3 Linear Equations in Two Variables

LINEAR EQUATIONS IN TWO VARIABLES

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers ($a \neq 0, b \neq 0$), is called a linear equation in two variables x and y.

Examples Each of the equations

(i) $3x - 4y + 2 = 0$ (ii) $2x + 5y = 9$

(iii) $0.4x + 0.3y = 2.7$ (iv) $\sqrt{2}x - \sqrt{3}y = 0$

is a linear equation in x and y.

SOLUTION OF A LINEAR EQUATION

We say that $x = \alpha$ and $y = \beta$ is a solution of $ax + by + c = 0$ if $a\alpha + b\beta + c = 0$.

EXAMPLE *Show that $x = 3$ and $y = 2$ is a solution of $5x - 3y = 9$.*

SOLUTION Substituting $x = 3$ and $y = 2$ in the given equation, we get

$$\text{LHS} = 5 \times 3 - 3 \times 2 = (15 - 6) = 9 = \text{RHS}.$$

$\therefore$ $x = 3$ and $y = 2$ is a solution of $5x - 3y = 9$.

SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES

Two linear equations in two unknowns x and y are said to form a system of simultaneous linear equations if each of them is satisfied by the same pair of values of x and y.

Example Consider the system of linear equations

$$x + y = 10, \; x - y = 2.$$

By substitution, you will find that each of these equations is satisfied by the values $x = 6$ and $y = 4$.

Hence, the given equations form a system of simultaneous linear equations in x and y.

SOLUTION OF A GIVEN SYSTEM OF TWO SIMULTANEOUS EQUATIONS

A pair of values of x and y satisfying each of the equations in a given system of two simultaneous equations in x and y is called a solution of the system.

EXAMPLE 1 *Show that* $x = 5, y = 2$ *is a solution of the system of linear equations*

$$2x + 3y = 16, x - 2y = 1.$$

SOLUTION The given equations are

$$2x + 3y = 16 \quad \ldots \text{(i)}$$

$$x - 2y = 1. \quad \ldots \text{(ii)}$$

Putting $x = 5$ and $y = 2$ in (i), we get

LHS = $(2 \times 5 + 3 \times 2) = 16$ = RHS.

Putting $x = 5$ and $y = 2$ in (ii), we get

LHS = $(5 - 2 \times 2) = 1$ = RHS.

Thus, $x = 5$ and $y = 2$ satisfy both (i) and (ii).

Hence, $x = 5, y = 2$ is a solution of the given system of equations.

EXAMPLE 2 *Show that* $x = 3, y = 2$ *is not a solution of the system of linear equations* $3x - 2y = 5, 2x + y = 7$.

SOLUTION The given equations are

$$3x - 2y = 5 \quad \ldots \text{(i)}$$

$$2x + y = 7. \quad \ldots \text{(ii)}$$

Putting $x = 3$ and $y = 2$ in (i), we get

LHS = $(3 \times 3 - 2 \times 2) = 5$ = RHS.

Putting $x = 3$ and $y = 2$ in (ii), we get

LHS = $(2 \times 3 + 2) = 8 \neq$ RHS.

Thus, the values $x = 3, y = 2$ do not satisfy (ii).

Hence, $x = 3, y = 2$ is not a solution of the given system of equations.

CONSISTENT AND INCONSISTENT SYSTEMS OF LINEAR EQUATIONS

CONSISTENT SYSTEM OF LINEAR EQUATIONS

A system of two linear equations in two unknowns is said to be consistent if it has at least one solution.

INCONSISTENT SYSTEM OF LINEAR EQUATIONS

A system of two linear equations in two unknowns is said to be inconsistent if it has no solution at all.

Example Consider the system of linear equations

$$x + y = 3,\ 2x + 2y = 7.$$

Clearly, we cannot find values of x and y which may satisfy both the given equations simultaneously.

Hence, the given system is inconsistent.

