2 Polynomials

POLYNOMIALS *An expression of the form* $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, *where* $a_n \neq 0$, *is called a polynomial in x of degree n.*

Here $a_0, a_1, a_2, \ldots, a_n$ are real numbers and each power of x is a non-negative integer.

EXAMPLES (i) $3x + 5$ is a polynomial in x of degree 1.

(ii) $8x^2 - 5x + 3$ is a polynomial in x of degree 2.

(iii) $2y^3 + \frac{4}{9}y^2 - 5y + \sqrt{3}$ is a polynomial in y of degree 3.

(iv) $3z^4 - 5z^3 + 2z^2 - 8z + 1$ is a polynomial in z of degree 4.

REMARK Note that $(\sqrt{x} + 3)$, $\frac{1}{(x+5)}$, $\frac{6}{(x^2 - 3x + 1)}$, etc., are not polynomials.

LINEAR POLYNOMIAL *A polynomial of degree* 1 *is called a linear polynomial.*

A linear polynomial is of the form $p(x) = ax + b$, *where* $a \neq 0$.

Thus, $(3x - 5)$, $(\sqrt{2}x + 3)$, $\left(x - \frac{5}{8}\right)$, etc., are all linear polynomials.

QUADRATIC POLYNOMIAL *A polynomial of degree* 2 *is called a quadratic polynomial.*

A quadratic polynomial is of the form $p(x) = ax^2 + bx + c$, *where* $a \neq 0$.

Thus, $(3x^2 + 5x - 8)$, $(2x^2 - 2\sqrt{2}x + 6)$, $(y^2 - 3y + \sqrt{3})$, etc., are all quadratic polynomials.

CUBIC POLYNOMIAL *A polynomial of degree* 3 *is called a cubic polynomial. A cubic polynomial is of the form* $p(x) = ax^3 + bx^2 + cx + d$, *where* $a \neq 0$.

Thus, $(2x^3 - 3x^2 + 8x + 1)$, $(\sqrt{2}y^3 - 2y^2 + y - 8)$, $(z^3 + 2z^2 - \sqrt{3}z + 3)$, etc., are all cubic polynomials.

BIQUADRATIC POLYNOMIAL *A polynomial of degree* 4 *is called a biquadratic polynomial.*

A biquadratic polynomial is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, *where* $a \neq 0$.

Thus, $(2x^4 + 3x^3 - 5x^2 + 9x + 1)$, $(4y^4 - 5y^3 + 6y^2 - 8y + 3)$, etc., are all biquadratic polynomials.

VALUE OF A POLYNOMIAL AT A GIVEN POINT

If $p(x)$ is a polynomial in x and if α is any real number then the value obtained by putting $x = \alpha$ in $p(x)$ is called the value of $p(x)$ at $x = \alpha$.

The value of $p(x)$ at $x = \alpha$ is denoted by $p(\alpha)$.

EXAMPLE Let $p(x) = 2x^2 - 3x + 5$. Then,

$$p(2) = \{2 \times 2^2 - 3 \times 2 + 5\} = (8 - 6 + 5) = 7,$$

$$p(-1) = \{2 \times (-1)^2 - 3 \times (-1) + 5\} = (2 + 3 + 5) = 10.$$

ZEROS OF A POLYNOMIAL A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0$.

EXAMPLE *Let $p(x) = x^2 - 2x - 3$. Find (i) $p(3)$ and (ii) $p(-1)$. What do you conclude?*

SOLUTION We have $p(x) = x^2 - 2x - 3$.

$\therefore$ (i) $p(3) = (3^2 - 2 \times 3 - 3) = (9 - 6 - 3) = 0$

and (ii) $p(-1) = \{(-1)^2 - 2 \times (-1) - 3\} = (1 + 2 - 3) = 0$.

This shows that 3 and –1 are the zeros of the polynomial $p(x)$.

RELATION BETWEEN THE ZEROS AND COEFFICIENTS OF A QUADRATIC POLYNOMIAL

Let α and β be the zeros of a quadratic polynomial

$p(x) = ax^2 + bx + c$, where $a \neq 0$.

Then, $(x - \alpha)$ and $(x - \beta)$ are the factors of $p(x)$.

$$\therefore \quad (ax^2 + bx + c) = k(x - \alpha)(x - \beta), \text{ where } k \text{ is a constant}$$

$$= k \cdot \{x^2 - (\alpha + \beta)x + \alpha\beta\}$$

$$= kx^2 - k(\alpha + \beta)x + k(\alpha\beta).$$

On comparing coefficients of like powers of x on both sides, we get

$k = a, -k(\alpha + \beta) = b$ and $k(\alpha\beta) = c$

$\Rightarrow$ $-a(\alpha + \beta) = b$ and $a(\alpha\beta) = c$ $\quad [\because k = a]$

$\Rightarrow$ $(\alpha + \beta) = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

$$\therefore \quad \text{sum of zeros} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)},$$

$$\text{product of zeros} = \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

SUMMARY

I. If α and β are the zeros of $p(x) = ax^2 + bx + c$, $a \neq 0$ then

(i) $\alpha + \beta = \frac{-b}{a}$

(ii) $\alpha\beta = \frac{c}{a}$.

II. A quadratic polynomial whose zeros are α and β is given by

$p(x) = \{x^2 - (\alpha + \beta)x + \alpha\beta\}$.

SOLVED EXAMPLES

EXAMPLE 1 *Find the zeros of the polynomial* $2x^2 + 5x - 12$ *and verify the relationship between its zeros and coefficients.*

SOLUTION Let the given polynomial be denoted by $f(x)$. Then,

$$\begin{aligned} f(x) &= 2x^2 + 5x - 12 \\ &= 2x^2 + 8x - 3x - 12 \\ &= 2x(x+4) - 3(x+4) \\ &= (x+4)(2x-3). \end{aligned}$$

$$\therefore \quad f(x) = 0 \Rightarrow (x+4)(2x-3) = 0$$
$$\Rightarrow x + 4 = 0 \text{ or } 2x - 3 = 0$$
$$\Rightarrow x = -4 \text{ or } x = \frac{3}{2}.$$

So, the zeros of $f(x)$ are –4 and $\frac{3}{2}$.

Sum of the zeros $= \left(-4 + \frac{3}{2}\right) = \frac{-5}{2} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$,

product of the zeros $= (-4) \times \frac{3}{2} = \frac{-12}{2} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$.

EXAMPLE 2 *Find the zeros of the polynomial* $6x^2 - 3 - 7x$ *and verify the relationship between the zeros and the coefficients.* [CBSE 2008]

SOLUTION Let the given polynomial be denoted by $f(x)$. Then,

$$\begin{aligned} f(x) &= 6x^2 - 3 - 7x \\ &= 6x^2 - 7x - 3 \quad \text{[in standard form]} \\ &= 6x^2 - 9x + 2x - 3 \\ &= 3x(2x-3) + (2x-3) \\ &= (2x-3)(3x+1). \end{aligned}$$

$$\therefore \quad f(x) = 0 \Rightarrow (2x-3)(3x+1) = 0$$
$$\Rightarrow 2x-3 = 0 \text{ or } 3x+1 = 0$$
$$\Rightarrow x = \frac{3}{2} \text{ or } x = \frac{-1}{3}.$$

So, the zeros of $f(x)$ are $\frac{3}{2}$ and $\frac{-1}{3}$.

Sum of zeros $= \left\{\frac{3}{2} + \left(\frac{-1}{3}\right)\right\} = \left(\frac{3}{2} - \frac{1}{3}\right) = \frac{7}{6} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$.

Product of zeros $= \frac{3}{2} \times \left(\frac{-1}{3}\right) = \frac{-3}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

EXAMPLE 3 *Find the zeros of the polynomial* $f(x) = x^2 - 2$ *and verify the relationship between its zeros and coefficients.*

SOLUTION We have

$$f(x) = (x^2-2) = \{x^2 - (\sqrt{2})^2\} = (x+\sqrt{2})(x-\sqrt{2}).$$

$$\therefore \quad f(x) = 0 \Rightarrow (x+\sqrt{2})(x-\sqrt{2}) = 0$$
$$\Rightarrow x+\sqrt{2} = 0 \text{ or } x-\sqrt{2} = 0$$
$$\Rightarrow x = -\sqrt{2} \text{ or } x = \sqrt{2}.$$

So, the zeros of $f(x)$ are $-\sqrt{2}$ and $\sqrt{2}$.

