Mean, Median, Mode of Grouped Data, Cumulative Frequency Graph and Ogive

INTRODUCTION

Suppose we want to compare the wage distribution of workers in two factories and determine which factory pays more to its workers. If we compare on the basis of individual workers, we cannot conclude anything. However, if for the given data, we get a representative value that signifies the characteristics of the data, the comparison becomes easy.

A certain value representative of the whole data and signifying its characteristics is called an average of the data.

An average tends to lie centrally with the values of the variable arranged in ascending order of magnitude. So, we call an average a *measure of central tendency* of the data.

Three measures of central tendency are useful for analysing the data, namely

(a) Mean (b) Median (c) Mode.

MEAN OF GROUPED DATA

We already know that

$$\text{mean} = \frac{\text{sum of observations}}{\text{number of observations}}.$$

Thus, if $x_1, x_2, \ldots, x_n$ are n observations with respective frequencies $f_1, f_2, \ldots, f_n$ then the mean is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\Sigma f_i},$$

where the Greek letter Σ (read as, sigma) stands for summation of the terms.

There are three methods for computing the mean of grouped data.

I. DIRECT METHOD In this method, the midpoint or class mark of each class interval is taken to represent the observations falling in the class. We

proceed stepwise as follows:

Step 1. For each class interval, find the class mark x_i as

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}.$$

Step 2. Calculate $f_i x_i$ for each i.

Step 3. Calculate the mean using the formula

$$\text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}.$$

EXAMPLE 1 *Find the mean of the following data:*

Class interval	0–10	10–20	20–30	30–40	40–50
Frequency	8	12	10	11	9

[CBSE 2007]

SOLUTION We may prepare the table as shown:

Class interval	Frequency f_i	Class mark x_i	$(f_i \times x_i)$
0–10	8	5	40
10–20	12	15	180
20–30	10	25	250
30–40	11	35	385
40–50	9	45	405
	$\Sigma f_i = 50$		$\Sigma (f_i \times x_i) = 1260$

$$\therefore \quad \text{mean} = \frac{\Sigma (f_i \times x_i)}{\Sigma f_i} = \frac{1260}{50} = 25.2.$$

EXAMPLE 2 *The arithmetic mean of the following frequency distribution is* 25. *Determine the value of p.* [CBSE 2006]

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	18	15	*p*	6

SOLUTION We have

Class interval	Frequency f_i	Midvalue x_i	$(f_i \times x_i)$
0–10	5	5	25
10–20	18	15	270

20–30	15	25	375
30–40	p	35	$35p$
40–50	6	45	270
	$\Sigma f_i = (44 + p)$		$\Sigma (f_i \times x_i) = (940 + 35p)$

$$\therefore \quad \text{mean, } \overline{x} = \frac{\Sigma (f_i \times x_i)}{\Sigma f_i} \Rightarrow \frac{(940 + 35p)}{(44 + p)} = 25$$

$$\Rightarrow (940 + 35p) = 25(44 + p)$$

$$\Rightarrow (35p - 25p) = (1100 - 940)$$

$$\Rightarrow 10p = 160 \Rightarrow p = 16.$$

Hence, $p = 16$.

EXAMPLE 3 *If the mean of the following frequency distribution is* 65.6, *find the missing frequencies* f_1 *and* f_2. [CBSE 2010]

Class	10–30	30–50	50–70	70–90	90–110	110–130	*Total*
Frequency	5	8	f_1	20	f_2	2	50

SOLUTION We have

$$5 + 8 + f_1 + 20 + f_2 + 2 = 50 \Rightarrow f_2 = (15 - f_1).$$

Now, we may prepare the table given below:

Class interval	Frequency f_i	Class mark x_i	$(f_i \times x_i)$
10–30	5	20	100
30–50	8	40	320
50–70	f_1	60	$60f_1$
70–90	20	80	1600
90–110	$15 - f_1$	100	$1500 - 100f_1$
110–130	2	120	240
	$\Sigma f_i = 50$		$\Sigma (f_i \times x_i) = (3760 - 40f_1)$

$$\therefore \quad \text{mean} = \frac{\Sigma (f_i \times x_i)}{\Sigma f_i} = \frac{(3760 - 40f_1)}{50}.$$

But, mean = 65.6 (given).

$$\therefore \quad \frac{(3760 - 40f_1)}{50} = 65.6 \Rightarrow 3760 - 40f_1 = 3280$$

$$\Rightarrow 40f_1 = 480 \Rightarrow f_1 = 12.$$

Thus, $f_1 = 12$ and $f_2 = (15 - 12) = 3$.

II. ASSUMED-MEAN METHOD This method is usually used when the numerical values of x_i and f_i are large due to which calculations become tedious and time-consuming.

We proceed stepwise as follows:

Step 1. For each class interval, calculate the class mark x_i by using the formula,

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}.$$

Step 2. Choose a suitable value of x_i in the middle as the assumed mean and denote it by A.

Step 3. Calculate the deviations $d_i = (x_i - A)$ for each i.

Step 4. Calculate the product $(f_i d_i)$ for each i.

Step 5. Find $n = \Sigma f_i$.

Step 6. Calculate the mean, $\overline{x}$, by using the formula,

$$\overline{x} = A + \frac{\Sigma f_i d_i}{n}.$$

MATHEMATICAL DERIVATION OF ASSUMED MEAN FORMULA

We have

mean of the deviations, $\overline{d} = \dfrac{\Sigma f_i d_i}{\Sigma f_i}$.

But, $d_i = (x_i - A)$, where A is the assumed mean.

$$\therefore \quad \overline{d} = \frac{\Sigma f_i (x_i - A)}{\Sigma f_i} = \frac{\Sigma f_i x_i}{\Sigma f_i} - \frac{\Sigma f_i A}{\Sigma f_i} = \overline{x} - A\frac{\Sigma f_i}{\Sigma f_i} = \overline{x} - A$$

$$\Rightarrow \quad \overline{x} = A + \overline{d}$$

$$\Rightarrow \quad \overline{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}.$$

EXAMPLE 4 *Data regarding the weights of students of Class X of a school is given below.*

Weight (in kg)	50–52	52–54	54–56	56–58	58–60	60–62	62–64
Number of students	18	21	17	28	16	35	15

Compute the mean weight of the students. [CBSE 2014]

SOLUTION Let $A = 57$ be the assumed mean. Then, we have

Class interval	Frequency f_i	Class mark x_i	Deviation $d_i = (x_i - A) = (x_i - 57)$	$(f_i \times d_i)$
50–52	18	51	–6	–108
52–54	21	53	–4	–84
54–56	17	55	–2	–34
56–58	28	$57 = A$	0	0
58–60	16	59	2	32
60–62	35	61	4	140
62–64	15	63	6	90
	$\Sigma f_i = 150$			$\Sigma (f_i \times d_i) = 36$

$$\therefore \quad \overline{x} = A + \frac{\Sigma (f_i \times d_i)}{\Sigma f_i} = \left(57 + \frac{36}{150}\right) = 57 + 0.24 = 57.24.$$

Hence, mean weight = 57.24 kg.

III. STEP-DEVIATION METHOD This method is also used to simplify calculations in the process of computing the mean. It is particularly used when the values of $(x_i - A)$ are large and divisible by the class size (= upper limit – lower limit).

We proceed stepwise as shown below.

Step 1. For each class interval, calculate the class mark x_i, where

$$x_i = \frac{\text{upper class limit} + \text{lower class limit}}{2}.$$

Step 2. Choose a suitable value of x_i in the middle as the assumed mean and denote it by A.

Step 3. Calculate h = (upper limit − lower limit), which is the same for all the classes.

Step 4. Calculate $u_i = \dfrac{(x_i - A)}{h}$ for each class.

Step 5. Calculate $f_i u_i$ for each class.

Step 6. Calculate Σf_i and $\Sigma f_i u_i$.

Step 7. Calculate the mean by using the formula

$$\overline{x} = A + \left\{h \times \frac{\Sigma f_i u_i}{\Sigma f_i}\right\}.$$

MATHEMATICAL DERIVATION OF STEP-DEVIATION FORMULA

We have $\overline{u} = \dfrac{\Sigma(f_i u_i)}{\Sigma f_i}$.

But, $u_i = \dfrac{x_i - A}{h}$.

$$\therefore \quad \overline{u} = \frac{\Sigma f_i \dfrac{(x_i - A)}{h}}{\Sigma f_i} = \frac{1}{h}\left[\frac{\Sigma(f_i x_i - A\Sigma f_i)}{\Sigma f_i}\right]$$

$$= \frac{1}{h}\left[\frac{\Sigma(f_i x_i)}{\Sigma f_i} - A\frac{\Sigma f_i}{\Sigma f_i}\right]$$

$$= \frac{1}{h}(\overline{x} - A)$$

$$\Rightarrow \quad h\overline{u} = \overline{x} - A$$

$$\Rightarrow \quad \overline{x} = A + h\overline{u}$$

$$\Rightarrow \quad \overline{x} = A + h\left(\frac{\Sigma(f_i u_i)}{\Sigma f_i}\right).$$

EXAMPLE 5 *Calculate the arithmetic mean of the following frequency distribution, using the step-deviation method:* [CBSE 2001C]

Class interval	*Frequency*
0–50	17
50–100	35
100–150	43
150–200	40
200–250	21
250–300	24

SOLUTION Here, $h = 50$. Let the assumed mean be $A = 125$.

For calculating the mean, we prepare the table given as follows:

Class interval	Frequency f_i	Midvalue x_i	$u_i = \dfrac{(x_i - A)}{h}$	$(f_i \times u_i)$
0–50	17	25	–2	–34
50–100	35	75	–1	–35
100–150	43	125 = A	0	0

150–200	40	175	1	40
200–250	21	225	2	42
250–300	24	275	3	72
	$\Sigma f_i = 180$			$\Sigma(f_i u_i) = 154 - 69 = 85$

Thus, we have

$A = 125$, $h = 50$, $\Sigma f_i = 180$ and $\Sigma(f_i u_i) = 85$.

$$\text{Mean, } \overline{x} = A + \left\{h \times \frac{\Sigma(f_i u_i)}{\Sigma f_i}\right\}$$

$$= 125 + \left\{50 \times \frac{85}{180}\right\} = 125 + 23.61 = 148.61.$$

Hence, the mean of the given frequency is 148.61.

EXAMPLE 6 *The following table gives the distribution of expenditures of different families on education. Find the mean expenditure on education of a family.* [CBSE 2004C]

Expenditure (in ₹)	*Number of families*
1000–1500	24
1500–2000	10
2000–2500	33
2500–3000	28
3000–3500	30
3500–4000	22
4000–4500	16
4500–5000	7

SOLUTION Here, $h = 500$.

For calculating the mean, we prepare the table given as follows:

Class interval	Frequency f_i	Midvalue x_i	$u_i = \frac{(x_i - A)}{h}$	$(f_i \times u_i)$
1000–1500	24	1250	–3	–72
1500–2000	10	1750	–2	–20
2000–2500	33	2250	–1	–33
2500–3000	28	2750 = A	0	0

3000–3500	30	3250	1	30
3500–4000	22	3750	2	44
4000–4500	16	4250	3	48
4500–5000	7	4750	4	28
	$\Sigma f_i = 170$			$\Sigma (f_i \times u_i) = 25$

Thus, $A = 2750$, $h = 500$, $\Sigma f_i = 170$ and $\Sigma (f_i \times u_i) = 25$.

$$\therefore \quad \text{mean} = A + \left\{ h \times \frac{\Sigma (f_i \times u_i)}{\Sigma f_i} \right\}$$

$$= 2750 + \left(\frac{500 \times 25}{170} \right) = 2750 + 73.53 = 2823.53.$$

Hence, the required mean expenditure = ₹ 2823.53.

