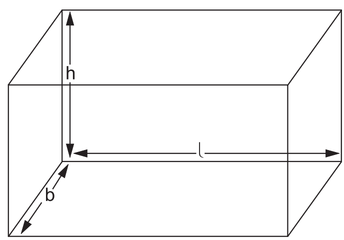
17 Volume and Surface Areas of Solids

SOLIDS *The objects having definite shape and size are called solids.*

A solid occupies a definite space.

CUBOID Solids like matchbox, chalkbox, a tile, a book, an almirah, a room, etc., are in the shape of a cuboid.



FORMULAE For a cuboid of length = l, breadth = b and height = h, we have

(i) *Volume* = $(l \times b \times h)$ *cubic units.*

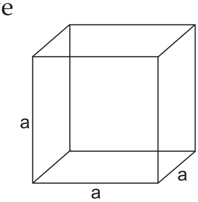
(ii) *Total surface area* = $2(lb + bh + lh)$ *sq units.*

(iii) *Lateral surface area* = $[2(l+b) \times h]$ *sq units.*

(iv) *Diagonal* = $\sqrt{l^2 + b^2 + h^2}$ *units.*

CUBE Solids like ice cubes, sugar cubes, dice, etc., are in the shape of a cube.

FORMULAE For a cube having each edge = a units, we have



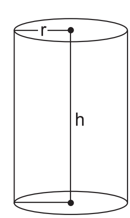
(i) *Volume* = a^3 *cubic units.*

(ii) *Total surface area* = $6a^2$ *sq units.*

(iii) *Lateral surface area* = $4a^2$ *sq units.*

(iv) *Diagonal* = $\sqrt{3}a$ *units.*

CYLINDER Solids like measuring jars, circular pillars, circular pencils, circular pipes, road rollers, gas cylinders, etc., are said to have a cylindrical shape.

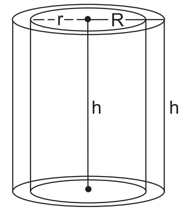


FORMULAE For a cylinder of base radius = r and height (or length) = h, we have

(i) *Volume* = $(\pi r^2 h)$ *cubic units.*

(ii) *Curved surface area* = $2\pi rh$ *sq units.*

(iii) *Total surface area* = $(2\pi rh + 2\pi r^2)$ *sq units* $= 2\pi r(h+r)$ *sq units.*



HOLLOW CYLINDERS Solids like iron pipes, rubber tubes are in the shape of hollow cylinders.

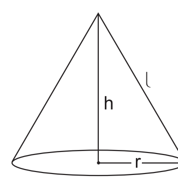
FORMULAE Consider a hollow cylinder having
external radius $= R$, internal radius $= r$
and height $= h$.

Then, we have

(i) *Volume of material* = (*external volume*) – (*internal volume*)
$= (\pi R^2 h - \pi r^2 h)$ *cubic units*
$= \pi h(R^2 - r^2)$ *cubic units.*

(ii) *Curved surface area of hollow cylinder*
= (*external surface area*) + (*internal surface area*)
$= (2\pi Rh + 2\pi rh)$ *sq units*
$= 2\pi h(R + r)$ *sq units.*

(iii) *Total surface area of hollow cylinder*
= (*curved surface area*) + (*area of the base rings*)
$= \{(2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)\}$ *sq units*
$= \{2\pi h(R + r) + 2\pi(R^2 - r^2)\}$ *sq units*
$= \{2\pi h(R + r) + 2\pi(R + r)(R - r)\}$ *sq units*
$= 2\pi(R + r)(h + R - r)$ *sq units.*



CONE Solids like ice-cream cones, conical tents, funnels, etc., are having the shape of a cone.

FORMULAE Consider a cone in which
base radius $= r$, height $= h$ and slant height, $l = \sqrt{h^2 + r^2}$.

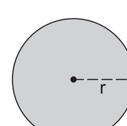
Then, we have

(i) *Volume of the cone* $= \frac{1}{3}\pi r^2 h$ *cubic units.*

(ii) *Curved surface area of the cone* $= \pi rl = \pi r\sqrt{r^2 + h^2}$ *sq units.*

(iii) *Total surface area of the cone* = (*curved surface area*) + (*area of the base*)
$= (\pi rl + \pi r^2) = \pi r(l + r)$ *sq units.*

SPHERE Objects like a football, a cricket ball, etc., are said to have the shape of a sphere.



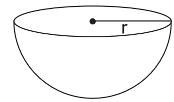
FORMULAE For a sphere of radius r, we have

(i) *Volume of the sphere* $= \left(\frac{4}{3}\pi r^3\right)$ *cubic units.*

(ii) *Surface area of the sphere* $= (4\pi r^2)$ *sq units.*

HEMISPHERE A plane through the centre of a sphere cuts it into two equal parts. Each part is called a hemisphere.

FORMULAE For a hemisphere of radius r, we have



(i) *Volume of the hemisphere* $= \frac{2}{3}\pi r^3$ *cubic units.*

(ii) *Curved surface area of the hemisphere* $= (2\pi r^2)$ *sq units.*

(iii) *Total surface area of the hemisphere* $= (3\pi r^2)$ *sq units.*

SPHERICAL SHELL Consider a spherical shell having external radius = R and internal radius = r. Then, we have

Volume of material = (*external volume*) – (*internal volume*)

$$= \left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3\right) \text{ cubic units}$$

$$= \frac{4}{3}\pi(R^3 - r^3) \text{ cubic units.}$$

VOLUME AND SURFACE AREA OF A COMBINATION OF SOLIDS

SOLVED EXAMPLES

EXAMPLE 1 *Three cubes each of volume* 216 cm^3 *are joined end to end to form a cuboid. Find the total surface area of the resulting cuboid.*

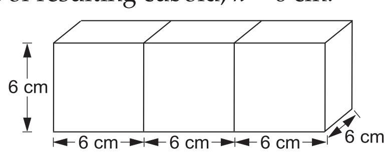
[CBSE 2012]

SOLUTION Edge of each cube = $\sqrt[3]{216}$ cm = 6 cm.

Length of resulting cuboid, l = 18 cm.

Breadth of resulting cuboid, b = 6 cm.

Height of resulting cuboid, h = 6 cm.



$\therefore$ total surface area of the cuboid

$$= 2(lb + bh + lh) = 2(18 \times 6 + 6 \times 6 + 18 \times 6) \text{ cm}^2$$

$$= 2(108 + 36 + 108) \text{ cm}^2$$

$$= (2 \times 252) \text{ cm}^2 = 504 \text{ cm}^2.$$

EXAMPLE 2 *A sphere and a cube have equal surface areas. Show that the ratio of the volume of sphere to that of the cube is* $\sqrt{6} : \sqrt{\pi}$. [CBSE 2011]

SOLUTION Let the radius of the sphere be r and the edge of the cube be a. Then,

surface area of the sphere = surface area of the cube

$$\Rightarrow \quad 4\pi r^2 = 6a^2 \Rightarrow a^2 = \frac{2}{3}\pi r^2$$

$$\Rightarrow \quad a = r\sqrt{\frac{2\pi}{3}}.$$

$\therefore$ required ratio = volume of sphere : volume of cube

$$= \frac{4}{3}\pi r^3 : a^3 = \frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3 \cdot \sqrt{\frac{2\pi}{3}}$$

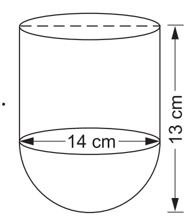
$$= 2 : \sqrt{\frac{2\pi}{3}} = \sqrt{2} : \sqrt{\frac{\pi}{3}} = \sqrt{6} : \sqrt{\pi}.$$

EXAMPLE 3 *A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the hemisphere is* 14 cm *and the total height of the vessel is* 13 cm. *Find*

(i) the capacity of the vessel, [CBSE 2006C]

(ii) the inner surface area of the vessel. [CBSE 2013]

SOLUTION Radius of the hemisphere, $r = 7$ cm.



Total height of the vessel, $H = 13$ cm.

Height of the cylinder, $h = (13 - 7)$ cm = 6 cm.

Radius of the cylinder

= radius of the hemisphere = 7 cm.

(i) Capacity of the vessel

= volume of the hemisphere + volume of the cylinder

$$= \frac{2}{3}\pi r^3 + \pi r^2 h$$

$$= \left[\left(\frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right) + \left(\frac{22}{7} \times 7 \times 7 \times 6\right)\right] \text{cm}^3$$

$$= \left(\frac{2156}{3} + 924\right) \text{cm}^3 = (718.67 + 924) \text{ cm}^3$$

$$= 1642.67 \text{ cm}^3.$$

(ii) Inner surface area of the vessel

= curved surface area of the hemisphere + curved surface area of the cylinder

$$= 2\pi r^2 + 2\pi rh$$

$$= \left[\left(2\times\frac{22}{7}\times7\times7\right)+\left(2\times\frac{22}{7}\times7\times6\right)\right]\text{ cm}^2$$

$$= (308+264)\text{ cm}^2 = 572\text{ cm}^2.$$

EXAMPLE 4 *Due to some floods, some welfare associations jointly requested the government to get* 100 *tents fixed immediately and offered to contribute* 50% *of the cost. If the lower part of each tent is of the form of a cylinder of diameter* 4.2 m *and height* 4 m *with the conical upper part of same diameter but of height* 2.8 m, *and the canvas to be used costs* ₹ 100 *per square metre, find the amount the associations will have to pay. What values are shown by these associations?*

SOLUTION Radius of the cylinder, $r = 2.1$ m.

Height of the cylinder, $h = 4$ m.

Radius of the cone = radius of the cylinder = 2.1 m.

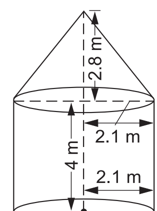
Height of the cone, $H = 2.8$ m.

Slant height of the cone, $l = \sqrt{r^2 + H^2}$

$= \sqrt{(2.1)^2 + (2.8)^2}$ m

$= \sqrt{4.41 + 7.84}$ m

$= \sqrt{12.25}$ m $= 3.5$ m.



Area of the canvas required for each tent

= curved surface area of the cylinder

+ curved surface area of the cone

$$= 2\pi rh + \pi rl = \left[\left(2\times\frac{22}{7}\times2.1\times4\right)+\left(\frac{22}{7}\times2.1\times3.5\right)\right]\text{ m}^2$$

$$= (52.8+23.1)\text{ m}^2 = 75.9\text{ m}^2.$$

Total area of the canvas required for 100 tents

$= (75.9\times100)\text{ m}^2 = 7590\text{ m}^2.$

Total cost of 100 tents = ₹ (7590×100) = ₹ 759000.

Amount to be paid by the associations

= 50% of ₹ 759000 = ₹ $\left(\frac{50}{100}\times759000\right)$ = ₹ 379500.

Hence, the associations will have to pay ₹ 379500.

Values shown: We must

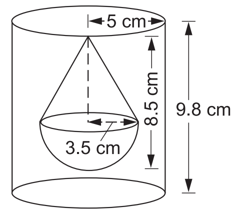
(a) help the people in need

(b) donate a part of our income to charity

(c) join hands together for a noble cause like community service.

EXAMPLE 5 *A cylindrical tub of radius* 5 cm *and length* 9.8 cm *is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed into the tub. If the radius of the hemisphere is* 3.5 cm *and the total height of the solid is* 8.5 cm, *find the volume of water left in the tub.*

SOLUTION Radius of the cylinder, $r = 5$ cm.



Height of the cylinder, $h = 9.8$ cm.

Radius of the hemisphere, $R = 3.5$ cm.

Radius of the cone

$= \text{radius of the hemisphere}$

$= 3.5 \text{ cm} = \frac{7}{2} \text{ cm}.$

Height of the cone, $H = (8.5 - 3.5)$ cm $= 5$ cm.

Volume of the water in the cylindrical tub

$$= \pi r^2 h = \left(\frac{22}{7} \times 5 \times 5 \times 9.8\right) \text{cm}^3 = 770 \text{ cm}^3.$$

Volume of the solid immersed in the tub

= volume of the hemisphere + volume of the cone

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H$$

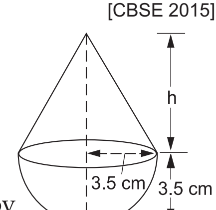
$$= \left[\left(\frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right) + \left(\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 5\right)\right] \text{cm}^3$$

$$= \left(\frac{539}{6} + \frac{385}{6}\right) \text{cm}^3 = \left(\frac{924}{6}\right) \text{cm}^3 = 154 \text{ cm}^3.$$

$\therefore$ volume of water left in the tub $= (770 - 154)$ cm^3 $= 616$ cm^3.

EXAMPLE 6 *A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is* 3.5 cm *and the total wood used in the making of toy is* $166\frac{5}{6}$ cm^3. *Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of* ₹ 10 *per* cm^2. [CBSE 2015]

SOLUTION Let the height of the conical part be h.



Radius of the hemisphere, r

$= \text{radius of the cone} = 3.5 \text{ cm} = \frac{7}{2} \text{ cm}.$

Volume of the wood used in making the toy

= volume of the hemisphere + volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 (2r + h)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(2 \times \frac{7}{2} + h \right) = \frac{77}{6}(7 + h).$$

But, the volume of the wood $= 166\frac{5}{6}\text{ cm}^3 = \frac{1001}{6}\text{ cm}^3$.

$$\therefore \quad \frac{77}{6}(7 + h) = \frac{1001}{6} \Rightarrow 7 + h = \frac{1001}{6} \times \frac{6}{77} = 13$$

$$\Rightarrow h = (13 - 7)\text{ cm} = 6\text{ cm}.$$

So, the height of the toy $= h + r = (6 + 3.5)\text{ cm} = 9.5\text{ cm}$.

Area to be painted = curved surface area of the hemisphere

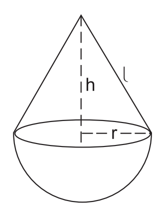
$$= 2\pi r^2 = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right)\text{ cm}^2$$

$$= 77\text{ cm}^2.$$

Hence, cost of painting = ₹ (77×10) = ₹ 770.

EXAMPLE 7 *A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal then find the ratio of the radius and the height of the conical part.* [CBSE 2012]

SOLUTION Let the radius of each of the cone and the hemisphere be r. Let the height of the cone be h and its slant height be l.



Then, curved surface area of the hemisphere

= curved surface area of the cone

$$\Rightarrow 2\pi r^2 = \pi r l \Rightarrow 2\pi r^2 = \pi r \sqrt{r^2 + h^2}$$

$$\Rightarrow 2r = \sqrt{r^2 + h^2} \Rightarrow 4r^2 = r^2 + h^2$$

[squaring both sides]

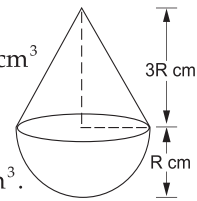
$$\Rightarrow 3r^2 = h^2 \Rightarrow \frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}.$$

Hence, the required ratio is $1 : \sqrt{3}$.

EXAMPLE 8 *A solid cylinder of diameter* 12 cm *and height* 15 cm *is melted and recast into* 12 *toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and total height of the toy, if the height of the cone is* 3 *times the radius.*

SOLUTION Radius of the cylinder, $r = 6$ cm and

height of the cylinder, $h = 15$ cm.



$\therefore$ volume of the cylinder $= \pi r^2 h$

$= (\pi \times 6 \times 6 \times 15)\ \text{cm}^3$

$= (540\pi)\ \text{cm}^3.$

Volume of 12 toys $= (540\pi)\ \text{cm}^3$.

$\therefore$ volume of 1 toy $= \left(\frac{540\pi}{12}\right)\ \text{cm}^3 = (45\pi)\ \text{cm}^3.$

Let the radius of each of the hemisphere and cone be R cm.

Then, height of the cone, $H = (3R)$ cm.

Volume of 1 toy = volume of the hemisphere + volume of the cone

$$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H$$

$$= \left(\frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 \times 3R\right)\ \text{cm}^3$$

$$= \left(\frac{5\pi R^3}{3}\right)\ \text{cm}^3.$$

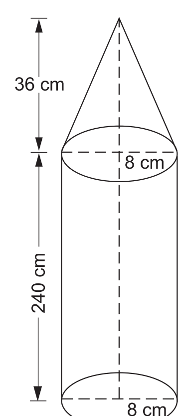
$$\therefore \quad \frac{5\pi R^3}{3} = 45\pi \Rightarrow R^3 = \left(45 \times \frac{3}{5}\right) = 27 = 3^3 \Rightarrow R = 3.$$

Total height of the toy $= (R + 3R)$ cm $= 4R$ cm

$= (4 \times 3)$ cm $= 12$ cm.

EXAMPLE 9 *An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is* 8 cm. *The cylindrical part is* 240 cm *high and the conical part is* 36 cm *high. Find the weight of the pillar if* 1 *cubic centimetre of iron weighs* 7.5 g.

SOLUTION Radius of the cylinder, $r = 8$ cm.



Radius of the cone, $r = 8$ cm.

Height of the cylinder, $h = 240$ cm.

Height of the cone, $H = 36$ cm.

Total volume of the iron

= volume of the cylinder + volume of the cone

$$= \pi r^2 h + \frac{1}{3}\pi r^2 H = \pi r^2\left(h + \frac{1}{3}H\right)$$

$$= \left[\frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36\right)\right]\ \text{cm}^3$$

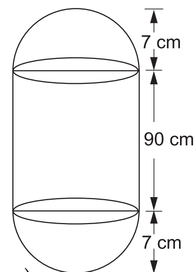
$$= 50688\ \text{cm}^3.$$

$\therefore$ weight of the pillar = volume in cm^3 × weight per cm^3

$$= \left(\frac{50688 \times 7.5}{1000}\right) \text{kg} = 380.16 \text{ kg}.$$

Hence, the weight of the pillar is 380.16 kg.

EXAMPLE 10 *A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is* 104 cm *and the radius of each of its hemispherical ends is* 7 cm, *find the cost of polishing its surface at the rate of* ₹ 10 *per* dm^2. [CBSE 2006C]



SOLUTION Radius of each hemispherical end = 7 cm.

Height of each hemispherical part

= its radius = 7 cm.

Height of the cylindrical part

$= (104 - 2 \times 7)$ cm = 90 cm.

Area of surface to be polished

= 2 (curved surface area of the hemisphere) + (curved surface area of the cylinder)

$= [2(2\pi r^2) + 2\pi rh]$ sq units

$$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7\right) + \left(2 \times \frac{22}{7} \times 7 \times 90\right)\right] \text{cm}^2$$

$= (616 + 3960) \text{ cm}^2 = 4576 \text{ cm}^2$

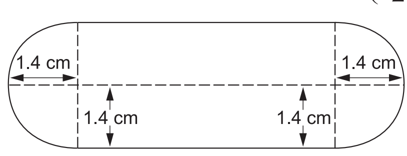
$$= \left(\frac{4576}{10 \times 10}\right) \text{dm}^2 = 45.76 \text{ dm}^2 \qquad [\because 10 \text{ cm} = 1 \text{ dm}].$$

$\therefore$ cost of polishing the surface of the solid

= ₹ (45.76×10) = ₹ 457.60.

EXAMPLE 11 *A gulabjamun contains sugar syrup up to about* 30% *of its volume. Find approximately how much syrup would be found in* 45 *gulab-jamuns, each shaped like a cylinder with two hemispherical ends of length* 5 cm *and diameter* 2.8 cm. [CBSE 2008]

SOLUTION Radius of each hemispherical part, $r = \left(\frac{2.8}{2}\right)$ cm = 1.4 cm.



Radius of the cylindrical part, $r = 1.4$ cm.

Length of the cylindrical part, $h = (5 - 2 \times 1.4)$ cm $= 2.2$ cm.

Volume of one gulabjamun

= volume of 2 hemispherical parts

+ volume of the cylindrical part

$$= \left(2 \times \frac{2}{3}\pi r^3 + \pi r^2 h\right) = \left(\frac{4}{3}\pi r^3 + \pi r^2 h\right)$$

$$= \pi r^2\left(\frac{4}{3}r + h\right) = \left[\frac{22}{7} \times 1.4 \times 1.4 \times \left(\frac{4}{3} \times 1.4 + 2.2\right)\right] \text{cm}^3$$

$$= \left(6.16 \times \frac{12.2}{3}\right) \text{cm}^3 = \left(\frac{75.152}{3}\right) \text{cm}^3.$$

Volume of 45 gulabjamuns

$$= \left(\frac{75.152}{3} \times 45\right) \text{cm}^3 = 1127.28 \text{ cm}^3.$$

Volume of the syrup = (30% of 1127.28) cm^3

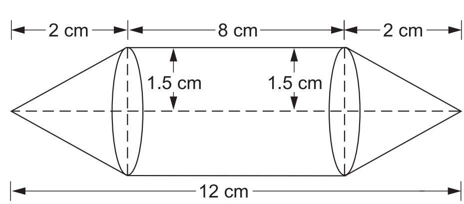
$$= \left(\frac{30}{100} \times 1127.28\right) \text{cm}^3 = 338.184 \text{ cm}^3.$$

Hence, 45 gulabjamuns contain approximately 338 cm^3 of syrup.

EXAMPLE 12 *Rahul, an engineering student, prepared a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is* 3 cm *and its length is* 12 cm. *If each conical part has a height of* 2 cm, *find the (i) volume of air contained in Rahul's model (assume the outer and inner dimensions of the model to be nearly the same), (ii) cost of painting the outer surface of the model at* ₹ 12.50 *per* cm^2.

SOLUTION Radius of each conical part, $r = \frac{3}{2}$ cm.

Radius of cylindrical part, $r = \frac{3}{2}$ cm.



Height of each conical part, $h = 2$ cm.

Length of the cylindrical part, $H = (12 - 2 \times 2)$ cm $= 8$ cm.

Slant height of each conical part,

$$l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{3}{2}\right)^2 + 2^2} \text{ cm} = \sqrt{\frac{9}{4} + 4} \text{ cm}$$

$$= \sqrt{\frac{25}{4}} \text{ cm} = \frac{5}{2} \text{ cm.}$$

(i) Volume of air contained in the model

= volume of 2 conical parts

+ volume of the cylindrical part

$$= 2 \times \frac{1}{3}\pi r^2 h + \pi r^2 H = \pi r^2\left(\frac{2}{3}h + H\right)$$

$$= \left[\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \left(\frac{2}{3} \times 2 + 8\right)\right] \text{cm}^3$$

$$= \left(\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{28}{3}\right) \text{cm}^3 = 66 \text{ cm}^3.$$

(ii) Total surface area of the model

= curved surface area of 2 conical parts

+ curved surface area of the cylindrical part

$$= 2 \times \pi rl + 2\pi rH = 2\pi r(l + H)$$

$$= \left[2 \times \frac{22}{7} \times \frac{3}{2} \times (2.5 + 8)\right] \text{cm}^2$$

$$= \left(2 \times \frac{22}{7} \times \frac{3}{2} \times 10.5\right) \text{cm}^2 = 99 \text{ cm}^2.$$

$\therefore$ cost of painting the model = ₹ (99×12.50)

= ₹ 1237.50.