SOLVING SIMULTANEOUS LINEAR EQUATIONS (GRAPHICAL METHOD)

METHOD Let the given system of linear equations be

$$a_1x + b_1y + c_1 = 0 \quad \text{... (i)}$$

$$a_2x + b_2y + c_2 = 0. \quad \text{... (ii)}$$

On the same graph paper, we draw the graph of each one of the given linear equations.

Each such graph is always a straight line.

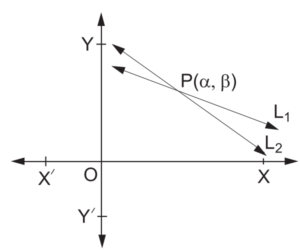
Let the lines L_1 and L_2 represent the graphs of (i) and (ii) respectively.

Now, the following cases arise.

Case I *When the lines L_1 and L_2 intersect at a point*

Let the graph lines L_1 and L_2 intersect at a point $P(\alpha, \beta)$, as shown in the adjoining figure.

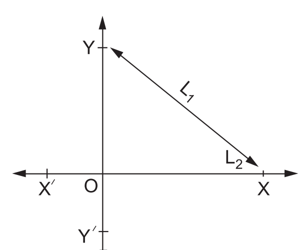
Then, $x = \alpha, y = \beta$ is the unique solution of the given system of equations.



(i)

Case II *When the lines L_1 and L_2 are coincident*

When the two graph lines L_1 and L_2 coincide, the given system of equations has *infinitely many solutions*.

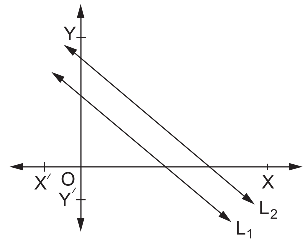


(ii)

Case III *When the lines L_1 and L_2 are parallel*

In this case, there is no common solution of the given system of equations.

Thus, in this case, the given system is *inconsistent*.



(iii)

SUMMARY

A system of two linear equations in x and y has

(i) *a unique solution if the graph lines intersect at a point*

(ii) *infinitely many solutions if the two graph lines coincide*

(iii) *no solution if the two graph lines are parallel*

SOLVED EXAMPLES

GRAPHS OF EQUATIONS HAVING UNIQUE SOLUTIONS

Note Such graph lines intersect at a point.

EXAMPLE 1 *Solve graphically the system of linear equations*

$$x + 2y = 3, \; 4x + 3y = 2.$$

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $x + 2y = 3$

$$x + 2y = 3 \Rightarrow 2y = (3 - x)$$

$$\Rightarrow \quad y = \frac{(3 - x)}{2}. \qquad \ldots \text{(i)}$$

Putting $x = -3$ in (i), we get $y = 3$.

Putting $x = -1$ in (i), we get $y = 2$.

Putting $x = 1$ in (i), we get $y = 1$.

Thus, we have the following table for the equation $x + 2y = 3$.

x	–3	–1	1
y	3	2	1

Now, plot the points $A(-3, 3)$, $B(-1, 2)$ and $C(1, 1)$ on the graph paper.

Join AB and BC to get the graph line ABC. Extend it on both ways.

Thus, the line ABC is the graph of $x + 2y = 3$.

Graph of $4x + 3y = 2$

$4x + 3y = 2 \Rightarrow 3y = (2 - 4x)$

$$\Rightarrow \quad y = \frac{(2-4x)}{3}. \qquad \ldots \text{(ii)}$$

Putting $x = -4$ in (ii), we get $y = 6$.

Putting $x = -1$ in (ii), we get $y = 2$.

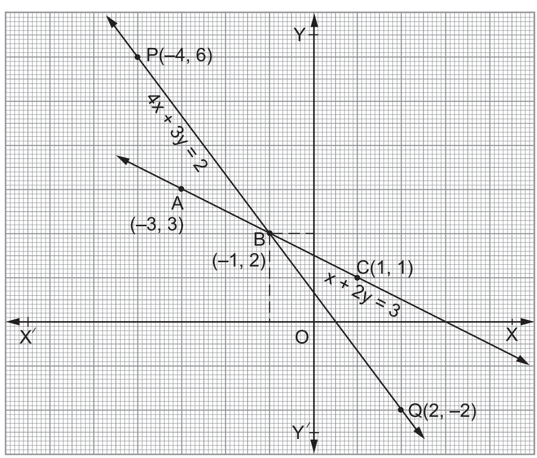
Putting $x = 2$ in (ii), we get $y = -2$.