EXAMPLE 7 *Compute the arithmetic mean for the following data:*

Marks obtained	*Number of students*
Less than 10	14
Less than 20	22
Less than 30	37
Less than 40	58
Less than 50	67
Less than 60	75

SOLUTION The above data may be written as:

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	14	8	15	21	9	8

Here, $h = 10$.

Let assumed mean = midpoint of (30–40) = 35.

Now, we form the table as under.

Class	Frequency f_i	Midvalue x_i	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 35)}{10}$	$(f_i \times u_i)$
0–10	14	5	–3	–42
10–20	8	15	–2	–16
20–30	15	25	–1	–15

30–40	21	35 = A	0	0
40–50	9	45	1	9
50–60	8	55	2	16
	$\Sigma f_i = 75$			$\Sigma(f_i \times u_i) = -48$

$$\therefore \quad \text{mean, } \overline{x} = \left\{A + \frac{h \times (\Sigma f_i \times u_i)}{\Sigma f_i}\right\} = \left\{35 + \left(\frac{10 \times (-48)}{75}\right)\right\}$$
$$= 35 - 6.4 = 28.6.$$

MEAN FOR AN INCLUSIVE SERIES

EXAMPLE 8 *Find the arithmetic mean of the following frequency distribution:*

Class	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Frequency	14	22	16	6	5	3	4

[CBSE 2006C]

SOLUTION The given series is an inclusive series. Making it an exclusive series, we get

Class	Frequency f_i	Midvalue x_i	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 42)}{5}$	$(f_i \times u_i)$
24.5–29.5	14	27	–3	–42
29.5–34.5	22	32	–2	–44
34.5–39.5	16	37	–1	–16
39.5–44.5	6	42 = A	0	0
44.5–49.5	5	47	1	5
49.5–54.5	3	52	2	6
54.5–59.5	4	57	3	12
	$\Sigma f_i = 70$			$\Sigma(f_i \times u_i) = -79$

Thus, $A = 42$, $h = 5$, $\Sigma f_i = 70$ and $\Sigma(f_i \times u_i) = -79$.

$$\therefore \quad \text{mean, } \overline{x} = A + \left\{h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right\} = 42 + \left\{5 \times \frac{(-79)}{70}\right\}$$
$$= 42 - 5.64 = 36.36.$$

Hence, the required arithmetic mean is 36.36.

REMEMBER The value of mean obtained by all the above three methods is the same.

EXERCISE 18A

1. If the mean of 5 observations x, $x+2$, $x+4$, $x+6$ and $x+8$ is 11, find the value of x. [CBSE 2014]
2. If the mean of 25 observations is 27 and each observation is decreased by 7, what will be the new mean? [CBSE 2014]
3. Compute the mean of the following data:

Class	1–3	3–5	5–7	7–9
Frequency	12	22	27	19

[CBSE 2013]

4. Find the mean of the following data, using direct method:

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	7	5	6	12	8	2

5. Calculate the mean of the following data, using direct method:

Class	25–35	35–45	45–55	55–65	65–75
Frequency	6	10	8	12	4

6. Compute the mean of the following data, using direct method:

Class	0–100	100–200	200–300	300–400	400–500
Frequency	6	9	15	12	8

[CBSE 2005C]

7. Using an appropriate method, find the mean of the following frequency distribution:

Class interval	84–90	90–96	96–102	102–108	108–114	114–120
Frequency	8	10	16	23	12	11

Which method did you use, and why?

8. If the mean of the following frequency distribution is 24, find the value of p.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	3	4	p	3	2

[CBSE 2013]

9. The following distribution shows the daily pocket allowance of children of a locality. If the mean pocket allowance is ₹ 18, find the missing frequency f.

Daily pocket allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Number of children	7	6	9	13	f	5	4

10. If the mean of the following frequency distribution is 54, find the value of p.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	7	p	10	9	13

[CBSE 2006C]

11. The mean of the following data is 42. Find the missing frequencies x and y if the sum of frequencies is 100.

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	10	x	13	y	10	14	9

[CBSE 2014]

12. The daily expenditure of 100 families are given below. Calculate f_1 and f_2 if the mean daily expenditure is ₹ 188.

Expenditure (in ₹)	140–160	160–180	180–200	200–220	220–240
Number of families	5	25	f_1	f_2	5

[CBSE 2014]

13. The mean of the following frequency distribution is 57.6 and the total number of observations is 50.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	7	f_1	12	f_2	8	5

Find f_1 and f_2.

14. During a medical check-up, the number of heartbeats per minute of 30 patients were recorded and summarised as follows:

Number of heart-beats per minute	65–68	68–71	71–74	74–77	77–80	80–83	83–86
Number of patients	2	4	3	8	7	4	2

Find the mean heartbeats per minute for these patients, choosing a suitable method.

15. Find the mean marks per student, using assumed-mean method:

Marks	0–10	10–20	20–30	30–40	40–50	50–60
Number of students	12	18	27	20	17	6

16. Find the mean of the following frequency distribution, using the assumed-mean method:

Class	100–120	120–140	140–160	160–180	180–200
Frequency	10	20	30	15	5

17. Find the mean of the following data, using the assumed-mean method:

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	20	35	52	44	38	31

18. The following table gives the literacy rate (in percentage) in 40 cities. Find the mean literacy rate, choosing a suitable method.

Literacy rate (%)	45–55	55–65	65–75	75–85	85–95
Number of cities	4	11	12	9	4

[CBSE 2014]

19. Find the mean of the following frequency distribution using step-deviation method.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	7	10	15	8	10

[CBSE 2014]

20. Find the mean of the following data, using step-deviation method:

Class	5–15	15–25	25–35	35–45	45–55	55–65	65–75
Frequency	6	10	16	15	24	8	7

[CBSE 2013]

21. The weights of tea in 70 packets are shown in the following table:

Weight (in grams)	200–201	201–202	202–203	203–204	204–205	205–206
Number of packets	13	27	18	10	1	1

Find the mean weight of packets using step-deviation method. [CBSE 2013]

22. Find the mean of the following frequency distribution using a suitable method:

Class	20–30	30–40	40–50	50–60	60–70
Frequency	25	40	42	33	10

[CBSE 2013]

23. In an annual examination, marks (out of 90) obtained by students of Class X in mathematics are given below:

Marks obtained	0–15	15–30	30–45	45–60	60–75	75–90
Number of students	2	4	5	20	9	10

Find the mean marks. [CBSE 2014]

24. Find the arithmetic mean of the following frequency distribution using step-deviation method:

Age (in years)	18–24	24–30	30–36	36–42	42–48	48–54
Number of workers	6	8	12	8	4	2

25. Find the mean of the following data using step-deviation method:

Class	500–520	520–540	540–560	560–580	580–600	600–620
Frequency	14	9	5	4	3	5

26. Find the mean age from the following frequency distribution:

Age (in years)	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Number of persons	4	14	22	16	6	5	3

HINT Change the given series to the exclusive series.

27. The following table shows the age distribution of patients of malaria in a village during a particular month:

Age (in years)	5–14	15–24	25–34	35–44	45–54	55–64
Number of cases	6	11	21	23	14	5

Find the average age of the patients.

28. Weight of 60 eggs were recorded as given below:

Weight (in grams)	75–79	80–84	85–89	90–94	95–99	100–104	105–109
Number of eggs	4	9	13	17	12	3	2

Calculate their mean weight to the nearest gram.

29. The following table shows the marks scored by 80 students in an examination:

Marks	Less than 5	Less than 10	Less than 15	Less than 20	Less than 25	Less than 30	Less than 35	Less than 40
Number of students	3	10	25	49	65	73	78	80

Calculate the mean marks correct to 2 decimal places.

ANSWERS (EXERCISE 18A)

1. $x = 7$ **2.** 20 **3.** 5.325 **4.** 28.75 **5.** 49.5
6. 264 **7.** 103.05 **8.** $p = 18$ **9.** $f = 20$ **10.** $p = 11$
11. $x = 12, y = 25$ **12.** $f_1 = 50, f_2 = 15$ **13.** $f_1 = 8, f_2 = 10$
14. 75.9 **15.** 28 **16.** 145 **17.** 62.55 **18.** 69.5%
19. 25.8 **20.** 40.81 **21.** 201.96 g **22.** 42.53 **23.** 55.5
24. 33.3 years **25.** 544 **26.** 39.36 years **27.** 34.87 years **28.** 90 g
29. 18.56

MEDIAN FOR GROUPED DATA

The *median* is a measure of central tendency which gives the value of the middlemost observation in the data.

As we have already studied, for an ungrouped data comprising n observations:

$$\text{Median} = \begin{cases} \left(\frac{n+1}{2}\right)\text{th observation, if } n \text{ is odd} \\ \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}, \text{ if } n \text{ is even} \end{cases}$$

METHOD FOR FINDING THE MEDIAN FOR GROUPED DATA

We proceed stepwise as follows:

Step 1. For the given frequency distribution and obtain $N = \Sigma f_i$.

Step 2. Find $\frac{N}{2}$.

Step 3. Look at the cumulative frequency just greater than $\frac{N}{2}$ and find the corresponding class, known as *median class* (as the middlemost observation lies in this class).

Step 4. Compute the median using the formula:

$$\text{Median, } M_e = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\},$$

where l = lower limit of median class;

h = width of median class;

f = frequency of median class;

cf = cumulative frequency of the class preceding the median class;

$N = \Sigma f_i$.

SOLVED EXAMPLES

EXAMPLE 1 *Find the median of the following data:*

Marks	20–30	30–40	40–50	50–60	60–70	70–80	80–90
Number of students	5	15	25	20	7	8	10

[CBSE 2013]

SOLUTION We prepare the cumulative frequency table, as given below.

Class	Frequency (f_i)	Cumulative frequency
20–30	5	5
30–40	15	20
40–50	25	45
50–60	20	65
60–70	7	72
70–80	8	80
80–90	10	90
	$N = \Sigma f_i = 90$	

Now, $N = 90 \Rightarrow \frac{N}{2} = 45$.

The cumulative frequency just greater than 45 is 65 and the corresponding class is 50–60.

Thus, the median class is 50–60.

$\therefore$ $l = 50$, $h = 10$, $f = 20$, cf = c.f. of preceding class = 45,

and $\frac{N}{2} = 45$.

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\} = 50 + \left\{10 \times \frac{(45 - 45)}{20}\right\}$$

$$= 50 + 0 = 50.$$

Hence, the median mark is 50.

EXAMPLE 2 *Find the median wage from the following data:*

Wages (in ₹)	800–820	820–840	840–860	860–880	880–900	900–920	920–940
Number of workers	7	14	19	25	20	10	5

[CBSE 2013]

SOLUTION We prepare the cumulative frequency table, as given below.

Class	Frequency (f_i)	Cumulative frequency
800–820	7	7
820–840	14	21
840–860	19	40
860–880	25	65
880–900	20	85
900–920	10	95
920–940	5	100
	$N = \Sigma f_i = 100$	

Now, $N = 100 \Rightarrow \frac{N}{2} = 50$.