EXAMPLE 13 *A spherical glass vessel has a cylindrical neck* 8 cm *long,* 2 cm *in diameter; the diameter of the spherical part is* 8.5 cm. *By measuring the amount of water it holds, Radhika finds its volume to be* 345 cm^3. *Check mathematically whether she is correct, taking the above as the inside measurements and* $\pi = 3.14$.

SOLUTION Radius of the spherical part, $R = \left(\frac{8.5}{2}\right)$ cm $= 4.25$ cm.

Radius of the cylindrical neck, $r = 1$ cm.

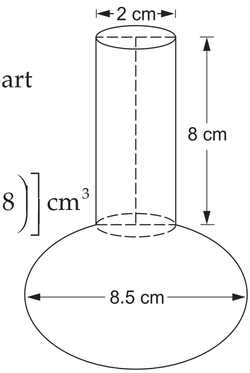
Height of the cylindrical neck, $h = 8$ cm.

Volume of the vessel

= volume of the spherical part
+ volume of the cylindrical part

$= \frac{4}{3}\pi R^3 + \pi r^2 h = \pi\left(\frac{4}{3}R^3 + r^2 h\right)$

$= \left[3.14 \times \left(\frac{4}{3} \times 4.25 \times 4.25 \times 4.25 + 1 \times 1 \times 8\right)\right] \text{cm}^3$



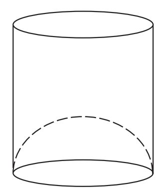
$= [3.14 \times (102.354 + 8)] \text{ cm}^3$

$= (3.14 \times 110.354) \text{ cm}^3$

$= 346.51156 \text{ cm}^3 \approx 346.51 \text{ cm}^3$.

Hence, the volume found by Radhika is almost correct.

EXAMPLE 14 *A juice seller was serving his customers using glasses as shown in the figure. The inner diameter of the cylindrical glass was* 5 cm, *but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was* 10 cm, *find the apparent capacity of the glass and its actual capacity.* [*Use* $\pi = 3.14$.] [CBSE 2009]



SOLUTION Radius of the cylindrical glass, $r = \frac{5}{2}$ cm.

Height of the cylindrical glass, $h = 10$ cm.

Radius of the hemispherical part, $r = \frac{5}{2}$ cm.

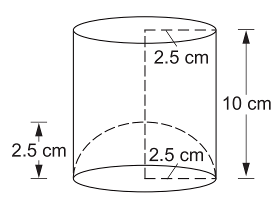
Apparent capacity of the glass

= volume of the cylinder

$= \pi r^2 h$

$= \left(3.14 \times \frac{5}{2} \times \frac{5}{2} \times 10\right) \text{cm}^3$

$= 196.25 \text{ cm}^3$.



Volume of the hemispherical part

$= \frac{2}{3}\pi r^3 = \left(\frac{2}{3} \times 3.14 \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}\right) \text{cm}^3$

$= \left(\frac{196.25}{6}\right) \text{cm}^3 = 32.708 \text{ cm}^3$

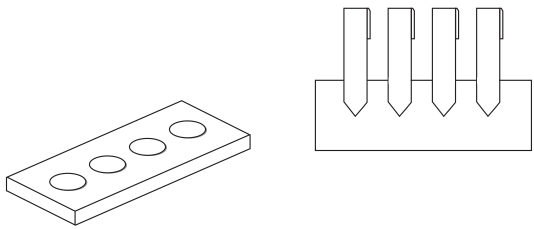
$\approx 32.71 \text{ cm}^3$.

$\therefore$ actual capacity of the glass

= apparent capacity of the glass – volume of the hemispherical part

$= (196.25 - 32.71)\ \text{cm}^3 = 163.54\ \text{cm}^3.$

EXAMPLE 15 *A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are* 15 cm *by* 10 cm *by* 3.5 cm*. The diameter of each of the depressions is* 1 cm *and the depth is* 1.4 cm*. Find the volume of wood in the entire stand.*



SOLUTION Radius of each conical depression, $r = \frac{1}{2}$ cm.

Depth of each conical depression, $h = 1.4$ cm.

Volume of the wood in the stand

= (volume of the cuboid – volume of 4 conical depressions)

$$= \left[(15\times 10\times 3.5) - \left(4\times\frac{1}{3}\pi r^2 h\right)\right]\text{cm}^3$$

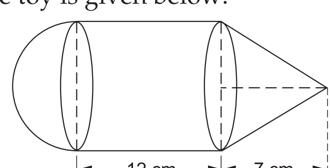
$$= \left[525 - \left(\frac{4}{3}\times\frac{22}{7}\times\frac{1}{2}\times\frac{1}{2}\times 1.4\right)\right]\text{cm}^3$$

$$= \left(525 - \frac{4.4}{3}\right)\text{cm}^3 = (525 - 1.47)\ \text{cm}^3$$

$$= 523.53\ \text{cm}^3.$$

EXAMPLE 16 *A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is* 4.2 cm*, and the heights of the cylindrical and conical portions are* 12 cm *and* 7 cm *respectively. Find the volume of the given toy.* [*Take* $\pi = 22/7$.]

SOLUTION The shape of the toy is given below.



Radius of the hemispherical part, $r = \left(\frac{4.2}{2}\right)$ cm = 2.1 cm.

Radius of the cylindrical part, $r = 2.1$ cm.

Radius of the conical part, $r = 2.1$ cm.

Height of the cylindrical part, $h = 12$ cm.

Height of the conical part, $H = 7$ cm.

Volume of the toy

= volume of the hemispherical part
+ volume of the cylindrical part
+ volume of the conical part

$$= \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 H = \pi r^2\left(\frac{2}{3}r + h + \frac{1}{3}H\right)$$

$$= \left[\frac{22}{7} \times 2.1 \times 2.1 \times \left(\frac{2}{3} \times 2.1 + 12 + \frac{7}{3}\right)\right] \text{cm}^3$$

$$= \left(13.86 \times \frac{47.2}{3}\right) \text{cm}^3 = 218.064 \text{ cm}^3.$$

Hence, the volume of the given toy is 218.064 cm^3.

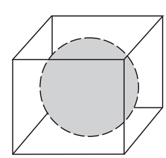
EXAMPLE 17 *The largest possible sphere is carved out from a solid wooden cube of side* 7 cm. *Find*

(i) the volume of the sphere, [CBSE 2011]

(ii) the percentage of wood wasted in the process.

SOLUTION Clearly, diameter of the largest possible sphere = 7 cm.

Radius of the sphere, $r = \frac{7}{2}$ cm.



(i) Volume of the sphere

$$= \frac{4}{3}\pi r^3 = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^3$$

$$= \left(\frac{539}{3}\right) \text{cm}^3 = 179.67 \text{ cm}^3.$$

(ii) Volume of the cube = $(7 \times 7 \times 7)$ cm^3 = 343 cm^3.

Volume of wood wasted

= volume of cube – volume of sphere

$$= \left(343 - \frac{539}{3}\right) \text{cm}^3 = \left(\frac{490}{3}\right) \text{cm}^3.$$

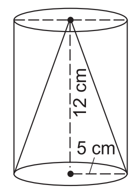
$\therefore$ percentage of wood wasted

$$= \left(\frac{490}{3} \times \frac{1}{343} \times 100\right)\% = \left(\frac{1000}{21}\right)\% = 47\frac{13}{21}\%.$$

EXAMPLE 18 *From a solid right circular cylinder with height* 12 cm *and radius of the base* 5 cm, *a right circular cone of the same height and the same base radius is removed. Find the volume and total surface area of the remaining solid.* [*Use* $\pi = 3.14$.] [CBSE 2014]

SOLUTION Radius of each of the cylinder and cone, $r = 5$ cm.

Height of each of the cylinder and cone, $h = 12$ cm.



Slant height of the cone,

$$l = \sqrt{r^2 + h^2} = \sqrt{5^2 + (12)^2} \text{ cm}$$

$$= \sqrt{169} \text{ cm} = 13 \text{ cm}.$$

Volume of the remaining solid

= volume of the cylinder – volume of the cone.

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \left(\frac{2}{3} \times 3.14 \times 5 \times 5 \times 12\right) \text{cm}^3 = 628 \text{ cm}^3.$$

Total surface area of the remaining solid

= curved surface area of the cylinder

+ curved surface area of the cone

+ area of the upper circular face of the cylinder

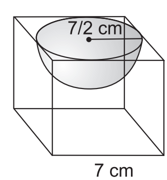
$$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$$

$$= [3.14 \times 5 \times (2 \times 12 + 13 + 5)] \text{ cm}^2$$

$$= (3.14 \times 5 \times 42) \text{ cm}^2 = 659.4 \text{ cm}^2.$$

EXAMPLE 19 *A hemispherical depression is cut out from one face of a cubical block of side* 7 cm, *such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.* [CBSE 2014]

SOLUTION Edge of the cube, $a = 7$ cm.



Radius of the hemisphere, $r = \frac{7}{2}$ cm.

Surface area of remaining solid

= total surface area of the cube

– area of the top of hemispherical part

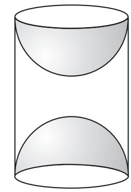
+ curved surface area of the hemisphere

$$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$$

$$= \left(6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$$

$$= (294 + 38.5) \text{ cm}^2 = 332.5 \text{ cm}^2.$$

EXAMPLE 20 *A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is* 12 cm *and its base is of radius* 4.2 cm, *find the total surface area of the article. Also, find the volume of the wood left in the article.* [CBSE 2009C]



SOLUTION Radius of the hemisphere, $r = 4.2$ cm.

Radius of the cylinder, $r = 4.2$ cm

Height of the cylinder, $h = 12$ cm

Total surface area of the article

= curved surface area of cylinder + curved surface area of 2 hemispheres

$= 2\pi rh + 2\times 2\pi r^2 = 2\pi rh + 4\pi r^2$

$= 2\pi r(h+2r) = \left[2\times\frac{22}{7}\times 4.2\times(12+2\times 4.2)\right] \text{cm}^2$

$= (26.4\times 20.4)\ \text{cm}^2 = 538.56\ \text{cm}^2.$

Total volume of the wood left in the article

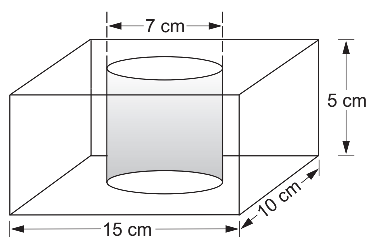
= volume of the cylinder – volume of 2 hemispheres

$= \pi r^2 h - 2\times\frac{2}{3}\pi r^3 = \pi r^2\left(h-\frac{4}{3}r\right)$

$= \left[\frac{22}{7}\times 4.2\times 4.2\times\left(12-\frac{4}{3}\times 4.2\right)\right] \text{cm}^3$

$= (55.44\times 6.4)\ \text{cm}^3 = 354.816\ \text{cm}^3.$

EXAMPLE 21 *In the given figure, from a cuboidal solid metallic block of dimensions* $15\text{ cm}\times 10\text{ cm}\times 5\text{ cm}$, *a cylindrical hole of diameter* 7 cm *is drilled out. Find the surface area of the remaining block.* [HOTS] [CBSE 2015]



SOLUTION Length of the cuboid, $l = 15$ cm.

Breadth of the cuboid, $b = 10$ cm.

Height of the cuboid, $h = 5$ cm.

Radius of the cylinder, $r = \frac{7}{2}$ cm.

Height of the cylinder, $h = 5$ cm.

Surface area of the remaining block

= total surface area of the cuboid
 – area of two circular faces of the cylinder
 + curved surface area of the cylinder

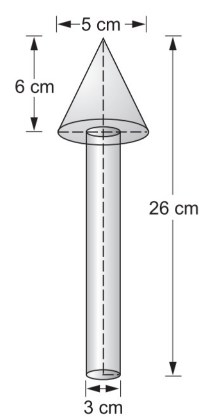
$= 2(lb + bh + lh) - 2 \times \pi r^2 + 2\pi rh$

$= \left[2(15 \times 10 + 10 \times 5 + 15 \times 5) - 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 \right] \text{cm}^2$

$= (550 - 77 + 110) \text{ cm}^2 = 583 \text{ cm}^2$.

EXAMPLE 22 *A wooden toy rocket is in the shape of a cone mounted on a cylinder. The height of the entire rocket is* 26 cm, *while the height of the conical part is* 6 cm. *The base of the conical portion has a diameter of* 5 cm, *while the base diameter of the cylindrical portion is* 3 cm. *If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours.* [*Take* $\pi = 3.14$.] [HOTS]

SOLUTION Radius of the conical part, $r = \frac{5}{2}$ cm.



Height of the conical part, $h = 6$ cm.

Radius of the cylindrical part, $R = \frac{3}{2}$ cm.

Height of cylindrical part, $H = (26 - 6)$ cm
 = 20 cm.

Slant height of the conical part,

$$l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{5}{2}\right)^2 + 6^2} \text{ cm}$$

$$= \sqrt{\frac{25}{4} + 36} \text{ cm} = \sqrt{\frac{169}{4}} \text{ cm} = \frac{13}{2} \text{ cm}.$$

Area to be painted orange

= curved surface area of the cone
 + base area of the cone – base area of the cylinder

$= \pi rl + \pi r^2 - \pi R^2 = \pi(rl + r^2 - R^2)$

$= \left[3.14 \times \left(\frac{5}{2} \times \frac{13}{2} + \frac{5}{2} \times \frac{5}{2} - \frac{3}{2} \times \frac{3}{2} \right) \right] \text{cm}^2$

$$= \left[3.14 \times \left(\frac{65}{4} + \frac{25}{4} - \frac{9}{4}\right)\right] \text{cm}^2 = \left(3.14 \times \frac{81}{4}\right) \text{cm}^2$$

$$= (3.14 \times 20.25) \text{ cm}^2 = 63.585 \text{ cm}^2.$$

Area to be painted yellow

= curved surface area of the cylinder + base area of the cylinder

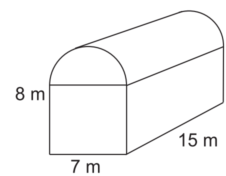
$$= 2\pi RH + \pi R^2 = \pi R(2H + R)$$

$$= \left[3.14 \times \frac{3}{2} \times \left(2 \times 20 + \frac{3}{2}\right)\right] \text{cm}^2$$

$$= \left(3.14 \times \frac{3}{2} \times \frac{83}{2}\right) \text{cm}^2 = \left(\frac{781.86}{4}\right) \text{cm}^2$$

$$= 195.465 \text{ cm}^2.$$

EXAMPLE 23 *Deepa runs an industry in a shed which is in the shape of a cuboid surmounted by a half-cylinder. The base of the shed is of dimensions* $7 \text{ m} \times 15 \text{ m}$ *and the height of the cuboidal portion is* 8 m.



(i) Find the volume of air that the shed can hold.

(ii) Suppose the machinery in the shed occupies a total space of 300 m^3 *and there are* 20 *workers, each of whom occupy about* 0.08 m^3 *space on an average. Then, how much air is in the shed?*

(iii) Find the internal surface area of the shed, excluding the floor.

[HOTS]

SOLUTION The shed consists of a cuboid with dimensions $l = 15$ m, $b = 7$ m, $h = 8$ m and a half-cylinder of radius, $r = \frac{7}{2}$ m and height, $H = 15$ m.

(i) Volume of air that the shed can hold

$$= \text{volume of the cuboid} + \frac{1}{2} \times \text{volume of the cylinder}$$

$$= lbh + \frac{1}{2}\pi r^2 H = \left(15 \times 7 \times 8 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15\right) \text{m}^3$$

$$= \left(840 + \frac{1155}{4}\right) \text{m}^3 = \left(\frac{4515}{4}\right) \text{m}^3$$

$$= 1128.75 \text{ m}^3.$$

(ii) Total space occupied by machinery and 20 workers

$= (300 + 0.08 \times 20)\ \text{m}^3 = 301.6\ \text{m}^3.$

Volume of air in the shed when there are machinery and workers inside it = $(1128.75 - 301.6)\ \text{m}^3 = 827.15\ \text{m}^3$

(iii) Total internal surface area of the shed (excluding floor)

= (surface area of walls) + (surface area of ceiling)

= (area of two walls each measuring 15 m × 8 m)

+ (area of two walls each measuring 7 m × 8 m)

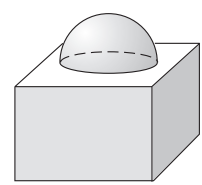
+ (area of two semicircles each of radius 3.5 m)

+ (curved surface area πrh of half-cylinder)

$$= \left[(2 \times 15 \times 8) + (2 \times 7 \times 8) + \left(2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \right) + \left(\frac{22}{7} \times \frac{7}{2} \times 15 \right) \right] \text{m}^2$$

$$= (240 + 112 + 38.5 + 165)\ \text{m}^2 = 555.5\ \text{m}^2.$$

EXAMPLE 24 *A solid is made up of a cube and a hemisphere attached on its top, as shown in the figure. Each edge of the cube measures* 5 cm *and the hemisphere has a diameter of* 4.2 cm. *Find the total area to be painted.* [*Take* $\pi = 22/7$.] [CBSE 2009]



SOLUTION Edge of the cube, $a = 5$ cm.

Radius of the hemisphere, $r = \left(\frac{4.2}{2} \right)$ cm = 2.1 cm.

Total area to be painted

= total surface area of the cube

− base area of the hemisphere

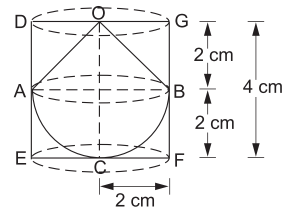
+ curved surface area of the hemisphere

$$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$$

$$= \left(6 \times 5 \times 5 + \frac{22}{7} \times 2.1 \times 2.1 \right) \text{cm}^2$$

$$= (150 + 13.86)\ \text{cm}^2 = 163.86\ \text{cm}^2.$$

EXAMPLE 25 *A solid toy is in the form of a hemisphere surmounted by a right circular cone of height* 2 cm *and diameter of base* 4 cm. *If a right circular cylinder circumscribes the toy, find how much more space than the toy it will cover.* [*Use* $\pi = 3.14$.]



SOLUTION Let OAB be the cone and ACB be the hemisphere, having the same base AB. Let the right circular cylinder $DEFG$ circumscribe the given solid.

Radius of each of the cone, cylinder and hemisphere, $r = 2$ cm.

Height of the cone, $h = 2$ cm.

Height of the cylinder,

$$H = h + r = (2+2) \text{ cm} = 4 \text{ cm}.$$

Volume of the toy

$$= (\text{volume of the hemisphere}) + (\text{volume of the cone})$$

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \left(\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2\right) \text{cm}^3$$

$$= \left(\frac{16\pi}{3} + \frac{8\pi}{3}\right) \text{cm}^3 = 8\pi \text{ cm}^3.$$

Volume of the cylinder $= \pi r^2 H$

$$= (\pi \times 2^2 \times 4) \text{ cm}^3 = (16\pi) \text{ cm}^3.$$

Required volume = volume of the cylinder – volume of the toy

$$= (16\pi - 8\pi) \text{ cm}^3 = 8\pi \text{ cm}^3$$

$$= (8 \times 3.14) \text{ cm}^3 = 25.12 \text{ cm}^3.$$

EXERCISE 17A

1. Two cubes each of volume 27 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid. [CBSE 2011, '14]

2. The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area. [CBSE 2012]

3. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [CBSE 2014]

4. (i) A 5-m-wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of ₹ 25 per metre. [CBSE 2014]

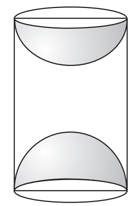
   (ii) The radius and height of a solid right-circular cone are in the ratio of 5 : 12. If its volume is 314 cm^3, find its total surface area. [Take $\pi = 3.14$.] [CBSE 2017]

5. If the volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5, find the ratio of their heights.
6. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km^2. Find the height of the mountain.
7. The sum of the radius of the base and the height of a solid cylinder is 37 metres. If the total surface area of the cylinder be 1628 sq metres, find its volume.
8. The surface area of a sphere is 2464 cm^2. If its radius be doubled, what will be the surface area of the new sphere?
9. A military tent of height 8.25 m is in the form of a right circular cylinder of base diameter 30 m and height 5.5 m surmounted by a right circular cone of same base radius. Find the length of canvas used in making the tent, if the breadth of the canvas is 1.5 m. [CBSE 2012]
10. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m and the radius of its base is 14 m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per square metre. [Take $\pi = 22/7$.] [CBSE 2005]
11. A circus tent is cylindrical to a height of 3 m and conical above it. If its base radius is 52.5 m and the slant height of the conical portion is 53 m, find the area of canvas needed to make the tent. [Take $\pi = 22/7$.] [CBSE 2004]
12. A rocket is in the form of a circular cylinder closed at the lower end and a cone of the same radius is attached to the top. The radius of the cylinder is 2.5 m, its height is 21 m and the slant height of the cone is 8 m. Calculate the total surface area of the rocket.
13. A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid. [CBSE 2012]
14. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy. [CBSE 2017]
15. A toy is in the shape of a cone mounted on a hemisphere of same base radius. If the volume of the toy is 231 cm^3 and its diameter is 7 cm, find the height of the toy. [CBSE 2012]
16. A cylindrical container of radius 6 cm and height 15 cm is filled with ice cream. The whole ice cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is 4 times the radius of its base, find the radius of the ice cream cone.
17. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder. The diameter of the hemisphere is 21 cm and the total height of the vessel is 14.5 cm. Find its capacity.

18. A toy is in the form of a cylinder with hemispherical ends. If the whole length of the toy is 90 cm and its diameter is 42 cm, find the cost of painting the toy at the rate of 70 paise per sq cm. [CBSE 2014]

19. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

20. A wooden article was made by scooping out a hemisphere from each end of a cylinder, as shown in the figure. If the height of the cylinder is 20 cm and its base is of diameter 7 cm, find the total surface area of the article when it is ready. [CBSE 2008C]



21. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 2.1 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub full of water in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 9.8 cm, find the volume of the water left in the tub. [CBSE 1996C, 2000C]

22. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume of the remaining solid. Also, find the total surface area of the remaining solid. [Take $\pi = 3.14$.] [HOTS] [CBSE 2009]

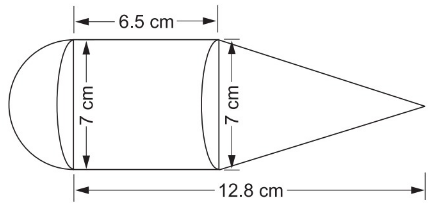
23. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [CBSE 2014]

24. From a solid cylinder of height 14 cm and base diameter 7 cm, two equal conical holes each of radius 2.1 cm and height 4 cm are cut off. Find the volume of the remaining solid. [CBSE 2011]

25. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole has a radius of $3/2$ cm and its depth is $8/9$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape. [CBSE 2015]

26. A spherical glass vessel has a cylindrical neck 7 cm long and 4 cm in diameter. The diameter of the spherical part is 21 cm. Find the quantity of water it can hold. [Use $\pi = 22/7$.] [CBSE 2009C]

27. The given figure represents a solid consisting of a cylinder surmounted by a cone at one end and a hemisphere at the other. Find the volume of the solid.