Sum of zeros $= (-\sqrt{2} + \sqrt{2}) = 0 = \frac{0}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$,

product of zeros $= (-\sqrt{2}) \times (\sqrt{2}) = \frac{-2}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

EXAMPLE 4 *Obtain the zeros of the quadratic polynomial* $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ *and verify the relation between its zeros and coefficients.* [CBSE 2008C]

SOLUTION We have

$$f(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3} = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$
$$= \sqrt{3}x(x-2\sqrt{3}) - 2(x-2\sqrt{3}) = (x-2\sqrt{3})(\sqrt{3}x-2).$$

$$\therefore \quad f(x) = 0 \Rightarrow (x-2\sqrt{3})(\sqrt{3}x-2) = 0$$
$$\Rightarrow (x-2\sqrt{3}) = 0 \text{ or } (\sqrt{3}x-2) = 0$$
$$\Rightarrow x = 2\sqrt{3} \text{ or } x = \frac{2}{\sqrt{3}}.$$

So, the zeros of $f(x)$ are $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$.

Sum of zeros $= \left(2\sqrt{3} + \frac{2}{\sqrt{3}}\right) = \frac{8}{\sqrt{3}} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$,

product of zeros $= \left(2\sqrt{3} \times \frac{2}{\sqrt{3}}\right) = \frac{4\sqrt{3}}{\sqrt{3}} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

EXAMPLE 5 *Find a quadratic polynomial, the sum and product of whose zeros are –5 and 6 respectively.*

SOLUTION Let α and β be the zeros of the required polynomial $f(x)$.

Then, $(\alpha + \beta) = -5$ and $\alpha\beta = 6$.

$$\therefore \quad f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (-5)x + 6$$
$$= x^2 + 5x + 6.$$

Hence, the required polynomial is $f(x) = x^2 + 5x + 6$.

EXAMPLE 6 *Find the quadratic polynomial, the sum of whose zeros is $\sqrt{2}$ and their product is –12. Hence, find the zeros of the polynomial.*

SOLUTION Let α and β be the zeros of the required polynomial $f(x)$.

Then, $(\alpha + \beta) = \sqrt{2}$ and $\alpha\beta = -12$.

$$\therefore \quad f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - \sqrt{2}x - 12.$$

So, the required polynomial is $f(x) = x^2 - \sqrt{2}x - 12$.

Now, $f(x) = x^2 - \sqrt{2}x - 12$

$$= x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12 \quad \text{[note it]}$$
$$= x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2})$$
$$= (x - 3\sqrt{2})(x + 2\sqrt{2}).$$

$$\therefore \quad f(x) = 0 \Rightarrow (x - 3\sqrt{2})(x + 2\sqrt{2}) = 0$$
$$\Rightarrow x - 3\sqrt{2} = 0 \text{ or } x + 2\sqrt{2} = 0$$
$$\Rightarrow x = 3\sqrt{2} \text{ or } x = -2\sqrt{2}.$$

Hence, the required polynomial is $f(x) = x^2 - \sqrt{2}x - 12$ whose zeros are $3\sqrt{2}$ and $-2\sqrt{2}$.

EXAMPLE 7 *If the product of the zeros of the polynomial $(ax^2 - 6x - 6)$ is 4, find the value of a.* [CBSE 2009]

SOLUTION Let α and β be the zeros of the polynomial $(ax^2 - 6x - 6)$.

Then, $\alpha\beta = \dfrac{\text{constant term}}{\text{coefficient of } x^2} = \dfrac{-6}{a}$.

But, $\alpha\beta = 4$ (given).

$$\therefore \quad \frac{-6}{a} = 4 \Rightarrow 4a = -6 \Rightarrow a = \frac{-6}{4} = \frac{-3}{2}.$$

Hence, $a = \dfrac{-3}{2}$.

EXAMPLE 8 *If one zero of the polynomial* $(a^2 + 9)x^2 + 13x + 6a$ *is reciprocal of the other, find the value of a.* [CBSE 2008]

SOLUTION Let one zero of the given polynomial be α.

Then, the other zero is $\frac{1}{\alpha}$.

$\therefore$ product of zeros $= \left(\alpha \times \frac{1}{\alpha}\right) = 1$.

But, product of zeros $= \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{6a}{(a^2+9)}$.

$$\therefore \quad \frac{6a}{(a^2+9)} = 1 \Rightarrow a^2 + 9 = 6a$$
$$\Rightarrow a^2 + 9 - 6a = 0$$
$$\Rightarrow (a-3)^2 = 0$$
$$\Rightarrow a - 3 = 0 \Rightarrow a = 3.$$

Hence, $a = 3$.

EXAMPLE 9 *Find a quadratic polynomial whose zeros are* 1 *and* –3. *Verify the relation between the coefficients and zeros of the polynomial.* [CBSE 2008C]

SOLUTION Let $\alpha = 1$ and $\beta = -3$.

Sum of zeros $= (\alpha + \beta) = 1 + (-3) = -2$.

Product of zeros $= \alpha\beta = 1 \times (-3) = -3$.

So, the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-2)x + (-3)$$
$$= x^2 + 2x - 3.$$

Sum of zeros $= -2 = \frac{-2}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$,

product of zeros $= -3 = \frac{-3}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

EXERCISE 2A

Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients:

1. $x^2 + 7x + 12$
2. $x^2 - 2x - 8$
3. $x^2 + 3x - 10$
4. $4x^2 - 4x - 3$ [CBSE 2008C]
5. $5x^2 - 4 - 8x$ [CBSE 2008]
6. $2\sqrt{3}x^2 - 5x + \sqrt{3}$ [CBSE 2011]
7. $2x^2 - 11x + 15$
8. $4x^2 - 4x + 1$

9. $x^2 - 5$

10. $8x^2 - 4$

11. $5y^2 + 10y$

12. $3x^2 - x - 4$

13. Find the quadratic polynomial whose zeros are 2 and –6. Verify the relation between the coefficients and the zeros of the polynomial.

14. Find the quadratic polynomial whose zeros are $\frac{2}{3}$ and $\frac{-1}{4}\cdot$ Verify the relation between the coefficients and the zeros of the polynomial.

15. Find the quadratic polynomial, sum of whose zeros is 8 and their product is 12. Hence, find the zeros of the polynomial. [CBSE 2008]

16. Find the quadratic polynomial, the sum of whose zeros is 0 and their product is –1. Hence, find the zeros of the polynomial.

17. Find the quadratic polynomial, the sum of whose zeros is $\left(\frac{5}{2}\right)$ and their product is 1. Hence, find the zeros of the polynomial.

18. Find the quadratic polynomial, the sum of whose roots is $\sqrt{2}$ and their product is $\frac{1}{3}\cdot$

19. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2 + 7x + b = 0$ then find the values of a and b. [CBSE 2011]

20. If $(x + a)$ is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of a. [CBSE 2009]

21. One zero of the polynomial $3x^3 + 16x^2 + 15x - 18$ is $\frac{2}{3}\cdot$ Find the other zeros of the polynomial.

ANSWERS (EXERCISE 2A)

1. –4, –3 **2.** 4, –2 **3.** –5, 2 **4.** $\frac{3}{2}, \frac{-1}{2}$ **5.** $2, \frac{-2}{5}$

6. $\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$ **7.** $3, \frac{5}{2}$ **8.** $\frac{1}{2}, \frac{1}{2}$ **9.** $\sqrt{5}, -\sqrt{5}$ **10.** $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

11. 0, –2 **12.** $\frac{4}{3}, -1$ **13.** $x^2 + 4x - 12$ **14.** $12x^2 - 5x - 2$

15. $x^2 - 8x + 12, \{6, 2\}$ **16.** $(x^2 - 1), \{1, -1\}$ **17.** $(2x^2 - 5x + 2), \left\{2, \frac{1}{2}\right\}$

18. $3x^2 - 3\sqrt{2}\,x + 1$ **19.** $a = 3, b = -6$ **20.** $a = 2$ **21.** –3, –3

HINTS TO SOME SELECTED QUESTIONS

5. $5x^2 - 8x - 4 = 5x^2 - 10x + 2x - 4$

$= 5x(x - 2) + 2(x - 2) = (x - 2)(5x + 2).$

6. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 2\sqrt{3}x^2 - 3x - 2x + \sqrt{3}$
$= \sqrt{3}x(2x - \sqrt{3}) - (2x - \sqrt{3})$
$= (2x - \sqrt{3})(\sqrt{3}x - 1)$.

8. $4x^2 - 4x + 1 = (2x - 1)^2$.

9. $(x^2 - 5) = (x - \sqrt{5})(x + \sqrt{5})$.

10. $8x^2 - 4 = 4(2x^2 - 1) = 4(\sqrt{2}x - 1)(\sqrt{2}x + 1)$.

11. $5y^2 + 10y = 5y(y + 2)$.