The cumulative frequency just greater than 50 is 65 and the corresponding class is 860–880.

Thus, the median class is 860–880.

$\therefore$ $l = 860$, $h = 20$, $f = 25$, cf = c.f. of preceding class = 40,

and $\frac{N}{2} = 50$.

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\} = 860 + \left\{20 \times \frac{(50-40)}{25}\right\}$$

$$= 860 + 20 \times \frac{10}{25} = 860 + 8 = 868.$$

Hence, the median wage is ₹ 868.

MEDIAN FOR INCLUSIVE SERIES

EXAMPLE 3 *Find the median for the following frequency distribution:*

Height (in cm)	160–162	163–165	166–168	169–171	172–174
Frequency	15	117	136	118	14

SOLUTION The given series is in inclusive form. Converting it to exclusive form and preparing the cumulative frequency table, we get

Class	Frequency (f_i)	Cumulative frequency
159.5–162.5	15	15
162.5–165.5	117	132
165.5–168.5	136	268
168.5–171.5	118	386
171.5–174.5	14	400
	$N = \Sigma f_i = 400$	

Now, $N = 400 \Rightarrow \frac{N}{2} = 200.$

The cumulative frequency just greater than 200 is 268 and the corresponding class is 165.5–168.5.

Thus, the median class is 165.5–168.5.

$\therefore$ $l = 165.5$, $h = 3$, $f = 136$, cf = c.f. of preceding class = 132,

and $\frac{N}{2} = 200.$

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$$

$$= 165.5 + \left\{3 \times \frac{(200 - 132)}{136}\right\} = 165.5 + \left(\frac{3 \times 68}{136}\right)$$
$$= 165.5 + 1.5 = 167.$$

Hence, median height is 167 cm.

EXAMPLE 4 *Given below is the distribution of IQ of* 100 *students. Find the median IQ.*

IQ	75–84	85–94	95–104	105–114	115–124	125–134	135–144
Frequency	8	11	26	31	18	4	2

SOLUTION The given series is in inclusive form. Converting it to exclusive form and preparing the cumulative frequency table, we get

Class	Frequency (f_i)	Cumulative frequency
74.5–84.5	8	8
84.5–94.5	11	19
94.5–104.5	26	45
104.5–114.5	31	76
114.5–124.5	18	94
124.5–134.5	4	98
134.5–144.5	2	100
	$N = \Sigma f_i = 100$	

Now, $N = 100 \Rightarrow \frac{N}{2} = 50.$

The cumulative frequency just greater than 50 is 76 and the corresponding class interval is 104.5–114.5.

Thus, the median class is 104.5–114.5.

$\therefore$ $l = 104.5$, $h = 10$, $f = 31$, cf = c.f. of preceding class = 45

and $\frac{N}{2} = 50.$

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$$

$$= 104.5 + \left\{10 \times \frac{(50-45)}{31}\right\} = 104.5 + \frac{50}{31}$$

$$= 104.5 + 1.6 = 106.1.$$

Hence, the median IQ is 106.1.

EXAMPLE 5 *Calculate the median for the following data:*

Marks obtained	*No. of students*
Below 10	6
Below 20	15
Below 30	29
Below 40	41
Below 50	60
Below 60	70

SOLUTION From the given table, we may get back frequencies and cumulative frequencies as shown below.

Class interval	Frequency (f_i)	Cumulative frequency
0–10	6	6
10–20	9	15
20–30	14	29
30–40	12	41
40–50	19	60
50–60	10	70
	$N = \Sigma f_i = 70$	

Now, $N = 70 \Rightarrow \frac{N}{2} = 35.$

The cumulative frequency just greater than 35 is 41 and the corresponding class is 30–40.

Thus, the median class is 30–40.

$\therefore$ $l = 30$, $h = 10$, $f = 12$, cf = c.f. of preceding class = 29,

and $\frac{N}{2} = 35.$

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\} = 30 + \left\{10 \times \frac{(35-29)}{12}\right\} = 35.$$

Hence, the required median is 35.

EXAMPLE 6 *Find the missing frequencies in the following frequency distribution table, if N = 100 and median is 32.*

Marks	0–10	10–20	20–30	30–40	40–50	50–60	*Total*
Number of students	10	?	25	30	?	10	100

[CBSE 2013]

SOLUTION Let f_1 and f_2 be the missing frequencies of class intervals 10–20 and 40–50 respectively. Then,

$$10 + f_1 + 25 + 30 + f_2 + 10 = 100 \Rightarrow f_1 + f_2 = 25.$$

Median is 32, which lies in 30–40. So, the median class is 30–40.

$\therefore$ $l = 30$, $h = 10$, $f = 30$, $N = 100$ and $cf = 10 + f_1 + 25 = f_1 + 35$.

Now, median, $M_e = l + \left\{h \times \dfrac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$$\Rightarrow \quad 30 + \left[10 \times \frac{\{50 - (f_1 + 35)\}}{30}\right] = 32$$

$$\Rightarrow \quad 30 + \frac{(15 - f_1)}{3} = 32 \Rightarrow (15 - f_1) = b \Rightarrow f_1 = 9.$$

$\therefore$ $f_1 = 9$ and $f_2 = 25 - 9 = 16$.

Hence, $f_1 = 9$ and $f_2 = 16$.

EXERCISE 18B

1. In a hospital, the ages of diabetic patients were recorded as follows. Find the median age.

Age (in years)	0–15	15–30	30–45	45–60	60–75
Number of patients	5	20	40	50	25

[CBSE 2014]

2. Compute the median from the following data:

Marks	0–7	7–14	14–21	21–28	28–35	35–42	42–49
Number of students	3	4	7	11	0	16	9

3. The following table shows the daily wages of workers in a factory:

Daily wages (in ₹)	0–100	100–200	200–300	300–400	400–500
Number of workers	40	32	48	22	8

Find the median daily wage income of the workers.

4. Calculate the median from the following frequency distribution:

Class	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	5	6	15	10	5	4	2	2

5. Given below is the number of units of electricity consumed in a week in a certain locality:

Consumption (in units)	65–85	85–105	105–125	125–145	145–165	165–185	185–205
Number of consumers	4	5	13	20	14	7	4

Calculate the median.

6. Calculate the median from the following data:

Height (in cm)	135–140	140–145	145–150	150–155	155–160	160–165	165–170	170–175
No. of boys	6	10	18	22	20	15	6	3

7. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	5	25	?	18	7

8. The median of the following data is 16. Find the missing frequencies a and b if the total of frequencies is 70.

Class	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	12	a	12	15	b	6	6	4

[CBSE 2013]

9. In the following data the median of the runs scored by 60 top batsmen of the world in one-day international cricket matches is 5000. Find the missing frequencies x and y.

Runs scored	2500–3500	3500–4500	4500–5500	5500–6500	6500–7500	7500–8500
Number of batsmen	5	x	y	12	6	2

10. If the median of the following frequency distribution is 32.5, find the values of f_1 and f_2.

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

11. Calculate the median for the following data:

Age (in years)	19–25	26–32	33–39	40–46	47–53	54–60
Frequency	35	96	68	102	35	4

HINT Convert it to exclusive form.

12. Find the median wages for the following frequency distribution:

Wages per day (in ₹)	61–70	71–80	81–90	91–100	101–110	111–120
No. of women workers	5	15	20	30	20	8

HINT Convert it to exclusive form.

13. Find the median from the following data:

Class	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45
Frequency	7	10	16	32	24	16	11	5	2

HINT Convert it to exclusive form.

14. Find the median from the following data:

Marks	No. of students
Below 10	12
Below 20	32
Below 30	57
Below 40	80
Below 50	92
Below 60	116
Below 70	164
Below 80	200

ANSWERS (EXERCISE 18B)

1. 46.5 years **2.** 28 **3.** ₹ 206.25 **4.** 19.5 **5.** 136.5 units
6. 153.64 cm **7.** 25 **8.** $a = 8, b = 7$ **9.** $x = 25, y = 10$
10. $f_1 = 3, f_2 = 6$ **11.** 36 years **12.** ₹ 93.50 **13.** 19.95 **14.** 53.33

MODE OF A GROUPED DATA

Mode is that value of a variate which occurs most often, i.e., the value of the observation having the maximum frequency.

More precisely, mode is that value of the variable at which the concentration of the data is maximum.

MODAL CLASS In a frequency distribution, the class having maximum frequency is called the modal class.

Formula for Calculating Mode:

We have

$$\text{mode, } M_o = x_k + h \cdot \left\{ \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\},$$

where x_k = lower limit of the modal class interval;

f_k = frequency of the modal class;

f_{k-1} = frequency of the class preceding the modal class;

f_{k+1} = frequency of the class succeeding the modal class;

h = width of the class interval.

SOLVED EXAMPLES

EXAMPLE 1 *Find the mode of the following data:*

Class	0–20	20–40	40–60	60–80	80–100	100–120	120–140
Frequency	6	8	10	12	6	5	3

[CBSE 2013]

SOLUTION Clearly, the modal class is 60–80, as it has the maximum frequency.

$\therefore \quad x_k = 60, h = 20, f_k = 12, f_{k-1} = 10, f_{k+1} = 6.$

$$\text{Mode, } M_o = x_k + \left\{ h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$$

$$= 60 + \left\{ 20 \times \frac{(12 - 10)}{(2 \times 12 - 10 - 6)} \right\}$$

$$= 60 + \left\{ 20 \times \frac{2}{(24 - 16)} \right\} = 60 + \left\{ 20 \times \frac{2}{8} \right\} = 60 + 5 = 65.$$

Hence, mode = 65.

EXAMPLE 2 *The distribution of sale of shirts sold in a month in a departmental store is as under.*

Size (in cm)	80–85	85–90	90–95	95–100	100–105	105–110	110–115
Number of shirts sold	33	27	85	155	110	45	15

Calculate the modal size of shirts sold. [CBSE 2014]

SOLUTION Clearly, the modal class is 95–100 as it has the maximum frequency.

$\therefore \quad x_k = 95,\ h = 5,\ f_k = 155,\ f_{k-1} = 85,\ f_{k+1} = 110.$

$$\text{Mode, } M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 95 + \left\{5 \times \frac{(155 - 85)}{(2 \times 155 - 85 - 110)}\right\}$$

$$= 95 + \left\{5 \times \frac{70}{(310 - 195)}\right\} = 95 + \left\{5 \times \frac{70}{115}\right\}$$

$$= 95 + \frac{70}{23} = 95 + 3.04 = 98.04.$$

$\therefore$ modal size = 98.04 cm.

EXAMPLE 3 *Given below is the frequency distribution of the heights of players in a school.*

Height (in cm)	160–162	163–165	166–168	169–171	172–174
Number of students	15	118	142	127	18

Find the modal height and interpret it.

SOLUTION The given data is an inclusive series. So, we convert it into an exclusive form, as given below.

Class	159.5–162.5	162.5–165.5	165.5–168.5	168.5–171.5	171.5–174.5
Frequency	15	118	142	127	18

Clearly, the class 165.5–168.5 has maximum frequency, so it is the modal class.