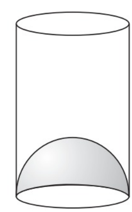
28. From a cubical piece of wood of side 21 cm, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece. Find the surface area and volume of the remaining piece. [CBSE 2014]

29. (i) A hemisphere of maximum possible diameter is placed over a cuboidal block of side 7 cm. Find the surface area of the solid so formed. [CBSE 2017]

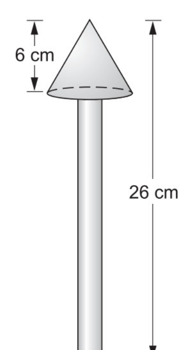
(ii) A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹ 5 per 100 sq cm. [Use $\pi = 3.14$.] [CBSE 2015]

30. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and the conical parts are the same as that of the cylindrical part. Find the surface area of the toy, if the total height of the toy is 30 cm.

31. The inner diameter of a glass is 7 cm and it has a raised portion in the bottom in the shape of a hemisphere, as shown in the figure. If the height of the glass is 16 cm, find the apparent capacity and the actual capacity of the glass.



32. A wooden toy is in the shape of a cone mounted on a cylinder, as shown in the figure. The total height of the toy is 26 cm, while the height of the conical part is 6 cm. The diameter of the base of the conical part is 5 cm and that of the cylindrical part is 4 cm. The conical part and the cylindrical part are respectively painted red and white. Find the area to be painted by each of these colours. [Take $\pi = 22/7$.]



ANSWERS (EXERCISE 17A)

1. 90 cm^2 **2.** 693 cm^2 **3.** 718.67 cm^2 **4.** (i) ₹ 2750 (ii) 282.6 cm^2
5. 25 : 64 **6.** 2.4 km **7.** 4620 m^3 **8.** 9856 cm^2
9. 825 m **10.** ₹ 82720 **11.** 9735 m^2 **12.** 412.5 m^2
13. 166.83 cm^3 **14.** 214.5 cm^2 **15.** 18 cm **16.** 3 cm
17. 3811.5 cm^3 **18.** ₹ 8316 **19.** 220 mm^2 **20.** 594 cm^2
21. 732.116 cm^3 **22.** 602.88 cm^2 **23.** 73.92 cm^2 **24.** 502.04 cm^3
25. 1463 : 22 **26.** 4939 cm^3 **27.** 376.016 m^3 **28.** 6835.5 cm^3, 2992.5 cm^2
29. (i) 332.5 cm^2 (ii) 10 cm, ₹ 53.93 **30.** 770 cm^2 **31.** 526.17 cm^3, 616 cm^3
32. Area painted red = 58.143 cm^2, Area painted white = 264 cm^2

HINTS TO SOME SELECTED QUESTIONS

4. (i) Area of the cloth required = curved surface area of the tent = $\pi rl = \pi r\sqrt{r^2 + h^2}$.

Length of the cloth required = $\dfrac{\text{area}}{\text{width}}$.

5. Let the radii of the cones be $4r$ and $5r$ and their heights be h_1 and h_2 respectively. Then,

$$\frac{\frac{1}{3}\pi\times(4r)^2\times h_1}{\frac{1}{3}\pi\times(5r)^2\times h_2} = \frac{1}{4} \Rightarrow \frac{16h_1}{25h_2} = \frac{1}{4} \Rightarrow \frac{h_1}{h_2} = \frac{1}{4}\times\frac{25}{16} = \frac{25}{64}.$$

6. $\pi r^2 = 1.54$. Find r.

Calculate l, using the formula, $l = \sqrt{r^2 + h^2}$.

7. $h + r = 37$ m and $2\pi r(h + r) = 1628$. Calculate r. Then, find h and calculate volume.

8. $4\pi r^2 = 2464$. Find r.

Now, calculate surface area using $2r$ as radius.

9. Total area of the canvas required

= curved surface area of the cylinder + curved surface area of the cone.

Length of the canvas = $\dfrac{\text{area}}{\text{width}}$.

13. Height of the conical part = (9.5 – 3.5) cm = 6 cm.

Volume of the solid = volume of the cone + volume of the hemisphere.

18. Length of the cylindrical part = $(90 - 2\times 21)$ cm = 48 cm.

Total area to be painted

= $2\times$ curved surface area of a hemisphere + curved surface area of the cylinder.

20. Total surface area

= curved surface area of the cylinder + $2\times$ curved surface area of each hemisphere.

22. Volume of the remaining solid = volume of the cylinder – volume of the cone.
Surface area of the remaining solid
= curved surface area of the cylinder + curved surface area of the cone
+ area of upper circular face of the cylinder.

28. Surface area of the remaining piece
= total surface area of the cube – area of the top of the hemispherical part
+ curved surface area of the hemisphere.
Volume of the remaining piece
= volume of the cube – volume of the hemisphere.

29. (ii) Largest possible diameter = edge of the cube.
Total surface area of the solid
= surface area of the cube – area of the top of the hemispherical part
+ curved surface area of the hemisphere.

CONVERSION OF SOLID FROM ONE SHAPE TO ANOTHER AND MIXED PROBLEMS

EXAMPLE 1 *The dimensions of a metallic cuboid are* $100 \text{ cm} \times 80 \text{ cm} \times 64 \text{ cm}$. *It is melted and recast into a cube. Find the surface area of the cube.* [CBSE 2011]

SOLUTION Let each edge of the cube be a.

Then, volume of the cube = a^3.

Volume of the cuboid = $(100 \times 80 \times 64) \text{ cm}^3$

$= 512000 \text{ cm}^3$.

Volume of the cube = volume of the cuboid

$\Rightarrow \quad a^3 = 512000 \text{ cm}^3$

$\Rightarrow \quad a = \sqrt[3]{512 \times 1000} \text{ cm} = (8 \times 10) \text{ cm} = 80 \text{ cm}$.

Surface area of the cube = $6a^2$

$= (6 \times 80 \times 80) \text{ cm}^2 = 38400 \text{ cm}^2$.

EXAMPLE 2 *How many spherical solid bullets can be made out of a solid cube of lead whose edge measures* 44 cm, *each bullet being* 4 cm *in diameter?* [CBSE 2014]

SOLUTION Edge of the cube, $a = 44$ cm.

Volume of the cube = $a^3 = (44 \times 44 \times 44) \text{ cm}^3$.

Radius of each spherical bullet, $r = 2$ cm.

Volume of each spherical bullet = $\frac{4}{3}\pi r^3$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 \right) \text{cm}^3 = \left(\frac{704}{21} \right) \text{cm}^3$$

$\therefore$ required number of bullets

$$= \frac{\text{volume of the cube}}{\text{volume of each spherical bullet}}$$

$$= \frac{44 \times 44 \times 44}{704/21} = 44 \times 44 \times 44 \times \frac{21}{704} = 2541.$$

EXAMPLE 3 *How many silver coins,* 1.75 cm *in diameter and of thickness* 2 mm, *must be melted to form a cuboid of dimensions*

$$5.5 \text{ cm} \times 10 \text{ cm} \times 3.5 \text{ cm?}$$

SOLUTION Radius of each coin, $r = \left(\frac{1.75}{2} \right) \text{cm} = \left(\frac{175}{200} \right) \text{cm} = \frac{7}{8} \text{cm}.$

Thickness of each coin, $h = 2 \text{ mm} = \left(\frac{2}{10} \right) \text{cm} = \frac{1}{5} \text{cm}.$

Volume of each coin $= \pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{8} \times \frac{7}{8} \times \frac{1}{5} \right) \text{cm}^3 = \left(\frac{77}{160} \right) \text{cm}^3.$$

Volume of the cuboid $= (5.5 \times 10 \times 3.5) \text{ cm}^3$

$$= 192.5 \text{ cm}^3.$$

$$\therefore \quad \text{number of coins} = \frac{\text{volume of the cuboid}}{\text{volume of each coin}}$$

$$= 192.5 \times \frac{160}{77} = 400.$$

EXAMPLE 4 *A metallic sphere of radius* 10.5 cm *is melted and then recast into smaller cones, each of radius* 3.5 cm *and height* 3 cm. *How many cones are obtained?* [CBSE 2004, '12, '17]

SOLUTION Radius of the sphere, $R = \frac{21}{2}$ cm.

Volume of the sphere $= \frac{4}{3} \pi R^3 = \left(\frac{4}{3} \pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \right) \text{cm}^3$

$$= \left(\frac{3087\pi}{2} \right) \text{cm}^3.$$

Radius of each cone, $r = \frac{7}{2}$ cm.

Height of each cone, $h = 3$ cm.

Volume of each cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times \frac{7}{2} \times \frac{7}{2} \times 3\right) \text{cm}^3$

$= \left(\frac{49\pi}{4}\right) \text{cm}^3.$

$\therefore$ required number of cones $= \frac{\text{volume of the sphere}}{\text{volume of each cone}}$

$= \left(\frac{3087\pi}{2} \times \frac{4}{49\pi}\right) = 126.$

EXAMPLE 5 *The internal and external radii of a hollow sphere are* 3 cm *and* 5 cm *respectively. The sphere is melted to form a solid cylinder of height* $2\frac{2}{3}$ cm. *Find the diameter and the curved surface area of the cylinder.*

SOLUTION External radius of the sphere, $r_1 = 5$ cm.

Internal radius of the sphere, $r_2 = 3$ cm.

Volume of metal in the hollow sphere

$= \frac{4}{3}\pi(r_1^3 - r_2^3) = \left[\frac{4}{3}\pi \times (5^3 - 3^3)\right] \text{cm}^3$

$= \left(\frac{392\pi}{3}\right) \text{cm}^3.$

Let the radius of the solid cylinder be R cm.

Height of the solid cylinder, $h = \frac{8}{3}$ cm.

Volume of the solid cylinder

$= \pi R^2 h = \left(\pi R^2 \times \frac{8}{3}\right) \text{cm}^3 = \left(\frac{8\pi R^2}{3}\right) \text{cm}^3.$

Volume of cylinder = volume of metal in the hollow sphere

$\Rightarrow \frac{8\pi R^2}{3} = \frac{392\pi}{3} \Rightarrow R^2 = 49 \Rightarrow R = 7.$

$\therefore$ diameter of the cylinder formed = (2×7) cm = 14 cm.

Curved surface area of the cylinder

$= 2\pi rh = \left(2 \times \frac{22}{7} \times 7 \times \frac{8}{3}\right) \text{cm}^2$

$= \frac{352}{3} \text{cm}^2 = 117\frac{1}{3} \text{cm}^2.$

EXAMPLE 6 *A hollow sphere of internal and external diameters* 4 cm *and* 8 cm *is melted to form a cone of base diameter* 8 cm. *Find the height and the slant height of the cone.* [CBSE 2011]

SOLUTION External radius of the sphere, $r_1 = 4$ cm.

Internal radius of the sphere, $r_2 = 2$ cm.

Volume of metal in the hollow sphere

$$= \frac{4}{3}\pi(r_1^3 - r_2^3) = \left[\frac{4}{3}\pi \times (4^3 - 2^3)\right] \text{cm}^3 = \left(\frac{224\pi}{3}\right) \text{cm}^3.$$

Radius of the cone formed, $R = 4$ cm.

Let the height of the cone be h.

Volume of the cone

$$= \frac{1}{3}\pi R^2 h = \left(\frac{1}{3}\pi \times 4 \times 4 \times h\right) \text{cm}^3 = \left(\frac{16\pi h}{3}\right) \text{cm}^3.$$

Volume of the cone = volume of metal in the sphere

$$\Rightarrow \frac{16\pi h}{3} = \frac{224\pi}{3} \Rightarrow h = 14 \text{ cm}.$$

Slant height, $l = \sqrt{R^2 + h^2} = \sqrt{4^2 + 14^2}$ cm

$$= \sqrt{212} \text{ cm} = 2\sqrt{53} \text{ cm}.$$

Hence, the height of the cone is 14 cm and its slant height is $2\sqrt{53}$ cm.

EXAMPLE 7 *A girl empties a cylindrical bucket full of sand, of base radius* 18 cm *and height* 32 cm, *on the floor to form a conical heap of sand. If the height of this conical heap is* 24 cm *then find its slant height correct to one place of decimal.* [CBSE 2014]

SOLUTION Radius of the cylindrical bucket, $R = 18$ cm.

Height of the cylindrical bucket, $H = 32$ cm.

Volume of sand in the bucket $= \pi R^2 H = (\pi \times 18 \times 18 \times 32)$ cm^3.

Let the radius of the conical heap be r.

Height of the conical heap, $h = 24$ cm.

Volume of the conical heap $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times r^2 \times 24\right)$ cm^3

$$= (8\pi r^2) \text{ cm}^3.$$

Volume of the conical heap = volume of sand in the bucket

$$\Rightarrow 8\pi r^2 = \pi \times 18 \times 18 \times 32 \Rightarrow r^2 = \frac{18 \times 18 \times 32}{8} = 1296$$

$$\Rightarrow r = \sqrt{1296} = 36 \text{ cm}.$$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(36)^2 + (24)^2} \text{ cm} = \sqrt{1296 + 576} \text{ cm}$$

$$= \sqrt{1872} \text{ cm} = 43.26 \text{ cm}.$$

$$\approx 43.3 \text{ cm (correct to one decimal place).}$$

EXAMPLE 8 504 *cones, each of diameter* 3.5 cm *and height* 3 cm, *are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area.* [CBSE 2015]

SOLUTION Radius of each cone, $r = \left(\frac{3.5}{2}\right) \text{cm} = \left(\frac{35}{20}\right) \text{cm} = \frac{7}{4}$ cm.

Height of each cone, $h = 3$ cm.

Volume of each cone $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \pi \times \frac{7}{4} \times \frac{7}{4} \times 3\right) \text{cm}^3 = \left(\frac{49\pi}{16}\right) \text{cm}^3.$$

Total volume of 504 cones

$$= \left(\frac{49\pi}{16} \times 504\right) \text{cm}^3 = \left(\frac{49\pi \times 63}{2}\right) \text{cm}^3.$$

Let the radius of the sphere be R.

Then, volume of the sphere $= \frac{4}{3}\pi R^3$.

Volume of the sphere = total volume of 504 cones

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{49\pi \times 63}{2} \Rightarrow R^3 = \left(\frac{49 \times 63}{2} \times \frac{3}{4}\right)$$

$$\Rightarrow R^3 = \left(\frac{7 \times 7 \times 7 \times 9 \times 3}{2 \times 4}\right) \Rightarrow R = \left(\frac{7 \times 3}{2}\right) \text{cm} = \frac{21}{2} \text{ cm}.$$

Diameter of the sphere $= \left(\frac{21}{2} \times 2\right)$ cm $= 21$ cm.

Surface area of the sphere $= 4\pi R^2 = \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2$

$$= 1386 \text{ cm}^2.$$

EXAMPLE 9 *Two spheres of same metal weigh* 1 kg *and* 7 kg. *The radius of the smaller sphere is* 3 cm. *The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.* [HOTS] [CBSE 2015]

SOLUTION Since both the spheres are made of same metal, their weights are directly proportional to their volumes, or the ratio of their volumes is equal to the ratio of their weights.

Radius of first small sphere, $r_1 = 3$ cm.

Let the radius of second small sphere be r_2 cm.

$$\therefore \quad \frac{\text{volume of first small sphere}}{\text{volume of second small sphere}} = \frac{1}{7}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{1}{7} \Rightarrow \frac{3^3}{r_2^3} = \frac{1}{7} \Rightarrow r_2^3 = 27 \times 7 = 189.$$

Sum of volumes of two small spheres

$$= \frac{4}{3}\pi\,(r_1^3 + r_2^3) = \left[\frac{4}{3}\pi(3^3 + 189)\right] \text{cm}^3$$

$$= \left(\frac{4}{3}\pi \times 216\right) \text{cm}^3.$$

Let the radius of the new sphere be R.

Volume of new sphere = sum of volumes of two small spheres

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 216 \Rightarrow R^3 = 216$$

$$\Rightarrow \quad R = \sqrt[3]{216} = 6 \text{ cm} \Rightarrow \text{diameter} = 2R = 12 \text{ cm}.$$

EXAMPLE 10 *The diameter of a copper sphere is* 6 cm. *The sphere is melted and drawn into a long wire of uniform circular cross section. If the length of the wire is* 36 m, *find its thickness.*

SOLUTION Radius of the sphere, $R = 3$ cm.

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

$$= \left(\frac{4}{3}\pi \times 3 \times 3 \times 3\right) \text{cm}^3$$

$$= (36\pi) \text{ cm}^3.$$

Length of the wire, $h = 36$ m $= 3600$ cm.

Let the radius of the wire be r.

Volume of the wire $= \pi r^2 h = (\pi r^2 \times 3600)$ cm^3.

But, volume of the wire = volume of the sphere

$$\Rightarrow \quad 3600\pi r^2 = 36\pi \Rightarrow r^2 = \frac{1}{100}$$

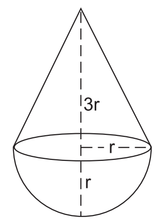
$$\Rightarrow \quad r = \sqrt{\frac{1}{100}} \text{ cm} = \frac{1}{10} \text{ cm} = 1 \text{ mm}.$$

Hence, thickness of the wire = its diameter = 2 mm.

EXAMPLE 11 *A solid cylinder of diameter* 12 cm *and height* 15 cm *is melted and recast into* 12 *toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and the total height of the toy, if the height of the conical part is thrice its radius.*

[CBSE 2005C]

SOLUTION Radius of the cylinder, $R = 6$ cm.



Height of the cylinder, $H = 15$ cm

Volume of the cylinder $= \pi R^2 H$

$$= (\pi \times 6 \times 6 \times 15) \text{ cm}^3$$
$$= (540\pi) \text{ cm}^3.$$

Let the radius of each of the conical and hemispherical parts be r.

Then, height of the conical part, $h = 3r$.

Volume of each toy

$$= \text{volume of the hemisphere} + \text{volume of the cone}$$
$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 \times 3r\right) \text{cm}^3$$
$$= \left(\frac{2}{3}\pi r^3 + \pi r^3\right) \text{cm}^3 = \left(\frac{5\pi r^3}{3}\right) \text{cm}^3.$$

Now, total volume of 12 toys = volume of the cylinder

$$\Rightarrow \quad 12 \times \frac{5\pi r^3}{3} = 540\pi \;\Rightarrow\; r^3 = \frac{540}{20} = 27 \;\Rightarrow\; r = \sqrt[3]{27} = 3 \text{ cm}.$$

Hence, radius of the hemisphere = 3 cm.

Total height of the toy $= (r + h) = (r + 3r) = 4r$

$$= (4 \times 3) \text{ cm} = 12 \text{ cm}.$$

EXAMPLE 12 *Find the number of coins,* 1.5 cm *in diameter and* 2 mm *thick, to be melted to form a right circular cylinder of height* 10 cm *and diameter* 4.5 cm.

SOLUTION Radius of each cylindrical coin, $r = \frac{1.5}{2} \text{ cm} = \frac{15}{20} \text{ cm} = \frac{3}{4} \text{ cm}.$

Thickness of each cylindrical coin, $h = 2 \text{ mm} = \frac{2}{10} \text{ cm} = \frac{1}{5} \text{ cm}.$

Volume of each coin $= \pi r^2 h = \left(\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{5}\right) \text{cm}^3$

$$= \left(\frac{9\pi}{80}\right) \text{cm}^3.$$

Radius of the required cylinder, $R = \left(\frac{4.5}{2}\right)$ cm = 2.25 cm.

Height of the required cylinder, $H = 10$ cm.

Volume of the required cylinder

$$= \pi R^2 H = (\pi \times 2.25 \times 2.25 \times 10)\text{ cm}^3$$

$$= \left(\pi \times \frac{225}{100} \times \frac{225}{100} \times 10\right)\text{ cm}^3 = \left(\frac{405\pi}{8}\right)\text{ cm}^3.$$

Required number of coins

$$= \frac{\text{volume of the required cylinder}}{\text{volume of each coin}} = \left(\frac{405\pi}{8} \times \frac{80}{9\pi}\right) = 450.$$

EXAMPLE 13 *A solid iron rectangular block of dimensions* 4.4 m , 2.6 m *and* 1 m *is cast into a hollow cylindrical pipe of internal radius* 30 cm *and thickness* 5 cm. *Find the length of the pipe.* [CBSE 2017]

SOLUTION Volume of iron = $(440 \times 260 \times 100)\text{ cm}^3$.

Internal radius of the pipe = 30 cm.

External radius of the pipe = $(30 + 5)$ cm = 35 cm.

Let the length of the pipe be h cm.

Volume of iron in the pipe

$$= (\text{external volume}) - (\text{internal volume})$$

$$= [\pi(35)^2 h - \pi(30)^2 h]\text{ cm}^3 = \pi h[(35)^2 - (30)^2]\text{ cm}^3$$

$$= (65 \times 5)\pi h\text{ cm}^3 = (325\pi h)\text{ cm}^3.$$

$\therefore \quad 325\pi h = 440 \times 260 \times 100$

$$\Rightarrow \quad \text{length} = h\text{ cm} = \left(\frac{440 \times 260 \times 100 \times 7}{325 \times 22}\right)\text{ cm}$$

$$= 11200\text{ cm} = 112\text{ m}.$$

Hence, the length of the pipe is 112 m.

EXAMPLE 14 *A hemispherical bowl of internal radius* 9 cm *is full of liquid. The liquid is to be filled into cylindrical-shaped small bottles, each of diameter* 3 cm *and height* 4 cm. *How many bottles are needed to empty the bowl?* [CBSE 2005, '12]

SOLUTION Radius of the hemispherical bowl, $R = 9$ cm.

Volume of liquid in the bowl $= \frac{2}{3}\pi R^3$

$$= \left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right)\text{ cm}^3$$

$$= (486\pi)\text{ cm}^3.$$

Radius of each cylindrical bottle, $r = \frac{3}{2}$ cm.

Height of each cylindrical bottle, $h = 4$ cm.

Volume of each cylindrical bottle

$$= \pi r^2 h = \left(\pi \times \frac{3}{2} \times \frac{3}{2} \times 4\right) \text{cm}^3 = (9\pi) \text{ cm}^3.$$

Required number of bottles

$$= \frac{\text{volume of liquid in the bowl}}{\text{volume of each bottle}} = \frac{486\pi}{9\pi} = 54.$$

EXAMPLE 15 *A hemispherical bowl of internal diameter* 36 cm *contains liquid. This liquid is filled into* 72 *cylindrical bottles of diameter* 6 cm. *Find the height of each bottle if* 10% *liquid is wasted in this transfer.*

[CBSE 2015]

SOLUTION Radius of the hemispherical bowl, $R = 18$ cm.

Volume of liquid in the bowl,

$$r = \frac{2}{3}\pi R^3 = \left(\frac{2}{3}\pi \times 18 \times 18 \times 18\right) \text{cm}^3$$

$$= (3888\pi) \text{ cm}^3.$$

Volume of liquid filled into bottles

$$= (90\% \text{ of } 3888\pi) \text{ cm}^3$$

$$= \left(\frac{90}{100} \times 3888\pi\right) \text{cm}^3.$$

Let the height of each cylindrical bottle be h.

