cm = 14\ cm.$

$\therefore$ radius of each circle $= \frac{14}{2}\ cm = 7\ cm.$

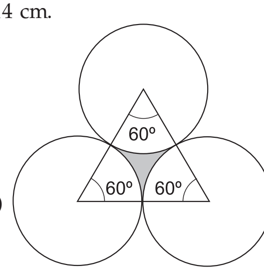
Clearly, each angle of the equilateral triangle is 60°.

Area of the shaded region

= ar($\triangle ABC$) – 3(area of a sector with $r = 7$ cm and $\theta = 60°$)

$= \left\{49\sqrt{3} - 3\left(\frac{22}{7} \times 7 \times 7 \times \frac{60}{360}\right)\right\}\ cm^2$

$= (49 \times 1.73 - 77)\ cm^2 = (84.77 - 77)\ cm^2 = 7.77\ cm^2.$



53. Required area = 2 × ar(||gm $ABCD$) + ar($\triangle EDF$) + ar(square $DCIF$) + ar(semcircle IKC).

$EL^2 = DE^2 - DL^2 = 5^2 - 4^2 = 25 - 16 = 9 \Rightarrow EL = 3\ cm.$

$\therefore$ required area $= \left\{2 \times (8 \times 4) + \left(\frac{1}{2} \times 8 \times 3\right) + (8 \times 8) + \left(\frac{1}{2} \times 3.14 \times 4 \times 4\right)\right\}\ cm^2$

$= (64 + 12 + 64 + 25.12)\ cm^2 = 165.12\ cm^2.$

54. $\frac{\text{Area of the sector with } \theta = 150°}{\text{Area of the circle}} = \frac{\pi \times (6)^2 \times \frac{150}{360}}{\pi \times (6)^2} = \frac{150}{360} = \frac{5}{12}.$

Required ratio $= \left(36\pi \times \frac{90}{360}\right) : \left(36\pi \times \frac{120}{360}\right) : \left(36\pi \times \frac{150}{360}\right)$

$= \frac{1}{4} : \frac{1}{3} : \frac{5}{12} = 3 : 4 : 5.$

55. Area of the shaded region

= (area of the circle) – (area of the hexagon)

$= \pi r^2 - 6 \times$ ar($\triangle OAB$ with each side r cm)

$= \left[(3.14 \times 35 \times 35) - \left(6 \times \frac{\sqrt{3}}{4} \times 35 \times 35\right)\right]\ cm^2$

$= (35 \times 35)\left[3.14 - \frac{3}{2} \times 1.732\right]\ cm^2$

$= 1225 \times (3.14 - 2.598)\ cm^2 = (1225 \times 0.542)\ cm^2$

$= \left(1225 \times \frac{542}{1000}\right)\ cm^2 = 663.95\ cm^2.$

56. Clearly, $\angle QPR = 90°$ [angle in a semicircle].

$QR^2 = PQ^2 + PR^2 = [(24)^2 + 7^2]\ cm^2 = 625\ cm^2 \Rightarrow QR = \sqrt{625}\ cm = 25\ cm.$

$\therefore$ radius of the circle $(r) = \frac{25}{2}$ cm.

Area of the shaded region

$$= \text{ar}\left(\text{semicircle with } r = \frac{25}{2}\right) - \text{ar}(\Delta PQR)$$

$$= \left\{\left(\frac{1}{2} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\right) - \left(\frac{1}{2} \times 7 \times 24\right)\right\} \text{ cm}^2$$

$$= (245.31 - 84) \text{ cm}^2 = 161.31 \text{ cm}^2.$$

57. $AC^2 = BC^2 - AB^2 = 100 - 36 = 64 \Rightarrow AC = 8$ cm.

Draw $OD \perp AB$, $OE \perp AC$ and $OF \perp CB$.

Let $OD = OE = OF = r$ cm.

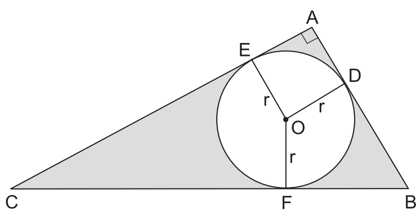
Then, $AEOD$ is a square such that $AD = AE = r$.

[$\because$ tangents to a circle from a point are equal.]

$CF = CE = (8 - r)$ cm and $BF = BD = (6 - r)$ cm.

$BF + CF = 10 \Rightarrow (6 - r) + (8 - r) = 10 \Rightarrow r = 2.$

ar(shaded region) = ar(ΔABC) – (area of the incircle)



$$= \left\{\frac{1}{2} \times AB \times AC - \pi \times 2^2\right\} \text{ cm}^2 = \left(\frac{1}{2} \times 6 \times 8 - 3.14 \times 4\right) \text{ cm}^2 = 11.44 \text{ cm}^2.$$

58. Area of the shaded region

$$= \{\text{ar}(\Delta ABC) + \text{ar(semicircle } APB) + \text{ar(semicircle } AQC)\} - \text{ar(semicircle } BAC)$$

$$= \left[\left(\frac{1}{2} \times 3 \times 4\right) + \left(\frac{1}{2}\pi \times \frac{3}{2} \times \frac{3}{2}\right) + \left(\frac{1}{2}\pi \times 2 \times 2\right) - \left(\frac{1}{2}\pi \times \frac{5}{2} \times \frac{5}{2}\right)\right] \text{cm}^2$$

$$= \left\{6 + \frac{1}{2}\pi\left(\frac{9}{4} + 4 - \frac{25}{4}\right)\right\} \text{ cm}^2 = (6 + 0) \text{ cm}^2 = 6 \text{ cm}^2.$$

59. $PQ = QR = RS = 4$ cm, $QS = 8$ cm.

Perimeter = arc PTS + arc PBQ + arc QES

$$= (\pi \times 6 + \pi \times 2 + \pi \times 4) \text{ cm} = 12\pi \text{ cm}.$$

Area = (area of the semicircle PBQ) + (area of the semicircle PTS)

– (area of the semicircle QES)

$$= \left[\frac{1}{2}\pi \times (2)^2 + \frac{1}{2}\pi \times (6)^2 - \frac{1}{2}\pi \times (4)^2\right] \text{cm}^2 = (12\pi) \text{ cm}^2.$$

60. $90 + 90 + 2\pi r = 400 \Rightarrow 2 \times \frac{22}{7} \times r = 220 \Rightarrow r = 35$ m.

So, $R = (35 + 14)$ m $= 49$ m.

Area of the track $= \left[(90 \times 14) + \frac{22}{7} \times \{(49)^2 - (35)^2\}\right] \text{m}^2 = 6216 \text{ m}^2.$

Length of the outer boundary = $(90 + 90 + 2\pi R)$ m, where $R = 49$ m

= 488 m.

. . . .