$\therefore \quad x_k = 165.5,\ f_k = 142,\ f_{k-1} = 118,\ f_{k+1} = 127$ and $h = 3$.

$$\text{Mode, } M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 165.5 + \left\{3 \times \frac{(142 - 118)}{(2 \times 142 - 118 - 127)}\right\}$$

$$= 165.5 + \left\{\frac{3 \times 24}{39}\right\} = 165.5 + \frac{24}{13}$$

$$= 165.5 + 1.85 = 167.35.$$

Thus, modal height = 167.35 cm.

This means that the height of maximum number of players in the school is 167.35 cm (approx.).

EXAMPLE 4 *The following table shows the ages of the patients admitted in a hospital during a month:*

Age (in years)	6–15	16–25	26–35	36–45	46–55	56–65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

SOLUTION The given data is an inclusive series. Making it an exclusive series, we get

Class	Frequency f_i	Class mark x_i	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 40.5)}{10}$	$f_i u_i$
5.5–15.5	6	10.5	–3	–18
15.5–25.5	11	20.5	–2	–22
25.5–35.5	21	30.5	–1	–21
35.5–45.5	23	40.5 = A	0	0
45.5–55.5	14	50.5	1	14
55.5–65.5	5	60.5	2	10
	$\Sigma f_i = 80$			$\Sigma f_i u_i = -37$

Thus, $A = 40.5$, $h = 10$, $\Sigma f_i = 80$, $\Sigma f_i u_i = -37$.

$$\text{Mean, } \overline{x} = A + \left\{h \times \frac{\Sigma f_i u_i}{\Sigma f_i}\right\}$$

$$= 40.5 + \left\{10 \times \frac{(-37)}{80}\right\} = 40.5 - \frac{37}{8}$$

$$= 40.5 - 4.63 = 35.87.$$

Also, the modal class of the given data is 35.5–45.5, as it has the maximum frequency.

$\therefore \quad x_k = 35.5$, $f_k = 23$, $f_{k-1} = 21$, $f_{k+1} = 14$ and $h = 10$.

$$\text{Mode, } M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 35.5 + \left\{10 \times \frac{(23 - 21)}{(2 \times 23 - 21 - 14)}\right\} = 35.5 + \left(\frac{10 \times 2}{11}\right)$$

$$= 35.5 + 1.82 = 37.32.$$

Clearly, mode > mean.

Interpretation The modal age is 37.32 years. This means that the maximum number of patients admitted in the hospital during the given month are of the age 37.32 years (approx.).

The mean age is 35.87 years. This means that on an average the age of a patient admitted to the hospital is 35.87 years.

EXAMPLE 5 *The mode of the following series is* 36. *Find the missing frequency in it.*

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	8	10	…	16	12	6	7

SOLUTION Since the mode of the given series is 36 and maximum frequency 16 lies in the class 30–40, so the modal class is 30–40.

Let the missing frequency be x. Then,

$\therefore \quad x_k = 30,\ f_k = 16,\ f_{k-1} = x,\ f_{k+1} = 12$ and $h = 10$.

Also, $m_o = 36$.

Using the formula, $M_o = x_k + \left\{h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$, we get

$$30 + \left\{10 \times \frac{(16 - x)}{(32 - x - 12)}\right\} = 36$$

$$\Rightarrow \quad \frac{10 \times (16 - x)}{(20 - x)} = 6 \Rightarrow 160 - 10x = 120 - 6x$$

$$\Rightarrow \quad 4x = 40 \Rightarrow x = 10.$$

Hence, the missing frequency is 10.

EXAMPLE 6 *Compare the modal ages of two groups of students appearing for an entrance examination.*

Age (in years)	16–18	18–20	20–22	22–24	24–26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

SOLUTION **Case I.** *Computation of Modal Age of Group A:*

In group A, class 18–20 has the maximum frequency.

So, 18–20 is the modal class and its frequency is 78.

$\therefore \quad x_k = 18,\ f_k = 78,\ f_{k-1} = 50,\ f_{k+1} = 46$ and $h = 2$.

$\therefore \quad$ mode, $M_o = x_k + \left\{h \times \dfrac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$

$$= 18 + \left\{2 \times \frac{(78-50)}{(2\times 78 - 50 - 46)}\right\} = 18 + \left\{\frac{2\times 28}{60}\right\}$$

$$= 18 + \frac{14}{15} = 18 + 0.93 = 18.93.$$

Case II. *Computation of Modal Age of Group B:*

In group B, class 18–20 has the maximum frequency.

So, 18–20 is the modal class and its frequency is 89.

$\therefore \quad x_k = 18,\ f_k = 89,\ f_{k-1} = 54,\ f_{k+1} = 40$ and $h = 2$.

$$\therefore \quad \text{mode, } M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 18 + \left\{2 \times \frac{(89-54)}{(2\times 89 - 54 - 40)}\right\} = 18 + \frac{(2\times 35)}{(178-94)}$$

$$= 18 + \frac{70}{84} = 18 + \frac{5}{6} = 18 + 0.83 = 18.83.$$

Hence, the modal ages of students in groups A and B are 18.93 years and 18.83 years respectively.

$\therefore$ (modal age in group A) > (modal age in group B).

EXAMPLE 7 *The following table shows the marks obtained by* 100 *students of Class X in a school during a particular academic session. Find the mode of this distribution.*

Marks	*Less than* 10	*Less than* 20	*Less than* 30	*Less than* 40	*Less than* 50	*Less than* 60	*Less than* 70	*Less than* 80
Number of students	7	21	34	46	66	77	92	100

[CBSE 2013]

SOLUTION Clearly, the above data can be written as:

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	14	13	12	20	11	15	8

Clearly, the modal class is 40–50, as it has the maximum frequency.

$\therefore \quad x_k = 40,\ f_k = 20,\ f_{k-1} = 12,\ f_{k+1} = 11,\ h = 10.$

$$\text{Mode, } M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 40 + \left\{10 \times \frac{(20-12)}{(2\times 20 - 12 - 11)}\right\}$$

$$= 40 + \left\{10 \times \frac{8}{(40-23)}\right\} = 40 + \left\{10 \times \frac{8}{17}\right\}$$

$$= 40 + \frac{80}{17} = 40 + 4.71 = 44.71.$$

Hence, mode = 44.71.

EXERCISE 18C

1. Find the mode of the following frequency distribution:

Marks	10–20	20–30	30–40	40–50	50–60
Frequency	12	35	45	25	13

[CBSE 2014]

2. Compute the mode of the following data:

Class	0–20	20–40	40–60	60–80	80–100
Frequency	25	16	28	20	5

[CBSE 2013]

3. Heights of students of Class X are given in the following frequency distribution:

Height (in cm)	150–155	155–160	160–165	165–170	170–175
Number of students	15	8	20	12	5

Find the modal height. [CBSE 2014]

Also, find the mean height. Compare and interpret the two measures of central tendency.

4. Find the mode of the following distribution:

Class interval	10–14	14–18	18–22	22–26	26–30	30–34	34–38	38–42
Frequency	8	6	11	20	25	22	10	4

5. Given below is the distribution of total household expenditure of 200 manual workers in a city.

Expenditure (in ₹)	No. of manual workers
1000–1500	24
1500–2000	40
2000–2500	31

2500–3000	28
3000–3500	32
3500–4000	23
4000–4500	17
4500–5000	5

Find the expenditure done by maximum number of manual workers.

6. Calculate the mode from the following data:

Monthly salary (in ₹)	No. of employees
0–5000	90
5000–10000	150
10000–15000	100
15000–20000	80
20000–25000	70
25000–30000	10

7. Compute the mode from the following data:

Age (in years)	0–5	5–10	10–15	15–20	20–25	25–30	30–35
Number of patients	6	11	18	24	17	13	5

8. Compute the mode from the following series:

Size	45–55	55–65	65–75	75–85	85–95	95–105	105–115
Frequency	7	12	17	30	32	6	10

9. Compute the mode of the following data:

Class interval	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
Frequency	3	8	13	18	28	20	13	8	6	4

10. The agewise participation of students in the Annual Function of a school is shown in the following distribution. [CBSE 2014]

Age (in years)	5–7	7–9	9–11	11–13	13–15	15–17	17–19
Number of students	x	15	18	30	50	48	x

Find the missing frequencies when the sum of frequencies is 181. Also, find the mode of the data.

ANSWERS (EXERCISE 18C)

1. 33.33 **2.** 52

3. Modal height = 163 cm. This means that the height of maximum number of students is 163 cm. Mean height = 161.17 cm. This means that on an average the height of a student of the class is 161.17 cm.

4. 28.5 **5.** ₹ 1820 **6.** ₹ 7727.27 **7.** 17.3 years

8. 81.5 **9.** 23.28

10. Each of two missing frequencies is 10; 11.18 years

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COMPARATIVE STUDY OF THE THREE MEASURES OF CENTRAL TENDENCY

The mean is the most frequently used measure of central tendency because it takes into account all the observations and lies between the largest and the smallest observations of the entire data. It also enables us to compare two or more distributions, e.g., by comparing the mean marks of students of different classes in a particular examination, we can conclude which class has a better performance. However, the mean is useful when there are no extreme values in the data set, i.e., if the data are normally distributed.

However, the median is considered a better measure of central tendency when there are a few extreme values that could greatly influence the mean and distort what might be considered typical, as in finding the typical productivity rate of workers, average wage in a country, etc., e.g., in a group of employees, if the salary of one is 10 times the mean salary of others then the mean salary of the group will be high. In such a case, the median may better represent the typical salary level of the group.

The mode is particularly useful for dealing with categorical data, where most popular item is to be ascertained, e.g., to find the most popular TV programme being watched, the consumer item in greatest demand, the colour of the vehicle used by most of the people, etc. It is particularly useful for the manufacturers. Thus, a shirt manufacturer is more interested in the modal size of shirts sold, so as to produce it in large numbers.

EMPIRICAL RELATIONSHIP BETWEEN THE THREE MEASURES OF CENTRAL TENDENCY

$$3(\text{Median}) = \text{Mode} + 2(\text{Mean})$$

MISCELLANEOUS QUESTIONS ON MEAN, MODE AND MEDIAN

EXAMPLE 100 *surnames were randomly picked up from a telephone directory and the distribution of the number of letters of the English alphabet in the surnames are obtained as follows:*

Number of letters	1–4	4–7	7–10	10–13	13–16	16–19
Number of surnames	6	30	40	16	4	4

Determine the median and mean number of letters in the surnames. Also, find the modal size of surnames. [CBSE 2014]

SOLUTION We have

Class	Class mark x_i	Frequency f_i	Cumulative frequency	$u_i = \frac{(x_i - A)}{h} = \frac{(x_i - 11.5)}{3}$	$(f_i u_i)$
1–4	2.5	6	6	–3	–18
4–7	5.5	30	36	–2	–60
7–10	8.5	40	76	–1	–40
10–13	11.5 = A	16	92	0	0
13–16	14.5	4	96	1	4
16–19	17.5	4	100	2	8
		$\Sigma f_i = 100$			$\Sigma(f_i u_i) = -106$

Clearly, $A = 11.5$, $h = 3$, $\Sigma f_i = 100$ and $\Sigma(f_i u_i) = -106$.

$$\text{Mean, } \overline{x} = A + \left\{h \times \frac{\Sigma(f_i u_i)}{\Sigma f_i}\right\} = 11.5 + \left\{3 \times \frac{(-106)}{100}\right\}$$

$$= 11.5 - 3.18 = 8.32.$$

Here, $N = 100 \Rightarrow \frac{N}{2} = 50$.