Radius of each cylindrical bottle, $r = 3$ cm.

Volume of each cylindrical bottle

$$= \pi r^2 h = (\pi \times 3^2 \times h) \text{ cm}^3 = (9\pi h) \text{ cm}^3.$$

Now, volume of 72 cylindrical bottles

= volume of liquid filled into bottles

$$\Rightarrow \quad 72 \times 9\pi h = \frac{90}{100} \times 3888\pi$$

$$\Rightarrow \quad h = \left(\frac{90 \times 3888}{100 \times 72 \times 9}\right) \text{cm} = \frac{27}{5} \text{ cm} = 5.4 \text{ cm}.$$

EXAMPLE 16 *A conical vessel whose internal radius is* 5 cm *and height* 24 cm, *is full of water. The water is emptied into a cylindrical vessel with internal radius* 10 cm. *Find the height to which the water rises in the cylindrical vessel.*

SOLUTION Radius of the conical vessel, $r = 5$ cm.

Height of the conical vessel, $h = 24$ cm.

Volume of the conical vessel

$$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times 5 \times 5 \times 24\right) \text{cm}^3 = (200\pi) \text{ cm}^3.$$

Radius of the cylindrical vessel, $R = 10$ cm.

Let the height to which water rises in the vessel be H. Then, volume of the water in the cylindrical vessel

$$= \pi R^2 H = (\pi \times 10 \times 10 \times H) \text{ cm}^3 = (100\pi H) \text{ cm}^3.$$

Volume of the water in cylindrical vessel

= volume of the water in the conical vessel

$$\Rightarrow \quad 100\pi H = 200\pi \quad \Rightarrow \quad H = \frac{200}{200} = 2 \text{ cm}.$$

Hence, the required height is 2 cm.

EXAMPLE 17 *A sphere of diameter* 6 cm *is dropped into a right circular cylindrical vessel, partly filled with water. The diameter of the cylindrical vessel is* 12 cm. *If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?*

SOLUTION Radius of the sphere, $R = 3$ cm.

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi \times 3^3\right) \text{cm}^3 = (36\pi) \text{ cm}^3.$$

Radius of the cylindrical vessel, $r = 6$ cm.

Let the rise in water level be h.

Increase in volume of water when the sphere is submerged

$$= \pi r^2 h = (\pi \times 6 \times 6 \times h) \text{ cm}^3 = (36\pi h) \text{ cm}^3.$$

But this volume must be equal to the volume of the sphere.

$$\therefore \quad 36\pi h = 36\pi \quad \Rightarrow \quad h = 1 \text{ cm}.$$

Hence, rise in the water level is 1 cm.

EXAMPLE 18 *Sushant has a vessel of the form of an inverted cone, open at the top, of height* 11 cm *and radius of the top as* 2.5 cm *and is full of water. Metallic spherical balls each of diameter* 0.5 cm *are put in the vessel due to which two-fifths of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?* [CBSE 2014]

SOLUTION Height of the conical vessel, $h = 11$ cm.

Radius of the conical vessel, $r = 2.5$ cm.

Volume of the conical vessel

$$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times 2.5 \times 2.5 \times 11\right) \text{cm}^3$$

$$= \left(\frac{1}{3}\pi \times \frac{5}{2} \times \frac{5}{2} \times 11\right) \text{cm}^3 = \left(\frac{275\pi}{12}\right) \text{cm}^3.$$

Volume of water flown out after putting spherical balls

$$= \left(\frac{2}{5} \text{ of } \frac{275\pi}{12}\right) \text{cm}^3 = \left(\frac{55\pi}{6}\right) \text{cm}^3.$$

Radius of each spherical ball,

$$R = \frac{0.5}{2} \text{ cm} = 0.25 \text{ cm} = \left(\frac{25}{100}\right) \text{cm} = \frac{1}{4} \text{ cm}.$$

Volume of each spherical ball

$$= \frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) \text{cm}^3 = \left(\frac{\pi}{48}\right) \text{cm}^3.$$

$\therefore$ required number of balls

$$= \frac{\text{volume of water flown out}}{\text{volume of each spherical ball}} = \left(\frac{55\pi}{6}\right) \times \frac{48}{\pi} = 440.$$

Value shown: We must not waste water and try to put every drop of it to optimum use.

EXAMPLE 19 *A vessel full of water is in the form of an inverted cone of height* 8 cm *and the radius of its top, which is open, is* 5 cm. 100 *spherical lead balls are dropped into the vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.* [CBSE 2015]

SOLUTION Height of the conical vessel, $h = 8$ cm.

Radius of the conical vessel, $r = 5$ cm.

Volume of the conical vessel

$$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi \times 5 \times 5 \times 8\right) \text{cm}^3 = \left(\frac{200\pi}{3}\right) \text{cm}^3.$$

Volume of water flown out on dropping spherical balls

$$= \left(\frac{1}{4} \text{ of } \frac{200\pi}{3}\right) \text{cm}^3 = \left(\frac{50\pi}{3}\right) \text{cm}^3.$$

Let the radius of each spherical ball be R.

Volume of each spherical ball $= \frac{4}{3}\pi R^3$.

Now, volume of 100 balls = volume of water flow out

$$\Rightarrow \quad 100 \times \frac{4}{3}\pi R^3 = \frac{50\pi}{3} \Rightarrow R^3 = \left(\frac{50}{3} \times \frac{3}{4} \times \frac{1}{100}\right) = \frac{1}{8}$$

$$\Rightarrow \quad R = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \text{ cm} = 0.5 \text{ cm}.$$

EXAMPLE 20 *A housing society used to collect rain water from the roof of its building* 22 m × 20 m *to a cylindrical vessel having diameter of base* 2 m *and height* 3.5 m *and then pump this water into the main water tank so that all members can use it. On a particular day the rain water collected from the roof just filled the cylindrical vessel. Then, find the rainfall in centimetre.* [CBSE 2013C]

SOLUTION Length of the roof = 22 m. Breadth of the roof = 20 m.

Let the rainfall be x cm.

Volume of water on the roof

$$= \left(22 \times 20 \times \frac{x}{100}\right) \text{m}^3 = \left(\frac{22x}{5}\right) \text{m}^3.$$

Radius of the cylindrical vessel, $r = 1$ m.

Height of the cylindrical vessel, $h = 3.5$ m.

Volume of water in the cylindrical vessel when just full

$$= \pi r^2 h = \left(\frac{22}{7} \times 1 \times 1 \times \frac{7}{2}\right) \text{m}^3 = 11 \text{ m}^3.$$

Now, volume of water on the roof

= volume of water in the cylindrical vessel

$$\Rightarrow \quad \frac{22x}{5} = 11 \Rightarrow x = \left(\frac{11 \times 5}{22}\right) = 2.5.$$

Hence, the rainfall is 2.5 cm.

EXAMPLE 21 *Water flows through a circular pipe whose internal diameter is* 2 cm, *at the rate of* 0.7 m *per second into a cylindrical tank, the radius of whose base is* 40 cm. *By how much will the level of water rise in the tank in half an hour?* [CBSE 2006C]

SOLUTION Internal radius of the pipe, $r = 1$ cm.

Length of water flowing in 1 second, $h = 0.7$ m = 70 cm.

Volume of water flowing in 1 second

$$= \pi r^2 h = (\pi \times 1 \times 1 \times 70) \text{ cm}^3 = (70\pi) \text{ cm}^3.$$

Volume of water flowing in 30 minutes

$$= (70\pi \times 60 \times 30) \text{ cm}^3 = (126000\pi) \text{ cm}^3.$$

Radius of cylindrical tank, $R = 40$ cm.

Let the rise in level of water be H cm.

Volume of water in the tank

$$= \pi R^2 H = (\pi \times 40 \times 40 \times H)\ \text{cm}^3$$
$$= (1600\pi H)\ \text{cm}^3.$$

Volume of water in the tank

= volume of water flown through the pipe

$$\Rightarrow \quad 1600\pi H = 126000\pi \ \Rightarrow H = \frac{126000}{1600} = \frac{315}{4} = 78.75.$$

Hence, rise in level = 78.75 cm.

EXAMPLE 22 *Water is flowing at the rate of* 15 km *per hour through a pipe of diameter* 14 cm *into a rectangular tank which is* 50 m *long and* 44 m *wide. Find the time in which the level of water in the tank will rise by* 21 cm. [CBSE 2006, '11]

SOLUTION Length of the tank = 50 m.

Width of the tank = 44 m.

Desired rise in level = 21 cm = $\frac{21}{100}$ m.

Desired volume of water in the tank

$$= \left(50 \times 44 \times \frac{21}{100}\right) \text{m}^3 = 462\ \text{m}^3.$$

Radius of the pipe, $r = 7$ cm $= \frac{7}{100}$ m.

Length of water flowing in 1 hour, h = 15 km = 15000 m.

Volume of water flowing from the pipe in 1 hour

$$= \pi r^2 h = \left(\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000\right) \text{m}^3 = 231\ \text{m}^3.$$

$$\therefore \text{ required time} = \frac{\text{desired volume}}{\text{volume of water flown in 1 hour}}$$
$$= \left(\frac{462}{231}\right) \text{hours} = 2 \text{ hours}.$$

EXAMPLE 23 *A farmer connects a pipe of internal diameter* 20 cm *from a canal into a cylindrical tank in his field, which is* 10 m *in diameter and* 2 m *deep. If water flows through the pipe at the rate of* 3 km/hr, *in how much time will the tank be filled?* [CBSE 2008C]

SOLUTION Radius of the pipe, r = 10 cm $= \left(\frac{10}{100}\right)$ m $= \frac{1}{10}$ m.

Length of water flowing through the pipe in 1 hour,

$h = 3 \text{ km} = 3000 \text{ m}$.

Volume of water that flows through the pipe in 1 hour

$$= \pi r^2 h = \left(\pi \times \frac{1}{10} \times \frac{1}{10} \times 3000\right) \text{m}^3 = (30\pi) \text{ m}^3.$$

Radius of the cylindrical tank, $R = 5$ m.

Depth of the tank, $H = 2$ m.

Volume of the tank $= \pi R^2 H = (\pi \times 5 \times 5 \times 2) \text{ m}^3$

$= (50\pi) \text{ m}^3$.

Time taken to fill the tank

$$= \frac{\text{volume of the tank}}{\text{volume of water flown in 1 hour}}$$

$$= \left(\frac{50\pi}{30\pi}\right) \text{hours} = \frac{5}{3} \text{hours} = 1 \text{ hr } 40 \text{ min.}$$

Hence, the required time is 1 hr 40 min.

EXAMPLE 24 *Water is flowing at the rate of* 2.52 km/hr *through a cylindrical pipe into a cylindrical tank, the radius of whose base is* 40 cm. *If the increase in the level of water in the tank, in half an hour is* 3.15 m, *find the internal diameter of the pipe.* [CBSE 2015]

SOLUTION Let the internal radius of the pipe be r.

Length of water flowing rhrough the pipe in 1 hour,

$h = 2.52 \text{ km} = 2520 \text{ m}$.

Volume of water flowing through the pipe in 1 hour

$= \pi r^2 h = (\pi \times r^2 \times 2520) \text{ m}^3$.

Volume of water flowing through the pipe in half an hour

$$= \left(\frac{1}{2} \times \pi r^2 \times 2520\right) \text{m}^3 = (1260\pi r^2) \text{ m}^3.$$

Radius of the cylindrical tank, $R = 40 \text{ cm} = \frac{40}{100} \text{ m} = \frac{2}{5} \text{ m}$.

Increase in level of water, $H = 3.15$ m.

Volume of water filled in the tank

$$= \pi R^2 H = \left(\pi \times \frac{2}{5} \times \frac{2}{5} \times \frac{315}{100}\right) \text{m}^3 = \left(\frac{63\pi}{125}\right) \text{m}^3.$$

Now, volume of water flown in half an hour

= volume of water filled in the tank

$$\Rightarrow \quad 1260\pi r^2 = \frac{63\pi}{125} \Rightarrow r^2 = \frac{63}{125 \times 1260} = \frac{1}{2500}$$

$$\Rightarrow \quad r = \sqrt{\frac{1}{2500}} = \frac{1}{50} \text{ m} = \left(\frac{1}{50} \times 100\right) \text{ cm} = 2 \text{ cm}.$$

Hence, the internal diameter of the pipe = (2 × 2) cm = 4 cm.

EXAMPLE 25 *A* 21-m-*deep well with diameter* 6 m *is dug and the earth from digging is evenly spread to form a platform* 27 m × 11 m. *Find the height of the platform.* [CBSE 2015]

SOLUTION Depth of the well, $h = 21$ m.

Radius of the well, $r = 3$ m.

Volume of the earth taken out

$$= \pi r^2 h = \left(\frac{22}{7} \times 3 \times 3 \times 21\right) \text{m}^3 = 594 \text{ m}^3.$$

Area of the platform = $(27 \times 11) \text{ m}^2 = 297 \text{ m}^2$.

Height of the platform

$$= \frac{\text{volume of the earth}}{\text{area of the platform}} = \left(\frac{594}{297}\right) \text{m} = 2 \text{m}.$$

EXAMPLE 26 *A well of diameter* 4 m *is dug* 14 m *deep. The earth taken out of it is spread evenly all around the well to form a* 40-cm-*high embankment. Find the width of the embankment.* [HOTS] [CBSE 2015]

SOLUTION Radius of the well, $r = 2$ m.

Depth of the well, $h = 14$ m.

Volume of the earth taken out

$$= \pi r^2 H = \left(\frac{22}{7} \times 2 \times 2 \times 14\right) \text{m}^3 = 176 \text{ m}^3.$$

Let the width of the embankment be x metres.

Internal radius of the embankment, $r = 2$ m.

External radius of the embankment, $R = (2 + x)$ m.

Height of the embankment, $H = 40 \text{ cm} = \left(\frac{40}{100}\right)$ m.

Volume of the embankment

$$= \pi H(R^2 - r^2) = \left\{\frac{40\pi}{100}[(2+x)^2 - 2^2]\right\} \text{m}^3$$

$$= \left[\frac{2\pi}{5}(x^2 + 4x)\right] \text{m}^3.$$

Now, volume of the embankment = volume of the earth dug out

$$\Rightarrow \frac{2}{5}\times\frac{22}{7}\times(x^2+4x)=176 \quad \Rightarrow \quad x^2+4x=\frac{176\times5\times7}{2\times22}=140$$

$$\Rightarrow x^2+4x-140=0 \Rightarrow x^2+14x-10x-140=0$$

$$\Rightarrow x(x+14)-10(x+14)=0 \Rightarrow (x+14)(x-10)=0$$

$$\Rightarrow x=10 \quad [\because x\neq -14].$$

Hence, width of the embankment = 10 m.

EXAMPLE 27 *From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is* 10 cm *and its base is of radius* 4.2 cm. *The rest of the cylinder is melted and converted into a cylindrical wire of* 1.4 cm *thickness. Find the length of the wire.* [HOTS] [CBSE 2015]

SOLUTION Height of the cylinder, h = 10 cm.

Radius of each of the cylinder and hemisphere, r = 4.2 cm.

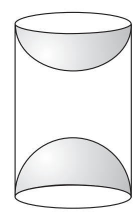
Volume of the solid

= volume of the cylinder – volume of 2 hemispheres

$$=\pi r^2h-2\times\frac{2}{3}\pi r^3=\pi r^2\left(h-\frac{4}{3}r\right)$$

$$=\left[\pi\times4.2\times4.2\times\left(10-\frac{4}{3}\times4.2\right)\right]\text{ cm}^3$$

$$=(\pi\times4.2\times4.2\times4.4)\text{ cm}^3.$$



Let the length of the wire be H.

Radius of the wire, $R=\left(\frac{1.4}{2}\right)$ cm = 0.7 cm = $\frac{7}{10}$ cm.

Volume of the wire = $\pi R^2H=\left(\pi\times\frac{7}{10}\times\frac{7}{10}\times H\right)\text{ cm}^3$.

Now, volume of the wire = volume of the solid

$$\Rightarrow \pi\times\frac{7}{10}\times\frac{7}{10}\times H=\pi\times4.2\times4.2\times4.4$$

$$\Rightarrow H=\left(\frac{4.2\times4.2\times4.4\times10\times10}{7\times7}\right)\text{ cm}=\left(\frac{42\times42\times4.4}{7\times7}\right)\text{ cm}$$

$$=158.4\text{ cm}.$$

EXAMPLE 28 50 *circular plates each of radius* 7 cm *and thickness* 5 mm *are placed one above another to form a solid right circular cylinder. Find the total surface area of the cylinder so formed.* [HOTS] [CBSE 2013C]

SOLUTION Clearly, we have

radius of the cylinder so formed, $r = 7$ cm, and

height of the cylinder so formed, $h = (50 \times 5)$ mm

$= 250$ mm $= 25$ cm.

$\therefore$ total surface area of the cylinder so formed

$$= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times 7 \times (25 + 7)\right] \text{cm}^2$$

$$= (2 \times 22 \times 32) \text{ cm}^2 = 1408 \text{ cm}^2.$$

EXAMPLE 29 *A copper wire,* 3 mm *in diameter, is wound about a cylinder whose length is* 12 cm *and diameter* 10 cm, *so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be* 8.88 g *per* cm^3. [*Use* $\pi = 3.14$.] [HOTS]

SOLUTION Diameter of the wire = 3 mm = 0.3 cm.

Length of the cylinder = 12 cm.

Radius of the cylinder = 5 cm.

$$\text{Number of turns} = \frac{\text{length of the cylinder}}{\text{diameter of the wire}} = \frac{12}{0.3} = 40.$$

Length of one turn = circumference of the base of the cylinder

$$= 2\pi r = (2 \times 3.14 \times 5) \text{ cm} = 31.4 \text{ cm}.$$

Length of 40 turns = (31.4×40) cm = 1256 cm.

Total length of the wire wrapped, $l = 1256$ cm.

Radius of the wire, $r = \left(\frac{0.3}{2}\right)$ cm $= 0.15$ cm.

Volume of the wire $= \pi r^2 l = (3.14 \times 0.15 \times 0.15 \times 1256) \text{ cm}^3$

$$= 88.7364 \text{ cm}^3 \approx 88.74 \text{ cm}^3.$$

$\therefore$ mass of the wire = volume $\times$ density = (88.74×8.88) g

$$= 788.01 \text{ g} \approx 788 \text{ g}.$$

EXAMPLE 30 *A cistern, internally measuring* $150 \text{ cm} \times 120 \text{ cm} \times 110 \text{ cm}$, *has* 129600 cm^3 *of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being* $22.5 \text{ cm} \times 7.5 \text{ cm} \times 6.5 \text{ cm}$?

SOLUTION Total volume of the cistern = $(150 \times 120 \times 110) \text{ cm}^3$

$$= 1980000 \text{ cm}^3.$$

Volume of water in the cistern = 129600 cm^3.

Volume of empty space in the cistern

$$= (1980000 - 129600) \text{ cm}^3 = 1850400 \text{ cm}^3.$$

Total volume of each brick = $(22.5 \times 7.5 \times 6.5)\ \text{cm}^3$
$= 1096.875\ \text{cm}^3$.

Since each brick absorbs water equal to $\frac{1}{17}$ of its own volume, so the effective volume of each brick may be taken as $\left(1-\frac{1}{17}\right)$, i.e., $\frac{16}{17}$ of its actual volume.

So, the effective volume of each brick $=\left(\frac{16}{17}\times 1096.875\right)\ \text{cm}^3$.

$\therefore$ required number of bricks

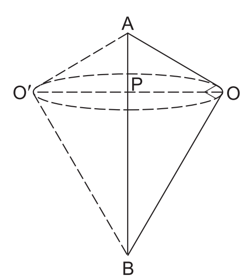
$$= \frac{\text{volume of empty space in the cistern}}{\text{effective volume of each brick}}$$

$$= \frac{1850400}{\left(\frac{16}{17}\times 1096.875\right)} = \frac{17\times 1850400}{16\times 1096.875}$$

$$= \frac{17\times 1850400}{17550} = 1792.41 \approx 1792.$$

EXAMPLE 31 *A right triangle whose sides are* 3 cm *and* 4 cm *(other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose the value of* π *as found appropriate).* [HOTS]

SOLUTION Let ΔAOB be the given right triangle in which $\angle AOB = 90°$. When ΔAOB is rotated about the hypotenuse AB, the two cones generated are $AO'O$ and BOO'.



Clearly, $OA = 3$ cm, $OB = 4$ cm.

In right-angled ΔAOB, we have

$$AB = \sqrt{(OA)^2+(OB)^2} = \sqrt{3^2+4^2}\ \text{cm}$$

$$= \sqrt{25}\ \text{cm} = 5\ \text{cm}.$$

Let $OP = x$ cm. Then,

$$\frac{1}{2}\times OA\times OB = \frac{1}{2}\times AB\times OP$$

$$\Rightarrow \frac{1}{2}\times 3\times 4 = \frac{1}{2}\times 5\times OP \Rightarrow OP = \left(\frac{3\times 4}{5}\right)\ \text{cm} = 2.4\ \text{cm}.$$

In right-angled ΔAPO, we have

$$AP = \sqrt{(OA)^2-(OP)^2} = \sqrt{3^2-(2.4)^2}\ \text{cm}$$

$$= \sqrt{9-5.76}\ \text{cm} = \sqrt{3.24}\ \text{cm} = 1.8\ \text{cm}.$$

$BP = AB - AP = (5 - 1.8)$ cm $= 3.2$ cm.

$\therefore$ radius of each of the cones $AO'O$ and BOO', $r = OP = 2.4$ cm.

Height of the cone $AO'O$, $h = AP = 1.8$ cm.

Height of the cone BOO', $H = BP = 3.2$ cm.

Volume of the double cone

$= $ volume of the the cone $AO'O$ + volume of the cone BOO'

$$= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi r^2 (h + H)$$

$$= \left(\frac{1}{3} \times 3.14 \times 2.4 \times 2.4 \times 5\right) \text{cm}^3 = 30.144 \text{ cm}^3.$$

Slant height of the cone $AO'O$, $l_1 = OA = 3$ cm.

Slant height of the cone $BO'O$, $l_2 = OB = 4$ cm.