12. $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4 = x(3x - 4) + (3x - 4) = (3x - 4)(x + 1)$.

19. Sum of the roots $= \left(\frac{2}{3} - 3\right) = \frac{-7}{3}$.

Product of roots $= \frac{2}{3} \times (-3) = -2$.

$\therefore \quad \left\{\frac{-7}{a} = \frac{-7}{3} \text{ and } \frac{b}{a} = -2\right\} \Rightarrow a = 3 \text{ and } b = -6.$

20. Let $f(x) = 2x^2 + 2ax + 5x + 10$.

Since $(x + a)$ is a factor of $f(x)$, we have $f(-a) = 0$.

$\therefore \quad 2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow \quad 2a^2 - 2a^2 - 5a + 10 = 0 \Rightarrow 5a = 10 \Rightarrow a = 2.$

21. $\left(x - \frac{2}{3}\right)$ is a factor of the given polynomial and therefore, $(3x - 2)$ is also its factor.

On dividing the given polynomial by $(3x - 2)$, we get $(x^2 + 6x + 9)$ as quotient, i.e., $(x + 3)^2 = 0 \Rightarrow x = -3$.

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RELATION BETWEEN THE ZEROS AND COEFFICIENTS OF A CUBIC POLYNOMIAL

Let α, β and γ be the zeros of a cubic polynomial

$$p(x) = ax^3 + bx^2 + cx + d, \text{ where } a \neq 0.$$

Then, $(x - \alpha)$, $(x - \beta)$ and $(x - \gamma)$ are the factors of $p(x)$.

$\therefore \quad (ax^3 + bx^2 + cx + d) = k(x - \alpha)(x - \beta)(x - \gamma)$ for some constant k

$= k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - (\alpha\beta\gamma)\}$

$= kx^3 - k(\alpha + \beta + \gamma)x^2 + k(\alpha\beta + \beta\gamma + \gamma\alpha)x - k(\alpha\beta\gamma)$.

Comparing coefficients of like powers of x on both sides, we get

$k = a, -k(\alpha + \beta + \gamma) = b, k(\alpha\beta + \beta\gamma + \gamma\alpha) = c, -k(\alpha\beta\gamma) = d$

$\Rightarrow \quad -a(\alpha + \beta + \gamma) = b, a(\alpha\beta + \beta\gamma + \gamma\alpha) = c, -a(\alpha\beta\gamma) = d \quad [\because\ k = a]$

$\Rightarrow \quad (\alpha + \beta + \gamma) = \frac{-b}{a}, (\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}, \alpha\beta\gamma = \frac{-d}{a}.$

SUMMARY

I. If α, β and γ are the zeros of $p(x) = ax^3 + bx^2 + cx + d$ then

(i) $(\alpha + \beta + \gamma) = \frac{-b}{a}$ (ii) $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}$ (iii) $\alpha\beta\gamma = \frac{-d}{a}$.

II. A cubic polynomial whose zeros are α, β and γ is given by

$$p(x) = \{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}.$$

SOLVED EXAMPLES

EXAMPLE 1 *Verify that 3, –1 and* $\frac{-1}{3}$ *are the zeros of the cubic polynomial* $p(x) = 3x^3 - 5x^2 - 11x - 3$ *and verify the relation between its zeros and coefficients.*

SOLUTION The given polynomial is $p(x) = 3x^3 - 5x^2 - 11x - 3$.

$$\therefore \quad p(3) = \{3 \times 3^3 - 5 \times 3^2 - 11 \times 3 - 3\} = (81 - 45 - 33 - 3) = 0;$$

$$p(-1) = \{3 \times (-1)^3 - 5 \times (-1)^2 - 11 \times (-1) - 3\}$$
$$= (-3 - 5 + 11 - 3) = 0;$$

$$\text{and } p\left(\frac{-1}{3}\right) = \left\{3 \times \left(\frac{-1}{3}\right)^3 - 5 \times \left(\frac{-1}{3}\right)^2 - 11 \times \left(\frac{-1}{3}\right) - 3\right\}$$

$$= \left\{3 \times \left(\frac{-1}{27}\right) - 5 \times \frac{1}{9} + \frac{11}{3} - 3\right\} = \left(\frac{-1}{9} - \frac{5}{9} + \frac{11}{3} - 3\right)$$

$$= \frac{(-1 - 5 + 33 - 27)}{9} = 0.$$

$\therefore$ 3, –1 and $\frac{-1}{3}$ are the zeros of $p(x)$.

Let $\alpha = 3, \beta = -1$ and $\gamma = \frac{1}{3}$. Then,

$$(\alpha + \beta + \gamma) = \left(3 - 1 - \frac{1}{3}\right) = \frac{5}{3} = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)};$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(-3 + \frac{1}{3} - 1\right) = \frac{-11}{3} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } x^3)};$$

$$\alpha\beta\gamma = \left\{3 \times (-1) \times \left(\frac{-1}{3}\right)\right\} = 1 = \frac{3}{3} = \frac{-(\text{constant term})}{(\text{coefficient of } x^3)}.$$

EXAMPLE 2 *Find a cubic polynomial with the sum of its zeros, sum of the products of its zeros taken two at a time and the product of its zeros as 2, –7 and –14 respectively.*

SOLUTION Let α, β, γ be the zeros of the required polynomial. Then,

$$\alpha + \beta + \gamma = 2,\ \alpha\beta + \beta\gamma + \gamma\alpha = -7 \text{ and } \alpha\beta\gamma = -14.$$

So, the required polynomial is

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$
$$= x^3 - 2x^2 - 7x - (-14) = x^3 - 2x^2 - 7x + 14.$$

EXAMPLE 3 *If the zeros of the polynomial* $x^3 - 3x^2 + x + 1$ *are* $(a - b), a, (a + b)$, *find a and b.*

SOLUTION Given polynomial is $f(x) = x^3 - 3x^2 + x + 1$.

Let $\alpha = (a - b), \beta = a$ and $\gamma = (a + b)$. Then,

$$\alpha + \beta + \gamma = 3 \Rightarrow (a - b) + a + (a + b) = 3 \Rightarrow 3a = 3 \Rightarrow a = 1.$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = 1 \Rightarrow a(a - b) + a(a + b) + (a + b)(a - b) = 1$$
$$\Rightarrow 3a^2 - b^2 = 1 \Rightarrow (3 \times 1^2) - b^2 = 1$$
$$\Rightarrow b^2 = 2 \Rightarrow b = \pm\sqrt{2}.$$

$\therefore$ $a = 1$ and $b = \pm\sqrt{2}$.

EXAMPLE 4 *Find a cubic polynomial whose zeros are* $3, \frac{1}{2}$ *and* -1.

SOLUTION Let $\alpha = 3, \beta = \frac{1}{2}$ and $\gamma = -1$. Then,

$$(\alpha + \beta + \gamma) = \left(3 + \frac{1}{2} - 1\right) = \frac{5}{2},$$
$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(\frac{3}{2} - \frac{1}{2} - 3\right) = \frac{-4}{2} = -2$$

and $\alpha\beta\gamma = \left\{3 \times \frac{1}{2} \times (-1)\right\} = \frac{-3}{2}$.

Hence, the required polynomial is

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = x^3 - \frac{5}{2}x^2 - 2x + \frac{3}{2}.$$

Thus, $2x^3 - 5x^2 - 4x + 3$ is the desired polynomial.

DIVISION ALGORITHM FOR POLYNOMIALS

If $f(x)$ *and* $g(x)$ *are any two polynomials with* $g(x) \neq 0$ *then we can find polynomials* $q(x)$ *and* $r(x)$ *such that*

$$f(x) = q(x) \times g(x) + r(x),$$

where $r(x) = 0$ *or* $\{\text{degree of } r(x)\} < \{\text{degree of } g(x)\}$.

We may write it as

$$\textit{Dividend} = (\textit{Quotient} \times \textit{Divisor}) + \textit{Remainder}.$$

SOME MORE EXAMPLES

EXAMPLE 5 *Divide* $3 - x + 2x^2$ *by* $(2 - x)$ *and verify the division algorithm.*

SOLUTION First we write the terms of dividend and divisor in decreasing order of their degrees and then perform the division as shown below.

$$
\begin{array}{r}
-x+2\overline{)\,2x^2 - x + 3\,}(-2x-3 \\
\underline{2x^2 - 4x} \\
3x + 3 \\
\underline{3x - 6} \\
\underline{9}
\end{array}
$$

Clearly, degree (9) = 0 < degree $(-x + 2)$.

$\therefore$ quotient = $(-2x - 3)$ and remainder = 9

$\Rightarrow$ (quotient × divisor) + remainder

$= (-2x - 3)(-x + 2) + 9$

$= 2x^2 - 4x + 3x - 6 + 9$

$= 2x^2 - x + 3$

= dividend.