Cumulative frequency just greater than 50 is 76 and the corresponding class is 7–10. Thus, the median class is 7–10.

$\therefore \quad l = 7, h = 3, f = 40, cf$ = c.f. of preceding class = 36, $\frac{N}{2} = 50$.

$$\text{Median, } M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$$

$$= 7 + \left\{3 \times \frac{(50 - 36)}{40}\right\} = 7 + \left(3 \times \frac{14}{40}\right)$$

$$= 7 + \frac{21}{20} = 7 + 1.05 = 8.05.$$

$$\text{Mode} = 3(\text{median}) - 2(\text{mean})$$
$$= 3 \times 8.05 - 2 \times 8.32 = 24.15 - 16.64 = 7.51.$$

EXERCISE 18D

1. Find the mean, mode and median of the following frequency distribution:

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	4	4	7	10	12	8	5

[CBSE 2010]

2. Find the mean, median and mode of the following data:

Class	0–20	20–40	40–60	60–80	80–100	100–120	120–140
Frequency	6	8	10	12	6	5	3

[CBSE 2008]

3. Find the mean, median and mode of the following data:

Class	0–50	50–100	100–150	150–200	200–250	250–300	300–350
Frequency	2	3	5	6	5	3	1

[CBSE 2013]

4. Find the mode, median and mean for the following data:

Marks obtained	25–35	35–45	45–55	55–65	65–75	75–85
Number of students	7	31	33	17	11	1

[CBSE 2009]

5. A survey regarding the heights (in cm) of 50 girls of a class was conducted and the following data was obtained:

Height (in cm)	120–130	130–140	140–150	150–160	160–170	Total
Number of girls	2	8	12	20	8	50

Find the mean, median and mode of the above data. [CBSE 2008]

6. The following table gives the daily income of 50 workers of a factory:

Daily income (in ₹)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Find the mean, mode and median of the above data. [CBSE 2009]

7. The table below shows the daily expenditure on food of 30 households in a locality:

Daily expenditure (in ₹)	Number of households
100–150	6
150–200	7
200–250	12
250–300	3
300–350	2

Find the mean and median daily expenditure on food. [CBSE 2009C]

ANSWERS (EXERCISE 18D)

1. Mean = 38.2, median = 40, mode = 43.6
2. Mean = 62.4, median = 61.67, mode = 60.21
3. Mean = 169, median = 170.83, mode = 174.49
4. Mean = 49.70, median = 48.64, mode = 46.52
5. Mean = 149.8 cm, median = 152.5 cm, mode = 157.9 cm
6. Mean = ₹ 145.2, median = ₹ 137.5, mode = ₹ 122.1
7. Mean = ₹ 205, median = ₹ 208.33, mode = ₹ 214.99

CUMULATIVE FREQUENCY CURVE (OR OGIVE)

An ogive is a freehand graph showing the curve of a cumulative frequency distribution. The term 'ogive' is derived from the word 'ogee' meaning a shape consisting of a concave arc flowing into a convex arc, so forming an S-shaped curve with vertical ends.

Let a grouped frequency distribution be given to us.

FOR A 'LESS THAN' SERIES:

On a graph paper, we mark the upper class limits along the x-axis and the corresponding *cumulative frequencies* along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called *cumulative frequency polygon*.

(ii) On joining these points successively by smooth curves, we get a curve, known as *cumulative frequency curve,* or an *ogive*.

FOR A 'GREATER THAN' SERIES

On a graph paper, we mark the lower class limits along the x-axis and the corresponding *cumulative frequencies* along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called *cumulative frequency polygon*.

(ii) On joining these points successively by smooth curves, we get a curve, known as *cumulative frequency curve*, or an *ogive*.

HOW TO OBTAIN MEDIAN FROM CUMULATIVE FREQUENCY CURVE?

METHOD 1 We proceed stepwise as follows:

Step 1. Draw either the less than type or the more than type cumulative frequency curve for the given frequency distribution.

Step 2. Locate $\frac{N}{2}$ on the y-axis. Let it be P.

Step 3. From point P, draw a line PQ parallel to the x-axis cutting the curve at a point Q.

Step 4. From Q, draw QM perpendicular to the x-axis cutting the x-axis at M.

The point of intersection of this perpendicular with the x-axis, i.e., the abscissa or x-coordinate of M gives the median of the data.

METHOD 2 We proceed stepwise as follows:

Step 1. Draw both ogives (i.e., of the 'less than' type and of the 'more than' type) for the given frequency distribution on the same axis.

Step 2. The two ogives intersect each other at a point, say P.

Step 3. From P, draw PQ perpendicular to the x-axis meeting the x-axis at Q.

Clearly, the abscissa or x-coordinate of point Q gives the median of the data.

SOLVED EXAMPLES

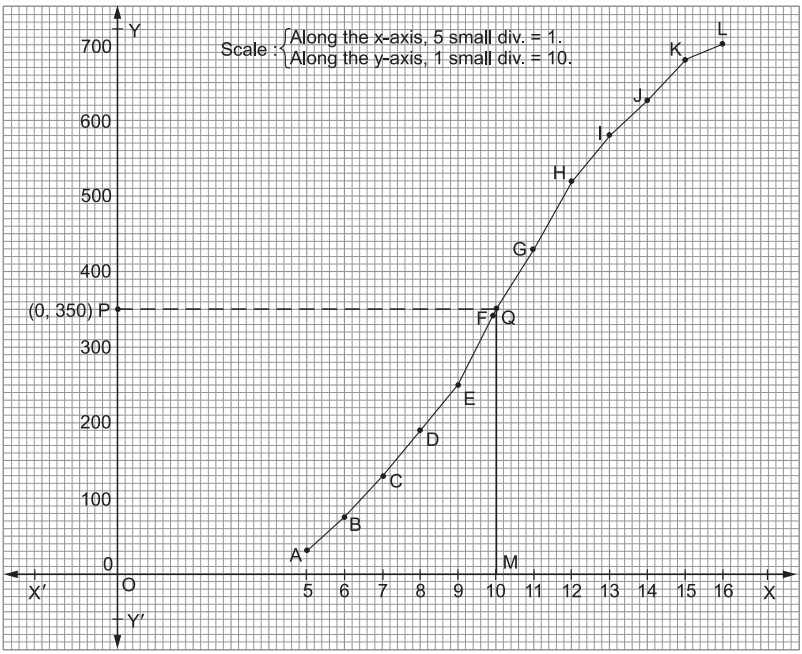
EXAMPLE 1 *Following is the age distribution of a group of students. Draw the cumulative frequency curve of 'less than' type and hence obtain the median value.*

Age (in years)	*Frequency*
4–5	36
5–6	42
6–7	52
7–8	60
8–9	68
9–10	84
10–11	96
11–12	82
12–13	66
13–14	48
14–15	50
15–16	16

SOLUTION From the given table, we may prepare the ('less than' type) cumulative frequency table as shown below:

Age (in years)	*cf*
Less than 5	36
Less than 6	78
Less than 7	130
Less than 8	190
Less than 9	258
Less than 10	342
Less than 11	438
Less than 12	520
Less than 13	586
Less than 14	634
Less than 15	684
Less than 16	700

On a graph paper, we take the scale.



Scale: $\begin{cases} \text{Along the } x\text{-axis, 5 small div.} = 1. \\ \text{Along the } y\text{-axis, 1 small div.} = 10. \end{cases}$

And, plot the points $A(5, 36)$, $B(6, 78)$, $C(7, 130)$, $D(8, 190)$, $E(9, 258)$, $F(10, 342)$, $G(11, 438)$, $H(12, 520)$, $I(13, 586)$, $J(14, 634)$, $K(15, 684)$ and $L(16, 700)$.

We join freehand these points successively a to get the cumulative frequency curve, or an ogive.

Here, $N = 700 \Rightarrow \frac{N}{2} = 350$.

Take a point $P(0, 350)$ on the y-axis and draw $PQ \parallel x$-axis, meeting the curve at Q. Draw $QM \perp x$-axis, intersecting the x-axis at M whose coordinates are (10, 0).

Hence, median = 10 years.

EXAMPLE 2 *For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.*

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	5	15	20	23	17	11	9

SOLUTION From the given table, we may prepare the 'more than' series as shown below:

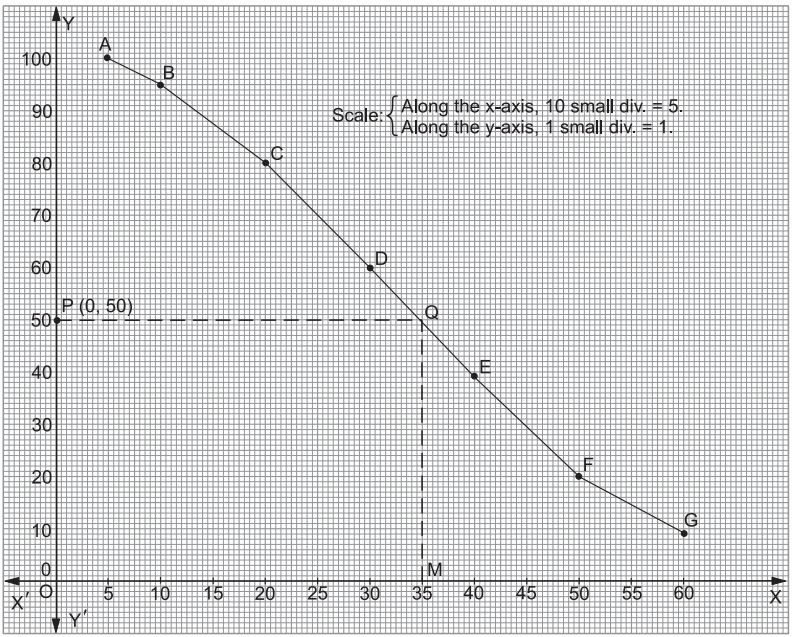
More than 60	9
More than 50	20
More than 40	37
More than 30	60
More than 20	80
More than 10	95
More than 5	100

Scale: $\begin{cases} \text{Along the } x\text{-axis, 10 small div.} = 5. \\ \text{Along the } y\text{-axis, 1 small div.} = 1. \end{cases}$

Now plot the points $A(5, 100)$, $B(10, 95)$, $C(20, 80)$, $D(30, 60)$, $E(40, 37)$, $F(50, 20)$ and $G(60, 9)$ on a graph paper.

Join AB, BC, CD, DE, EF and FG freehand to get the required curve, as shown below.

Here, $N = 100 \Rightarrow \frac{N}{2} = 50$.



From $P(0, 50)$ draw $PQ \parallel x$-axis, meeting the curve at Q. Draw $QM \perp OX$, meeting x-axis at M whose coordinates are $(35, 0)$.

Hence, median = 35.