EXERCISE 16B

Very-Short-Answer Questions

1. The difference between the circumference and radius of a circle is 37 cm. Using $\pi = \frac{22}{7}$, find the circumference of the circle. [CBSE 2013]
2. The circumference of a circle is 22 cm. Find the area of its quadrant. [CBSE 2012]
3. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm? [CBSE 2012]
4. If the area of a circle is numerically equal to twice its circumference, then what is the diameter of the circle? [CBSE 2011]
5. What is the perimeter of a square which circumscribes a circle of radius a cm? [CBSE 2011]
6. Find the length of the arc of a circle of diameter 42 cm which subtends an angle of 60° at the centre. [CBSE 2012]
7. Find the diameter of the circle whose area is equal to the sum of the areas of two circles having radii 4 cm and 3 cm. [CBSE 2011]
8. Find the area of a circle whose circumference is 8π. [CBSE 2014]
9. Find the perimeter of a semicircular protractor whose diameter is 14 cm. [CBSE 2009]
10. Find the radius of a circle whose perimeter and area are numerically equal.
11. The radii of two circles are 19 cm and 9 cm. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
12. The radii of two circles are 8 cm and 6 cm. Find the radius of the circle having area equal to the sum of the areas of the two circles.
13. Find the area of the sector of a circle having radius 6 cm and of angle 30°. [Take $\pi = 3.14$.]
14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.
15. The circumferences of two circles are in the ratio 2 : 3. What is the ratio between their areas?

16. The areas of two circles are in the ratio 4 : 9. What is the ratio between their circumferences?

Short-Answer Questions

17. A square is inscribed in a circle. Find the ratio of the areas of the circle and the square.

18. The circumference of a circle is 8 cm. Find the area of the sector whose central angle is 72°.

19. A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum.

20. The minute hand of a clock is 15 cm long. Calculate the area swept by it in 20 minutes. [Take $\pi = 3.14$.]

21. A sector of $56°$, cut out from a circle, contains $17.6\ \text{cm}^2$. Find the radius of the circle.

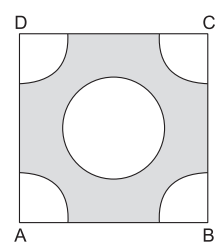
22. The area of the sector of a circle of radius 10.5 cm is $69.3\ \text{cm}^2$. Find the central angle of the sector.

23. The perimeter of a certain sector of a circle of radius 6.5 cm is 31 cm. Find the area of the sector.

24. The radius of a circle is 17.5 cm. Find the area of the sector enclosed by two radii and an arc 44 cm in length.

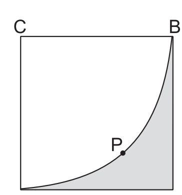
25. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular cardboard of dimensions $14\ \text{cm} \times 7\ \text{cm}$. Find the area of the remaining cardboard. [CBSE 2013]

26. In the given figure, *ABCD* is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region. [Use $\pi = 3.14$.] [CBSE 2012]

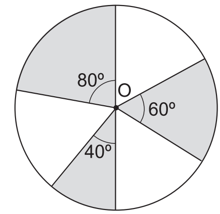


27. From a rectangular sheet of paper *ABCD* with $AB = 40$ cm and $AD = 28$ cm, a semicircular portion with *BC* as diameter is cut off. Find the area of the remaining paper. [CBSE 2012]

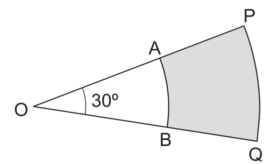
28. In the given figure, *OABC* is a square of side 7 cm. If *COPB* is a quadrant of a circle with centre *C* find the area of the shaded region. [CBSE 2012]



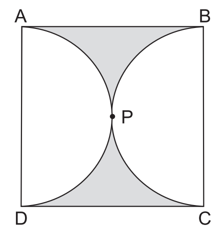
29. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. Find the area of the shaded region. [CBSE 2012]



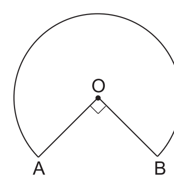
30. In the given figure, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm with centre O. If $\angle POQ = 30°$, find the area of the shaded region. [CBSE 2012]



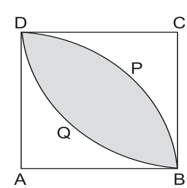
31. In the given figure, find the area of the shaded region, if $ABCD$ is a square of side 14 cm and APD and BPC are semicircles. [CBSE 2012]



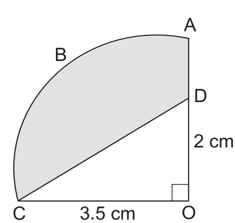
32. In the given figure, the shape of the top of a table is that of a sector of a circle with centre O and $\angle AOB = 90°$. If $AO = OB = 42$ cm then find the perimeter of the top of the table. [CBSE 2012]



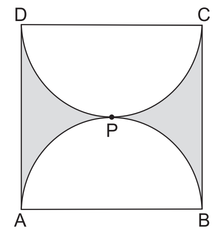
33. In the given figure, $ABCD$ is a square of side 7 cm, $DPBA$ and $DQBC$ are quadrants of circles each of the radius 7 cm. Find the area of shaded region.



34. In the given figure, $OABC$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ cm, find the area of the shaded region. [CBSE 2017]

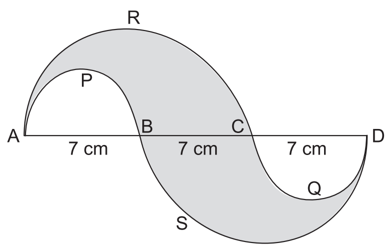


35. Find the perimeter of the shaded region in the figure, if *ABCD* is a square of side 14 cm and *APB* and *CPD* are semicircles. [CBSE 2011]

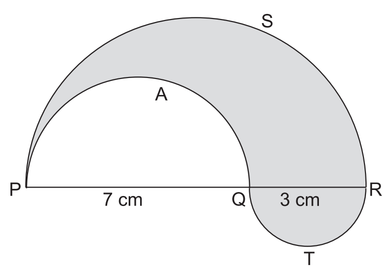


36. In a circle of radius 7 cm, a square *ABCD* is inscribed. Find the area of the circle which is outside the square. [CBSE 2011]

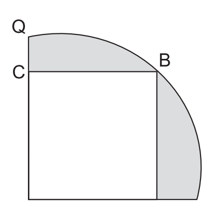
37. In the given figure, *APB* and *CQD* are semicircles of diameter 7 cm each, while *ARC* an *BSD* are semicircles of diameter 14 cm each. Find the (i) perimeter, (ii) area of the shaded region. [CBSE 2011]



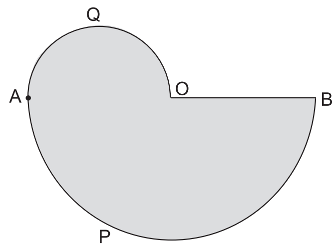
38. In the given figure, *PSR*, *RTQ* and *PAQ* are three semicircles of diameter 10 cm, 3 cm and 7 cm respectively. Find the perimeter of shaded region. [Use $\pi = 3.14$.] [CBSE 2014]