Thus, (quotient × divisor) + remainder = dividend.

Hence, the division algorithm is verified.

EXAMPLE 6 *Divide* $5x^3 - 13x^2 + 21x - 14$ *by* $(3 - 2x + x^2)$ *and verify the division algorithm.*

SOLUTION First we write the given polynomials in standard form in decreasing order of degrees and then perform the division as shown below.

$$
\begin{array}{r}
x^2-2x+3\overline{)\,5x^3 - 13x^2 + 21x - 14\,}(5x-3 \\
\underline{5x^3 - 10x^2 + 15x} \\
-3x^2 + 6x - 14 \\
\underline{-3x^2 + 6x - 9} \\
\underline{-5}
\end{array}
$$

Clearly, degree (-5) = 0 < degree $(x^2 - 2x + 3)$.

$\therefore$ quotient = $(5x - 3)$ and remainder = –5

$\Rightarrow$ (quotient × divisor) + remainder

$= (5x - 3)(x^2 - 2x + 3) - 5$

$$= 5x^3 - 10x^2 + 15x - 3x^2 + 6x - 9 - 5$$
$$= 5x^3 - 13x^2 + 21x - 14$$
$$= \text{dividend.}$$

Thus, (quotient × divisor) + remainder = dividend.

Hence, the division algorithm is verified.

EXAMPLE 7 *What real number should be subtracted from the polynomial* $(3x^3 + 10x^2 - 14x + 9)$ *so that* $(3x - 2)$ *divides it exactly?* [CBSE 2009C]

SOLUTION On dividing $(3x^3 + 10x^2 - 14x + 9)$ by $(3x - 2)$, we get

$$
\begin{array}{r|l}
3x-2 & 3x^3 + 10x^2 - 14x + 9 \;(\; x^2 + 4x - 2 \\
 & 3x^3 - 2x^2 \\ \hline
 & 12x^2 - 14x + 9 \\
 & 12x^2 - 8x \\ \hline
 & -6x + 9 \\
 & -6x + 4 \\ \hline
 & 5 \\ \hline
\end{array}
$$

Required number to be subtracted = 5.

EXAMPLE 8 *If the polynomial* $(x^4 + 2x^3 + 8x^2 + 12x + 18)$ *is divided by another polynomial* $(x^2 + 5)$, *the remainder comes out to be* $(px + q)$. *Find the values of p and q.* [CBSE 2009]

SOLUTION Let $f(x) = (x^4 + 2x^3 + 8x^2 + 12x + 18)$ and $g(x) = (x^2 + 5)$.

On dividing $f(x)$ by $g(x)$, we get

$$
\begin{array}{r|l}
x^2+5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \;(\; x^2 + 2x + 3 \\
 & x^4 + 5x^2 \\ \hline
 & 2x^3 + 3x^2 + 12x + 18 \\
 & 2x^3 + 10x \\ \hline
 & 3x^2 + 2x + 18 \\
 & 3x^2 + 15 \\ \hline
 & 2x + 3 \\ \hline
\end{array}
$$

Now, $px + q = 2x + 3 \Rightarrow p = 2$ and $q = 3$.

EXAMPLE 9 *On dividing* $(x^3 - 3x^2 + x + 2)$ *by a polynomial* $g(x)$, *the quotient and remainder are* $(x - 2)$ *and* $(-2x + 4)$ *respectively. Find* $g(x)$.

[CBSE 2009C]

SOLUTION Let $f(x) = (x^3 - 3x^2 + x + 2)$, $q(x) = (x - 2)$ *and* $r(x) = (-2x + 4)$.

Then, $f(x) = g(x) \cdot q(x) + r(x)$

$$\Rightarrow \quad g(x) = \frac{\{f(x) - r(x)\}}{q(x)}. \qquad \ldots \text{(i)}$$

Now, $\{f(x) - r(x)\} = (x^3 - 3x^2 + x + 2) - (-2x + 4)$

$= (x^3 - 3x^2 + 3x - 2)$.

$$\therefore \quad g(x) = \frac{(x^3 - 3x^2 + 3x - 2)}{(x - 2)}. \qquad \text{[using (i)]}$$

On dividing $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$, we get $g(x)$.

$$\begin{array}{l} x - 2\,\big)\, x^3 - 3x^2 + 3x - 2\,\big(\, x^2 - x + 1 \\ \qquad \underline{x^3 - 2x^2} \\ \qquad\quad -x^2 + 3x - 2 \\ \qquad\quad \underline{-x^2 + 2x} \\ \qquad\qquad\quad x - 2 \\ \qquad\qquad\quad \underline{x - 2} \\ \qquad\qquad\quad\;\; \times \end{array}$$

$$\therefore \quad g(x) = (x^2 - x + 1).$$

AN IMPORTANT NOTE

If α is a zero of the polynomial $f(x)$ then $(x - \alpha)$ is a divisor of $f(x)$.

EXAMPLE 10 *It being given that* 1 *is a zero of the polynomial* $(7x - x^3 - 6)$, *find its other zeros.*

SOLUTION Let $f(x) = -x^3 + 7x - 6$.

Since 1 is a zero of $f(x)$, so $(x - 1)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x - 1)$, we get

$$\begin{array}{l} x - 1\,\big)\, -x^3 \quad + 7x - 6\,\big(\, -x^2 - x + 6 \\ \qquad \underline{-x^3 + x^2} \\ \qquad\quad -x^2 + 7x - 6 \\ \qquad\quad \underline{-x^2 + x} \\ \qquad\qquad\quad 6x - 6 \\ \qquad\qquad\quad \underline{6x - 6} \\ \qquad\qquad\quad\;\; \times \end{array}$$

$$\therefore \quad f(x) = (-x^3 + 7x - 6) = (x - 1)(-x^2 - x + 6)$$

$$= -(x - 1)(x^2 + x - 6) = -(x - 1)(x^2 + 3x - 2x - 6)$$

$$= (1 - x)[x(x + 3) - 2(x + 3)] = (1 - x)(x + 3)(x - 2).$$

$\therefore \quad f(x) = 0 \Rightarrow (1-x)(x+3)(x-2) = 0$

$\Rightarrow (1-x) = 0$ or $(x+3) = 0$ or $(x-2) = 0$

$\Rightarrow x = 1$ or $x = -3$ or $x = 2.$

Thus, the other zeros are –3 and 2.

EXAMPLE 11 *Obtain all zeros of the polynomial* $(2x^3 - 4x - x^2 + 2)$, *if two of its zeros are* $\sqrt{2}$ *and* $-\sqrt{2}$. [CBSE 2008C]

SOLUTION The given polynomial is $f(x) = 2x^3 - x^2 - 4x + 2.$

Since $\sqrt{2}$ and $-\sqrt{2}$ are the zeros of $f(x)$, it follows that each one of $(x-\sqrt{2})$ and $(x+\sqrt{2})$ is a factor of $f(x)$.

Consequently, $(x-\sqrt{2})(x+\sqrt{2}) = (x^2-2)$ is a factor of $f(x)$.

On dividing $f(x) = 2x^3 - x^2 - 4x + 2$ by (x^2-2), we get

$$\begin{array}{r|l} x^2-2 \;)\; 2x^3 - x^2 - 4x + 2 \;(& 2x-1 \\ 2x^3 \qquad - 4x & \\ \hline -x^2 \qquad + 2 & \\ -x^2 \qquad + 2 & \\ \hline \times & \\ \hline \end{array}$$

$\therefore \quad f(x) = 0 \Rightarrow (x^2-2)(2x-1) = 0$

$\Rightarrow (x-\sqrt{2})(x+\sqrt{2})(2x-1) = 0$

$\Rightarrow (x-\sqrt{2}) = 0$ or $(x+\sqrt{2}) = 0$ or $(2x-1) = 0$

$\Rightarrow x = \sqrt{2}$ or $x = -\sqrt{2}$ or $x = \frac{1}{2}$.

Hence, all zeros of $f(x)$ are $\sqrt{2}, -\sqrt{2}$ and $\frac{1}{2}$.