Thus, we have the following table for the equation $4x + 3y = 2$.

x	–4	–1	2
y	6	2	–2

Now, on the same graph paper as above, plot the points $P(-4, 6)$ and $Q(2, -2)$. The point $B(-1, 2)$ has already been plotted. Join PB and BQ to get the line PBQ. Extend it on both ways.

Thus, the line PBQ is the graph of $4x + 3y = 2$.



The two graph lines intersect at the point $B(-1, 2)$.

$\therefore$ $x = -1$ and $y = 2$ is the solution of the given system of equations.

EXAMPLE 2 *Solve graphically the system of linear equations*

$$4x - 5y + 16 = 0 \textit{ and } 2x + y - 6 = 0.$$

Determine the vertices of the triangle formed by these lines and the x-axis. [CBSE 2006]

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $4x - 5y + 16 = 0$

$4x - 5y + 16 = 0 \Rightarrow 5y = 4x + 16$

$$\Rightarrow \quad y = \frac{(4x + 16)}{5}. \qquad \ldots \text{(i)}$$

Putting $x = -4$ in (i), we get $y = 0$.

Putting $x = 1$ in (i), we get $y = 4$.

Putting $x = 6$ in (i), we get $y = 8$.

Thus, we have the following table for $4x - 5y + 16 = 0$.

x	–4	1	6
y	0	4	8

Now, plot the points $A(-4, 0)$, $B(1, 4)$ and $C(6, 8)$ on the graph paper.

Join AB and BC to get the graph line ABC. Extend it on both ways.

Thus, the line ABC is the graph of $4x - 5y + 16 = 0$.

Graph of $2x + y - 6 = 0$

$2x + y - 6 = 0 \Rightarrow y = (6 - 2x)$. … (ii)

Putting $x = 0$ in (ii), we get $y = 6$.

Putting $x = 2$ in (ii), we get $y = 2$.

Putting $x = 3$ in (ii), we get $y = 0$.

Thus, we have the following table for $2x + y - 6 = 0$.

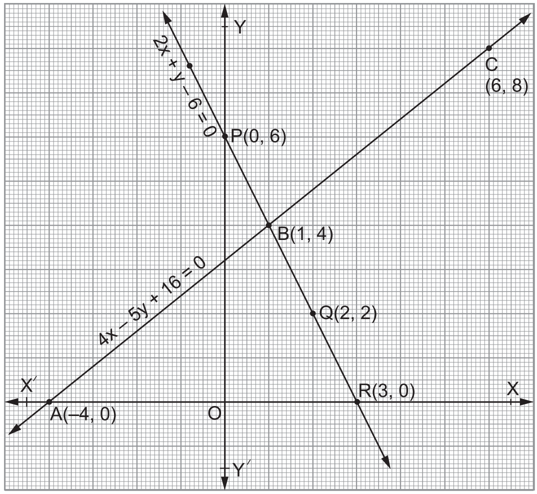
x	0	2	3
y	6	2	0

On the same graph paper as above, plot the points

$P(0, 6)$, $Q(2, 2)$ and $R(3, 0)$.

Join PQ and QR to get the graph line PQR. Extend it on both ways.

Thus, the line PQR is the graph of $2x + y - 6 = 0$.



The two graph lines ABC and PQR intersect at the point $B(1, 4)$.

$\therefore$ $x = 1$ and $y = 4$ is the solution of the given system of equations.

These lines form $\triangle BAR$ with the x-axis, whose vertices are $B(1, 4)$, $A(-4, 0)$ and $R(3, 0)$.