EXAMPLE 3 *The following table gives production yield per hectare of wheat of* 100 *farms of a village:*

Production yield (kg/ha)	40–45	45–50	50–55	55–60	60–65	65–70
Number of farms	4	6	16	20	30	24

Change the distribution to a 'more than' type distribution and draw its ogive. [CBSE 2009C, '13]

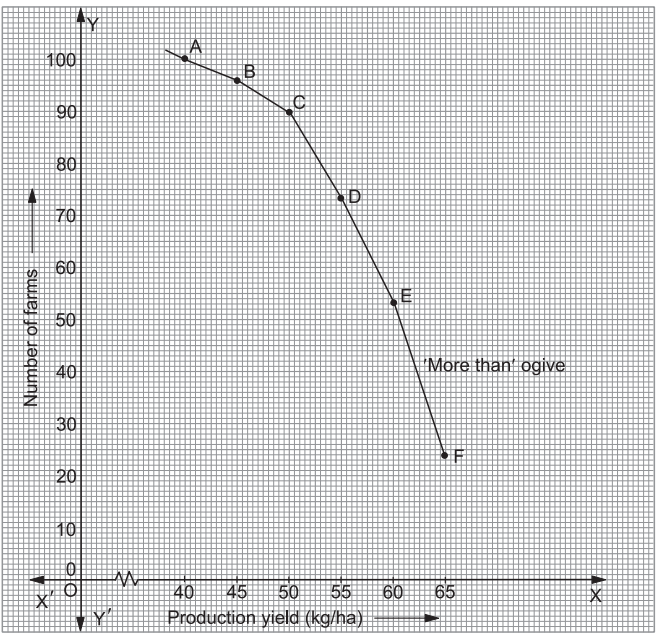
SOLUTION We may prepare the 'more than' series as shown below:

More than 65	24
More than 60	54
More than 55	74
More than 50	90
More than 45	96
More than 40	100

Scale: $\begin{cases}\text{Along the } x\text{-axis, 1 small div.} = 1.\\ \text{Along the } y\text{-axis, 1 small div.} = 1.\end{cases}$

On a graph paper, we plot the points $A(40, 100)$, $B(45, 96)$, $C(50, 90)$, $D(55, 74)$, $E(60, 54)$ and $F(65, 24)$.

Join AB, BC, CD, DE and EF freehand to get a '*More Than Ogive*'.



EXAMPLE 4 *During the medical check-up of* 35 *students of a class their weights were recorded as follows:*

Weight (in kg*)*	38–40	40–42	42–44	44–46	46–48	48–50	50–52
No. of students	3	2	4	5	14	4	3

Draw a less than type and a more than type ogive from the given data. Hence, obtain the median weight from the graph. [CBSE 2009]

SOLUTION (i) Less Than Series:

We may prepare the less than series as under.

Weight (in kg)	Number of students
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Scale: $\begin{cases} \text{Along the } x\text{-axis, 5 small div.} = 1 \text{ kg.} \\ \text{Along the } y\text{-axis, 10 small div.} = 5 \text{ students.} \end{cases}$

Now, plot the points $A(40, 3)$, $B(42, 5)$, $C(44, 9)$, $D(46, 14)$, $E(48, 28)$, $F(50, 32)$ and $G(52, 35)$ on a graph paper.

Join AB, BC, CD, DE, EF and FG freehand to get the curve representing '*Less Than Series*'.

(ii) More Than Series:

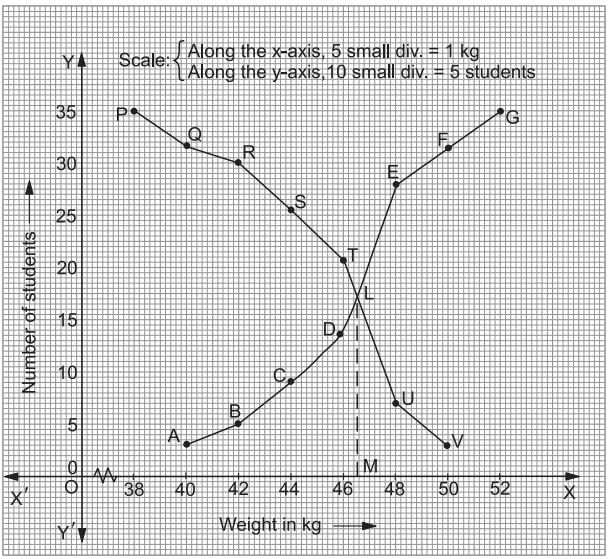
We may prepare the more than series as under.

Weight (in kg)	Number of students
More than 38	35
More than 40	32
More than 42	30
More than 44	26

More than 46	21
More than 48	7
More than 50	3

On the same graph paper as above, we plot the points $P(38, 35)$, $Q(40, 32)$, $R(42, 30)$, $S(44, 26)$, $T(46, 21)$, $U(48, 7)$ and $V(50, 3)$.

Join *PQ, QR, RS, ST, TU* and *UV* freehand to get the curve representing '*More Than Series*'.



The two curves intersect at the point *L*. Draw $LM \perp OX$.

$\therefore$ median weight = OM = 46.5 kg.

EXAMPLE 5 *The table given below shows the frequency distribution of the scores obtained by* 200 *candidates in a BCA entrance examination.*

Score	200–250	250–300	300–350	350–400	400–450	450–500	500–550	550–600
No. of candidates	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) 'less than' series and (ii) 'more than' series.

Hence, find the median.

SOLUTION (i) Less Than Series:

We may prepare the less than series as under.

Score	Number of candidates
Less than 250	30
Less than 300	45
Less than 350	90
Less than 400	110
Less than 450	135
Less than 500	175
Less than 550	185
Less than 600	200

Scale: $\begin{cases}\text{Along the } x\text{-axis, 1 small div. = 5.}\\ \text{Along the } y\text{-axis, 1 small div. = 2.}\end{cases}$

We plot the points $A(250, 30)$, $B(300, 45)$, $C(350, 90)$, $D(400, 110)$, $E(450, 135)$, $F(500, 175)$, $G(550, 185)$ and $H(600, 200)$ on a graph paper.

Join AB, BC, CD, DE, EF, FG and GH freehand to get the curve representing '*Less Than Series*'.

(ii) More Than Series:

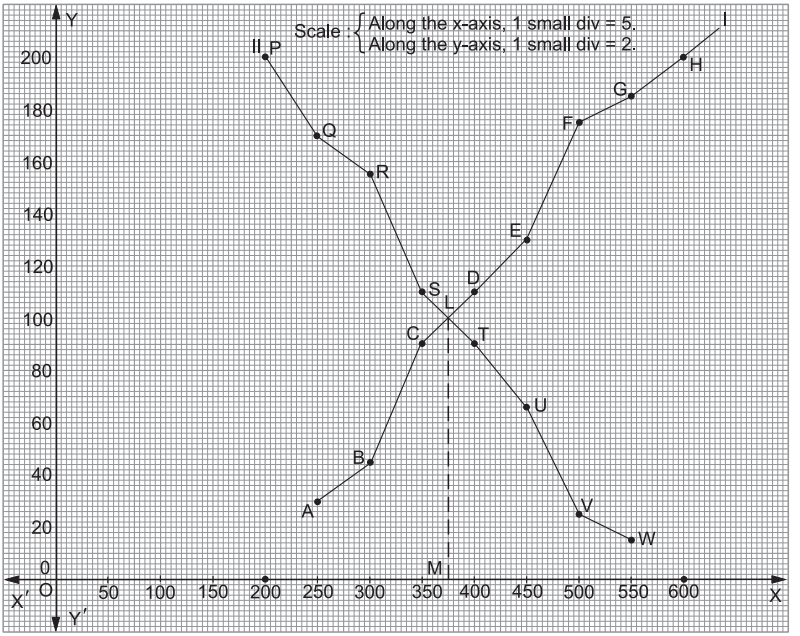
We may prepare the more than series as under.

Score	Number of candidates
Less than 200	200
More than 250	170
More than 300	155
More than 350	110
More than 400	90
More than 450	65
More than 500	25
More than 550	15

Now, on the same graph paper as above, we plot the points $P(200, 200)$, $Q(250, 170)$, $R(300, 155)$, $S(350, 110)$, $T(400, 90)$, $U(450, 65)$, $V(500, 25)$, $W(550, 15)$.

Join PQ, QR, RS, ST, TU, UV and VW freehand to get the curve representing '*More Than Series*'.

The two curves intersect at the point L. Draw $LM \perp OX$, cutting the x-axis at M.



Clearly, M represents 375.

Hence, median = 375.

EXERCISE 18E

1. Find the median of the following data by making a 'less than ogive'.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Number of students	5	3	4	3	3	4	7	9	7	8

[CBSE 2014]

2. The given distribution shows the number of wickets taken by the bowlers in one-day international cricket matches:

Number of wickets	Less than 15	Less than 30	Less than 45	Less than 60	Less than 75	Less than 90	Less than 105	Less than 120
Number of bowlers	2	5	9	17	39	54	70	80

Draw a 'less than type' ogive from the above data. Find the median.

[CBSE 2014]

3. Draw a 'more than' ogive for the data given below which gives the marks of 100 students.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Number of students	4	6	10	10	25	22	18	5

[CBSE 2013]

4. The heights of 50 girls of Class X of a school are recorded as follows:

Height (in cm)	135–140	140–145	145–150	150–155	155–160	160–165
Number of girls	5	8	9	12	14	2

Draw a 'more than type' ogive for the above data. [CBSE 2014]

5. The monthly consumption of electricity (in units) of some families of a locality is given in the following frequency distribution:

Monthly consumption (in units)	140–160	160–180	180–200	200–220	220–240	240–260	260–280
Number of families	3	8	15	40	50	30	10

Prepare a 'more than type' ogive for the given frequency distribution.

[CBSE 2014]

6. The following table gives the production yield per hectare of wheat of 100 farms of a village.

Production yield (kg/ha)	50–55	55–60	60–65	65–70	70–75	75–80
Number of farms	2	8	12	24	38	16

Change the distribution to a 'more than type' distribution and draw its ogive. Using ogive, find the median of the given data. [CBSE 2013]

7. The table given below shows the weekly expenditures on food of some households in a locality.

Weekly expenditure (in ₹)	Number of households
100–200	5
200–300	6
300–400	11
400–500	13
500–600	5

600–700	4
700–800	3
800–900	2

Draw a 'less than type ogive' and a 'more than type ogive' for this distribution.

8. From the following frequency distribution, prepare the 'more than' ogive.

Score	Number of candidates
400–450	20
450–500	35
500–550	40
550–600	32
600–650	24
650–700	27
700–750	18
750–800	34
Total	230

Also, find the median.

9. The marks obtained by 100 students of a class in an examination are given below:

Marks	Number of students
0–5	2
5–10	5
10–15	6
15–20	8
20–25	10
25–30	25
30–35	20
35–40	18
40–45	4
45–50	2

Draw cumulative frequency curves by using (i) 'less than' series and (ii) 'more than' series.

Hence, find the median.

10. From the following data, draw the two types of cumulative frequency curves and determine the median.

Height (in cm)	Frequency
140–144	3
144–148	9
148–152	24
152–156	31
156–160	42
160–164	64
164–168	75
168–172	82
172–176	86
176–180	34

.....

SUMMARY OF THE RESULTS AND FORMULAE

1. MEAN OF THE GROUPED DATA

(i) DIRECT METHOD

$\textbf{\textit{Mean}}\ (\overline{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$, where $x_i = \dfrac{1}{2}$ (lower limit + upper limit) of ith class interval and f_i is its frequency.

(ii) $\Sigma f_i (x_i - \overline{x}) = 0$.

(iii) ASSUMED-MEAN METHOD

$\textbf{\textit{Mean}}\ (\overline{x}) = A + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$, where A is the assumed mean and $d_i = (x_i - A)$ for each i.