EXAMPLE 12 *If two zeros of the polynomial* $(x^4 - 6x^3 - 26x^2 + 138x - 35)$ *are* $(2+\sqrt{3})$ *and* $(2-\sqrt{3})$, *find other zeros.*

SOLUTION Let $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35.$

Let $\alpha = (2+\sqrt{3})$ and $\beta = (2-\sqrt{3})$. Then,

$(\alpha + \beta) = 4$ and $\alpha\beta = (4-3) = 1.$

So, the quadratic polynomial whose roots are α and β is given by

$x^2 - (\alpha+\beta)x + \alpha\beta = (x^2 - 4x + 1).$

$\therefore \quad (x^2 - 4x + 1)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 4x + 1)$, we get

$$
\begin{array}{r|l}
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \;(x^2-2x-35 \\
 & \underline{x^4-4x^3+\quad x^2} \\
 & \quad -2x^3-27x^2+138x-35 \\
 & \quad \underline{-2x^3+\;8x^2-\quad 2x} \\
 & \qquad -35x^2+140x-35 \\
 & \qquad \underline{-35x^2+140x-35} \\
 & \qquad\qquad \times
\end{array}
$$

$\therefore$ $f(x) = (x^2-4x+1)(x^2-2x-35)$.

$\therefore$ the other two zeros of $f(x)$ are given by $(x^2-2x-35) = 0$.

Now, $x^2-2x-35 = 0 \Rightarrow x^2-7x+5x-35 = 0$

$\Rightarrow x(x-7)+5(x-7) = 0$

$\Rightarrow (x-7)(x+5) = 0$

$\Rightarrow x-7 = 0$ or $x+5 = 0$

$\Rightarrow x = 7$ or $x = -5$.

Hence, the other two zeros of $f(x)$ are 7 and –5.

EXAMPLE 13 *Obtain all zeros of* $(3x^4-15x^3+13x^2+25x-30)$, *if two of its zeros are* $\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$. [CBSE 2009C]

SOLUTION The given polynomial is $f(x) = (3x^4-15x^3+13x^2+25x-30)$.

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the zeros of $f(x)$, it follows that each one of $\left(x-\sqrt{\frac{5}{3}}\right)$ and $\left(x+\sqrt{\frac{5}{3}}\right)$ is a factor of $f(x)$.

$\therefore$ $\left(x-\frac{\sqrt{5}}{\sqrt{3}}\right)\left(x+\frac{\sqrt{5}}{\sqrt{3}}\right) = \left(x^2-\frac{5}{3}\right) = \frac{(3x^2-5)}{3}$ is a factor of $f(x)$.

Consequently, $(3x^2-5)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(3x^2-5)$, we get

$$
\begin{array}{r|l}
3x^2-5 & 3x^4-15x^3+13x^2+25x-30 \;(x^2-5x+6 \\
 & \underline{3x^4 \qquad\quad -\;5x^2} \\
 & \quad -15x^3+18x^2+25x-30 \\
 & \quad \underline{-15x^3 \qquad\quad +25x} \\
 & \qquad\qquad 18x^2 \qquad -30 \\
 & \qquad\qquad \underline{18x^2 \qquad -30} \\
 & \qquad\qquad\quad \times
\end{array}
$$

$\therefore \quad f(x) = 3x^4 - 15x^3 + 13x^2 + 25x - 30$

$= (3x^2 - 5)(x^2 - 5x + 6)$

$= (\sqrt{3}x + \sqrt{5})(\sqrt{3}x - \sqrt{5})(x - 2)(x - 3).$

$\therefore \quad f(x) = 0 \Rightarrow (\sqrt{3}x + \sqrt{5}) = 0$ or $(\sqrt{3}x - \sqrt{5}) = 0$

or $(x - 2) = 0$ or $(x - 3) = 0$

$\Rightarrow x = -\sqrt{\frac{5}{3}}$ or $x = \sqrt{\frac{5}{3}}$ or $x = 2$ or $x = 3.$

Hence, all zeros of $f(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, 2 and 3.

EXERCISE 2B

1. Verify that 3, –2, 1 are the zeros of the cubic polynomial $p(x) = x^3 - 2x^2 - 5x + 6$ and verify the relation between its zeros and coefficients.
2. Verify that $5, -2$ and $\frac{1}{3}$ are the zeros of the cubic polynomial $p(x) = 3x^3 - 10x^2 - 27x + 10$ and verify the relation between its zeros and coefficients.
3. Find a cubic polynomial whose zeros are 2, –3 and 4.
4. Find a cubic polynomial whose zeros are $\frac{1}{2}$, 1 and -3.
5. Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and the product of its zeros as 5, –2 and –24 respectively.

Find the quotient and the remainder when

6. $f(x) = x^3 - 3x^2 + 5x - 3$ is divided by $g(x) = x^2 - 2$.
7. $f(x) = x^4 - 3x^2 + 4x + 5$ is divided by $g(x) = x^2 + 1 - x$.
8. $f(x) = x^4 - 5x + 6$ is divided by $g(x) = 2 - x^2$.
9. By actual division, show that $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$.
10. On dividing $3x^3 + x^2 + 2x + 5$ by a polynomial $g(x)$, the quotient and remainder are $(3x - 5)$ and $(9x + 10)$ respectively. Find $g(x)$.

    HINT $g(x) = \frac{(3x^3 + x^2 + 2x + 5) - (9x + 10)}{(3x - 5)}$.
11. Verify division algorithm for the polynomials $f(x) = 8 + 20x + x^2 - 6x^3$ and $g(x) = 2 + 5x - 3x^2$.
12. It is given that –1 is one of the zeros of the polynomial $x^3 + 2x^2 - 11x - 12$. Find all the zeros of the given polynomial.

13. If 1 and –2 are two zeros of the polynomial $(x^3 - 4x^2 - 7x + 10)$, find its third zero.
14. If 3 and –3 are two zeros of the polynomial $(x^4 + x^3 - 11x^2 - 9x + 18)$, find all the zeros of the given polynomial.
15. If 2 and –2 are two zeros of the polynomial $(x^4 + x^3 - 34x^2 - 4x + 120)$, find all the zeros of the given polynomial. [CBSE 2008]
16. Find all the zeros of $(x^4 + x^3 - 23x^2 - 3x + 60)$, if it is given that two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$. [CBSE 2009C]
17. Find all the zeros of $(2x^4 - 3x^3 - 5x^2 + 9x - 3)$, it being given that two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$.
18. Obtain all other zeros of $(x^4 + 4x^3 - 2x^2 - 20x - 15)$ if two of its zeros are $\sqrt{5}$ and $-\sqrt{5}$. [CBSE 2009C]
19. Find all the zeros of the polynomial $(2x^4 - 11x^3 + 7x^2 + 13x - 7)$, it being given that two of its zeros are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

ANSWERS (EXERCISE 2B)

3. $x^3 - 3x^2 - 10x + 24$ **4.** $2x^3 + 3x^2 - 8x + 3$ **5.** $x^3 - 5x^2 - 2x + 24$
6. $q(x) = (x - 3)$, $r(x) = (7x - 9)$ **7.** $q(x) = x^2 + x - 3$, $r(x) = 8$
8. $q(x) = -x^2 - 2$, $r(x) = -5x + 10$ **10.** $g(x) = x^2 + 2x + 1$
12. $-4, -1, 3$ **13.** 5 **14.** $1, -2, 3, -3$ **15.** $2, -2, -6, 5$
16. $\sqrt{3}, -\sqrt{3}, 4, -5$ **17.** $\sqrt{3}, -\sqrt{3}, 1, \frac{1}{2}$ **18.** $-1, -3$
19. $(3 + \sqrt{2}), (3 - \sqrt{2}), \frac{1}{2}, -1$

EXERCISE 2C

Very-Short-Answer Questions

1. If one zero of the polynomial $x^2 - 4x + 1$ is $(2 + \sqrt{3})$, write the other zero. [CBSE 2010]
2. Find the zeros of the polynomial $x^2 + x - p(p + 1)$. [CBSE 2011]
3. Find the zeros of the polynomial $x^2 - 3x - m(m + 3)$. [CBSE 2011]
4. If α, β are the zeros of a polynomial such that $\alpha + \beta = 6$ and $\alpha\beta = 4$ then write the polynomial. [CBSE 2010]
5. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2 then find the value of k.
6. If 3 is a zero of the polynomial $2x^2 + x + k$, find the value of k. [CBSE 2010]

7. If -4 is a zero of the polynomial $x^2 - x - (2k + 2)$ then find the value of k. [CBSE 2009]

8. If 1 is a zero of the polynomial $ax^2 - 3(a-1)x - 1$ then find the value of a.

9. If -2 is a zero of the polynomial $3x^2 + 4x + 2k$ then find the value of k. [CBSE 2010]

10. Write the zeros of the polynomial $x^2 - x - 6$. [CBSE 2008]

11. If the sum of the zeros of the quadratic polynomial $kx^2 - 3x + 5$ is 1, write the value of k.

12. If the product of the zeros of the quadratic polynomial $x^2 - 4x + k$ is 3 then write the value of k.

13. If $(x + a)$ is a factor of $(2x^2 + 2ax + 5x + 10)$, find the value of a. [CBSE 2010]

14. If $(a - b)$, a and $(a + b)$ are zeros of the polynomial $2x^3 - 6x^2 + 5x - 7$, write the value of a.

15. If $x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$, write the values of a and b.

16. If α and β are the zeros of the polynomial $2x^2 + 7x + 5$, write the value of $\alpha + \beta + \alpha\beta$. [CBSE 2010]

17. State division algorithm for polynomials.

18. The sum of the zeros and the product of zeros of a quadratic polynomial are $\frac{-1}{2}$ and -3 respectively. Write the polynomial.