39. In the given figure, a square *OABC* is inscribed in a quadrant *OPBQ* of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$.] [CBSE 2014]

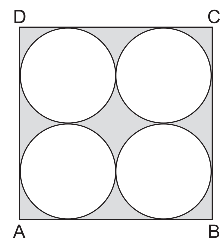


40. In the given figure, APB and AQO are semicircles and $AO = OB$. If the perimeter of the figure is 40 cm, find the area of the shaded region. [CBSE 2015]

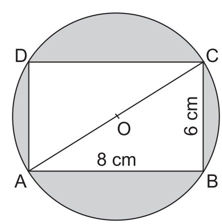


41. Find the area of a quadrant of a circle whose circumference is 44 cm. [CBSE 2011]

42. In the given figure, find the area of the shaded region, where $ABCD$ is a square of side 14 cm and all circles are of the same diameter. [CBSE 2014]



43. Find the area of the shaded region in the given figure, if $ABCD$ is a rectangle with sides 8 cm and 6 cm and O is the centre of the circle. [CBSE 2014]

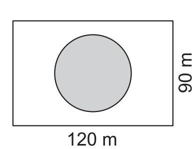


44. A wire is bent to form a square enclosing an area of 484 m^2. Using the same wire, a circle is formed. Find the area of the circle. [CBSE 2014]

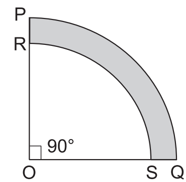
45. A square $ABCD$ is inscribed in a circle of radius r. Find the area of the square.

46. The cost of fencing a circular field at the rate of ₹ 25 per metre is ₹ 5500. The field is to be ploughed at the rate of 50 paise per m^2. Find the cost of ploughing the field. [Take $\pi = \frac{22}{7}$.]

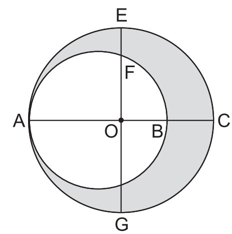
47. A park is in the form of a rectangle 120 m by 90 m. At the centre of the park, there is a circular lawn as shown in the figure. The area of the park excluding the lawn is 2950 m^2. Find the radius of the circular lawn. [Given, $\pi = 3.14$.]



48. In the given figure, $PQSR$ represents a flower bed. If $OP = 21$ m and $OR = 14$ m, find the area of the flower bed.

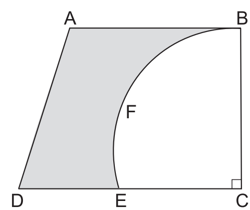


49. In the given figure, O is the centre of the bigger circle, and AC is its diameter. Another circle with AB as diameter is drawn. If $AC = 54$ cm and $BC = 10$ cm, find the area of the shaded region.

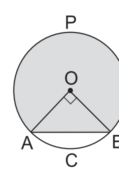


50. From a thin metallic piece in the shape of a trapezium $ABCD$ in which $AB \parallel CD$ and $\angle BCD = 90°$, a quarter circle $BFEC$ is removed. Given, $AB = BC = 3.5$ cm and $DE = 2$ cm, calculate the area of remaining (shaded) part of metal sheet.

[CBSE 2011]



51. Find the area of the major segment APB of a circle of radius 35 cm and $\angle AOB = 90°$, as shown in the given figure.



ANSWERS (EXERCISE 16B)

1. 44 cm	**2.** $\frac{77}{8}$ cm^2	**3.** 26 cm	**4.** 8 cm	**5.** 8a cm
6. 22 cm	**7.** 10 cm	**8.** 16π	**9.** 36 cm	**10.** 2 units
11. 28 cm	**12.** 10 cm	**13.** 9.42 cm^2	**14.** 22 cm	**15.** 4 : 9
16. 2 : 3	**17.** π : 2	**18.** 123.2 cm^2	**19.** 16.8 cm	**20.** 235.5 cm^2
21. 6 cm	**22.** 72°	**23.** 58.5 cm^2	**24.** 385 cm^2	**25.** 21 cm^2
26. 9.72 cm^2	**27.** 812 cm^2	**28.** 10.5 cm^2	**29.** 77 cm^2	**30.** $\frac{77}{8}$ cm^2
31. 42 cm^2	**32.** 282 cm	**33.** 28 cm^2	**34.** 6.125 cm^2	**35.** 72 cm
36. 63 cm^2	**37.** (i) 66 cm (ii) 115.5 cm^2		**38.** 31.4 cm	**39.** 228 cm^2
40. 96.25 cm^2	**41.** 38.5 cm^2	**42.** 42 cm^2	**43.** 30.57 cm^2	**44.** 616 m^2

45. $(2r^2)$ sq units **46.** ₹ 1925 **47.** 50 m **48.** 192.5 m^2

49. 770 cm^2 **50.** 6.125 cm^2 **51.** 3500 cm^2

HINTS TO SOME SELECTED QUESTIONS

1. $(2\pi R - R) = 37 \Rightarrow R(2\pi - 1) = 37$

$$\Rightarrow R\left(2\times\frac{22}{7} - 1\right) = 37 \Rightarrow R = 7 \text{ cm.}$$

$\therefore$ circumference $= 2\pi R = \left(2\times\frac{22}{7}\times 7\right)$ cm $= 44$ cm.

2. $2\pi R = 22 \Rightarrow 2\times\frac{22}{7}\times R = 22 \Rightarrow R = \frac{7}{2}$ cm.

Area of the quadrant $= \frac{1}{4}\times\pi R^2 = \left(\frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\text{ cm}^2 = \frac{77}{8}\text{ cm}^2.$

3. $\pi R^2 = \{\pi\times 5^2 + \pi\times(12)^2\} = (25\pi + 144\pi) = 169\pi$

$\Rightarrow R^2 = 169 = (13)^2 \Rightarrow R = 13 \text{ cm} \Rightarrow 2R = 26$ cm.

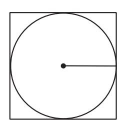
4. $\pi R^2 = 4\pi R \Rightarrow R = 4 \Rightarrow$ diameter $= 8$ cm.

5. Side of the square

$= 2\times$ radius of circle $= 2a$.

Perimeter of the square

$= (4\times 2a) = 8a$ cm.

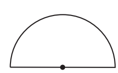


6. Length of the arc $= \frac{2\pi R\theta}{360} = \left(2\times\frac{22}{7}\times 21\times\frac{60}{360}\right)$ cm $= 22$ cm.

7. $\pi R^2 = \pi\times(4)^2 + \pi\times(3)^2 = 25\pi \Rightarrow R^2 = 25 \Rightarrow R = 5 \Rightarrow 2R = 10$ cm.

9. Perimeter $= (\pi R + 2R) = R(\pi + 2)$

$= 7\left(\frac{22}{7} + 2\right)$ cm $= 36$ cm.