Surface area of the double cone

$= $ curved surface area of the cone $AO'O$

+ curved surface area of the cone BOO'

$$= \pi r l_1 + \pi r l_2 = \pi r(l_1 + l_2) = \left[\frac{22}{7} \times 2.4 \times (3 + 4)\right] \text{cm}^2$$

$$= \left(\frac{22}{7} \times 2.4 \times 7\right) \text{cm}^2 = 52.8 \text{ cm}^2.$$

EXERCISE 17B

1. A solid metallic cuboid of dimensions 9 m × 8 m × 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed. [CBSE 2017]

2. A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere. [CBSE 2011]

3. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere. [CBSE 2012]

4. A solid metal cone with radius of base 12 cm and height 24 cm is melted to form solid spherical balls of diameter 6 cm each. Find the number of balls thus formed. [CBSE 2005C]

5. The radii of internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respctively. It is melted and recast into a solid cylinder of diameter 14 cm. Find the height of the cylinder. [CBSE 2012]

6. The internal and external diameters of a hollow hemispherical shell are 6 cm and 10 cm respectively. It is melted and recast into a solid cone of base diameter 14 cm. Find the height of the cone so formed. [CBSE 2005C]

7. A copper rod of diameter 2 cm and length 10 cm is drawn into a wire of uniform thickness and length 10 m. Find the thickness of the wire. [CBSE 2012]

8. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical-shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl. [CBSE 2004, '11]

9. A solid metallic sphere of diameter 21 cm is melted and recast into a number of smaller cones, each of diameter 3.5 cm and height 3 cm. Find the number of cones so formed. [CBSE 2004]

10. A spherical cannon ball 28 cm in diameter is melted and recast into a right circular conical mould, base of which is 35 cm in diameter. Find the height of the cone. [CBSE 2001C]

11. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5 cm and 2 cm. Find the radius of the third ball.

12. A spherical shell of lead whose external and internal diameters are respectively 24 cm and 18 cm, is melted and recast into a right circular cylinder 37 cm high. Find the diameter of the base of the cylinder.

13. A hemisphere of lead of radius 9 cm is cast into a right circular cone of height 72 cm. Find the radius of the base of the cone.

14. A spherical ball of diameter 21 cm is melted and recast into cubes, each of side 1 cm. Find the number of cubes so formed.

15. How many lead balls, each of radius 1 cm, can be made from a sphere of radius 8 cm?

16. A solid sphere of radius 3 cm is melted and then cast into small spherical balls, each of diameter 0.6 cm. Find the number of small balls so obtained.

17. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of diameter 2.8 cm. Find the length of the wire.

18. The diameter of a copper sphere is 18 cm. It is melted and drawn into a long wire of uniform cross section. If the length of the wire is 108 m, find its diameter.

19. A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied into a cylindrical vessel of internal radius 6 cm. Find the height of water in the cylindrical vessel. [CBSE 2012]

20. A hemispherical tank, full of water, is emptied by a pipe at the rate of $25/7$ litres per second. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m? [CBSE 2012]

21. The rain water from a roof of 44 m x 20 m drains into a cylindrical tank having diameter of base 4 m and height 3.5 m. If the tank is just full, find the rainfall in cm. [CBSE 2014]

22. The rain water from a 22 m $\times$ 20 m roof drains into a cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4}{5}$th of the cylindrical vessel then find the rainfall in centimetre. [CBSE 2015, '17]

23. A solid right circular cone of height 60 cm and radius 30 cm is dropped in a right circular cylinder full of water, of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder, in cubic metres. [CBSE 2015]

24. Water is flowing through a cylindrical pipe of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m per second. Determine the rise in level of water in the tank in half an hour. [CBSE 2013]

25. Water is flowing at the rate of 6 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 60 m long and 22 m wide. Determine the time in which the level of water in the tank will rise by 7 cm. [CBSE 2011]

26. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hr. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation? [CBSE 2017]

27. A farmer connects a pipe of internal diameter 25 cm from a canal into a cylindrical tank in his field, which is 12 m in diameter and 2.5 m deep. If water flows through the pipe at the rate of 3.6 km/hr, in how much time will the tank be filled? Also, find the cost of water if the canal department charges at the rate of ₹ 0.07 per m^3. [CBSE 2009C]

28. Water running in a cylindrical pipe of inner diameter 7 cm, is collected in a container at the rate of 192.5 litres per minute. Find the rate of flow of water in the pipe in km/hr. [CBSE 2013]

29. 150 spherical marbles, each of diameter 14 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel. [CBSE 2014]

30. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm, containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

31. In a village, a well with 10 m inside diameter, is dug 14 m deep. Earth taken out of it is spread all around to a width of 5 m to form an embankment. Find the height of the embankment. What value of the villagers is reflected here? [CBSE 2014]

32. In a corner of a rectangular field with dimensions 35 m × 22 m, a well with 14 m inside diameter is dug 8 m deep. The earth dug out is spread evenly over the remaining part of the field. Find the rise in the level of the field. [HOTS]

33. A copper wire of diameter 6 mm is evenly wrapped on a cylinder of length 18 cm and diameter 49 cm to cover its whole surface. Find the length and the volume of the wire. If the density of copper be 8.8 g per cu-cm, find the weight of the wire. [HOTS]

34. A right triangle whose sides are 15 cm and 20 cm (other than hypotenuse), is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate) [HOTS]

35. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If the tank is filled completely then what will be the height of standing water used for irrigating the park? Write your views on recycling of water. [CBSE 2017]

ANSWERS (EXERCISE 17B)

1. 18 **2.** 10 cm **3.** 12 cm **4.** 32 **5.** $2\frac{2}{3}$ cm

6. 4 cm **7.** 2 mm **8.** 60 **9.** 504 **10.** 35.84 cm

11. 2.5 cm **12.** 12 cm **13.** 4.5 cm **14.** 4851 **15.** 512

16. 1000 **17.** 63 m **18.** 0.6 cm **19.** 13.5 cm

20. 16 minutes 30 seconds **21.** 5 cm **22.** 2 cm **23.** 1.98 m^3

24. 180 cm **25.** 1 hour **26.** 1620000 m^2 **27.** 1 hr 36 min, ₹ 19.80

28. 3 km/hr **29.** 56 m **30.** 150

31. 2.8 m;
Value: We must labour hard to make maximum use of the available resources.

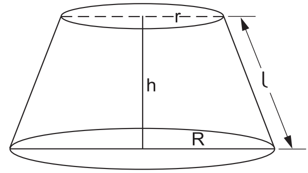
32. 2 m **33.** 46.2 m, 1306.8 cm^3, 11.5 kg **34.** 3768 cm^3, 1320 cm^2

VOLUME AND SURFACE AREA OF A FRUSTUM OF A CONE

FRUSTUM OF A CONE

When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.

FORMULAE Let R and r be the radii of the base and the top of the frustum of a cone.



Let h be its height and l be its slant height.

Then,

(i) *Volume of the frustum of the cone*

$$= \frac{\pi h}{3}[R^2 + r^2 + Rr] \text{ cubic units.}$$

(ii) *Lateral surface area of the frustum of the cone*

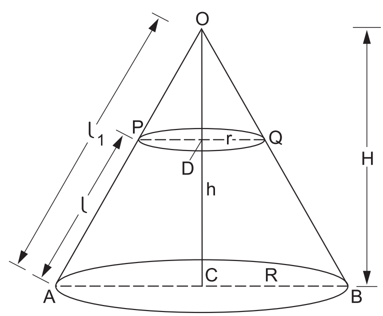
$$= \pi l(R + r), \text{ where } l^2 = h^2 + (R - r)^2 \text{ sq units.}$$

(iii) *Total surface area of the frustum of the cone*

= (*area of the base*) + (*area of the top*) + (*lateral surface area*)

$$= [\pi R^2 + \pi r^2 + \pi l(R + r)] = \pi[R^2 + r^2 + l(R + r)] \text{ sq units.}$$

DERIVATION OF THE ABOVE FORMULAE



Let H, R and l_1 be the height, radius of the base and slant height respectively of a cone OAB. Suppose it is cut by a plane PQ parallel to its base at a height h from the base to form the frustum $ABQP$.

Let the radius of the base of cone OPQ be r and l be the slant height of the frustum so formed.

Clearly, we have $OC = H$, $DC = h$, $OD = OC - DC = (H - h)$, $DQ = r$,

$CB = R$, $OB = l_1$, $BQ = l$, $OQ = OB - BQ = (l_1 - l)$.

Clearly, $\triangle ODQ \sim \triangle OCB$.

$$\therefore \quad \frac{OD}{OC} = \frac{DQ}{CB} = \frac{OQ}{OB} \Rightarrow \frac{H-h}{H} = \frac{r}{R} = \frac{l_1 - l}{l_1}. \qquad \text{... (i)}$$

$$\frac{H-h}{H} = \frac{r}{R} \Rightarrow R(H - h) = Hr \Rightarrow HR - hR = Hr$$

$$\Rightarrow \quad H(R - r) = hR \Rightarrow H = \frac{hR}{R - r}. \qquad \text{... (ii)}$$

And, $\dfrac{l_1 - l}{l_1} = \dfrac{r}{R} \Rightarrow (l_1 - l)R = l_1 r \Rightarrow l_1 R - lR = l_1 r$

$$\Rightarrow \quad l_1(R-r) = lR \Rightarrow l_1 = \frac{lR}{R-r}. \qquad \text{... (iii)}$$

Now, volume of the frustum $ABQP$

= volume of the cone OAB – volume of the cone OPQ

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 (H-h) = \frac{1}{3}\pi\left[R^2\left(\frac{hR}{R-r}\right) - r^2\left\{\frac{hR}{R-r} - h\right\}\right]$$

$$= \frac{1}{3}\pi\left[\left(\frac{hR^3}{R-r}\right) - r^2\left\{\frac{hR - hR + hr}{R-r}\right\}\right] = \frac{1}{3}\pi\left[\frac{hR^3}{R-r} - \frac{hr^3}{R-r}\right]$$

$$= \frac{1}{3}\pi\left[\frac{h(R^3-r^3)}{(R-r)}\right] = \frac{1}{3}\pi\left[\frac{h(R-r)(R^2+Rr+r^2)}{(R-r)}\right] = \frac{1}{3}\pi h(R^2+Rr+r^2).$$

And, curved surface area of the frustum $ABQP$

= curved surface area of the cone OAB

– curved surface area of the cone OPQ

$$= \pi R l_1 - \pi r(l_1 - l) = \pi\left[R\left(\frac{lR}{R-r}\right) - r\left\{\frac{lR}{R-r} - l\right\}\right]$$

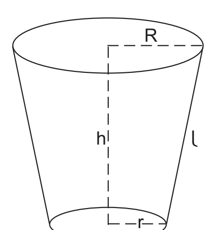
$$= \pi\left[\frac{lR^2}{R-r} - r\left\{\frac{lR - lR + lr}{R-r}\right\}\right] = \pi\left[\frac{lR^2}{R-r} - \frac{lr^2}{R-r}\right]$$

$$= \pi\left[\frac{l(R^2-r^2)}{(R-r)}\right] = \pi\left[\frac{l(R-r)(R+r)}{(R-r)}\right] = \pi(R+r)l.$$

SOLVED EXAMPLES

EXAMPLE 1 *An open metal bucket is in the shape of a frustum of a cone of height* 21 cm *with radii of its lower and upper ends as* 10 cm *and* 20 cm *respectively. Find the cost of milk which can completely fill the bucket at* ₹ 30 *per litre.* [CBSE 2012]

SOLUTION Let R and r be the radii of the top and the base of the bucket respectively, and let h be its height.



Then, $R = 20$ cm, $r = 10$ cm and $h = 21$ cm.

Capacity of the bucket

= volume of frustum of the cone

$$= \frac{1}{3}\pi h[R^2 + r^2 + Rr]$$

$$= \left[\left(\frac{1}{3} \times \frac{22}{7} \times 21\right) \cdot \{(20)^2 + (10)^2 + 20 \times 10\}\right] \text{cm}^3$$

$$= (22 \times 700) \text{ cm}^3 = 15400 \text{ cm}^3.$$

Volume of milk that the bucket can hold

$$= \left(\frac{15400}{1000}\right) \text{ litres} = 15.4 \text{ litres.}$$

$\therefore$ cost of milk = ₹ (15.4×30) = ₹ 462.

EXAMPLE 2 *A bucket open at the top, and made up of a metal sheet is in the form of the frustum of a cone. The depth of the bucket is* 24 cm *and the diameters of its upper and lower circular ends are* 30 cm *and* 10 cm *respectively. Find the cost of metal sheet used in it at the rate of* ₹ 10 *per* 100 m^2. [*Use* π = 3.14.] [CBSE 2013]

SOLUTION Radius of the upper end, $R = 15$ cm.

Radius of the lower end, $r = 5$ cm.

Depth of the bucket, $h = 24$ cm.

Slant height of the bucket, $l = \sqrt{h^2 + (R-r)^2}$

$$= \sqrt{(24)^2 + (15-5)^2} \text{ cm}$$

$$= \sqrt{676} \text{ cm} = 26 \text{ cm.}$$

Area of metal sheet used

$$= \text{curved surface area + area of the bottom}$$

$$= \pi l(R+r) + \pi r^2$$

$$= [3.14 \times 26 \times (15+5) + 3.14 \times 5 \times 5] \text{ cm}^2$$

$$= (3.14 \times 26 \times 20 + 3.14 \times 25) \text{ cm}^2$$

$$= [3.14 \times (520 + 25)] \text{ cm}^2 = (3.14 \times 545) \text{ cm}^2 = 1711.3 \text{ cm}^2.$$

$\therefore$ cost of metal sheet used = ₹ $\left(\frac{1711.3}{100} \times 10\right)$ = ₹ 171.13.

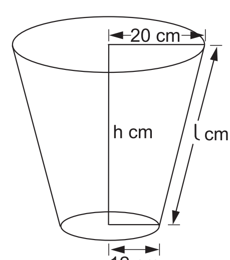
EXAMPLE 3 *A bucket is in the form of a frustum of a cone with a capacity of* 12308.8 cm^3 *of water. The radii of the top and bottom circular ends are* 20 cm *and* 12 cm *respectively. Find the height of the bucket and the area of the metal sheet used in its making.* [*Use* π = 3.14.]

[CBSE 2006C]

SOLUTION Here, $R = 20$ cm, $r = 12$ cm

and volume = 12308.8 cm^3

Let the height of the bucket be h cm.



Volume of the bucket

= volume of frustum of the cone.

$$\therefore \quad \frac{1}{3}\pi h(R^2 + r^2 + Rr) = 12308.8$$

$$\Rightarrow \quad \frac{1}{3} \times 3.14 \times h[(20)^2 + (12)^2 + 20 \times 12] = 12308.8$$

$$\Rightarrow \quad 784h = \frac{12308.8 \times 3}{3.14}$$

$\Rightarrow\ h = \left(\dfrac{12308.8 \times 3}{3.14 \times 784}\right) = 15.$

Slant height of the bucket

$l = \sqrt{h^2 + (R - r)^2}$ units

$= \sqrt{(15)^2 + (20 - 12)^2}$ cm $= \sqrt{(15)^2 + 8^2}$ cm

$= \sqrt{225 + 64}$ cm $= \sqrt{289}$ cm $= 17$ cm.

Area of the metal sheet used

= (curved surface area) + (area of the bottom)

$= [\pi l(R + r) + \pi r^2]$

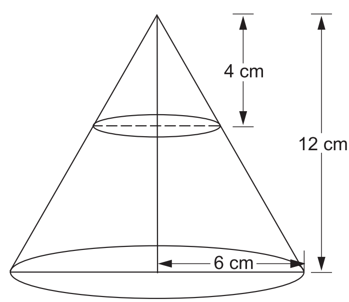
$= [3.14 \times 17 \times (20 + 12) + 3.14 \times 12 \times 12]\ \text{cm}^2$

$= [3.14 \times (17 \times 32) + 3.14 \times 144]\ \text{cm}^2$

$= [3.14 \times (544 + 144)]\ \text{cm}^2$

$= (3.14 \times 688)\ \text{cm}^2 = 2160.32\ \text{cm}^2.$

EXAMPLE 4 *In the figure, from the top of a solid cone of height* 12 cm *and base radius* 6 cm, *a cone of height* 4 cm *is removed by a plane parallel to the base. Find the total surface area of the remaining solid.* [*Use* $\pi = 22/7$ *and* $\sqrt{5} = 2.236$.] [CBSE 2015]



SOLUTION Clearly, the remaining solid is the frustum *CDBA* of cone *OPD*.

Radius of lower end of the frustum, $R = CD = 6$ cm.

Height of the frustum,

$h = AC = OC - OA$

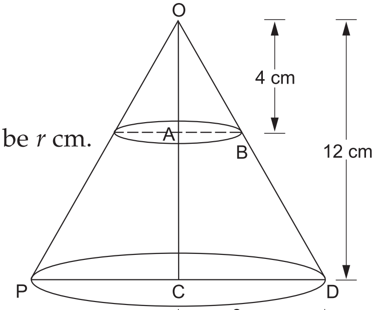
$= (12 - 4)$ cm $= 8$ cm.

Let the radius of upper face be r cm.

Now, $\Delta OAB \sim \Delta OCD$.

So, $\dfrac{OA}{OC} = \dfrac{AB}{CD} \Rightarrow \dfrac{4}{12} = \dfrac{r}{6}$

$\Rightarrow r = \left(\dfrac{4}{12} \times 6\right)$ cm = 2 cm.



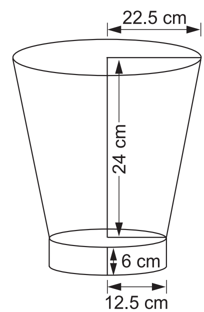
Slant height of the frustum,

$$l = \sqrt{h^2 + (R-r)^2} = \sqrt{8^2 + (6-2)^2} = \sqrt{80} \text{ cm} = 4\sqrt{5} \text{ cm}.$$

Total surface area of the frustum

$$= \text{area of base + area of top + curved surface area}$$
$$= \pi R^2 + \pi r^2 + \pi l(R+r) = \pi[R^2 + r^2 + l(R+r)]$$
$$= \frac{22}{7} \times [6^2 + 2^2 + 4 \times 2.236 \times (6+2)] \text{ cm}^2$$
$$= \left[\frac{22}{7} \times (36 + 4 + 71.552)\right] \text{cm}^2 = \left(\frac{22}{7} \times 11.552\right) \text{cm}^2$$
$$= (22 \times 15.936) \text{ cm}^2 = 350.592 \text{ cm}^2.$$

EXAMPLE 5 *An open metallic bucket is in the shape of a frustum of a cone mounted on hollow cylindrical base made of metallic sheet. If the diameters of the two circular ends of the bucket are* 45 cm *and* 25 cm, *the total vertical height of the bucket is* 30 cm *and that of the cylindrical portion is* 6 cm, *find the area of the metallic sheet used to make the bucket. Also, find the volume of water it can hold.* [*Take* $\pi = 22/7$.]



SOLUTION Here $R = \frac{45}{2}$ cm, $r = \frac{25}{2}$ cm $= 12.5$ cm.

Height of the frustum of the cone, $h = (30 - 6)$ cm $= 24$ cm.

$\therefore \quad h = 24$ cm.

Slant height of the frustum of the cone,

$$l = \sqrt{h^2 + (R-r)^2} \text{ units} = \sqrt{(24)^2 + (22.5 - 12.5)^2} \text{ cm}$$
$$= \sqrt{(24)^2 + (10)^2} \text{ cm} = \sqrt{576 + 100} \text{ cm}$$
$$= \sqrt{676} \text{ cm} = 26 \text{ cm}.$$

Area of metallic sheet used

= (curved surface area of the frustum of the cone)
+ (area of the base) + (curved surface area of the cylinder)

$$= \pi l(R+r) + \pi r^2 + 2\pi rH, \text{ where } H = 6 \text{ cm}$$
$$= \pi \cdot \{l(R+r) + r^2 + 2rH\} \text{ sq units}$$
$$= \frac{22}{7} \cdot \{26(22.5 + 12.5) + (12.5)^2 + 2 \times 12.5 \times 6\} \text{ cm}^2$$
$$= \frac{22}{7} \cdot \{(26 \times 35) + (12.5 \times 12.5) + 150\} \text{ cm}^2$$

$$= \frac{22}{7} \cdot \{910 + 156.25 + 150\} \text{ cm}^2 = \left(\frac{22}{7} \times 1216.25 \right) \text{cm}^2$$

$$= (22 \times 173.75) \text{ cm}^2 = 3822.5 \text{ cm}^2.$$

Volume of water which the bucket can hold

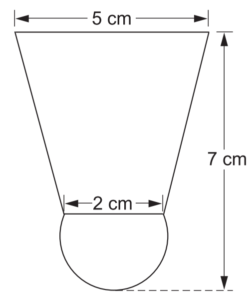
$$= \frac{1}{3} \pi h [R^2 + r^2 + Rr] \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left[\left(\frac{45}{2} \right)^2 + \left(\frac{25}{2} \right)^2 + \left(\frac{45}{2} \times \frac{25}{2} \right) \right] \text{cm}^3$$

$$= \frac{176}{7} \times \left(\frac{2025}{4} + \frac{625}{4} + \frac{1125}{4} \right) \text{cm}^3 = \left(\frac{176}{7} \times \frac{3775}{4} \right) \text{cm}^3$$

$$= \frac{166100}{7} \text{ cm}^3 = 23728.57 \text{ cm}^3 = 23.73 \text{ litres}.$$

EXAMPLE 6 *A shuttlecock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are* 5 cm *and* 2 cm, *and the height of the entire shuttlecock is* 7 cm. *Find its external surface area.*



SOLUTION Here, $R = \frac{5}{2}$ cm, $r = 1$ cm and $h = (7 - 1)$ cm $= 6$ cm.

Let l be the slant height of the frustum of the cone.

$$\therefore \quad l^2 = h^2 + (R - r)^2 = \left\{ 6^2 + \left(\frac{5}{2} - 1 \right)^2 \right\} \text{cm}^2$$

$$= \left(36 + \frac{9}{4} \right) \text{cm}^2 = \frac{153}{4} \text{cm}^2$$

$$\Rightarrow \quad l = \frac{\sqrt{153}}{2} \text{ cm} = \frac{12.36}{2} \text{ cm} = 6.18 \text{ cm}.$$

External surface area of the shuttlecock

= (its lateral surface area + area of the base)

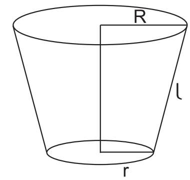
$$= [\pi l (R + r) + 2\pi r^2] \text{ sq units}$$

$$= \left\{ \frac{22}{7} \times 6.18 \times \left(\frac{5}{2} + 1 \right) + 2 \times \frac{22}{7} \times 1 \times 1 \right\} \text{cm}^2$$

$$= \left(11 \times 6.18 + \frac{44}{7} \right) \text{cm}^2 = (67.98 + 6.28) \text{ cm}^2 = 74.26 \text{ cm}^2.$$

EXAMPLE 7 *The slant height of the frustum of a cone is* 4 cm *and the perimeters of its circular ends are* 18 cm *and* 6 cm. *Find the curved surface area of the frustum.* [CBSE 2017]

SOLUTION Let the radii of its circular ends be R cm and r cm, and its slant height be l cm. Then,



$$2\pi R = 18 \Rightarrow \pi R = 9.$$

$$2\pi r = 6 \Rightarrow \pi r = 3.$$

Also, $l = 4$ cm (given).

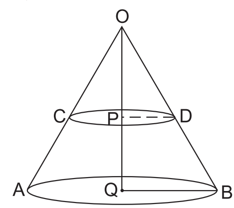
Curved surface area of the frustum

$$= \pi l(R + r) \text{ sq units}$$

$$= l \times (\pi R + \pi r) = 4 \times (9 + 3) \text{ cm}^2 = 48 \text{ cm}^2.$$

EXAMPLE 8 *The height of a cone is* 10 cm. *The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.* [CBSE 2017]

SOLUTION Let OAB be the cone and OQ be its axis, and let P be the midpoint of OQ.