Short-Answer Questions

19. Write the zeros of the quadratic polynomial $f(x) = 6x^2 - 3$.

20. Write the zeros of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$.

21. If α and β are the zeros of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

22. If α and β are the zeros of the polynomial $f(x) = 6x^2 + x - 2$, find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

23. If α and β are the zeros of the polynomial $f(x) = 5x^2 - 7x + 1$, find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$.

24. If α and β are the zeros of the polynomial $f(x) = x^2 + x - 2$, find the value of $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)$.

25. If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, find a and b.

ANSWERS (EXERCISE 2C)

1. $(2-\sqrt{3})$ **2.** $-(p+1)$ and p **3.** $(m+3)$ and $-m$ **4.** x^2-6x+4
5. $k=\frac{-6}{5}$ **6.** $k=-21$ **7.** $k=9$ **8.** $a=1$ **9.** $k=-2$
10. 3 and –2 **11.** $k=3$ **12.** $k=3$ **13.** $a=2$ **14.** $a=1$
15. $a=2$ and $b=0$ **16.** –1 **18.** $x^2+\frac{1}{2}x-3$ **19.** $x=\frac{1}{\sqrt{2}}$ or $x=\frac{-1}{\sqrt{2}}$
20. $x=\frac{-2}{\sqrt{3}}$ or $x=\frac{\sqrt{3}}{4}$ **21.** $k=6$ **22.** $\frac{-25}{12}$ **23.** 7 **24.** $\frac{-3}{2}$
25. $a=1$ and $b=\pm\sqrt{2}$

HINTS TO SOME SELECTED QUESTIONS

1. Let the other zero be α.

   Then, sum of zeros $= \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)} = \frac{-(-4)}{1} = 4.$

   $\therefore \quad (2+\sqrt{3})+\alpha = 4 \Rightarrow \alpha = (2-\sqrt{3}).$

2. $x^2+x-p(p+1) = x^2+(p+1)x-px-p(p+1)$
   $= x\{x+(p+1)\}-p\{x+(p+1)\}$
   $= \{x+(p+1)\}\{x-p\}.$

   So, its zeros are $-(p+1)$ and p.

3. $x^2-3x-m(m+3) = x^2-(m+3)x+mx-m(m+3)$
   $= x\{x-(m+3)\}+m\{x-(m+3)\}$
   $= \{x-(m+3)\}\{x+m\}.$

   So, its zeros are $(m+3)$ and $-m$.

4. Required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta = x^2-6x+4.$

5. Since 2 is a zero of kx^2+3x+k, we have $4k+6+k=0$.

   $\therefore \quad 5k=-6 \Rightarrow k=\frac{-6}{5}.$

6. Since 3 is a zero of $2x^2+x+k$, we have $18+3+k=0 \Rightarrow k=-21$.

7. Since –4 is a zero of $x^2-x-(2k+2)$, we have $16+4-2k-2=0$.

   $\therefore \quad 2k=18 \Rightarrow k=9.$

8. Since 1 is a zero of $ax^2-3(a-1)x-1$, we have $a-3(a-1)-1=0$.

   $\therefore \quad a-3a+3-1=0 \Rightarrow 2a=2 \Rightarrow a=1.$

9. Since –2 is a zero of $3x^2+4x+2k$, we have $12-8+2k=0$.

   $\therefore \quad 2k=-4 \Rightarrow k=-2.$

10. $x^2 - x - 6 = x^2 - 3x + 2x - 6 = x(x-3) + 2(x-3) = (x-3)(x+2)$.

So, its zeros are 3 and –2.

11. Sum of the zeros $= \dfrac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)} = \dfrac{3}{k}$.

$\therefore \quad \dfrac{3}{k} = 1 \Rightarrow k = 3.$

12. Product of the zero $= \dfrac{\text{constant term}}{\text{coefficient of } x^2} = \dfrac{k}{1}$. So, $k = 3$.

13. Let $f(x) = 2x^2 + 2ax + 5x + 10$. Then, $f(-a) = 0$.

$\therefore \quad 2a^2 - 2a^2 - 5a + 10 = 0 \Rightarrow 5a = 10 \Rightarrow a = 2.$

14. $(\alpha + \beta + \gamma) = \dfrac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)} = \dfrac{-(-6)}{2} = 3.$

$\therefore \quad (a-b) + a + (a+b) = 3 \Rightarrow 3a = 3 \Rightarrow a = 1.$

15. On dividing $x^3 + x^2 - ax + b$ by $x^2 - x$, we get

$$
\begin{array}{r|l}
x^2 - x & x^3 + x^2 - ax + b \;(\; x + 2 \\
 & \underline{x^3 - x^2 \qquad\qquad} \\
 & \quad 2x^2 - ax \\
 & \quad \underline{2x^2 - 2x \qquad} \\
 & \quad (2-a)x + b
\end{array}
$$

$\therefore \quad (2 - a = 0 \text{ and } b = 0) \Rightarrow a = 2 \text{ and } b = 0.$

16. Clearly, $\alpha + \beta = \dfrac{-7}{2}$ and $\alpha\beta = \dfrac{5}{2}$.

$\therefore \quad \alpha + \beta + \alpha\beta = \left(\dfrac{-7}{2} + \dfrac{5}{2}\right) = \dfrac{(-7+5)}{2} = \dfrac{-2}{2} = -1.$

18. $\alpha + \beta = \dfrac{-1}{2}$ and $\alpha\beta = -3$.

So, the polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 + \dfrac{1}{2}x - 3$.

19. $6x^2 - 3 = 3(2x^2 - 1) = 3(\sqrt{2}x - 1)(\sqrt{2}x + 1)$.

So, its zeros are $\dfrac{1}{\sqrt{2}}$ and $\dfrac{-1}{\sqrt{2}}$.

20. $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (\sqrt{3}x + 2)(4x - \sqrt{3})$.

So, its zeros are $\dfrac{-2}{\sqrt{3}}, \dfrac{\sqrt{3}}{4}$.

21. $(\alpha + \beta) = 5$ and $\alpha\beta = k$. Also, $\alpha - \beta = 1$ (given).

$\therefore \quad (\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta \Rightarrow 5^2 - 1^2 = 4\alpha\beta$

$\therefore \quad 4\alpha\beta = (25 - 1) = 24 \Rightarrow \alpha\beta = 6.$

Hence, $k = 6$.

22. $\alpha+\beta=\frac{-1}{6}$ and $\alpha\beta=\frac{-2}{6}=\frac{-1}{3}$.

$$\therefore \quad \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{(\alpha^2+\beta^2)}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(\frac{1}{36}+\frac{2}{3}\right)}{\left(\frac{-1}{3}\right)}=\left(\frac{25}{36}\times\frac{3}{-1}\right)=\frac{-25}{12}.$$

23. $\alpha+\beta=\frac{7}{5}$ and $\alpha\beta=\frac{1}{5}$.

$$\therefore \quad \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{(\alpha+\beta)}{\alpha\beta}=\left(\frac{7}{5}\times\frac{5}{1}\right)=7.$$

24. $\alpha+\beta=-1$ and $\alpha\beta=-2$.