11. $2\pi R = (2\pi\times 19) + (2\pi\times 9) \Rightarrow 2\pi R = 2\pi\times(19 + 9) = 2\pi\times 28$

$\Rightarrow R = 28$ cm.

12. $\pi R^2 = (\pi\times 8^2) + (\pi\times 6^2) = \pi(64 + 36) = 100\pi$

$\Rightarrow R^2 = 100 \Rightarrow R = 10$ cm.

13. Area of the sector $= \frac{\pi R^2\theta}{360} = \left(3.14\times 36\times\frac{30}{360}\right)\text{ cm}^2 = 9.42\text{ cm}^2.$

14. Length of the arc $= \frac{2\pi R\theta}{360} = \left(2\times\frac{22}{7}\times 21\times\frac{60}{360}\right)$ cm $= 22$ cm.

15. $\frac{2\pi R_1}{2\pi R_2} = \frac{2}{3} \Rightarrow \frac{R_1}{R_2} = \frac{2}{3}.$

$\therefore \frac{\pi R_1^2}{\pi R_2^2} = \frac{R_1^2}{R_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$

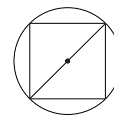
Required ratio = 4 : 9.

16. $\dfrac{\pi R_1^{\,2}}{\pi R_2^{\,2}} = \dfrac{4}{9} \Rightarrow \dfrac{R_1^{\,2}}{R_2^{\,2}} = \dfrac{4}{9} \Rightarrow \left(\dfrac{R_1}{R_2}\right)^2 = \left(\dfrac{2}{3}\right)^2 \Rightarrow \dfrac{R_1}{R_2} = \dfrac{2}{3}$.

$\dfrac{2\pi R_1}{2\pi R_2} = \dfrac{R_1}{R_2} = \dfrac{2}{3}$.

Required ratio = 2 : 3.

17. Let the radius of the circle be r cm.



Then, diagonal of the square = diameter of the circle = $2r$ cm.

Area of the circle $= (\pi r^2)$ cm^2.

Area of the square $= \dfrac{1}{2} \times (\text{diagonal})^2 = \dfrac{1}{2} \times 4r^2 = (2r^2)$ cm^2.

$\therefore$ required ratio $= \pi r^2 : 2r^2 = \pi : 2$.

18. $2\pi R = 88 \Rightarrow 2 \times \dfrac{22}{7} \times R = 88 \Rightarrow R = 14$ cm.

Area of the sector $= \dfrac{\pi R^2 \theta}{360} = \left(\dfrac{22}{7} \times 14 \times 14 \times \dfrac{72}{360}\right)$ cm^2 = 123.2 cm^2.

19. Length of the pendulum = radius of the sector = r cm.

Arc length = 8.8 $\Rightarrow 2 \times \dfrac{22}{7} \times r \times \dfrac{30}{360} = 8.8 \Rightarrow r = \dfrac{168}{10} = 16.8$ cm.

20. Angle described by the minute hand in 20 minutes $= \left(\dfrac{360}{60} \times 20\right)^\circ = 120^\circ$.

Now, $\theta = 120^\circ$ and $r = 15$ cm.

$\therefore$ area swept by minute hand in 20 min $= \left(\dfrac{\pi r^2 \theta}{360}\right)$ cm^2

$= \left(3.14 \times 15 \times 15 \times \dfrac{120}{360}\right)$ cm^2

= 235.5 cm^2.

21. $\dfrac{\pi r^2 \theta}{360} = 17.6 \Rightarrow \dfrac{22}{7} \times r^2 \times \dfrac{56}{360} = 17.6 \Rightarrow r^2 = 36 \Rightarrow r = 6$ cm.

22. $\dfrac{\pi r^2 \theta}{360} = 69.3 \Rightarrow \dfrac{22}{7} \times \dfrac{21}{2} \times \dfrac{21}{2} \times \theta = \dfrac{693}{10} \times 360 \Rightarrow \theta = 72^\circ$.

23. Perimeter of the sector $= \left(2r + \dfrac{2\pi r\theta}{360}\right) = \left(2 \times 6.5 + \dfrac{2\pi \times 6.5 \times \theta}{360}\right)$ cm

$\therefore \quad 13 + \dfrac{13\pi\theta}{360} = 31 \Rightarrow \dfrac{13\pi\theta}{360} = 18 \Rightarrow \theta = \left(\dfrac{18 \times 360}{13\pi}\right)^0$.

Area $= \dfrac{\pi r^2 \theta}{360} = \left(\pi \times \dfrac{13}{2} \times \dfrac{13}{2} \times \dfrac{18 \times 360}{13\pi} \times \dfrac{1}{360}\right)$ cm^2 = 58.5 cm^2.

24. $\dfrac{2\pi r\theta}{360} = 44 \Rightarrow 2\pi \times \dfrac{35}{2} \times \dfrac{\theta}{360} = 44 \Rightarrow \theta = \dfrac{44 \times 72}{7\pi}$.

Area $= \dfrac{\pi r^2 \theta}{360} = \left(\pi \times \dfrac{35}{2} \times \dfrac{35}{2} \times \dfrac{44 \times 72}{7\pi} \times \dfrac{1}{360}\right)$ cm^2 = 385 cm^2.

25. Clearly, the diameter of each circle is 7 cm.

Required area $= \left\{(14\times 7) - \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\right\}$ cm^2

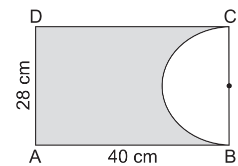
$= (98 - 77)$ cm^2 $= 21$ cm^2.

26. Required area $= \left\{(4\times 4) - 4\times\frac{1}{4}\pi\times(1)^2 + \pi\times(1)^2\right\}$ cm^2

$= (16 - 2\times 3.14\times 1)$ cm^2 $= 9.72$ cm^2.

27. Required area $= \left\{(40\times 28) - \left(\frac{1}{2}\times\frac{22}{7}\times 14\times 14\right)\right\}$ cm^2

$= (1120 - 308)$ cm^2 $= 812$ cm^2.



28. Required area $= \left\{(7\times 7) - \left(\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right)\right\}$ cm^2

$= \left(49 - \frac{77}{2}\right)$ cm^2 $= \frac{21}{2}$ cm^2 $= 10.5$ cm^2.

29. Required area $= \left\{\pi\times 7^2\times\frac{(60^\circ + 80^\circ\times 40^\circ)}{360^\circ}\right\}$ cm^2

$= \left(\frac{22}{7}\times 49\times\frac{180}{360}\right) = 77$ cm^2.

30. Required area = (area of sector *OQP*) – (area of sector *OAB*)

$= \left[\left(\frac{\pi\times 7^2\times 30}{360}\right) - \left(\pi\times\frac{7}{2}\times\frac{7}{2}\times\frac{30}{360}\right)\right]$ cm^2

$= \left[\left(\frac{22}{7}\times\frac{49}{12}\right) - \left(\frac{22}{7}\times\frac{49}{48}\right)\right]$ cm^2 $= \left(\frac{77}{6} - \frac{77}{24}\right)$ cm^2 $= \frac{77}{8}$ cm^2.