(iv) STEP-DEVIATION METHOD

$\textbf{\textit{Mean}}\ (\bar{x}) = A + \left\{h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right\}$, where A = assumed mean, h = class size and $u_i = \frac{(x_i - A)}{h}$.

2. (i) MEDIAN CLASS

Let $N = \Sigma f_i$. Then, the class whose cumulative frequency is just greater than $\left(\frac{N}{2}\right)$ is the median class.

(ii) MEDIAN FOR GROUPED DATA

$\textbf{\textit{Median}}\ (M_e) = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$, where l = lower limit of the median class; h = width of the median class; f = frequency of the median class; $N = \Sigma f_i$; cf = cumulative frequency of the class preceding the median class.

3. *Mode* = 3(*median*) – 2(*mean*).

4. (i) MODAL CLASS

The class having maximum frequency is called the modal class.

(ii) $\textbf{\textit{Mode}}\ (M_o) = x_k + h \cdot \left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$, where x_k = lower limit of the modal class interval; f_k = frequency of the modal class; f_{k-1} = frequency of the class preceding the modal class; f_{k+1} = frequency of the class succeeding the modal class; h = width of the class interval.

5. OGIVES (OR CUMULATIVE FREQUENCY CURVES)

(i) FOR 'LESS THAN' SERIES

On a graph paper, we mark the ***upper class limits*** along x-axis and the corresponding ***cumulative frequencies*** along y-axis. Join these points successively to obtain a curve, called ***ogive for less than series***.

(ii) FOR 'GREATER THAN' SERIES

On the same graph paper as above, we mark the lower class limits along x-axis and the corresponding cumulative frequencies along y-axis. Join these points successively to obtain a curve, called ***ogive for greater than series***.

(iii) *The abscissa of the point of intersection of the two ogives gives us the* ***median.***

EXERCISE 18F

Very-Short-Answer Questions

1. Write the median class of the following distribution:

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	4	4	8	10	12	8	4

[CBSE 2009]

2. What is the lower limit of the modal class of the following frequency distribution?

Age (in years)	0–10	10–20	20–30	30–40	40–50	50–60
Number of patients	16	13	6	11	27	18

[CBSE 2009]

3. The monthly pocket money of 50 students of a class are given in the following distribution:

Monthly pocket money (in ₹)	0–50	50–100	100–150	150–200	200–250	250–300
Number of students	2	7	8	30	12	1

Find the modal class and also give class mark of the modal class.

[CBSE 2014]

4. A data has 25 observations arranged in a descending order. Which observation represents the median? [CBSE 2014]
5. For a certain distribution, mode and median were found to be 1000 and 1250 respectively. Find mean for this distribution using an empirical relation. [CBSE 2014]
6. In a class test, 50 students obtained marks as follows:

Marks obtained	0–20	20–40	40–60	60–80	80–100
Number of students	4	6	25	10	5

Find the modal class and the median class. [CBSE 2014]

7. Find the class marks of classes 10–25 and 35–55. [CBSE 2008]

8. While calculating the mean of a given data by the assumed-mean method, the following values were obtained:
   $A = 25,\ \Sigma f_i d_i = 110,\ \Sigma f_i = 50.$
   Find the mean.
9. The distributions X and Y with total number of observations 36 and 64, and mean 4 and 3 respectively are combined. What is the mean of the resulting distribution $X + Y$?
10. In a frequency distribution table with 12 classes, the class-width is 2.5 and the lowest class boundary is 8.1, then what is the upper class boundary of the highest class?
11. The observations 29, 32, 48, 50, x, $x + 2$, 72, 78, 84, 95 are arranged in ascending order. What is the value of x if the median of the data is 63?
12. The median of 19 observations is 30. Two more observations are made and the values of these are 8 and 32. Find the median of the 21 observations taken together.
    HINT Since 8 is less than 30 and 32 is more than 30, so the value of median (middle value) remains unchanged.
13. If the median of $\frac{x}{5}, \frac{x}{4}, \frac{x}{2}, x$ and $\frac{x}{3}$, where $x > 0$, is 8, find the value of x.
    HINT Arranging the observations in ascending order, we have $\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$.
    Median $= \frac{x}{3} = 8$.
14. What is the cumulative frequency of the modal class of the following distribution?

Class	3–6	6–9	9–12	12–15	15–18	18–21	21–24
Frequency	7	13	10	23	4	21	16

Short-Answer Questions

15. Find the mode of the given data:

Class interval	0–20	20–40	40–60	60–80
Frequency	15	6	18	10

[CBSE 2015]

16. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
Number of patients	60	42	55	70	53	20

Form a 'less than type' cumulative frequency distribution. [CBSE 2013]

17. In the following data, find the values of p and q. Also, find the median class and modal class.

Class	Frequency (f)	Cumulative frequency (cf)
100–200	11	11
200–300	12	p
300–400	10	33
400–500	q	46
500–600	20	66
600–700	14	80

[CBSE 2013]

18. The following frequency distribution gives the monthly consumption of electricity of 64 consumers of a locality.

Monthly consumption (in units)	65–85	85–105	105–125	125–145	145–165	165–185
Number of consumers	4	5	13	20	14	8

Form a 'more than type' cumulative frequency distribution.

19. The following table gives the life-time (in days) of 100 electric bulbs of a certain brand.

Life-time (in days)	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300
Number of bulbs	7	21	52	79	91	100

From this table, construct the frequency distribution table.

20. The following table gives the frequency distribution of the percentage of marks obtained by 2300 students in a competitive examination.

Marks obtained (in per cent)	11–20	21–30	31–40	41–50	51–60	61–70	71–80
Number of students	141	221	439	529	495	322	153

(a) Convert the given frequency distribution into the continuous form.

(b) Find the median class and write its class mark.

(c) Find the modal class and write its cumulative frequency.

21. If the mean of the following distribution is 27, find the value of p.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	8	p	12	13	10

22. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Age (in years)	0–10	10–20	20–30	30–40	40–50
Number of persons	5	25	?	18	7

ANSWERS (EXERCISE 18F)

1. 30–40 **2.** 40 **3.** 150–200, 175 **4.** 13th **5.** 1375
6. Modal class → 40–60, median class → 40–60 **7.** 17.5, 45 **8.** 27.2
9. 3.36 **10.** 38.1 **11.** 62 **12.** 30 **13.** 24 **14.** 53 **15.** 52
17. $p = 23$, $q = 13$; median class → 400–500, modal class → 500–600
20. (b) 40.5–50.5, 45.5 (c) 40.5–50.5, 1330 **21.** $p = 7$ **22.** 25

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. Which of the following is not a measure of central tendency? [CBSE 2013]
   (a) Mean (b) Mode (c) Median (d) Range
2. Which of the following cannot be determined graphically?
   (a) Mean (b) Median (c) Mode (d) None of these
3. Which of the following measures of central tendency is influenced by extreme values?
   (a) Mean (b) Median (c) Mode (d) None of these
4. The mode of a frequency distribution is obtained graphically from
   (a) a frequency curve (b) a frequency polygon
   (c) a histogram (d) an ogive
5. The median of a frequency distribution is found graphically with the help of
   (a) a histogram (b) a frequency curve
   (c) a frequency polygon (d) ogives

6. The cumulative frequency table is useful in determining the
   (a) mean (b) median
   (c) mode (d) all of these
7. The abscissa of the point of intersection of the Less Than Type and of the More Than Type cumulative frequency curves of a grouped data gives its
   (a) mean (b) median
   (c) mode (d) none of these
8. If x_i's are the midpoints of the class intervals of a grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean then $\Sigma f_i(x_i - \overline{x}) = ?$
   (a) 1 (b) 0 (c) –1 (d) 2
9. For finding the mean by using the formula, $\overline{x} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$, we have $u_i = ?$
   (a) $\frac{(A - x_i)}{h}$ (b) $\frac{(x_i - A)}{h}$ (c) $\frac{(A + x_i)}{h}$ (d) $h(x_i - A)$
10. In the formula, $\overline{x} = \left\{A + \frac{\Sigma f_i d_i}{\Sigma f_i}\right\}$ for finding the mean of the grouped data, the d_i's are the deviations from A of
    (a) lower limits of the classes (b) upper limits of the classes
    (c) midpoints of the classes (d) none of these
11. While computing the mean of the grouped data, we assume that the frequencies are
    (a) evenly distributed over the classes
    (b) centred at the class marks of the classes
    (c) centred at the lower limits of the classes
    (d) centred at the upper limits of the classes
12. The relation between mean, mode and median is
    (a) mode = (3 × mean) – (2 × median)
    (b) mode = (3 × median) – (2 × mean)
    (c) median = (3 × mean) – (2 × mode)
    (d) mean = (3 × median) – (2 × mode)
13. If the 'less than type' ogive and 'more than type' ogive intersect each other at (20.5, 15.5) then the median of the given data is [CBSE 2013]
    (a) 5.5 (b) 15.5 (c) 20.5 (d) 36.0

14. Consider the frequency distribution of the heights of 60 students of a class:

Height (in cm)	No. of Students	Cumulative Frequency
150–155	16	16
155–160	12	28
160–165	9	37
165–170	7	44
170–175	10	54
175–180	6	60

The sum of the lower limit of the modal class and the upper limit of the median class is

(a) 310 (b) 315 (c) 320 (d) 330

15. Consider the following frequency distribution:

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	3	9	15	30	18	5

The modal class is

(a) 10–20 (b) 20–30 (c) 30–40 (d) 50–60

16. Mode = ?

(a) $x_k + h \cdot \left\{ \frac{(f_{k-1} - f_k)}{(2f_k - f_{k-1} - f_{k+1})} \right\}$ (b) $x_k + h \cdot \left\{ \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})} \right\}$

(c) $x_k + h \cdot \left\{ \frac{(f_k - f_{k-1})}{(f_k - 2f_{k-1} - f_{k+1})} \right\}$ (d) $x_k + h \cdot \left\{ \frac{(f_k - f_{k-1})}{(f_k - f_{k-1} - 2f_{k+1})} \right\}$

17. Median = ?

(a) $l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$ (b) $l + \left\{ h \times \frac{\left(cf - \frac{N}{2}\right)}{f} \right\}$

(c) $l - \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$ (d) none of these

18. If the mean and median of a set of numbers are 8.9 and 9 respectively then the mode will be

(a) 7.2 (b) 8.2 (c) 9.2 (d) 10.2

19. Look at the frequency distribution table given below:

Class interval	35–45	45–55	55–65	65–75
Frequency	8	12	20	10

The median of the above distribution is

(a) 56.5 (b) 57.5 (c) 58.5 (d) 59

20. Consider the following table:

Class interval	10–14	14–18	18–22	22–26	26–30
Frequency	5	11	16	25	19

The mode of the above data is

(a) 23.5 (b) 24 (c) 24.4 (d) 25

21. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is

(a) 22 (b) 23.5 (c) 24 (d) 24.5

22. The median and mode of a frequency distribution are 26 and 29 respectively. Then, the mean is

(a) 27.5 (b) 24.5 (c) 28.4 (d) 25.8

23. For a symmetrical frequency distribution, we have

(a) mean < mode < median (b) mean > mode > median

(c) mean = mode = median (d) mode = $\frac{1}{2}$ (mean + median)

24. Look at the cumulative frequency distribution table given below:

Monthly income	Number of families
More than ₹ 10000	100
More than ₹ 14000	85
More than ₹ 18000	69
More than ₹ 20000	50
More than ₹ 25000	37
More than ₹ 30000	15

Number of families having income range 20000 to 25000 is

(a) 19 (b) 16 (c) 13 (d) 22

25. The median of first 8 prime numbers is

(a) 7 (b) 9 (c) 11 (d) 13

26. The mean of 20 numbers is zero. Of them, at the most, how many may be greater than zero?

(a) 0 (b) 1 (c) 10 (d) 19

27. If the median of the data $4, 7, x-1, x-3, 16, 25$, written in ascending order, is 13 then x is equal to

(a) 13 (b) 14 (c) 15 (d) 16

28. The mean of 2, 7, 6 and x is 15 and the mean of 18, 1, 6, x and y is 10. What is the value of y?

(a) 5 (b) 10 (c) 20 (d) 30

Matching of columns

29. Match the following columns:

Column I	Column II
(a) The most frequent value in a data is known as …… .	(p) standard deviation
(b) Which of the following cannot be determined graphically out of mean, mode and median?	(q) median
(c) An ogive is used to determine …… .	(r) mean
(d) Out of mean, mode, median and standard deviation, which is not a measure of central tendency?	(s) mode

The correct answer is:

(a) –……, (b) –……, (c) –……, (d) –…… .