Let $OQ = h$ cm.

Then, $OP = PQ = \frac{h}{2}$ cm and $QB = 10$ cm.

Also, $\triangle OPD \sim \triangle OQB$.

$$\therefore \quad \frac{OP}{OQ} = \frac{PD}{QB} \Rightarrow \frac{(h/2)}{h} = \frac{PD}{10} \Rightarrow PD = 5 \text{ cm}.$$

The plane CD divides the cone into two parts, namely

(i) a smaller cone of radius = 5 cm and height $(h/2)$ cm,

(ii) frustum of a cone in which

$$R = 10 \text{ cm}, r = 5 \text{ cm and height} = \left(\frac{h}{2}\right) \text{cm}.$$

Volume of the smaller cone

$$= \left(\frac{1}{3} \times \pi \times 5 \times 5 \times \frac{h}{2}\right) \text{cm}^3 = \left(\frac{25\pi h}{6}\right) \text{cm}^3.$$

Volume of the frustum of the cone

$$= \frac{1}{3}\pi \times \frac{h}{2} \cdot [(10)^2 + (5)^2 + 10 \times 5] \text{ cm}^3 = \left(\frac{175\pi h}{6}\right) \text{cm}^3.$$

$$\text{Ratio of the required volumes} = \frac{25\pi h}{6} : \frac{175\pi h}{6} = 25 : 175 = 1 : 7.$$

EXAMPLE 9 *The height of a cone is* 30 cm. *A small cone is cut off at the top by a plane parallel to the base. If its volume be* 1/27 *of the volume of the given cone, at what height above the base is the section made?*

[HOTS] [CBSE 2005C]

SOLUTION Height of the given cone = 30 cm.

Let the radius of its base be R cm.

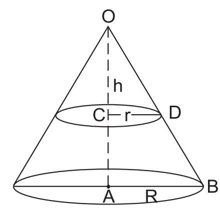
Volume of the given cone = $\left(\frac{1}{3}\pi R^2 \times 30\right)$ cm^3 = $(10\pi R^2)$ cm^3.

Let the radius and height of the smaller cone be r cm and h cm respectively.

Then, volume of the smaller cone = $\left(\frac{1}{3}\pi r^2 h\right)$ cm^3.

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{1}{27}\cdot(10\pi R^2) \quad \text{[given]}$$

$$\Rightarrow \quad \left(\frac{R}{r}\right)^2 = \frac{9h}{10}. \quad \text{... (i)}$$



Now, $\triangle OAB \sim \triangle OCD$.

$$\therefore \quad \frac{AB}{CD} = \frac{OA}{OC} \Rightarrow \frac{R}{r} = \frac{30}{h}. \quad \text{... (ii)}$$

From (i) and (ii), we get

$$\left(\frac{30}{h}\right)^2 = \frac{9h}{10} \Rightarrow \frac{30 \times 30}{h \times h} = \frac{9h}{10}$$

$$\Rightarrow \quad h^3 = \frac{30 \times 30 \times 10}{9} = 1000 \Rightarrow h^3 = (10)^3 \Rightarrow h = 10.$$

$\therefore$ height of the smaller cone = 10 cm.

Height of the section from the base = $(30 - 10)$ cm = 20 cm.

EXAMPLE 10 *A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9 of the curved surface of the whole cone, find the ratio of the line segments into which the altitude of the cone is divided by the plane.*

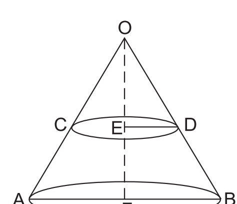
[CBSE 2004, '04C]

SOLUTION Let OAB be the given hollow cone cut by the plane CD parallel to base AB and let cone OCD be removed. Then, the remainder is the frustum $CABD$ of the given cone.

Let $OE = h$ units, $OF = H$ units,

$OD = l$ units, $OB = L$ units,

$ED = r$ units, $FB = R$ units.



In $\triangle OED$ and $\triangle OFB$, we have

$\angle EOD = \angle FOB$ (common),

$\angle OED = \angle OFB = 90°$.

$\therefore\quad \triangle OED \sim \triangle OFB$

$\Rightarrow\quad \dfrac{OE}{OF} = \dfrac{OD}{OB} = \dfrac{ED}{FB} \Rightarrow \dfrac{h}{H} = \dfrac{l}{L} = \dfrac{r}{R}\cdot$ ... (i) [by Thales' theorem]

Now, (curved surface area of the frustum $CABD$)

$= \dfrac{8}{9}$ (curved surface area of the cone OAB)

$\Rightarrow$ (curved surface area of the cone OCD)

= (curved surface of the cone OAB)
– (curved surface of the frustum $CABD$)

= (curved surface of the cone OAB)
$-\dfrac{8}{9}$ (curved surface of the cone OAB)

$= \dfrac{1}{9}$ (curved surface of the cone OAB)

$\Rightarrow\quad \pi rl = \dfrac{1}{9}\pi RL$

$\Rightarrow\quad \left(\dfrac{r}{R}\right)\left(\dfrac{l}{L}\right) = \dfrac{1}{9} \quad \Rightarrow \quad \left(\dfrac{h}{H} \times \dfrac{h}{H}\right) = \dfrac{1}{9} \Rightarrow \dfrac{h}{H} = \dfrac{1}{3}$ [using (i)]

$\Rightarrow\quad H = 3h.$ ... (ii)

Now, $EF = (OF - OE) = (H - h) = (3h - h) = 2h.$

$\therefore\quad \dfrac{OE}{EF} = \dfrac{h}{2h} = \dfrac{1}{2}\cdot$

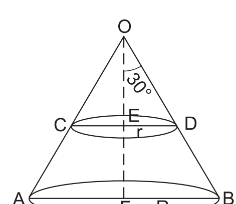
Hence, $OE : EF = 1 : 2.$

EXAMPLE 11 *A metallic right circular cone is* 20 cm *high and has a vertical angle of* 60°. *This is cut into two parts at the middle of its height by a plane parallel to the base. If the frustum so obtained is drawn into a wire of diameter* $\frac{1}{16}$ cm, *find the length of the wire.* [CBSE 2017]

SOLUTION Let OAB be the cone in which $\angle AOB = 60°$.

Clearly $\angle DOE = 30°$, $OE = 10$ cm, $OF = 20$ cm.

Let $ED = r$ cm and $FB = R$ cm.



$\therefore\quad \dfrac{ED}{OE} = \tan 30° \Rightarrow \dfrac{ED}{10} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow\quad ED = \left(10 \times \dfrac{1}{\sqrt{3}}\right)$ cm $\Rightarrow r = \dfrac{10}{\sqrt{3}}$ cm

and, $\dfrac{FB}{OF} = \tan 30° \Rightarrow \dfrac{FB}{20} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad FB = \left(20 \times \frac{1}{\sqrt{3}}\right) \text{cm} \Rightarrow R = \frac{20}{\sqrt{3}} \text{cm}.$

Also, $EF = 10$ cm.

Thus, $ABCD$ is the frustum of a cone in which

$$R = \frac{20}{\sqrt{3}} \text{ cm}, r = \frac{10}{\sqrt{3}} \text{ cm and } h = 10 \text{ cm}.$$

$$\text{Volume of this frustum} = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \pi \times 10 \left\{\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right\} \text{cm}^3$$

$$= \left(\frac{\pi \times 10}{3} \times \frac{700}{3}\right) \text{cm}^3 = \left(\frac{7000\pi}{9}\right) \text{cm}^3.$$

Let the length of the wire be l.

Radius of the wire, $r_1 = \frac{1}{32}$ cm.

Volume of the wire $= \pi r_1^2 l = \pi \times \left(\frac{1}{32}\right)^2 \times l.$

$$\therefore \quad \frac{7000\pi}{9} = \frac{\pi l}{32 \times 32} \Rightarrow l = \left(\frac{7000 \times 32 \times 32}{9}\right) \text{cm}$$

$$\Rightarrow \quad l = \left(\frac{7000 \times 32 \times 32}{9 \times 100}\right) \text{m} = \left(\frac{71680}{9}\right) \text{m}$$

$$= 7964.44 \text{ m}.$$

EXERCISE 17C

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 16 cm and 12 cm. Find the capacity of the glass. [CBSE 2012]
2. The radii of the circular ends of a solid frustum of a cone are 18 cm and 12 cm and its height is 8 cm. Find its total surface area. [Use $\pi = 3.14$.] [CBSE 2011]
3. A metallic bucket, open at the top, of height 24 cm is in the form of the frustum of a cone, the radii of whose lower and upper circular ends are 7 cm and 14 cm respectively. Find
   (i) the volume of water which can completely fill the bucket;
   (ii) the area of the metal sheet used to make the bucket. [CBSE 2014]

4. A container, open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹ 21 per litre. [CBSE 2014]
5. A container made of a metal sheet open at the top is of the form of frustum of cone, whose height is 16 cm and the radii of its lower and upper circular edges are 8 cm and 20 cm respectively. Find
   (i) the cost of metal sheet used to make the container if it costs ₹ 10 per 100 cm^2 [CBSE 2013]
   (ii) the cost of milk at the rate of ₹ 35 per litre which can fill it completely. [CBSE 2017]
6. The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm, and its slant height is 10 cm. Find its capacity and total surface area. [Take $\pi = 22/7$.] [CBSE 2005]
7. A bucket is in the form of a frustum of a cone. Its depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many litres of water the bucket can hold. [Take $\pi = 22/7$.] [CBSE 2005C]
8. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket if the cost of metal sheet used is ₹ 15 per 100 cm^2. [Use $\pi = 3.14$.] [CBSE 2006, '08, '08C]
9. A bucket made up of a metal sheet is in the form of frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹ 20 per litre and the cost of metal sheet used if it costs ₹ 10 per 100 cm^2. [Use $\pi = 3.14$.] [CBSE 2006, '09C]
10. A container in the shape of a frustum of a cone having diameters of its two circular faces as 35 cm and 30 cm and vertical height 14 cm, is completely filled with oil. If each cm^3 of oil has mass 1.2 g then find the cost of oil in the container if it costs ₹ 40 per kg. [CBSE 2009C]
11. A bucket is in the form of a frustum of a cone and it can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket. [CBSE 2012]
12. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm ($r < 14$). If the volume of bucket is 5390 cm^3, find the value of r. [CBSE 2011]
13. The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area. [Use $\pi = 3.14$.] [CBSE 2012]

14. A tent is made in the form of a frustum of a cone surmounted by another cone. The diameters of the base and the top of the frustum are 20 m and 6 m respectively, and the height is 24 m. If the height of the tent is 28 m and the radius of the conical part is equal to the radius of the top of the frustum, find the quantity of canvas required. [Take $\pi = 22/7$.]

15. A tent consists of a frustum of a cone, surmounted by a cone. If the diameters of the upper and lower circular ends of the frustum be 14 m and 26 m respectively, the height of the frustum be 8 m and the slant height of the surmounted conical portion be 12 m, find the area of the canvas required to make the tent. (Assume that the radii of the upper circular end of the frustum and the base of the surmounted conical portion are equal.) [CBSE 2008]

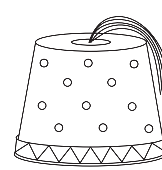
16. The perimeters of the two circular ends of a frustum of a cone are 48 cm and 36 cm. If the height of the frustum is 11 cm, find its volume and curved surface area. [CBSE 2014]

17. A solid cone of base radius 10 cm is cut into two parts through the midpoint of its height, by a plane parallel to its base. Find the ratio of the volumes of the two parts of the cone. [CBSE 2013]

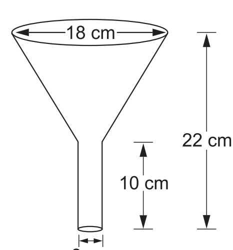
18. The height of a right circular cone is 20 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $1/8$ of the volume of the given cone, at what height above the base is the section made? [HOTS] [CBSE 2014]

19. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $1/12$ cm, find the length of the wire. [HOTS] [CBSE 2014]

20. A fez, the cap used by the Turks, is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it. [Use $\pi = 22/7$.]



21. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel.



22. A right circular cone is divided into three parts by trisecting its height by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1 : 7 : 19. [CBSE 2017]

ANSWERS (EXERCISE 17C)

1. 2170.67 cm^3 **2.** 2411.52 cm^2 **3.** (i) 8624 cm^3 (ii) 1804 cm^2 **4.** ₹ 329.49
5. (i) ₹ 145.83 (ii) ₹ 366.08 **6.** 22704 cm^3, 7599.43 cm^2 **7.** 28.49 litres
8. ₹ 293.90 **9.** ₹ 163.28, ₹ 171.13 **10.** ₹ 558.80 **11.** 15 cm **12.** $r = 7$
13. 7592.52 cm^2 **14.** 1068.57 m^2 **15.** 892.6 m^2 **16.** 1554 cm^3, 468.91 cm^2
17. 1 : 7 **18.** 10 cm **19.** 4480 m **20.** 710.28 cm^2 **21.** 782.57 cm^2

HINTS TO SOME SELECTED QUESTIONS

10. $r = 15$ cm, $R = 17.5$ cm, $h = 14$ cm.

Volume of the container $= \frac{1}{3}\pi(R^2 + r^2 + Rr)h = \left(\frac{3175 \times 11}{3}\right)$ cm^3.

Mass of oil in the container $= \left(\frac{3175 \times 11}{3} \times 1.2\right)$ g = 13.97 kg.

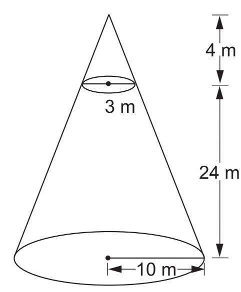
Cost of oil = ₹ (13.97 × 40) = ₹ 558.80.

11. $r = 21$ cm, $R = 28$ cm.

Volume of the bucket = (28.49 x 1000) cm^3.

$\therefore \quad \frac{1}{3}\pi[(28)^2 + (21)^2 + 28 \times 21]h = 28490.$ Solve for h.

14. $R = 10$ m, $r = 3$ m and $h = 24$ m.



Let l be the slant height of the frustum. Then,

$l = \sqrt{h^2 + (R - r)^2} = \sqrt{(24)^2 + 7^2}$ m

$= \sqrt{576 + 49}$ m $= \sqrt{625}$ m = 25 m.

Let L be the slant height of the conical part.
For this conical part, $r = 3$ m and $H = 4$ m.

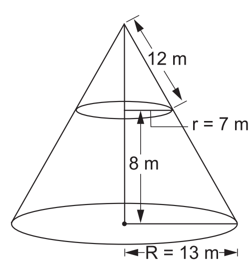
$\therefore \quad L = \sqrt{3^2 + 4^2}$ m $= \sqrt{25}$ m = 5 m.

Quantity of canvas

= (lateral surface area of the frustum)
+ (lateral surface area of the cone)

$= \{\pi l(R + r) + \pi rL\}$ m^2 $= \frac{22}{7} \times \{(25 \times 13) + (3 \times 5)\}$ m^2 = 1068.57 m^2.

15. $R = 13$ m, $r = 7$ m and $h = 8$ m.



$\therefore \quad l = \sqrt{h^2 + (R - r)^2} = 10$ m.

Required area

$= \left\{\frac{22}{7} \times 10 \times (13 + 7) + \frac{22}{7} \times 7 \times 12\right\}$ m^2.

16. $2\pi r = 36$ and $2\pi R = 48$. Find r and R.
20. Area of material = curved surface area + area of closed, small circular face
$= \pi l(r + R) + \pi r^2$.
21. Height of the cylindrical portion, $h = 10$ cm.
Height of the frustum, $H = (22 - 10)$ cm = 12 cm.
Radius of the cylindrical portion, $r = 4$ cm.
Radius of smaller end of the frustum, $r = 4$ cm.
Radius of bigger end of the frustum, $R = 9$ cm.
Area of tin sheet required
= curved surface area of the frustum + curved surface area of the cylindrical portion
$= \pi l(r + R) + 2\pi rh$.

EXERCISE 17D

Very-Short-Answer Questions

1. A river 1.5 m deep and 36 m wide is flowing at the rate of 3.5 km/hr. Find the amount of water (in cubic metres) that runs into the sea per minute.
2. The volume of a cube is 729 cm^3. Find its surface area.
3. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge?
4. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. Find the edge of the new cube formed.
5. Five identical cubes, each of edge 5 cm, are placed adjacent to each other. Find the volume of the resulting cuboid.
6. The volumes of two cubes are in the ratio 8 : 27. Find the ratio of their surface areas.
7. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder.
8. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If the volume of the cylinder is 12936 cm^3, find the radius of the base of the cylinder.
9. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. Find the ratio of their volumes.
10. 66 cubic cm of silver is drawn into a wire 1 mm in diameter. Calculate the length of the wire in metres.

11. If the area of the base of a right circular cone is 3850 cm^2 and its height is 84 cm, find the slant height of the cone.
12. A cylinder with base radius 8 cm and height 2 cm is melted to form a cone of height 6 cm. Calculate the radius of the base of the cone.
13. A right cylinderical vessel is full of water. How many right cones having the same radius and height as those of the right cylinder will be needed to store that water?
14. The volume of a sphere is 4851 cm^3. Find its curved surface area.
15. The curved surface area of a sphere is 5544 cm^3. Find its volume.
16. The surface areas of two spheres are in the ratio of 4 : 25. Find the ratio of their volumes.
17. A solid metallic sphere of radius 8 cm is melted and recast into spherical balls each of radius 2 cm. Find the number of spherical balls obtained.
18. How many lead shots each 3 mm in diameter can be made from a cuboid of dimensions 9 cm $\times$ 11 cm $\times$ 12 cm?
19. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are formed?
20. A hemisphere of lead of radius 6 cm is cast into a right circular cone of height 75 cm. Find the radius of the base of the cone.
21. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Find the length of the wire.
22. The radii of the circular ends of a frustum of height 6 cm are 14 cm and 6 cm respectively. Find the slant height of the frustum.
23. Find the ratio of the volume of a cube to that of a sphere which will fit inside it.
24. Find the ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height?

Short-Answer Questions

25. Two cubes each of volume 125 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid. [CBSE 2013C]
26. Three metallic cubes whose edges are 3 cm, 4 cm and 5 cm, are melted and recast into a single large cube. Find the edge of the new cube formed. [CBSE 2011]
27. A solid metallic sphere of diameter 8 cm is melted and drawn into a cylindrical wire of uniform width. If the length of the wire is 12 m, find its width. [CBSE 2013]

28. A 5-m-wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used, at the rate of ₹ 25 per metre. [CBSE 2014]

29. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the volume of wood in the toy. [CBSE 2013]

30. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes. [CBSE 2013C]

HINT $\sqrt{3} \times \text{edge} = 12\sqrt{3} \Rightarrow \text{edge} = 12$ cm.

$\therefore \quad (3x)^3 + (4x)^3 + (5x)^3 = (12)^3$. Solve to find x.

31. A hollow sphere of external and internal diameters 8 cm and 4 cm respectively is melted into a solid cone of base diameter 8 cm. Find the height of the cone. [CBSE 2013C]

32. A bucket of height 24 cm is in the form of frustum of a cone whose circular ends are of diameter 28 cm and 42 cm. Find the cost of milk at the rate of ₹ 30 per litre, which the bucket can hold. [CBSE 2013C]

33. The interior of a building is in the form of a right circular cylinder of diameter 4.2 m and height 4 m surmounted by a cone of same diameter. The height of the cone is 2.8 m. Find the outer surface area of the building. [CBSE 2014]

34. A metallic solid right circular cone is of height 84 cm and the radius of its base is 21 cm. It is melted and recast into a solid sphere. Find the diameter of the sphere. [CBSE 2014]

35. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. [CBSE 2012]

36. If the radii of the circular ends of a bucket 28 cm high, are 28 cm and 7 cm, find its capacity and total surface area. [CBSE 2011]

37. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket. (Use $\pi = 3.14$.) [CBSE 2006C]

38. A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7}\ \text{cm}^3$. The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of ₹ 1.40 per cm^2. [CBSE 2010]

39. A solid metallic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

40. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of water (i) displaced out of the cylinder (ii) left in the cylinder. [CBSE 2009]

ANSWERS (EXERCISE 17D)

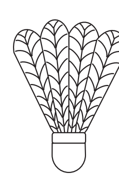
1. 3150 m^3	**2.** 486 cm^2	**3.** 1000	**4.** 12 cm	**5.** 625 cm^3
6. 4 : 9	**7.** 2 cm	**8.** 14 cm	**9.** 20 : 27	**10.** 84 m
11. 91 cm	**12.** 8 cm	**13.** 3	**14.** 1386 cm^2	**15.** 38808 cm^3
16. 8 : 125	**17.** 64	**18.** 84000	**19.** 108	**20.** 2.4 cm
21. 243 m	**22.** 10 cm	**23.** $6 : \pi$	**24.** 3 : 1 : 2	**25.** 250 cm^2
26. 6 cm	**27.** $\frac{8}{15}$ cm	**28.** ₹ 2750	**29.** 205.33 cm^3	
30. 6 cm, 8 cm, 10 cm		**31.** 14 cm	**32.** ₹ 702.24	**33.** 75.9 cm^2
34. 42 cm	**35.** 231 cm^2	**36.** 30184 cm^3, 6468 cm^2		**37.** 15 cm
38. ₹ 4224	**39.** 126	**40.** 77 cm^3, 748 cm^3		

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following:

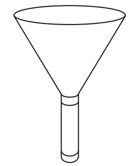
1. A **cylindrical pencil** sharpened at one edge is the combination of
   (a) a cylinder and a cone
   (b) a cylinder and frustum of a cone
   (c) a cylinder and a hemisphere
   (d) two cylinders

2. A **shuttlecock** used for playing badminton is the combination of
   (a) cylinder and a hemisphere
   (b) frustum of a cone and a hemisphere
   (c) a cone and a hemisphere
   (d) a cylinder and a sphere



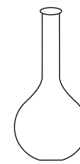
Shuttlecock

3. A **funnel** is the combination of
   (a) a cylinder and a cone
   (b) a cylinder and a hemisphere
   (c) a cylinder and frustum of a cone
   (d) a cone and a hemisphere



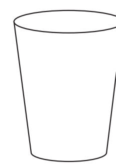
Funnel

4. A **surahi** is a combination of
   (a) a sphere and a cylinder
   (b) a hemisphere and a cylinder
   (c) a cylinder and a cone
   (d) two hemispheres



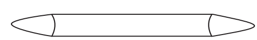
Surahi

5. The shape of a **glass (tumbler)** is usually in the form of
   (a) a cylinder
   (b) frustum of a cone
   (c) a cone
   (d) a sphere



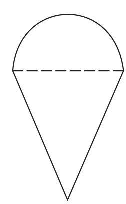
Glass

6. The shape of a **gilli** in the **gilli-danda** game is a combination of
   (a) a cone and a cylinder
   (b) two cylinders
   (c) two cones and a cylinder
   (d) two cylinders and a cone



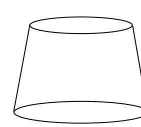
Gilli

7. A **plumbline (sahul)** is the combination of
   (a) a hemisphere and a cone
   (b) a cylinder and a cone
   (c) a cylinder and frustum of a cone
   (d) a cylinder and a sphere



Plumbline

8. A cone is cut by a plane parallel to its base and the upper part is removed. The part that is left over is called
   (a) a cone　　(b) a sphere
   (c) a cylinder　　(d) frustum of a cone



9. During conversion of a solid from one shape to another, the volume of the new shape will
   (a) decrease　　(b) increase
   (c) remain unaltered　　(d) be doubled

10. In a right circular cone, the cross section made by a plane parallel to the base is a
    (a) sphere (b) hemisphere (c) circle (d) a semicircle
11. A solid piece of iron in the form of a cuboid of dimensions $(49\text{ cm} \times 33\text{ cm} \times 24\text{ cm})$ is moulded to form a solid sphere. The radius of the sphere is
    (a) 19 cm (b) 21 cm (c) 23 cm (d) 25 cm
12. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is [CBSE 2011]
    (a) 2.1 (b) 4.2 (c) 8.4 (d) 1.05
13. A metallic solid sphere of radius 9 cm is melted to form a solid cylinder of radius 9 cm. The height of the cylinder is [CBSE 2014]
    (a) 12 cm (b) 18 cm (c) 36 cm (d) 96 cm
14. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is [CBSE 2014]
    (a) 3.5 (b) 7 (c) $\frac{80}{7}$ (d) 5
15. The number of solid spheres, each of diameter 6 cm, that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm, is [CBSE 2014]
    (a) 2 (b) 4 (c) 5 (d) 6
16. The surface areas of two spheres are in the ratio 16 : 9. The ratio of their volumes is [CBSE 2013C]
    (a) 64 : 27 (b) 16 : 9 (c) 4 : 3 (d) $16^3 : 9^3$
17. If the surface area of a sphere is 616 cm^2, its diameter (in cm) is [CBSE 2012]
    (a) 7 (b) 14 (c) 28 (d) 56
18. If the radius of a sphere becomes 3 times then its volume will become [CBSE 2011]
    (a) 3 times (b) 6 times (c) 9 times (d) 27 times
19. If the height of a bucket in the shape of frustum of a cone is 16 cm and the diameters of its two circular ends are 40 cm and 16 cm then its slant height is [CBSE 2013C]
    (a) 20 cm (b) $12\sqrt{5}$ cm (c) $8\sqrt{13}$ cm (d) 16 cm
20. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged then the water level rises by [CBSE 2011]
    (a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

21. A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. The ratio of the volume of the smaller cone to the whole cone is [CBSE 2012]

(a) 1 : 2 (b) 1 : 4 (c) 1 : 6 (d) 1 : 8

22. The radii of the circular ends of a bucket of height 40 cm are 24 cm and 15 cm. The slant height (in cm) of the bucket is [CBSE 2012]

(a) 41 (b) 43 (c) 49 (d) 51

23. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is [CBSE 2011]

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

24. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is [CBSE 2012]

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

25. A cubical ice-cream brick of edge 22 cm is to be distributed among some children by filling ice-cream cones of radius 2 cm and height 7 cm up to its brim. How many children will get the ice-cream cones?