31. Required area = (area of sq *ABCD*) – (area of 2 semicircles)

$= \left[(14\times 14) - \left(\frac{22}{7}\times 7\times 7\right)\right]$ cm^2

$= (196 - 154)$ cm^2 $= 42$ cm^2.

32. Required perimeter = (circumference – arc *AB*) + (*OA* + *OB*)

$= \left\{\left(2\times\frac{22}{7}\times 42\right) - \left(2\times\frac{22}{7}\times 42\times\frac{90}{360}\right) + (42 + 42)\right\}$ cm

$= (264 - 66 + 83)$ cm $= 282$ cm.

33. Required area = (area of quadrant *ABPD* + area of quadrant *CDQB*)

– ar(area of square *ABCD*)

$= \left\{\left(\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right) + \left(\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right) - (7\times 7)\right\}$ cm^2

$= \left(\frac{77}{2} + \frac{77}{2} - 49\right)$ cm^2 $= (77 - 49)$ cm^2 $= 28$ cm^2.

34. Required area = ar(quadrant $OABC$) – ar(ΔCOD)

$$= \left\{\left(\frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)-\left(\frac{1}{2}\times\frac{7}{2}\times 2\right)\right\}\text{ cm}^2$$

$$= \left(\frac{77}{8}-\frac{7}{2}\right)\text{ cm}^2 = \frac{(77-28)}{8}\text{ cm}^2$$

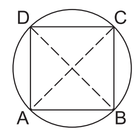
$$= \frac{49}{8}\text{ cm}^2 = 6.125\text{ cm}^2.$$

35. Perimeter of the shaded region

= perimeter of semicircle DPC + perimeter of semicircle APB + AD + BC

$$= \left\{\left(\frac{22}{7}\times 7\right)+\left(\frac{22}{7}\times 7\right)+14+14\right\}\text{ cm} = (22+22+28)\text{ cm} = 72\text{ cm}.$$

36. Diagonal of the square = (2×7) cm = 14 cm.



Area of the square $= \frac{1}{2}d^2 = \left(\frac{1}{2}\times 14\times 14\right)\text{ cm}^2 = 91\text{ cm}^2.$

Area of the circle $= \left(\frac{22}{7}\times 7\times 7\right)\text{ cm}^2 = 154\text{ cm}^2.$

Required area $= (154-91)\text{ cm}^2 = 63\text{ cm}^2.$

37. (i) Perimeter of the shaded region $= \left\{2\left(\frac{22}{7}\times\frac{7}{2}\right)+2\left(\frac{22}{7}\times 7\right)\right\}$ cm

$$= (22+44)\text{ cm} = 66\text{ cm}.$$

(ii) Area of the shaded region $= 2\left\{\left(\frac{1}{2}\times\frac{22}{7}\times 7\times 7\right)-\left(\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\right\}\text{ cm}^2$

$$= 2\left(77-\frac{77}{4}\right)\text{ cm}^2 = 115.5\text{ cm}^2.$$

38. PSR is a semicircle of radius 5 cm, PAQ is a semicircle of radius 3.5 cm and QRT is a semicircle of radius 1.5 cm.

Perimeter $= (5\pi + 3.5\pi + 1.5\pi)\text{ cm} = (10\pi)\text{ cm} = (10\times 3.14)\text{ cm} = 31.4\text{ cm}.$

39. Side of the square = 20 cm.

$\therefore$ area of the square $= (20\times 20)\text{ cm}^2 = 400\text{ cm}^2.$

Diagonal of the square $= \sqrt{(20)^2+(20^2)}\text{ cm} = \sqrt{800}\text{ cm} = 20\sqrt{2}\text{ cm}.$

Radius of the quadrant $= 20\sqrt{2}$ cm.

Area of the shaded region $= \left\{\frac{1}{4}\times 3.14\times(20\sqrt{2})^2 - 400\right\}\text{ cm}^2$

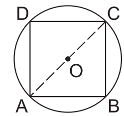
$$= (628-400)\text{ cm}^2 = 228\text{ cm}^2.$$

40. Let $AO = OB = r$.

Perimeter = (arc APB + OB + arc OQA)

$$= \left(\pi r + r + \frac{\pi r}{2}\right) = \left(\frac{3\pi}{2}+1\right)r = \frac{40r}{7}.$$

$\therefore \quad \frac{40r}{7} = 40\text{ cm} \Rightarrow r = 7\text{ cm}.$



Area of the shaded region $= \left[\left(\frac{1}{2}\times\frac{22}{7}\times 7\times 7\right)+\left(\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\right]\text{cm}^2$

$= \left(77+\frac{77}{4}\right)\text{cm}^2 = \frac{385}{4}\text{ cm}^2 = 96.25\text{ cm}^2.$

41. $2\pi R = 44 \Rightarrow 2\times\frac{22}{7}\times R = 44 \Rightarrow R = 7$ cm.

Area of the quadrant $= \left(\frac{1}{4}\times\frac{22}{7}\times 7\times 7\right)\text{cm}^2$

$= \frac{77}{2}\text{ cm}^2 = 38.5\text{ cm}^2.$

42. Radius of each circle $= \frac{7}{2}$ cm $= 3.5$ cm.

Shaded area = ar(square $ABCD$) – 4ar(circle with $r = 3.5$ cm)

$= \left\{(14\times 14) - \left(4\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\right\}\text{ cm}^2$

$= (196 - 154)\text{ cm}^2 = 42\text{ cm}^2.$

43. $AC^2 = AB^2 + BC^2 = (8^2+6^2) = (64+36) = 100 \Rightarrow AC = \sqrt{100} = 10$ cm.

$\therefore\quad r = 5$ cm.

Shaded area $= \left\{\left(\frac{22}{7}\times 5\times 5\right) - (8\times 6)\right\}\text{ cm}^2 = \left(\frac{550}{7} - 48\right)\text{cm}^2$

$= \frac{(550-336)}{7}\text{ cm}^2 = \frac{214}{7}\text{ cm}^2$

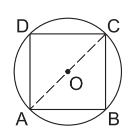
$= 30.57\text{ cm}^2.$

44. Side of the square $= \sqrt{484}$ m $= 22$ m.

Circumference of the circle = perimeter of square $= (22\times 4)$ m $= 88$ m

$\Rightarrow\quad 2\times\frac{22}{7}\times R = 88 \Rightarrow R = \left(88\times\frac{7}{44}\right) = 14$ m.

Area of the circle $= \pi R^2 = \left(\frac{22}{7}\times 14\times 14\right)\text{m}^2 = 616\text{ m}^2.$



45. Let O be the centre of the given circle and let $ABCD$ be the square inscribed in it.
So, diag. $AC = 2r$.
ar(square $ABCD$) $= \frac{1}{2}\times(\text{diagonal})^2 = \left\{\frac{1}{2}\times(2r)^2\right\}$ sq units $= (2r^2)$ sq units.