$(\beta+\alpha)^2-(\beta-\alpha)^2=4\alpha\beta$

$\Rightarrow \quad (\beta-\alpha)^2=(\alpha+\beta)^2-4\alpha\beta=(-1)^2-4\times(-2)=9$

$\Rightarrow \quad \beta-\alpha=3.$

$$\therefore \quad \left(\frac{1}{\alpha}-\frac{1}{\beta}\right)=\frac{(\beta-\alpha)}{\alpha\beta}=\frac{3}{-2}=\frac{-3}{2}.$$

25. $\alpha+\beta+\gamma=3$, $\alpha\beta+\beta\gamma+\gamma\alpha=1$ and $\alpha\beta\gamma=-1$

$\therefore \quad (a-b)+a+(a+b)=3 \Rightarrow 3a=3 \Rightarrow a=1$

and $(a-b)\times a\times(a+b)=-1 \Rightarrow a(a^2-b^2)=-1$

$\Rightarrow 1-b^2=-1 \Rightarrow b^2=2 \Rightarrow b=\pm\sqrt{2}.$

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. Which of the following is a polynomial?
   (a) $x^2-5x+4\sqrt{x}+3$ (b) $x^{3/2}-x+x^{1/2}+1$
   (c) $\sqrt{x}+\frac{1}{\sqrt{x}}$ (d) $\sqrt{2}x^2-3\sqrt{3}x+\sqrt{6}$

2. Which of the following is not a polynomial?
   (a) $\sqrt{3}x^2-2\sqrt{3}x+5$ (b) $9x^2-4x+\sqrt{2}$
   (c) $\frac{3}{2}x^3+6x^2-\frac{1}{\sqrt{2}}x-8$ (d) $x+\frac{3}{x}$

3. The zeros of the polynomial x^2-2x-3 are
   (a) –3, 1 (b) –3, –1 (c) 3, –1 (d) 3, 1

4. The zeros of the polynomial $x^2-\sqrt{2}x-12$ are
   (a) $\sqrt{2}, -\sqrt{2}$ (b) $3\sqrt{2}, -2\sqrt{2}$ (c) $-3\sqrt{2}, 2\sqrt{2}$ (d) $3\sqrt{2}, 2\sqrt{2}$

5. The zeros of the polynomial $4x^2+5\sqrt{2}x-3$ are
   (a) $-3\sqrt{2}, \sqrt{2}$ (b) $-3\sqrt{2}, \frac{\sqrt{2}}{2}$ (c) $\frac{-3\sqrt{2}}{2}, \frac{\sqrt{2}}{4}$ (d) none of these

6. The zeros of the polynomial $x^2 + \frac{1}{6}x - 2$ are
   (a) –3, 4 (b) $\frac{-3}{2}, \frac{4}{3}$ (c) $\frac{-4}{3}, \frac{3}{2}$ (d) none of these
7. The zeros of the polynomial $7x^2 - \frac{11x}{3} - \frac{2}{3}$ are
   (a) $\frac{2}{3}, \frac{-1}{7}$ (b) $\frac{2}{7}, \frac{-1}{3}$ (c) $\frac{-2}{3}, \frac{1}{7}$ (d) none of these
8. The sum and the product of the zeros of a quadratic polynomial are 3 and –10 respectively. The quadratic polynomial is
   (a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$ (c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$
9. A quadratic polynomial whose zeros are 5 and –3, is
   (a) $x^2 + 2x - 15$ (b) $x^2 - 2x + 15$ (c) $x^2 - 2x - 15$ (d) none of these
10. A quadratic polynomial whose zeros are $\frac{3}{5}$ and $\frac{-1}{2}$, is
    (a) $10x^2 + x + 3$ (b) $10x^2 + x - 3$ (c) $10x^2 - x + 3$ (d) $10x^2 - x - 3$
11. The zeros of the quadratic polynomial $x^2 + 88x + 125$ are
    (a) both positive (b) both negative
    (c) one positive and one negative (d) both equal
12. If α and β are the zeros of $x^2 + 5x + 8$ then the value of $(\alpha + \beta)$ is
    (a) 5 (b) –5 (c) 8 (d) –8
13. If α and β are the zeros of $2x^2 + 5x - 9$ then the value of αβ is
    (a) $\frac{-5}{2}$ (b) $\frac{5}{2}$ (c) $\frac{-9}{2}$ (d) $\frac{9}{2}$
14. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2 then the value of k is
    (a) $\frac{5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{6}{5}$ (d) $\frac{-6}{5}$
15. If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -4 then the value of k is
    (a) $\frac{-5}{4}$ (b) $\frac{5}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$
16. If –2 and 3 are the zeros of the quadratic polynomial $x^2 + (a+1)x + b$ then
    (a) $a = -2, b = 6$ (b) $a = 2, b = -6$
    (c) $a = -2, b = -6$ (d) $a = 2, b = 6$
17. If one zero of $3x^2 + 8x + k$ be the reciprocal of the other then k = ?
    (a) 3 (b) –3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

18. If the sum of the zeros of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeros then $k = ?$

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

19. If α, β are the zeros of the polynomial $x^2 + 6x + 2$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = ?$

(a) 3 (b) –3 (c) 12 (d) –12

20. If α, β, γ are the zeros of the polynomial $x^3 - 6x^2 - x + 30$ then $(\alpha\beta + \beta\gamma + \gamma\alpha) = ?$

(a) –1 (b) 1 (c) –5 (d) 30

21. If α, β, γ are the zeros of the polynomial $2x^3 + x^2 - 13x + 6$ then $\alpha\beta\gamma = ?$

(a) –3 (b) 3 (c) $\frac{-1}{2}$ (d) $\frac{-13}{2}$

22. If α, β, γ be the zeros of the polynomial $p(x)$ such that $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$ then $p(x) = ?$

(a) $x^3 + 3x^2 - 10x + 24$ (b) $x^3 + 3x^2 + 10x - 24$
(c) $x^3 - 3x^2 - 10x + 24$ (d) None of these

23. If two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0 then the third zero is

(a) $\frac{-b}{a}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $\frac{-d}{a}$

24. If one of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ is 0 then the product of the other two zeros is

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

25. If one of the zeros of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1 then the product of the other two zeros is

(a) $a - b - 1$ (b) $b - a - 1$ (c) $1 - a + b$ (d) $1 + a - b$

26. If α, β be the zeros of the polynomial $2x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$ then $k = ?$

(a) 3 (b) –3 (c) –2 (d) 2

27. On dividing a polynomial $p(x)$ by a nonzero polynomial $q(x)$, let $g(x)$ be the quotient and $r(x)$ be the remainder then $p(x) = q(x) \cdot g(x) + r(x)$, where

(a) $r(x) = 0$ always

(b) $\deg r(x) < \deg g(x)$ always

(c) either $r(x) = 0$ or $\deg r(x) < \deg g(x)$

(d) $r(x) = g(x)$

28. Which of the following is a true statement?
   (a) x^2+5x-3 is a linear polynomial.
   (b) x^2+4x-1 is a binomial.
   (c) $x+1$ is a monomial.
   (d) $5x^3$ is a monomial.

ANSWERS (MCQ)

1. (d) **2.** (d) **3.** (c) **4.** (b) **5.** (c) **6.** (b) **7.** (a) **8.** (c) **9.** (c)
10. (d) **11.** (b) **12.** (b) **13.** (c) **14.** (d) **15.** (b) **16.** (c) **17.** (a) **18.** (d)
19. (b) **20.** (a) **21.** (a) **22.** (c) **23.** (a) **24.** (b) **25.** (c) **26.** (d) **27.** (c)
28. (d)

HINTS TO SOME SELECTED QUESTIONS

1. Clearly, $\sqrt{2}x^2-3\sqrt{3}x+\sqrt{6}$ is a polynomial.

2. Clearly, $x+\dfrac{3}{x}$ is not a polynomial.

3. $x^2-2x-3=x^2-3x+x-3$

$$=x(x-3)+(x-3)=(x-3)(x+1).$$

$\therefore\quad (x-3)(x+1)=0 \Rightarrow x=3$ or $x=-1$.

4. $x^2-\sqrt{2}x-12=x^2-3\sqrt{2}x+2\sqrt{2}x-12$

$$=x(x-3\sqrt{2})+2\sqrt{2}(x-3\sqrt{2})=(x-3\sqrt{2})(x+2\sqrt{2}).$$

$\therefore\quad x=3\sqrt{2}$ or $x=-2\sqrt{2}$.

5. $4x^2+5\sqrt{2}x-3=4x^2+6\sqrt{2}x-\sqrt{2}x-3$

$$=2\sqrt{2}x(\sqrt{2}x+3)-(\sqrt{2}x+3)=(\sqrt{2}x+3)(2\sqrt{2}x-1).$$

$\therefore\quad x=\dfrac{-3}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{-3\sqrt{2}}{2}$ or $x=\dfrac{1}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{2}}{4}$.

6. $x^2+\dfrac{1}{6}x-2=\dfrac{6x^2+x-12}{6}$.

$6x^2+x-12=6x^2+9x-8x-12=3x(2x+3)-4(2x+3)$

$$=(2x+3)(3x-4)$$

$\therefore\quad$ the zeros are $\dfrac{-3}{2}$ and $\dfrac{4}{3}$.

7. $7x^2-\dfrac{11x}{3}-\dfrac{2}{3}=\dfrac{21x^2-11x-2}{3}$.

Now, $21x^2-11x-2=21x^2-14x+3x-2$

$$=7x(3x-2)+(3x-2)=(3x-2)(7x+1).$$

$\therefore\quad$ the zeros are $\dfrac{2}{3},\dfrac{-1}{7}$.