(a) 163 (b) 263 (c) 363 (d) 463

26. A mason constructs a wall of dimensions (270 cm $\times$ 300 cm $\times$ 350 cm) with bricks, each of size (22.5 cm $\times$ 11.25 cm $\times$ 8.75 cm) and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Number of bricks used to construct the wall is

(a) 11000 (b) 11100 (c) 11200 (d) 11300

27. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

28. The diameters of two circular ends of a bucket are 44 cm and 24 cm, and the height of the bucket is 35 cm. The capacity of the bucket is

(a) 31.7 litres (b) 32.7 litres (c) 33.7 litres (d) 34.7 litres

29. The slant height of a bucket is 45 cm and the radii of its top and bottom are 28 cm and 7 cm respectively. The curved surface area of the bucket is

(a) 4953 cm^2 (b) 4952 cm^2 (c) 4951 cm^2 (d) 4950 cm^2

30. The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

(a) 9 : 16 (b) 16 : 9 (c) 3 : 4 (d) 4 : 3

31. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and $\frac{1}{8}$ space of the cube remains unfilled. Number of marbles required is

(a) 142296 (b) 142396 (c) 142496 (d) 142596

32. A metallic spherical shell of internal and external diameters 4 cm and 8 cm respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is

(a) 12 cm (b) 14 cm (c) 15 cm (d) 8 cm

33. A medicine capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. The capacity of the capsule is

(a) 0.33 cm^3 (b) 0.34 cm^3 (c) 0.35 cm^3 (d) 0.36 cm^3

34. The length of the longest pole that can be kept in a room $(12 \text{ m} \times 9 \text{ m} \times 8 \text{ m})$ is

(a) 29 m (b) 21 m (c) 19 m (d) 17 m

35. The length of the diagonal of a cube is $6\sqrt{3}$ cm. Its total surface area is

(a) 144 cm^2 (b) 216 cm^2 (c) 180 cm^2 (d) 108 cm^2

36. The volume of a cube is 2744 cm^3. Its surface area is

(a) 196 cm^2 (b) 1176 cm^2 (c) 784 cm^2 (d) 588 cm^2

37. The total surface area of a cube is 864 cm^2. Its volume is

(a) 3456 cm^3 (b) 432 cm^3 (c) 1728 cm^3 (d) 3456 cm^3

38. How many bricks each measuring $(25 \text{ cm} \times 11.25 \text{ cm} \times 6 \text{ cm})$ will be required to construct a wall $(8 \text{ m} \times 6 \text{ m} \times 22.5 \text{ cm})$?

(a) 8000 (b) 6400 (c) 4800 (d) 7200

39. The area of the base of a rectangular tank is 6500 cm^2 and the volume of water contained in it is 2.6 m^3. The depth of water in the tank is

(a) 3.5 m (b) 4 m (c) 5 m (d) 8 m

40. The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 m^3. The breadth of the wall is

(a) 30 cm (b) 40 cm (c) 22.5 cm (d) 25 cm

41. If the areas of three adjacent faces of a cuboid are x, y, z respectively then the volume of the cuboid is

(a) xyz (b) $2xyz$ (c) $\sqrt{xyz}$ (d) $3\sqrt{xyz}$

42. The sum of length, breadth and height of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. Its surface area is

(a) 361 cm^2 (b) 125 cm^2 (c) 236 cm^2 (d) 486 cm^2

43. If each edge of a cube is increased by 50%, the percentage increase in the surface area is

(a) 50% (b) 75% (c) 100% (d) 125%

44. How many bags of grain can be stored in a cuboidal granary (8 m × 6 m × 3 m), if each bag occupies a space of 0.64 m^3?

(a) 8256 (b) 90 (c) 212 (d) 225

45. A cube of side 6 cm is cut into a number of cubes each of side 2 cm. The number of cubes formed is

(a) 6 (b) 9 (c) 12 (d) 27

46. In a shower, 5 cm of rain falls. The volume of the water that falls on 2 hectares of ground, is

(a) 100 m^3 (b) 10 m^3 (c) 1000 m^3 (d) 10000 m^3

47. Two cubes have their volumes in the ratio 1 : 27. The ratio of their surface areas is

(a) 1 : 3 (b) 1 : 8 (c) 1 : 9 (d) 1 : 18

48. The diameter of the base of a cylinder is 4 cm and its height is 14 cm. The volume of the cylinder is

(a) 176 cm^3 (b) 196 cm^3 (c) 276 cm^3 (d) 352 cm^3

49. The diameter of a cylinder is 28 cm and its height is 20 cm. The total surface area of the cylinder is

(a) 2993 cm^2 (b) 2992 cm^2 (c) 2292 cm^2 (d) 2229 cm^2

50. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is

(a) 308 cm^3 (b) 396 cm^3 (c) 1232 cm^3 (d) 1848 cm^3

51. The curved surface area of a cylinder is 1760 cm^2 and its base radius is 14 cm. The height of the cylinder is

(a) 10 cm (b) 15 cm (c) 20 cm (d) 40 cm

52. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is

(a) 2 : 1 (b) 3 : 1 (c) 4 : 1 (d) 5 : 1

53. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The height of the pillar is

(a) 4 m (b) 5 m (c) 6 m (d) 7 m

54. The ratio between the radius of the base and the height of the cylinder is 2 : 3. If its volume is 1617 cm^3, the total surface area of the cylinder is

(a) 308 cm^2 (b) 462 cm^2 (c) 540 cm^2 (d) 770 cm^2

55. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is

(a) 27 : 20 (b) 20 : 27 (c) 4 : 9 (d) 9 : 4

56. Two circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is

(a) $1 : \sqrt{2}$ (b) $\sqrt{2} : 1$ (c) 1 : 2 (d) 1 : 4

57. The radius of the base of a cone is 5 cm and its height is 12 cm. Its curved surface area is

(a) 60π cm^2 (b) 65π cm^2 (c) 30π cm^2 (d) none of these

58. The diameter of the base of a cone is 42 cm and its volume is 12936 cm^3. Its height is

(a) 28 cm (b) 21 cm (c) 35 cm (d) 14 cm

59. The area of the base of a right circular cone is 154 cm^2 and its height is 14 cm. Its curved surface area is

(a) $154\sqrt{5}$ cm^2 (b) $154\sqrt{7}$ cm^2 (c) $77\sqrt{7}$ cm^2 (d) $77\sqrt{5}$ cm^2

60. On increasing each of the radius of the base and the height of a cone by 20% its volume will be increased by

(a) 20% (b) 40% (c) 60% (d) 72.8%

61. The radii of the base of a cylinder and a cone are in the ratio 3 : 4. If they have their heights in the ratio 2 : 3, the ratio between their volumes is

(a) 9 : 8 (b) 3 : 4 (c) 8 : 9 (d) 4 : 3

62. A metallic cylinder of radius 8 cm and height 2 cm is melted and converted into a right circular cone of height 6 cm. The radius of the base of this cone is

(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

63. The height of a conical tent is 14 m and its floor area is 346.5 m^2. How much canvas, 1.1 m wide, will be required for it ?

(a) 490 m (b) 525 m (c) 665 m (d) 860 m

64. The diameter of a sphere is 14 cm. Its volume is

(a) 1428 cm^3 (b) 1439 cm^3 (c) $1437\frac{1}{3}$ cm^3 (d) 1440 cm^3

65. The ratio between the volumes of two spheres is 8 : 27. What is the ratio between their surface areas?

(a) 2 : 3 (b) 4 : 5 (c) 5 : 6 (d) 4 : 9

66. A hollow metallic sphere with external diameter 8 cm and internal diameter 4 cm is melted and moulded into a cone having base radius 4 cm. The height of the cone is

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

67. A metallic cone having base radius 2.1 cm and height 8.4 cm is melted and moulded into a sphere. The radius of the sphere is

(a) 2.1 cm (b) 1.05 cm (c) 1.5 cm (d) 2 cm

68. The volume of a hemisphere is 19404 cm^3. The total surface area of the hemisphere is

(a) 4158 cm^2 (b) 16632 cm^2 (c) 8316 cm^2 (d) 3696 cm^2

69. The surface area of a sphere is 154 cm^2. The volume of the sphere is

(a) $179\frac{2}{3}$ cm^3 (b) $359\frac{1}{3}$ cm^3 (c) $1437\frac{1}{3}$ cm^3 (d) none of these

70. The total surface area of a hemisphere of radius 7 cm is

(a) (588π) cm^2 (b) (392π) cm^2 (c) (147π) cm^2 (d) (98π) cm^2

71. The circular ends of a bucket are of radii 35 cm and 14 cm and the height of the bucket is 40 cm. Its volume is

(a) 60060 cm^3 (b) 80080 cm^3 (c) 70040 cm^3 (d) 80160 cm^3

72. If the radii of the ends of a bucket are 5 cm and 15 cm and it is 24 cm high then its surface area is

(a) 1815.3 cm^2 (b) 1711.3 cm^2

(c) 2025.3 cm^2 (d) 2360 cm^2

73. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, the total area of canvas required is

(a) 1760 m^2 (b) 2640 m^2

(c) 3960 m^2 (d) 7920 m^2

Matching of columns

74. Match the following columns:

Column I	Column II
(a) A solid metallic sphere of radius 8 cm is melted and the material is used to make solid right cones with height 4 cm and radius of the base 8 cm. How many cones are formed?	(p) 18
(b) A 20-m-deep well with diameter 14 m is dug up and the earth from digging is evenly spread out to form a platform 44 m by 14 m. The height of the platform is ...... m.	(q) 8
(c) A sphere of radius 6 cm is melted and recast into the shape of a cylinder of radius 4 cm. Then, the height of the cylinder is ...... cm.	(r) 16 : 9
(d) The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is	(s) 5

The correct answer is

(a)–......, (b)–......, (c)–......, (d)–...... .

75. Match the following columns:

Column I	Column II
(a) The radii of the circular ends of a bucket in the form of frustum of a cone of height 30 cm are 20 cm and 10 cm respectively. The capacity of the bucket is ...... cm^3. [Take $\pi = \frac{22}{7}$·]	(p) 2418π

(b) The radii of the circular ends of a conical bucket of height 15 cm are 20 cm and 12 cm respectively. The slant height of the bucket is ...... cm.	(q) 22000
(c) The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. The total surface area of the bucket is ...... cm^2.	(r) 12
(d) Three solid metallic spheres of radii 3 cm, 4 cm and 5 cm are melted to form a single solid sphere. The diameter of the resulting sphere is ...... cm.	(s) 17

The correct answer is

(a)–......, (b)–......, (c)–......, (d)–...... .

Assertion-and-Reason Type

Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

76.

Assertion (A)	Reason (R)
If the radii of the circular ends of a bucket 24 cm high are 15 cm and 5 cm respectively, then the surface area of the bucket is 545π cm^2.	If the radii of the circular ends of the frustum of a cone are R and r respectively and its height is h, then its surface area is $\pi\{R^2 + r^2 + l(R - r)\}$, where $l^2 = h^2 + (R + r)^2$.

The correct answer is (a)/(b)/(c)/(d).

77.

Assertion (A)	Reason (R)
A hemisphere of radius 7 cm is to be painted outside on the surface. The total cost of painting at ₹ 5 per cm^2 is ₹ 2300.	The total surface volume of a hemisphere is $3\pi r^2$.

The correct answer is (a)/(b)/(c)/(d).

78.

Assertion (A)	Reason (R)
The number of coins 1.75 cm in diameter and 2 mm thick from a melted cuboid (10 cm × 5.5 cm × 3.5 cm) is 400.	Volume of a cylinder of base radius r and height h is given by $V = (\pi r^2 h)$ cubic units. And, volume of a cuboid $= (l \times b \times h)$ cubic units.

The correct answer is (a)/(b)/(c)/(d).

79.

Assertion (A)	Reason (R)
If the volumes of two spheres are in the ratio 27 : 8 then their surface areas are in the ratio 3 : 2.	Volume of a sphere $= \frac{4}{3}\pi R^3$. Surface area of a sphere $= 4\pi R^2$.

The correct answer is (a)/(b)/(c)/(d).

80.

Assertion (A)	Reason (R)
The curved surface volume of a cone of base radius 3 cm and height 4 cm is (15π) cm^2.	Volume of a cone $= \pi r^2 h$.

The correct answer is (a)/(b)/(c)/(d).

ANSWERS (MCQ)

1. (a) **2.** (b) **3.** (c) **4.** (a) **5.** (b) **6.** (c) **7.** (a) **8.** (d) **9.** (c)
10. (c) **11.** (b) **12.** (a) **13.** (a) **14.** (a) **15.** (c) **16.** (a) **17.** (b) **18.** (d)
19. (a) **20.** (a) **21.** (d) **22.** (a) **23.** (a) **24.** (c) **25.** (c) **26.** (c) **27.** (a)
28. (b) **29.** (d) **30.** (b) **31.** (a) **32.** (b) **33.** (d) **34.** (d) **35.** (b) **36.** (b)
37. (c) **38.** (b) **39.** (b) **40.** (b) **41.** (c) **42.** (c) **43.** (d) **44.** (d) **45.** (d)
46. (c) **47.** (c) **48.** (a) **49.** (b) **50.** (b) **51.** (c) **52.** (d) **53.** (c) **54.** (d)
55. (b) **56.** (b) **57.** (b) **58.** (a) **59.** (a) **60.** (d) **61.** (a) **62.** (d) **63.** (b)
64. (c) **65.** (d) **66.** (b) **67.** (a) **68.** (a) **69.** (a) **70.** (c) **71.** (b) **72.** (b)
73. (d) **74.** (a)–(q), (b)–(s), (c)–(p), (d)–(r) **75.** (a)–(q), (b)–(s), (c)–(p), (d)–(r)
76. (d) **77.** (d) **78.** (a) **79.** (d) **80.** (c)

HINTS TO SOME SELECTED QUESTIONS

9. During conversion of a solid from one shape to another, the volume remains the same.
10. In a right circular cone, the cross section made by a plane parallel to the base is a
11. $\frac{4}{3}\pi r^3 = 49\times 33\times 24 \Rightarrow \frac{4}{3}\times\frac{22}{7}\times r^3 = 49\times 33\times 24$

    $\Rightarrow\ r^3 = 49\times \cancel{33}^3\times \cancel{24}^3\times\frac{21}{\cancel{88}_{\cancel{8}\,1}} = 21\times 21\times 21 \Rightarrow r = 21\text{ cm.}$
12. Diameter of the cone = edge of the cube = 4.2 cm.

    $\therefore$ radius = 2.1 cm.
13. $\frac{4}{3}\pi\times 9\times 9\times 9 = \pi\times 9\times 9\times h \Rightarrow h = \left(\frac{4}{3}\times 9\right)\text{cm} = 12\text{ cm.}$
14. $2\pi r = 22 \Rightarrow r = \left(22\times\frac{1}{2}\times\frac{7}{22}\right)\text{cm} = 3.5\text{ cm.}$
15. Number of spheres $= \frac{\text{volume of the cylinder}}{\text{volume of each sphere}}$

    $= \frac{\pi\times 2\times 2\times 45}{\frac{4}{3}\times\pi\times 3\times 3\times 3} = 5.$
16. $\frac{4\pi R^2}{4\pi r^2} = \frac{16}{9} \Rightarrow \frac{R^2}{r^2} = \frac{16}{9} \Rightarrow \frac{R}{r} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

    $\Rightarrow \frac{R^3}{r^3} = \frac{4^3}{3^3} = \frac{64}{27} \Rightarrow \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27}.$
17. $4\pi r^2 = 616 \Rightarrow r^2 = 616\times\frac{1}{4}\times\frac{7}{22} = 49 \Rightarrow r = 7\text{ cm.}$
18. Volume of a sphere of radius r, $V = \frac{4}{3}\pi r^3$.

    Volume of a sphere of radius $3r = \frac{4}{3}\pi\times(3r)^3 = 27\times\frac{4}{3}\pi r^3 = 27V.$
19. $l = \sqrt{h^2 + (R-r)^2} = \sqrt{(16)^2 + (20-8)^2}\text{ cm}$

    $= \sqrt{256 + 144}\text{ cm} = \sqrt{400}\text{ cm} = 20\text{ cm.}$
20. Increase in volume of water = volume of the sphere

    $\Rightarrow\ \pi\times 18\times 18\times h = \frac{4}{3}\pi\times 9\times 9\times 9 \Rightarrow h = \left(\frac{4}{3}\times\frac{9\times 9\times 9}{18\times 18}\right)\text{cm} = 3\text{ cm.}$
21. Ratio of volumes $= \frac{\frac{1}{3}\pi\times\left(\frac{r}{2}\right)^2\times\left(\frac{h}{2}\right)}{\frac{1}{3}\pi\times r^2\times h} = \frac{1}{8}.$
22. $l = \sqrt{h^2 + (R-r)^2} = \sqrt{(40)^2 + (24-15)^2}\text{ cm}$

    $= \sqrt{1600 + 81}\text{ cm} = \sqrt{1681}\text{ cm} = 41\text{ cm.}$

23. $2\pi r^2 = \pi rl \Rightarrow \frac{r}{l} = \frac{1}{2}\cdot$

24. Ratio of volumes $= \dfrac{\pi \times \left(\frac{r}{2}\right)^2 \times h}{\pi \times r^2 \times h} = \frac{1}{4}\cdot$

25. Let the number of cones be n. Then,

$$n \times \frac{1}{3}\pi r^2 h = 22 \times 22 \times 22 \Rightarrow n \times \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 7 = 22 \times 22 \times 22$$

$\therefore \quad n = \left(22 \times 22 \times 22 \times \frac{3}{88}\right) = 363.$

26. Volume of the wall filled with bricks $= \left(\frac{7}{8} \times 270 \times 300 \times 350\right) \text{cm}^3.$

Volume of each brick $= \left(\frac{\cancel{225}^{9}}{\cancel{10}_{2}} \times \frac{\cancel{1125}^{225}}{\cancel{100}_{4}} \times \frac{\cancel{875}^{35}}{\cancel{100}_{4}}\right) = \left(\frac{9 \times 225 \times 35}{32}\right) \text{cm}^3.$

Number of bricks $= \left\{\frac{7}{8} \times 270 \times 300 \times 350 \times \frac{32}{9 \times 225 \times 35}\right\} = 11200.$

27. Let the diameter of each sphere be d cm. Then,

$$12 \times \frac{4}{3}\pi r^3 = \pi R^2 h \Rightarrow 12 \times \frac{4}{3}\pi \times \left(\frac{d}{2}\right)^3 = \pi \times 1^2 \times 16$$

$\Rightarrow \quad 2d^3 = 16 \Rightarrow d^3 = 8 = 2^3 \Rightarrow d = 2 \text{ cm}.$

28. Here, $R = 22$ cm, $r = 12$ cm and $h = 35$ cm.

Capacity of the bucket $= \frac{1}{3}\pi h(R^2 + r^2 + Rr) \text{ cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 35 \times [(22)^2 + (12)^2 + 22 \times 12] \text{ cm}^3$

$= \left(\frac{110}{3} \times 892\right) \text{cm}^3 = \left(\frac{110 \times 892}{3 \times 1000}\right) \text{litres} = 32.7 \text{ litres}.$

29. Here, $l = 45$ cm, $R = 28$ cm and $r = 7$ cm.

$\therefore \quad$ curved surface area $= \pi l(R + r) = \frac{22}{7} \times 45 \times (28 + 7) \text{ cm}^2 = 4950 \text{ cm}^2.$

30. $\dfrac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \frac{R^3}{r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}\cdot$

Ratio of their surface areas $= \frac{4\pi R^2}{4\pi r^2} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 16:9.$

31. Space filled in the cube $= \left(\frac{7}{8} \times 22 \times 22 \times 22\right) \text{cm}^3 = (7 \times 1331) \text{ cm}^3.$

Radius of each marble $= \frac{0.5}{2} \text{ cm} = \frac{5}{20} \text{ cm} = \frac{1}{4} \text{ cm}.$

Volume of each marble $= \frac{4}{3}\pi r^3 = \left(\frac{4}{3}\times\frac{22}{7}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)\text{cm}^3 = \left(\frac{11}{24\times 7}\right)\text{cm}^3.$

Number of marbles $= \left(\frac{7\times 1331\times 24\times 7}{11}\right) = 142296.$

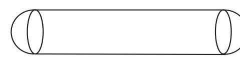
32. Here, $R = 4$ cm and $r = 2$ cm.