46. Rate of fencing = ₹ 25 per metre. Cost of fencing = ₹ 5500.

Perimeter $= \frac{\text{total cost}}{\text{rate/metre}} = \left(\frac{5500}{25}\right)$ m $= 220$ m.

Let R be the radius of the field. Then,

$2\pi R = 220 \Rightarrow 2\times\frac{22}{7}\times R = 220 \Rightarrow R = \left(220\times\frac{7}{44}\right)$ m $= 35$ m.

$\therefore$ area of the field $= \pi R^2 = \left(\frac{22}{7} \times 35 \times 35\right) = 3850 \text{ m}^2$.

Cost of ploughing at 50 P per $\text{m}^2 =$ ₹ $\left(3850 \times \frac{50}{100}\right) =$ ₹ 1925.

47. Area of the lawn $= [(120 \times 90) - 2950] \text{ m}^2 = 7850 \text{ m}^2$.

$\therefore$ $3.14 \times r^2 = 7850$. Find r.

48. Area of flower bed = ar(quadrant OPQ) – ar(quadrant ORS)

$$= \left\{\left(\frac{1}{4} \times \frac{22}{7} \times 21 \times 21\right) - \left(\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right)\right\} \text{ m}^2 = 192.5 \text{ m}^2.$$

49. Diameter of the larger circle $= AC = 54$ cm.

Diameter of the smaller circle $= AB = (AC - BC) = 44$ cm.

Area of the shaded region $= \{\pi \times (27)^2 - \pi \times (22)^2\} \text{ cm}^2$.

$$= \{\pi \times (27 + 22)(27 - 22)\} \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 49 \times 5\right) \text{ cm}^2.$$

50. Clearly, $AB = BC = CE = 3$ cm and $DE = 2$ cm.

Area of the shaded part = ar(trapezium $ABCD$) – ar(quadrant BCE)

$$= \left[\left\{\frac{1}{2}(AB + CD) \times BC\right\} - \left\{\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\}\right] \text{ cm}^2$$

$$= \left[\left\{\frac{1}{2}(3.5 + 5.5) \times 3.5\right\} - \frac{77}{8}\right] = \frac{49}{8} = 6.125 \text{ cm}^2.$$

51. Area of the minor segment $ACBA$ = ar(sector $OACBO$) – ar($\triangle OAB$)

$$= \left[\left(\frac{22}{7} \times 35 \times 35 \times \frac{90}{360}\right) - \left(\frac{1}{2} \times 35 \times 35\right)\right] \text{ cm}^2 = 350 \text{ cm}^2.$$

Area of the major segment $= \left\{\left(\frac{22}{7} \times 35 \times 35\right) - 350\right\} \text{ cm}^2$

$$= (3850 - 350) \text{ cm}^2 = 3500 \text{ cm}^2.$$

. . . .

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. The area of a circle is 38.5 cm^2. The circumference of the circle is

   (a) 6.2 cm (b) 12.1 cm (c) 11 cm (d) 22 cm

2. The area of a circle is $49\pi \text{ cm}^2$. Its circumference is

   (a) 7π cm (b) 14π cm (c) 21π cm (d) 28π cm

3. The difference between the circumference and radius of a circle is 37 cm. The area of the circle is

   (a) 111 cm^2 (b) 184 cm^2 (c) 154 cm^2 (d) 259 cm^2

4. The perimeter of a circular field is 242 m. The area of the field is
   (a) 9317 m^2 (b) 18634 m^2 (c) 4658.5 m^2 (d) none of these
5. On increasing the diameter of a circle by 40%, its area will be increased by
   (a) 40% (b) 80% (c) 96% (d) 82%
6. On decreasing the radius of a circle by 30%, its area is decreased by
   (a) 30% (b) 60% (c) 45% (d) none of these
7. The area of a circle is the same as the area of a square. Their perimeters are in the ratio
   (a) $1:1$ (b) $2:\pi$ (c) $\pi:2$ (d) $\sqrt{\pi}:2$
8. The circumference of a circle is equal to the sum of the circumferences of two circles having diameters 36 cm and 20 cm. The radius of the new circle is
   (a) 16 cm (b) 28 cm (c) 42 cm (d) 56 cm
9. The area of a circle is equal to the sum of the areas of two circles of radii 24 cm and 7 cm. The diameter of the new circle is
   (a) 25 cm (b) 31 cm (c) 50 cm (d) 62 cm
10. If the perimeter of a square is equal to the circumference of a circle then the ratio of their areas is
    (a) $4:\pi$ (b) $\pi:4$ (c) $\pi:7$ (d) $7:\pi$
11. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R then
    (a) $R_1 + R_2 = R$ (b) $R_1 + R_2 < R$ (c) $R_1^2 + R_2^2 < R^2$ (d) $R_1^2 + R_2^2 = R^2$
12. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R then
    (a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$ (c) $R_1 + R_2 < R$ (d) none of these
13. If the circumference of a circle and the perimeter of a square are equal then
    (a) area of the circle = area of the square
    (b) (area of the circle) > (area of the square)
    (c) (area of the circle) < (area of the square)
    (d) none of these
14. The radii of two concentric circles are 19 cm and 16 cm respectively. The area of the ring enclosed by these circles is
    (a) 320 cm^2 (b) 330 cm^2
    (c) 332 cm^2 (d) 340 cm^2

15. The areas of two concentric circles are 1386 cm^2 and 962.5 cm^2. The width of the ring is

(a) 2.8 cm (b) 3.5 cm (c) 4.2 cm (d) 3.8 cm

16. The circumferences of two circles are in the ratio 3 : 4. The ratio of their areas is

(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

17. The areas of two circles are in the ratio 9 : 4. The ratio of their circumferences is

(a) 3 : 2 (b) 4 : 9 (c) 2 : 3 (d) 81 : 16

18. The radius of a wheel is 0.25 m. How many revolutions will it make in covering 11 km?

(a) 2800 (b) 4000 (c) 5500 (d) 7000

19. The diameter of a wheel is 40 cm. How many revolutions will it make in covering 176 m?

(a) 140 (b) 150 (c) 160 (b) 166

20. In making 1000 revolutions, a wheel covers 88 km. The diameter of the wheel is

(a) 14 m (b) 24 m (c) 28 m (d) 40 m

21. The area of a sector of angle $\theta°$ of a circle with radius R is

(a) $\frac{2\pi R\theta}{180}$ (b) $\frac{\pi R^2\theta}{180}$ (c) $\frac{2\pi R\theta}{360}$ (d) $\frac{\pi R^2\theta}{360}$

22. The length of an arc of a sector of angle $\theta°$ of a circle with radius R is

(a) $\frac{2\pi R\theta}{180}$ (b) $\frac{2\pi R\theta}{360}$ (c) $\frac{\pi R^2\theta}{180}$ (d) $\frac{\pi R^2\theta}{360}$