8. The required polynomial is

$$x^2-(\alpha+\beta)x+\alpha\beta = x^2-3x-10.$$

9. $\alpha+\beta = 5+(-3) = 2$, $\alpha\beta = 5\times(-3) = -15$.

Required polynomial is $x^2-2x-15$.

10. $\alpha+\beta = \left(\frac{3}{5}-\frac{1}{2}\right) = \frac{1}{10}$, $\alpha\beta = \frac{3}{5}\times\left(\frac{-1}{2}\right) = \frac{-3}{10}\cdot$

Required polynomial is $x^2-\frac{1}{10}x-\frac{3}{10}$, i.e., $10x^2-x-3$.

11. Let α and β be the zeros of the given polynomial.

Then, $\alpha+\beta = -88$ and $\alpha\beta = 125$.

This happens when α and β are both negative.

12. For ax^2+bx+c, we have $\alpha+\beta = \frac{-b}{a}\cdot$

For x^2+5x+8, we have $\alpha+\beta = -5$.

13. For ax^2+bx+c, we have $\alpha\beta = \frac{c}{a}\cdot$

For $2x^2+5x-9$, we have $\alpha\beta = \frac{-9}{2}\cdot$

14. $x = 2$ satisfies $kx^2+3x+k = 0$.

$\therefore \quad 4k+6+k = 0 \Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}\cdot$

15. $x = -4$ satisfies $(k-1)x^2+kx+1 = 0$.

$\therefore \quad 16(k-1)-4k+1 = 0 \Rightarrow 12k = 15 \Rightarrow k = \frac{5}{4}\cdot$

16. $\alpha+\beta = 3+(-2) = 1$ and $\alpha\beta = 3\times(-2) = -6$.

$\therefore \quad -(a+1) = 1 \Rightarrow a+1 = -1 \Rightarrow a = -2$.

Also, $b = -6$.

17. $\alpha\beta = 1 \Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$.

18. $\alpha+\beta = \alpha\beta \Rightarrow \frac{-2}{k} = \frac{3k}{k} \Rightarrow \frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}\cdot$

19. $\alpha+\beta = -6$ and $\alpha\beta = 2$.

$\therefore \quad \left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-6}{2} = -3.$

Note For ax^3+bx^2+cx+d, we have

$$(\alpha+\beta+\gamma) = \frac{-b}{a}, (\alpha\beta+\beta\gamma+\gamma\alpha) = \frac{c}{a} \text{ and } \alpha\beta\gamma = \frac{-d}{a}\cdot$$

20. $(\alpha\beta+\beta\gamma+\gamma\alpha) = \frac{c}{a} = \frac{-1}{1} = -1$.

21. $\alpha\beta\gamma = \frac{-d}{a} = \frac{-6}{2} = -3$.

22. $p(x) = x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma$

$\quad = x^3-3x^2-10x+24.$

23. Let α, 0, 0 be the zeros of $ax^3 + bx^2 + cx + d$. Then,

sum of zeros $= \frac{-b}{a} \Rightarrow \alpha + 0 + 0 = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{a}$.

Hence, the third zero is $\frac{-b}{a}$.

24. Let α, β, 0 be the zeros of $ax^3 + bx^2 + cx + d$. Then,

sum of the products of zeros, taken two at a time is given by

$$(\alpha\beta + \beta \times 0 + \alpha \times 0) = \frac{c}{a} \Rightarrow \alpha\beta = \frac{c}{a}.$$

∴ the product of the other two zeros is $\frac{c}{a}$.

25. Since –1 is a zero of $x^3 + ax^2 + bx + c$, we have

$$(-1)^3 + a \times (-1)^2 + b \times (-1) + c = 0 \Rightarrow a - b + c - 1 = 0 \Rightarrow c = 1 - a + b.$$

Also, product of all zeros is given by

$$\alpha\beta \times (-1) = -c \Rightarrow \alpha\beta = c \Rightarrow \alpha\beta = 1 - a + b.$$

26. $\alpha + \beta = \frac{-5}{2}$ and $\alpha\beta = \frac{k}{2}$.

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{k}{2} = \left(\frac{25}{4} - \frac{21}{4}\right) = 1 \Rightarrow k = 2.$$

SUMMARY OF RESULTS

1. I. **Polynomial:** *An expression of the form* $p(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, *where* $a_n \neq 0$, *is called a polynomial in x of degree n.*

II. A polynomial is said to be *linear, quadratic, cubic* and *biquadratic* according as its degree is 1, 2, 3 and 4 respectively.

General form:

(i) linear equation: $\boldsymbol{ax + b}$.

(ii) quadratic equation: $\boldsymbol{ax^2 + bx + c}$.

(iii) cubic equation: $\boldsymbol{ax^3 + bx^2 + cx + d}$.

(iv) biquadratic equation: $\boldsymbol{ax^4 + bx^3 + cx^2 + dx + e}$.

III. **Value of a polynomial at a point:** *The value of a polynomial* $p(x)$ *at* $x = \alpha$ *is obtained by putting* $x = \alpha$ *and it is denoted by* $p(\alpha)$.

IV. **Zeros of a polynomial:** *A real number* α *is called a zero of* $p(x)$, *if* $p(\alpha) = 0$.

2. I. *If* α, β *are the zeros of* $p(x) = ax^2 + bx + c, a \neq 0$ *then*

(i) $\alpha + \beta = \frac{-b}{a}$, (ii) $\alpha\beta = \frac{c}{a}$.

II. *If* α *and* β *are the zeros of a quadratic polynomial* $p(x)$ *then*

$$p(x) = \{x^2 - (\alpha + \beta)x + \alpha\beta\}.$$

3. I. *If* α, β, γ *are the zeros of* $p(x) = ax^3 + bx^2 + cx + d, a \neq 0$ *then*

(i) $\alpha + \beta + \gamma = \frac{-b}{a}$, (ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$, (iii) $\alpha\beta\gamma = \frac{-d}{a}$.

II. *If* α, β *and* γ *be the zeros of a polynomial* $p(x)$ *then*

$$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma.$$

TEST YOURSELF

MCQ

1. Zeros of $p(x) = x^2 - 2x - 3$ are
   (a) 1, –3 (b) 3, –1 (c) –3, –1 (d) 1, 3
2. If α, β, γ are the zeros of the polynomial $x^3 - 6x^2 - x + 30$ then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is
   (a) –1 (b) 1 (c) –5 (d) 30
3. If α, β are the zeros of $kx^2 - 2x + 3k$ such that $\alpha + \beta = \alpha\beta$ then k = ?
   (a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$
4. It is given that the difference between the zeros of $4x^2 - 8kx + 9$ is 4 and $k > 0$. Then, k = ?
   (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

Short-Answer Questions

5. Find the zeros of the polynomial $x^2 + 2x - 195$.
6. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a.
7. Find a quadratic polynomial whose zeros are 2 and –5.
8. If the zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, find the values of a and b.
9. Verify that 2 is a zero of the polynomial $x^3 + 4x^2 - 3x - 18$.
10. Find the quadratic polynomial, the sum of whose zeros is –5 and their product is 6.

11. Find a cubic polynomial whose zeros are 3, 5 and –2.
12. Using remainder theorem, find the remainder when $p(x) = x^3 + 3x^2 - 5x + 4$ is divided by $(x - 2)$.
13. Show that $(x + 2)$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$.
14. If α, β, γ are the zeros of the polynomial $p(x) = 6x^3 + 3x^2 - 5x + 1$, find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$.
15. If α, β are the zeros of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.
16. Show that the polynomial $f(x) = x^4 + 4x^2 + 6$ has no zero.

Long-Answer Questions

17. If one zero of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ is 3, find the other two zeros.
18. If two zeros of the polynomial $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}$ and $-\sqrt{2}$, find its other two zeros.
19. Find the quotient when $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$ is divided by $(x^2 + 3x + 1)$.
20. Use remainder theorem to find the value of k, it being given that when $x^3 + 2x^2 + kx + 3$ is divided by $(x - 3)$ then the remainder is 21.

ANSWERS (TEST YOURSELF)

1. (a) **2.** (a) **3.** (c) **4.** (c) **5.** –15, 13 **6.** $a = 3$
7. $x^2 + 3x - 10$ **8.** $a = 1, b = \pm\sqrt{2}$ **10.** $x^2 + 5x + 6$ **11.** $x^3 - 6x^2 - x + 30$
12. 14 **14.** 5 **15.** $k = 6$ **17.** 1, 2 **18.** $1, \frac{1}{2}$
19. $3x^2 - 4x + 2$ **20.** $k = -9$

✿