Volume of spherical shell $= \frac{4}{3}\pi\{(4)^3 - (2^3)\}\text{ cm}^3 = \left(\frac{4}{3}\pi\times 56\right)\text{cm}^3.$

Let the height of the cone be h cm.

$\therefore \quad \frac{1}{3}\pi\times 4\times 4\times h = \frac{4}{3}\pi\times 56 \Rightarrow h = 14$ cm.

33. Radius of the capsule $= \frac{0.5}{2}$ cm $= 0.25$ cm.



Let the length of cylindrical part be x cm.

Then, $0.25 + x + 0.25 = 2 \Rightarrow x + 0.5 = 2 \Rightarrow x = 1.5$ cm.

Capacity of the capsule $= \left(\frac{2}{3}\pi r^3\times 2\right) + \pi r^2 h$

$$= \frac{4}{3}\times\frac{22}{7}\times(0.25)^3 + \frac{22}{7}\times(0.25)^2\times 1.5$$

$$= \left(\frac{4}{3}\times\frac{22}{7}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{\cancel{22}^{11}}{7}\times\frac{1}{4}\times\frac{1}{4}\times\frac{\cancel{15}^{3}}{\cancel{10}_{2}}\right)$$

$$= \frac{11}{168} + \frac{33}{112} = \left(\frac{22+99}{336}\right) = \frac{121}{336} = 0.36\text{ cm}^3.$$

34. Length of the longest pole = length of diagonal of the room

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(12)^2 + 9^2 + 8^2}\text{ m} = \sqrt{289}\text{ m} = 17\text{ m}.$$

35. $\sqrt{3}\,a = 6\sqrt{3} \Rightarrow a = 6.$

Total surface area $= 6a^2\text{ cm}^2 = (6\times 6\times 6)\text{ cm}^2 = 216\text{ cm}^2.$

36. $a^3 = 2744 = (2^3\times 7^3) \Rightarrow a = (2\times 7) = 14$ cm.

$\therefore$ surface area $= 6a^2\text{ cm}^2 = (6\times 14\times 14)\text{ cm}^2 = 1176\text{ cm}^2.$

37. $6a^2 = 864 \Rightarrow a^2 = 144 \Rightarrow a = 12$ cm.

$\therefore$ volume $= (12\times 12\times 12)\text{ cm}^3 = 1728\text{ cm}^3.$

38. Volume of the wall $= (800\times 600\times 22.5)\text{ cm}^3.$

Number of bricks $= \frac{\text{volume of the wall}}{\text{volume of 1 brick}}$

$$= \left(\frac{800\times 600\times 22.5}{25\times 11.25\times 6}\right) = 6400.$$

39. Area of the base $= \frac{6500}{100\times 100}\text{ m}^2 = \frac{13}{20}\text{ m}^2.$

Let the depth of water be d metres. Then,

$$\frac{13}{20}\times d = 2.6 \Rightarrow d = \left(\frac{26}{10}\times\frac{20}{13}\right) = 4.$$

40. Let breadth = x cm. Then, height = $5x$ cm and length = $40x$ cm

$\therefore \quad 40x \times x \times 5x = 12.8 \times 100 \times 100 \times 100$

$\Rightarrow \quad x^3 = 64000 = (40 \times 40 \times 40) \Rightarrow x = 40.$

41. Let length = 2, breadth = b and height = h. Then, $lb = x$, $bh = y$, and $lh = z$

$\Rightarrow \quad lb \times bh \times lh = xyz \Rightarrow lbh = \sqrt{xyz}$

$\Rightarrow \quad$ volume $= \sqrt{xyz}$.

42. $l + b + h = 19 \Rightarrow (l + b + h)^2 = (19)^2 = 361$

$\Rightarrow (l^2 + b^2 + h^2) + 2(lb + bh + lh) = 361$

$\Rightarrow (5\sqrt{5})^2 + 2(lb + bh + lh) = 361$

$\Rightarrow 2(lb + bh + lh) = 361 - 125 = 236 \text{ cm}^2.$

43. Let original edge be a. Original surface area = $6a^2$.

New edge = 150% of $a = \frac{150a}{100} = \frac{3a}{2}\cdot$

New surface area $= 6 \times \left(\frac{3a}{2}\right)^2 = \frac{27a^2}{2}\cdot$

Increase in area $= \left(\frac{27a^2}{2} - 6a^2\right) = \frac{15a^2}{2}\cdot$

Increase % $= \left(\frac{15a^2}{2} \times \frac{1}{6a^2} \times 100\right) = 125\%$.

44. Number of bags $= \frac{(8 \times 6 \times 3)}{0.64} = 225.$

45. Number of cubes formed $= \frac{\text{volume of given cube}}{\text{volume of each small cube}}$

$= \frac{(6 \times 6 \times 6)}{(2 \times 2 \times 2)} = 27.$

46. Required volume = (area × depth)

$= \left(2 \times 10000 \times \frac{5}{100}\right) \text{ m}^3 = 1000 \text{ m}^3.$

47. $\frac{a^3}{b^3} = \frac{1}{27} \Rightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \Rightarrow \frac{a}{b} = \frac{1}{3}\cdot$

$\frac{S_1}{S_2} = \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}\cdot$

Required ratio = 1 : 9.

48. Here, $r = 2$ cm and $h = 14$ cm.

$\therefore \quad$ volume $= \pi r^2 h = \left(\frac{22}{7} \times 2 \times 2 \times 14\right) \text{ cm}^3 = 176 \text{ cm}^3.$

49. Here, $r = 14$ cm and $h = 20$ cm.

Total surface area $= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 14 \times (20 + 14) \text{ cm}^2 = 2992 \text{ cm}^2.$

50. $2\times\frac{22}{7}\times r\times 14=264 \Rightarrow r=\frac{264}{88}=3.$

Volume $=\left(\frac{22}{7}\times 3\times 3\times 14\right)\text{cm}^3 = 396 \text{ cm}^3.$

51. $2\pi rh=1760 \Rightarrow 2\times\frac{22}{7}\times 14\times h=1760$

$\Rightarrow h=\frac{1760}{88}=20 \text{ cm}.$

52. $\frac{\text{Total surface area}}{\text{Lateral surface area}}=\frac{2\pi r(h+r)}{2\pi rh}=\frac{(h+r)}{h}$

$=\frac{(20+80)}{20}=\frac{100}{20}=\frac{5}{1}\cdot$

53. $2\pi rh=264$ and $\pi r^2h=924$

$\Rightarrow \frac{\pi r^2h}{2\pi rh}=\frac{924}{264} \Rightarrow r=\frac{2\times 924}{264}=7 \text{ m}.$

$\therefore \quad 2\times\frac{22}{7}\times 7\times h=264 \Rightarrow h=\frac{264}{44}=6 \text{ cm}.$

54. Let radius $=2x$ and height $=3x$. Then,

$\pi r^2h=1617 \Rightarrow \frac{22}{7}\times(2x)^2\times 3x=1617$

$\Rightarrow x^3=\left(1617\times\frac{7}{22}\times\frac{1}{12}\right)=\left(\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}\right)=\left(\frac{7}{2}\right)^3$

$\Rightarrow x=\frac{7}{2} \text{ cm}.$

$\therefore \quad r=7$ cm and $h=\frac{21}{2}$ cm.

Total surface area $=2\pi r(h+r)$

$=2\times\frac{22}{7}\times 7\times\left(\frac{21}{2}+7\right)\text{cm}^2=770 \text{ cm}^2.$

55. Let the radii be $2r$, $3r$ and heights be $5h$ and $3h$. Then, ratio of their volumes

$=\frac{\pi\times(2r)^2\times 5h}{\pi\times(3r)^2\times 3h}=\frac{20}{27}\cdot$

56. $\pi\times r^2\times h=\pi\times R^2\times 2h \Rightarrow \frac{r^2}{R^2}=\frac{2}{1} \Rightarrow \left(\frac{r}{R}\right)^2=2 \Rightarrow \frac{r}{R}=\sqrt{2}.$

$\therefore \quad$ required ratio $=\sqrt{2}:1.$

57. Here, $r=5$ cm and $h=12$ cm.

$\therefore \quad l^2=(r^2+h^2)=(5)^2+(12)^2=169 \Rightarrow l=\sqrt{169}=13 \text{ cm}.$

Curved surface area $=\pi rl=(\pi\times 5\times 13)\text{ cm}^2=(65\pi)\text{ cm}^2.$

58. $\frac{1}{3}\pi r^2h=12936 \Rightarrow \frac{1}{3}\times\frac{22}{7}\times 21\times 21\times h=12936 \Rightarrow h=\left(\frac{12936}{22\times 21}\right)\text{cm}=28 \text{ cm}.$

59. $\pi r^2 = 154 \Rightarrow \frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = \left(154 \times \frac{7}{22}\right) = 49 \Rightarrow r = 7.$

Now, $r = 7$ cm and $h = 14$ cm.

$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{49 + 196} = \sqrt{245} = 7\sqrt{5}$ cm.

Curved surface area = πrl

$$= \left(\frac{22}{7} \times 7 \times 7\sqrt{5}\right) \text{cm}^2 = 154\sqrt{5} \text{ cm}^2.$$

60. Let the original radius be r and height be h.

Then, original volume = $\frac{1}{3}\pi r^2 h = V.$

New radius = 120% of $r = \left(\frac{120r}{100}\right) = \frac{6r}{5}.$

New height = 120% of $h = \left(\frac{120h}{100}\right) = \frac{6h}{5}.$

New volume = $\frac{1}{3}\pi \times \left(\frac{6r}{5}\right)^2 \times \frac{6h}{5} = \frac{216}{125}\left(\frac{1}{3}\pi r^2 h\right) = \frac{216}{125}V.$

Increase = $\left(\frac{216}{125}V - V\right) = \frac{91V}{125}.$

Increase % $= \left(\frac{91V}{125} \times \frac{100}{V}\right)\% = 72.8\%.$

61. Let the radii of the cylinder and cone be $3r$ and $4r$ respectively and their heights be $2h$ and $3h$ respectively. Then,

$$\frac{V_1}{V_2} = \frac{\pi(3r)^2 \times (2h)}{\frac{1}{3}\pi(4r)^2 \times (3h)} = \frac{54}{48} = \frac{9}{8}.$$

62. $\pi \times 8 \times 8 \times 2 = \frac{1}{3}\pi R^2 \times 6 \Rightarrow R^2 = 64 \Rightarrow R = 8$ cm.

63. $\pi R^2 = 346.5 \Rightarrow R^2 = \left(\frac{3465}{10} \times \frac{7}{22}\right) = \frac{441}{4} = \left(\frac{21}{2}\right)^2 \Rightarrow R = \frac{21}{2}$ m.

$l = \sqrt{R^2 + h^2} = \sqrt{\frac{441}{4} + (14)^2} = \sqrt{\frac{1225}{4}} = \frac{35}{2}$ m.

Area of canvas = $\pi Rl = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right) \text{m}^2 = 577.5 \text{ m}^2.$

Length of canvas = $\frac{\text{area}}{\text{width}} = \frac{577.50}{1.1}$ m = 525 m.

64. Volume = $\left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right) \text{cm}^3 = 1437\frac{1}{3} \text{ cm}^3.$

65. $\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{8}{27} \Rightarrow \frac{R^3}{r^3} = \frac{8}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{2}{3}.$

Ratio between their surface areas $= \dfrac{4\pi R^2}{4\pi r^2} = \dfrac{R^2}{r^2} = \left(\dfrac{R}{r}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}\cdot$

66. $\dfrac{4}{3}\pi\times(4)^3 - \dfrac{4}{3}\pi\times(2)^3 = \dfrac{1}{3}\pi\times(4)^2\times h \Rightarrow 16h = 224 \Rightarrow h = 14$ cm.

67. $\dfrac{1}{3}\pi\times(2.1)^2\times 8.4 = \dfrac{4}{3}\pi\times r^3$

$\Rightarrow \quad r^3 = \left(\dfrac{1}{3}\times 4.41\times 8.4\times\dfrac{3}{4}\right) = (2.1)^3 \Rightarrow r = 2.1$ cm.

68. $\dfrac{2}{3}\times\dfrac{22}{7}\times R^3 = 19404 \Rightarrow R^3 = \left(19404\times\dfrac{21}{44}\right)$

$\Rightarrow \quad R^3 = (21)^3 \Rightarrow R = 21$ cm.

Total surface area $= 3\pi R^2 = \left(3\times\dfrac{22}{7}\times 21\times 21\right)$ cm^2 $= 4158$ cm^2.

69. $4\pi R^2 = 154 \Rightarrow 4\times\dfrac{22}{7}\times R^2 = 154$

$\Rightarrow R^2 = \left(154\times\dfrac{7}{88}\right) \Rightarrow R = \dfrac{7}{2}\cdot$

Volume of the sphere $= \dfrac{4}{3}\pi R^3$

$= \left(\dfrac{4}{3}\times\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}\times\dfrac{7}{2}\right)$ cm^3 $= 179\dfrac{2}{3}$ cm^3.

70. Total surface area $= 3\pi R^2 = (3\pi\times 7\times 7)$ cm^2 $= (147\pi)$ cm^2.

71. Volume of the bucket $= \dfrac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \dfrac{22}{7}\times\dfrac{1}{3}\times 40\times[(35)^2 + (14)^2 + (35\times 14)]$ cm^3

$= 80080$ cm^3.

72. $l = \sqrt{h^2 + (R-r)^2} = \sqrt{(24)^2 + (15-5)^2} = 26$ cm.

Surface area of the bucket $= \pi[l(R+r) + r^2]$

$= 3.14\times[26\times 20 + 25]$ cm^2 $= 1711.3$ cm^2.

73. Area of canvas $= \left(2\times\dfrac{22}{7}\times\dfrac{105}{2}\times 4 + \dfrac{22}{7}\times\dfrac{105}{2}\times 40\right)$ m^2

$= (1320 + 6600)$ m^2 $= 7920$ m^2.

74. (a) Let the number of cones be n. Then,

$\dfrac{4}{3}\pi R^3 = n\times\dfrac{1}{3}\pi r^2 h \Rightarrow 4R^3 = nr^2h \Rightarrow 4\times 8^3 = n\times 8^2\times 4 \Rightarrow n = 8.$

(b) Volume of the earth dug out $= \pi r^2 h = \left(\dfrac{22}{7}\times 7\times 7\times 20\right)$ m^3 $= (22\times 140)$ m^3.

Let the height of the platform be h metres.

Volume of the earth on platform $= (44\times 14\times h)$ m^3.

$\therefore \quad 44\times 14\times h = 22\times 140 \Rightarrow h = \dfrac{22\times 140}{44\times 14}$ m $= 5$ m.

(c) Volume of the sphere $= \frac{4}{3}\pi r^3$.

Volume of the cylinder $= \pi R^2 h$.

$\therefore \quad \frac{4}{3}\pi r^3 = \pi R^2 h \Rightarrow \frac{4}{3}r^3 = R^2 h$

$\therefore \quad \frac{4}{3}\times 6\times 6\times 6 = 4\times 4\times h \Rightarrow h = \frac{8\times 6\times 6}{4\times 4} = 18$ cm.

(d) $\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \frac{R^3}{r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}$.

Ratio of their surface areas $= \frac{4\pi R^2}{4\pi r^2} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 16:9$.

The correct answer is (a)–(q), (b)–(s), (c)–(p), (d)–(r).

75. (a) $R = 20$ cm, $r = 10$ cm and $h = 30$ cm.

Capacity $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \left\{\frac{1}{3}\times\frac{22}{7}\times 15\times(400 + 100 + 200)\right\}$ cm^3 = 11000 cm^3.

(b) $R = 20$ cm, $r = 12$ cm and $h = 15$ cm.

$l = \sqrt{h^2 + (R-r)^2} = \sqrt{(15)^2 + (20-12)^2}$ cm $= \sqrt{(15)^2 + 8^2}$ cm

$= \sqrt{225 + 64}$ cm $= \sqrt{289}$ cm = 17 cm.

(c) $R = 33$ cm, $r = 27$ cm and $l = 10$ cm.

Total surface area $= \pi[R^2 + r^2 + l(R + r)]$

$= \pi[(33)^2 + (27)^2 + 10\times(33 + 27)]$ cm^2

$= \pi\times(2418)$ cm^2 = (2418π) cm^2.

(d) $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\times[(3)^3 + (4)^3 + (5)^3] \Rightarrow R^3 = 216 = (6)^3 \Rightarrow R = 6$.

$\therefore$ diameter = 12 cm.

The correct answer is (a)–(q), (b)–(s), (c)–(p), (d)–(r).

76. The Reason (R) is clearly true.

Now, $h = 24$ cm, $R = 15$ cm and $r = 5$ cm.

$\therefore \quad l^2 = h^2 + (R-r)^2 = \{(24)^2 + (15-5)^2\}$ cm^2 = (576 + 100) cm^2 = 676 cm^2

$\Rightarrow \quad l = \sqrt{676}$ cm = 26 cm.

$\therefore$ surface area $= \pi\{R^2 + r^2 + l(R + r)\}$

$= \pi\{(15)^2 + 5^2 + 26\times 10\}$ cm^2 $= \pi\{225 + 25 + 260\}$ cm^2

$= (510\pi)$ cm^2, which is true.

$\therefore$ Assertion (A) is true and Reason (R) gives Assertion (A).

Hence, the correct answer is (a).

77. The Reason (R) is clearly true.

Total surface area $= 3\pi r^2 = \left(3\times\frac{22}{7}\times 7\times 7\right)$ cm^2 = 462 cm^2.

Cost of painting = ₹ (462×5) = ₹ 2310.

$\therefore$ Assertion (A) is false and Reason (R) is true.

So, the correct answer is (d).

78. The Reason (R) is clearly true.

Radius of each coin = $\frac{1.75}{2}$ cm, thickness = $\frac{2}{10}$ cm = $\frac{1}{5}$ cm.

$\therefore$ volume of each coin = $\pi r^2 h = \left(\frac{22}{7} \times \frac{1.75}{2} \times \frac{1.75}{2} \times \frac{1}{5}\right) \text{cm}^3$

$$= \left(\frac{11}{70} \times \frac{175}{100} \times \frac{175}{100}\right) \text{cm}^3 = \frac{77}{160} \text{cm}^3.$$

Volume of cuboid = $\left(10 \times \frac{55}{10} \times \frac{35}{10}\right) \text{cm}^3 = \frac{385}{2} \text{cm}^3$.

Number of coins = $\left(\frac{385}{2} \times \frac{160}{77}\right) = 400$, which is true.

Clearly, Reason (R) is true and Assertion (A) gives Reason (R).

So, the correct answer is (a).

79. We know that the volume of a sphere is $\frac{4}{3}\pi R^3$ and its surface area is $4\pi R^2$. So, the Reason (R) is clearly true.

Now, let R and r be the radii of two given spheres. Then,

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{27}{8} \Rightarrow \frac{R^3}{r^3} = \frac{27}{8} \Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{3}{2}\right)^3 \Rightarrow \frac{R}{r} = \frac{3}{2}.$$

$\therefore$ ratio of their surface areas $= \frac{4\pi R^2}{4\pi r^2} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} = 9:4.$

So, the given result is false.

Thus, Assertion (A) is false and Reason (R) is true.

So, the correct answer is (d).

80. We know that the volume of a cone is $\frac{1}{3}\pi r^2 h$.

So, the given result, i.e., Reason (R), is false.

Now, $r = 3$ cm and $h = 4$ cm.

$\therefore$ curved surface area of the cone

$$= \pi r \times \sqrt{r^2 + h^2} = \pi \times 3 \times \sqrt{3^2 + 4^2}$$

$$= 3\pi\sqrt{25} \text{ cm}^2 = (15\pi) \text{ cm}^2, \text{which is true.}$$

Thus, Assertion (A) is true and Reason (R) is false.

Hence, the correct answer is (c).

TEST YOURSELF

Very-Short-Answer Questions

1. Find the number of solid spheres, each of diameter 6 cm, that could be moulded to form a solid metallic cylinder of height 45 cm and diameter 4 cm.
2. Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. What is the ratio of their radii?
3. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, find the total area of the canvas required.
4. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm respectively. Find the curved surface area of the bucket.

Short-Answer Questions

5. A solid metal cone with radius of base 12 cm and height 24 cm is melted to form solid spherical balls of diameter 6 cm each. Find the number of balls formed.
6. A hemispherical bowl of internal diameter 30 cm is full of a liquid. This liquid is filled into cylindrical-shaped bottles each of diameter 5 cm and height 6 cm. How many bottles are required?
7. A solid metallic sphere of diameter 21 cm is melted and recast into small cones, each of diameter 3.5 cm and height 3 cm. Find the number of cones so formed.
8. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of diameter 2.8 cm. Find the length of the wire.
9. A drinking glass is in the shape of frustum of a cone of height 21 cm with 6 cm and 4 cm as the diameters of its two circular ends. Find the capacity of the glass.
10. Two cubes, each of volume 64 cm^3, are joined end to end. Find the total surface area of the resulting cuboid.
11. The radius of the base and the height of a solid right circular cylinder are in the ratio 2 : 3 and its volume is 1617 cm^3. Find the total surface area of the cylinder. [Take $\pi = \frac{22}{7}\cdot$]
12. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy.

13. A hemispherical bowl of internal radius 9 cm is full of water. This water is to be filled in cylindrical bottles of diameter 3 cm and height 4 cm. Find the number of bottles needed to fill the whole water of the bowl.

14. The surface areas of a sphere and a cube are equal. Find the ratio of their volumes. [Take $\pi = \frac{22}{7}$·]

15. The slant height of the frustum of a cone is 4 cm and the perimeters (i.e., circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

16. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each hemispherical end is 7 cm, find the surface area of the solid.

Long-Answer Questions

17. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Use $\pi = 3.14$.]

18. A solid rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

19. An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket. Also, find the volume of water the bucket can hold, in litres.

20. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km/hr, in how much time will the tank be filled completely?

ANSWERS (TEST YOURSELF)

1. 5	**2.** $\sqrt{2}:1$	**3.** 7920 m^2	**4.** 4950 cm^2	**5.** 32
6. 60	**7.** 504	**8.** 63 m	**9.** 418 cm^3	**10.** 160 cm^2
11. 770 cm^2	**12.** 858 cm^2	**13.** 54	**14.** 231 : 121	**15.** 710.3 cm^2
16. 4576 cm^2	**17.** 443.14 cm^2	**18.** 112 m	**19.** 4860.9 cm^2, 33.62 litres	
20. 1 hour 15 minutes				

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