23. The length of the minute hand of a clock is 21 cm. The area swept by the minute hand in 10 minutes is [CBSE 2012]

(a) 231 cm^2 (b) 210 cm^2 (c) 126 cm^2 (d) 252 cm^2

24. A chord of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor segments (given, π = 3.14) is

(a) 32.5 cm^2 (b) 34.5 cm^2 (c) 28.5 cm^2 (d) 30.5 cm^2

25. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is

(a) 21 cm (b) 22 cm (c) 18.16 cm (d) 23.5 cm

26. In a circle of radius 14 cm, an arc subtends an angle of 120° at the centre. If $\sqrt{3} = 1.73$ then the area of the segment of the circle is

(a) 120.56 cm^2 (b) 124.63 cm^2 (c) 118.24 cm^2 (d) 130.57 cm^2

ANSWERS (MCQ)

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (c)
10. (b)	**11.** (d)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (d)
19. (a)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (a)	

HINTS TO SOME SELECTED QUESTIONS

1. $\pi R^2 = 38.5 \Rightarrow \frac{22}{7} \times R^2 = \frac{385}{10} \Rightarrow R^2 = \left(\frac{385}{10} \times \frac{7}{22}\right) = \frac{49}{4} \Rightarrow R = \frac{7}{2}$ cm.

   Circumference $= 2\pi R = \left(2 \times \frac{22}{7} \times \frac{7}{2}\right)$ cm $= 22$ cm.

2. $\pi R^2 = 49\pi \Rightarrow R^2 = 49 \Rightarrow R = 7$ cm.

   Circumference $= 2\pi R = (2\pi \times 7)$ cm $= 14\pi$ cm.

3. $2\pi R - R = 37 \Rightarrow R\left(2 \times \frac{22}{7} - 1\right) = 37 \Rightarrow R = \left(37 \times \frac{7}{37}\right) = 7.$

   Area $= \pi R^2 = \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 $= 154$ cm^2.

4. $2\pi R = 242 \Rightarrow 2 \times \frac{22}{7} \times R = 242 \Rightarrow R = \left(242 \times \frac{7}{44}\right) = \frac{77}{2}\cdot$

   Area $= \pi R^2 = \left(\frac{22}{7} \times \frac{77}{2} \times \frac{77}{2}\right)$ m^2 $= 4658.5$ m^2.

5. Let original diameter be 100 units. Then, original radius = 50.

   $\therefore$ original area $= \pi \times (50)^2 = 2500\pi$.

   New diameter = 140. New radius = 70.

   New area $= \pi \times (70)^2 = 4900\pi$.

   Increase % $= \left(\frac{2400}{2500} \times 100\right)\% = 96\%$.

6. Let original radius be 100 units.

   Then, original area $= \pi \times (100)^2 = 10000\pi$.

   New radius = 70 units.

   New area $= \pi \times (70)^2 = 4900\pi$

   Decrease % $= \left(\frac{5100}{10000} \times 100\right)\% = 51\%$.

7. $a^2 = \pi R^2 \Rightarrow \frac{R^2}{a^2} = \frac{1}{\pi} \Rightarrow \frac{R}{a} = \frac{1}{\sqrt{\pi}}.$

   Ratio of their perimeters $= \frac{2\pi R}{4a} = \frac{\pi}{2} \times \left(\frac{R}{a}\right) = \frac{\pi}{2} \times \frac{1}{\sqrt{\pi}} = \frac{\sqrt{\pi}}{2} = \sqrt{\pi} : 2.$

8. Circumference of new circle $= (2\pi \times 18 + 2\pi \times 10) = 2\pi \times (18 + 10) = (2\pi \times 28)$ cm.

   $2\pi R = 2\pi \times 28$ cm $\Rightarrow R = 28$ cm.

   $\therefore$ radius of the new circle is 28 cm.

9. Area of the new circle $= \{\pi \times (24)^2 + \pi \times (7)^2\}\ \text{cm}^2$

$$= \{\pi \times (576 + 49)\}\ \text{cm}^2 = (625\pi)\ \text{cm}^2.$$

$\therefore \quad \pi R^2 = 625\pi \Rightarrow R^2 = 625 = (25)^2 \Rightarrow R = 25$ cm.

$\therefore$ radius of the new circle = 25 cm.

10. $4a = 2\pi r \Rightarrow \frac{a}{r} = \frac{\pi}{2}$.

$$\frac{\text{Area of square}}{\text{Area of circle}} = \frac{a^2}{\pi r^2} = \frac{1}{\pi} \times \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4\pi} = \frac{\pi}{4} = \pi : 4.$$

11. $\pi R_1^2 + \pi R_2^2 = \pi R^2 \Rightarrow R_1^2 + R_2^2 = R^2$.

12. $2\pi R_1 + 2\pi R_2 = 2\pi R \Rightarrow R_1 + R_2 = R$.

13. $2\pi R = 4a \Rightarrow \frac{R}{a} = \frac{2}{\pi} \Rightarrow \frac{R^2}{a^2} = \frac{4}{\pi^2}$

$$\Rightarrow \frac{\pi R^2}{a^2} = \frac{4}{\pi} > 1 \quad [\because \pi = 3.14 < 4]$$

$$\Rightarrow \pi R^2 > a^2$$

$\Rightarrow$ area of circle > area of square.

14. Required area $= \pi\{(19)^2 - (16)^2\}\ \text{cm}^2$

$$= \left(\frac{22}{7} \times 35 \times 3\right) \text{cm}^2 = 330\ \text{cm}^2.$$

15. $\pi R^2 = 1386 \Rightarrow R^2 = \left(1386 \times \frac{7}{22}\right) = 441 = (21)^2 \Rightarrow R = 21$ cm.

$$\pi r^2 = 962.5 \Rightarrow r^2 = \left(\frac{9625}{10} \times \frac{7}{22}\right) = \frac{(49 \times 25)}{4} \Rightarrow r = \left(\frac{7 \times 5}{2}\right) \text{cm} = \frac{35}{2}\ \text{cm}.$$

Width of the ring $= (R - r) = \left(21 - \frac{35}{2}\right)$ cm $= \frac{7}{2}$ cm = 3.5 cm.

16. $\frac{2\pi R_1}{2\pi R_2} = \frac{3}{4} \Rightarrow \frac{R_1}{R_2} = \frac{3}{4} \Rightarrow \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{3}{4}\right)^2$

$$\Rightarrow \frac{R_1^2}{R_2^2} = \frac{9}{16} \Rightarrow \frac{\pi R_1^2}{\pi R_2^2} = \frac{9}{16}.$$

17. $\frac{\pi R_1^2}{\pi R_2^2} = \frac{9}{4} \Rightarrow \frac{R_1^2}{R_2^2} = \frac{9}{4} \Rightarrow \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{3}{2}\right)^2$

$$\Rightarrow \frac{R_1}{R_2} = \frac{3}{2} \Rightarrow \frac{2\pi R_1}{2\pi R_2} = \frac{3}{2}.$$

18. Distance moved in 1 revolution $= 2\pi R = \left(2 \times \frac{22}{7} \times \frac{25}{100}\right)$ m $= \frac{11}{7}$ m.