Assertion-and-Reason Type

Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

30.

Assertion (A)	Reason (R)
If the median and mode of a frequency distribution are 150 and 154 respectively, then its mean is 148.	Mean, median and mode of a frequency distribution are related as: mode = 3 median – 2 mean.

The correct answer is: (a)/(b)/(c)/(d).

31.

Assertion (A)	Reason (R)
Consider the following frequency distribution: Class interval: 3–6, 6–9, 9–12, 12–15, 15–18, 18–21 Frequency: 2, 5, 21, 23, 10, 12 The mode of the above data is 12.4.	The value of the variable which occurs most often is the mode.

The correct answer is: (a)/(b)/(c)/(d).

ANSWERS (MCQ)

1. (d) **2.** (a) **3.** (a) **4.** (c) **5.** (d) **6.** (b) **7.** (b) **8.** (b) **9.** (b)
10. (c) **11.** (b) **12.** (b) **13.** (c) **14.** (b) **15.** (c) **16.** (b) **17.** (a) **18.** (c)
19. (b) **20.** (c) **21.** (c) **22.** (b) **23.** (c) **24.** (c) **25.** (b) **26.** (d) **27.** (c)
28. (c) **29.** (a)–(s), (b)–(r), (c)–(q), (d)–(p) **30.** (a) **31.** (b)

HINTS TO SOME SELECTED QUESTIONS

1. Range is not a measure of central tendency.
2. Mean cannot be determined graphically.
3. Only mean is affected by extreme values, while both median and mode remain unaffected.
4. Mode of a frequency distribution can be obtained graphically from a histogram.
6. The cumulative frequency table is useful in determining the median.
8. Clearly, we have $\Sigma f_i(x_i - \overline{x}) = 0$.
9. We have $u_i = \dfrac{(x_i - A)}{h}$.
10. d_i's are deviations from A of midpoints of the classes.
11. In computing the mean of the grouped data, we assume that the frequencies are centred at the class marks of the classes.
13. Clearly, the abscissa of the point of intersection of both the ogives gives the median.
14. The class having maximum frequency is the modal class.
So, the modal class is 150–155. Its lower limit is 150.
Also, $N = 60 \Rightarrow \dfrac{N}{2} = 30$ and the cumulative frequency just more than 30 is 37. Its class is 160–165, whose upper limit is 165.
Required sum = (150 + 165) = 315.

15. The class 30–40 has maximum frequency. So, the modal class is 30–40.

18. Mode = (3 median) – (2 mean) = $(3 \times 9 - 2 \times 8.9) = (27 - 17.8) = 9.2$.

19. We have:

Class interval	35–45	45–55	55–65	65–75
Frequency	8	12	20	10
Cumulative frequency	8	20	40	50

Here, $N = 50 \Rightarrow \frac{N}{2} = 25$, which lies in class interval 55–65.

$$\text{Median} = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\} = 55 + \frac{(65-55)}{20} \times (25 - 20) = 57.5.$$

20. The maximum frequency is 25 and the modal class is 22–26.

$\therefore \quad x_k = 22,\ f_k = 25,\ f_{k-1} = 16,\ f_{k+1} = 19$ and $h = 4$.

$$\therefore \quad \text{mode} = x_k + h \cdot \left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= \left\{22 + 4 \times \frac{(25-16)}{(50-16-19)}\right\} = \left(22 + 4 \times \frac{9}{15}\right) = \left(22 + \frac{12}{5}\right) = (22 + 2.4) = 24.4.$$

21. $3 \times$ median = (mode + 2 mean) = $(16 + 2 \times 28) = 72$

$\Rightarrow \quad \text{median} = \frac{72}{3} = 24.$

22. Mode = (3 median) – (2 mean)

$\Rightarrow$ 2 mean = (3 median) – (mode) = $(3 \times 26) - 29 = 49$

$\Rightarrow \quad \text{mean} = \frac{49}{2} = 24.5.$

23. We must have, mean = mode = median.

25. First 8 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19.

Required median = mean of fourth and fifth observations

$$= \frac{7+11}{2} = 9.$$

26. Mean of 20 numbers = 0.

$\therefore$ sum of 20 numbers = $0 \times 20 = 0$.

It is possible that 19 of these numbers may be positive and if their sum is a, the 20th number is $(-a)$.

27. Clearly, median of 6 observations

= mean of 3rd and 4th observations

$= \frac{(x-1)+(x-3)}{2} = \frac{2x-4}{2} = x-2.$

$\therefore \quad x-2=13$ or $x=13+2=15.$

28. We have $\left(\frac{2+7+6+x}{4}\right)=5$ or $15+x=20$ or $x=5.$

Also, $\left(\frac{18+1+6+x+y}{5}\right)=10$ or $25+5+y=50$ or $y=20.$

30. Reason (R) is clearly true.

Using the relation given in Reason (R), we have

2 mean = (3 median) – (mode) = $(3\times150)-(154)=450-154=296.$

$\therefore$ mean = 148, which is true.

Thus, Assertion (A) and Reason (R) are both true and Reason (R) is the correct explanation of Assertion (A).

Hence, the correct answer is (a).

31. Reason (R) is clearly true.

The maximum frequency is 23 and the modal class is 12–15.

$\therefore \quad x_k=12,\ f_k=23,\ f_{k-1}=21,\ f_{k+1}=10$ and $h=3.$

$\therefore$ mode $= \left\{12+3\times\frac{(23-21)}{(2\times23-21-10)}\right\} = \left(12+3\times\frac{2}{15}\right)=12.4.$

$\therefore$ Assertion (A) is true.

But, Reason (R) is not a correct explanation of Assertion (A).

Hence, the correct answer is (b).

TEST YOURSELF

MCQ

1. Which one of the following measures is determined only after the construction of cumulative frequency distribution?
   (a) Mean (b) Median (c) Mode (d) None of these
2. If the mean of a data is 27 and its median is 33 then the mode is
   (a) 30 (b) 43 (c) 45 (d) 47
3. Consider the following distribution:

Class	0–5	5–10	10–15	15–20	20–25
Frequency	10	15	12	20	9

The sum of the lower limits of the median class and the modal class is
(a) 15 (b) 25 (c) 30 (d) 35

4. Consider the following frequency distribution:

Class	0–5	6–11	12–17	18–23	24–29
Frequency	13	10	15	8	11

The upper limit of the median class is

(a) 16.5 (b) 18.5 (c) 18 (d) 17.5

Very-Short-Answer Questions

5. If the mean and mode of a frequency distribution be 53.4 and 55.2 respectively, find the median.
6. In the table given below, the times taken by 120 athletes to run a 100-m-hurdle race are given.

Class	13.8–14	14–14.2	14.2–14.4	14.4–14.6	14.6–14.8	14.8–15
Frequency	2	4	15	54	25	20

Find the number of athletes who completed the race in less than 14.6 seconds.

7. Consider the following frequency distribution:

Class	0–5	6–11	12–17	18–23	24–29
Frequency	13	10	15	8	11

Find the upper limit of the median class.

8. The annual profits earned by 30 shops of a shopping complex in a locality are recorded in the table shown below:

Profit (in lakhs ₹)	Number of shops
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

If we draw the frequency distribution table for the above data, find the frequency corresponding to the class 20–25.

Short-Answers Questions

9. Find the mean of the following frequency distribution:

Class	1–3	3–5	5–7	7–9
Frequency	9	22	27	18

10. The maximum bowling speeds (in km/hr) of 33 players at a cricket coaching centre are given below:

Speed in km/hr	85–100	100–115	115–130	130–145
No. of players	10	4	7	9

Calculate the median bowling speed.

11. The arithmetic mean of the following frequency distribution is 50.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	16	p	30	32	14

Find the value of p.

12. Find the median of the following frequency distribution:

Marks	0–10	10–20	20–30	30–40	40–50
Number of students	6	16	30	9	4

13. Following is the distribution of marks of 70 students in a periodical test:

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
Number of students	3	11	28	48	70

Draw a cumulative frequency curve for the above data.

14. Find the median of the following data.

Class interval	0–10	10–20	20–30	30–40	40–50	Total
Frequency	8	16	36	34	6	100

[CBSE 2014]

15. For the following distribution draw a 'less than type' ogive and from the curve find the median.

Marks obtained	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90	Less than 100
Number of students	2	7	17	40	60	82	85	90	100

[CBSE 2014]

16. The median value for the following frequency distribution is 35 and the sum of all the frequencies is 170. Using the formula for median, find the missing frequencies.

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	10	20	?	40	?	25	15

17. Find the missing frequencies f_1 and f_2 in the table given below, it being given that the mean of the given frequency distribution is 50.

Class	0–20	20–40	40–60	60–80	80–100	Total
Frequency	17	f_1	32	f_2	19	120

18. Find the mean of the following frequency distribution using step-deviation method:

Class	84–90	90–96	96–102	102–108	108–114	114–120
Frequency	15	22	20	18	20	25

Long-Answer Questions

19. Find the mean, median and mode of the following data:

Class	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	6	8	10	15	5	4	2

20. Draw 'less than ogive' and 'more than ogive' on a single graph paper and hence find the median of the following data:

Class interval	5–10	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	2	12	2	4	3	4	3

[CBSE 2014]

21. The production yield per hectare of wheat of some farms of a village are given in the following table:

Production yield (in kg/ha)	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80	80–85
Number of farms	1	9	15	18	40	26	16	14	10

Draw a less than type ogive and a more than type ogive for this data.

[CBSE 2014]

22. The following table gives the marks obtained by 50 students in a class test:

Marks	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
Number of students	2	3	6	7	14	12	4	2

Calculate the mean, median and mode for the above data.

ANSWERS (TEST YOURSELF)

1. (b) **2.** (c) **3.** (b) **4.** (d) **5.** 54

6. 75 **7.** 17.5 **8.** 4 **9.** 5.42

10. 117.1 km/hr **11.** 28 **12.** 24 **14.** 27.22 **16.** 35, 25

17. $f_1 = 28$, $f_2 = 24$ **18.** 102.75

19. Mean = 30, median = 30.67, mode = 32.01

22. Mean = 32, median = 33, mode = 35

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