Total distance covered = 11 km = 11000 m.

Number of revolutions $= \left(11000 \times \frac{7}{11}\right) = 7000$.

19. Distance moved in 1 revolution $= \pi d = \left(\frac{22}{7} \times \frac{40}{100}\right) \text{m} = \frac{44}{35}$ m.

Total distance covered = 176 m.

Number of revolutions $= \left(176 \times \frac{35}{44}\right) = 140.$

20. Distance moved in 1 revolution $= \frac{88000}{1000}$ m = 88 m.

$\pi d = 88 \Rightarrow \frac{22}{7} \times d = 88$

$\Rightarrow d = \left(88 \times \frac{7}{22}\right) = 28$ m.

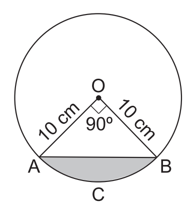
23. Area swept by minute hand in 60 minutes $= \pi R^2$.

Area swept by it in 10 minutes

$= \left(\frac{\pi R^2}{60} \times 10\right) \text{cm}^2 = \left(\frac{22}{7} \times 21 \times 21 \times \frac{1}{6}\right) \text{cm}^2$

$= 231 \text{ cm}^2.$

24. ar(minor segment $ACBA$) = ar(sector $OACBO$) – ar(ΔOAB)



$= \left(\frac{\pi r^2 \theta}{360} - \frac{1}{2} \times r \times r\right)$

$= \left(\frac{3.14 \times 10 \times 10 \times 90}{360} - \frac{1}{2} \times 10 \times 10\right) \text{cm}^2$

$= (78.5 - 50) \text{ cm}^2 = 28.5 \text{ cm}^2.$

25. Arc length $= \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right)$ cm = 22 cm.

26. ar(segment) $= \left(\frac{\pi r^2 \theta}{360} - r^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}\right)$

$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{120}{360}\right) - (14 \times 14 \times \sin 60° \cos 60°)$

$= \left(\frac{616}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \times 14 \times 14\right) \text{cm}^2$

$= (205.33 - 49 \times 1.73) \text{ cm}^2$

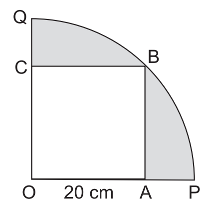
$= (205.33 - 84.77) \text{ cm}^2$

$= 120.56 \text{ cm}^2.$

TEST YOURSELF

MCQ

1. In the given figure, a square *OABC* has been inscribed in the quadrant *OPBQ*. If *OA* = 20 cm then the area of the shaded region is [take π = 3.14]

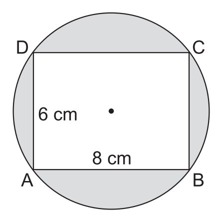


   (a) 214 cm^2 (b) 228 cm^2
   (c) 242 cm^2 (d) 248 cm^2

2. The diameter of a wheel is 84 cm. How many revolutions will it make to cover 792 m?
   (a) 200 (b) 250 (c) 300 (d) 350

3. The area of a sector of a circle with radius *r*, making an angle of $x°$ at the centre is
   (a) $\frac{x}{180} \times 2\pi r$ (b) $\frac{x}{180} \times \pi r^2$ (c) $\frac{x}{360} \times 2\pi r$ (d) $\frac{x}{360} \times \pi r^2$

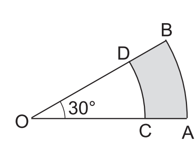
4. In the given figure, *ABCD* is a rectangle inscribed in a circle having length 8 cm and breadth 6 cm. If π = 3.14 then the area of the shaded region is



   (a) 264 cm^2 (b) 266 cm^2
   (c) 272 cm^2 (d) 254 cm^2

Short-Answer Questions

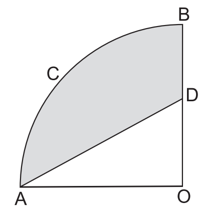
5. The circumference of a circle is 22 cm. Find its area. [Take $\pi = \frac{22}{7}$.]
6. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.
7. The minute hand of a clock is 12 cm long. Find the area swept by it in 35 minutes.
8. The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. Find the area of the sector.
9. A chord of a circle of radius 14 cm makes a right angle at the centre. Find the area of the sector.
10. In the give figure, the sectors of two concentric circles of radii 7 cm and 3.5 cm are shown. Find the area of the shaded region.



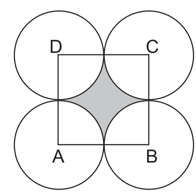
11. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}\text{ cm}^2$. If the same wire is bent into the form of a circle, what will be the area of the circle? [Take $\pi = \frac{22}{7}\cdot$]

12. The wheel of a cart is making 5 revolutions per second. If the diameter of the wheel is 84 cm, find its speed in km per hour. [Take $\pi = \frac{22}{7}\cdot$]

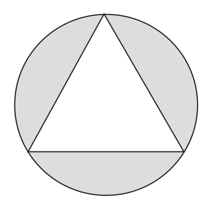
13. *OACB* is a quadrant of a circle with centre *O* and its radius is 3.5 cm. If *OD* = 2 cm, find the area of (i) the quadrant *OACB* (ii) the shaded region. [Take $\pi = \frac{22}{7}\cdot$]



14. In the given figure, *ABCD* is a square each of whose sides measures 28 cm. Find the area of the shaded region. [Take $\pi = \frac{22}{7}\cdot$]



15. In the given figure, an equilateral triangle has been inscribed in a circle of radius 4 cm. Find the area of the shaded region. [Take $\pi = 3.14$ and $\sqrt{3} = 1.73$.]



16. The minute hand of a clock is 7.5 cm long. Find the area of the face of the clock described by the minute hand in 56 minutes.

Long-Answer Questions

17. A racetrack is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width and the area of the track.

18. A chord of a circle of radius 30 cm makes an angle of 60° at the centre of the circle. Find the areas of the minor and major segments. [Take $\pi = 3.14$ and $\sqrt{3} = 1.732$.]

19. Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed? [Take $\pi = 3.14$.]

20. A square tank has an area of 1600 m^2. There are four semicircular plots around it. Find the cost of turfing the plots at ₹ 12.50 per m^2. [Take $\pi = 3.14$.]

ANSWERS (TEST YOURSELF)

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)
5. 38.5 cm^2	**6.** 22 cm	**7.** 264 cm^2	**8.** 44.8 cm^2
9. 154 cm^2	**10.** 9.625 cm^2	**11.** 346.5 cm^2	**12.** 47.52 km/hr
13. (i) 9.625 cm^2	(ii) 6.125 cm^2	**14.** 168 cm^2	**15.** 29.48 cm^2
16. 165 cm^2	**17.** 7 m, 2618 m^2	**18.** 81.27 cm^2, 2744.73 cm^2	
19. 537.5 m^2	**20.** ₹ 31400		

✿