EXAMPLE 3 *Solve the following system of linear equations graphically:*

$$4x - 5y - 20 = 0 \text{ and } 3x + 5y - 15 = 0.$$

Determine the vertices of the triangle formed by the lines representing the above equations and the y-axis. [CBSE 2004]

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $4x - 5y - 20 = 0$

$4x - 5y - 20 = 0 \Rightarrow 5y = (4x - 20)$

$$\Rightarrow \quad y = \frac{(4x - 20)}{5}. \qquad \ldots \text{(i)}$$

Putting $x = 0$ in (i), we get $y = -4$.

Putting $x = 2$ in (i), we get $y = -2.4$.

Putting $x = 5$ in (i), we get $y = 0$.

Thus, we have the following table for $4x - 5y - 20 = 0$.

x	0	2	5
y	–4	–2.4	0

Now, plot the points $A(0, -4)$, $B(2, -2.4)$ and $C(5, 0)$ on the graph paper.

Join AB and BC to get the graph line ABC. Extend it on both ways.

Thus, the line ABC is the graph of $4x - 5y - 20 = 0$.

Graph of $3x + 5y - 15 = 0$

$3x + 5y - 15 = 0 \Rightarrow 5y = (15 - 3x)$

$$\Rightarrow \quad y = \frac{(15 - 3x)}{5}. \qquad \text{... (ii)}$$

Putting $x = -5$ in (ii), we get $y = 6$.

Putting $x = 0$ in (ii), we get $y = 3$.

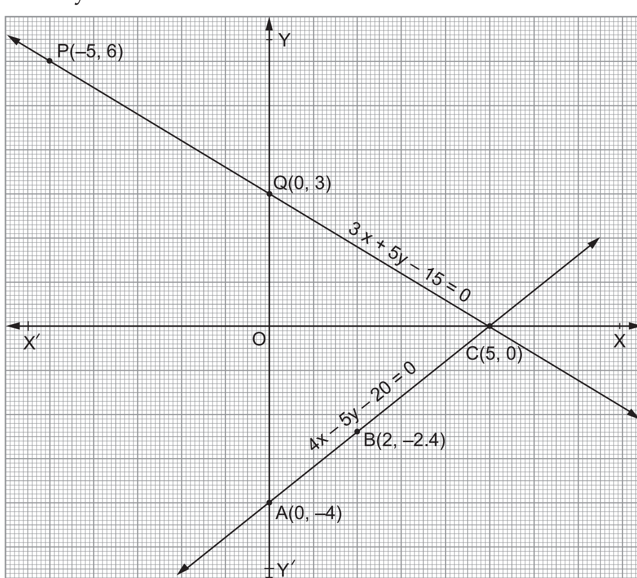
Putting $x = 5$ in (ii), we get $y = 0$.

Thus, we have the following table for $3x + 5y - 15 = 0$.

x	–5	0	5
y	6	3	0

On the same graph paper as above, plot the points $P(-5, 6)$ and $Q(0, 3)$. The third point $C(5, 0)$ has already been plotted.

Join PQ and QC to get the graph line PQC. Extend it on both ways.



Thus, the line PQC is the graph of $3x + 5y - 15 = 0$.

The two graph lines intersect at the point $C(5, 0)$.

$\therefore$ $x = 5, y = 0$ is the solution of the given system of equations.

Clearly, the given equations are represented by the graph lines ABC and PQC respectively.

The vertices of $\triangle AQC$ formed by these lines and the y-axis are $A(0, -4)$, $Q(0, 3)$ and $C(5, 0)$.

EXAMPLE 4 *Solve the following system of equations graphically:*

$3x + 2y - 11 = 0$ *and* $2x - 3y + 10 = 0$. [CBSE 2006C]

Shade the region bounded by these lines and the x-axis.

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $3x + 2y - 11 = 0$

$3x + 2y - 11 = 0 \Rightarrow 2y = (11 - 3x)$

$$\Rightarrow \quad y = \frac{(11 - 3x)}{2}. \qquad \ldots \text{(i)}$$

Putting $x = -1$ in (i), we get $y = 7$.

Putting $x = 1$ in (i), we get $y = 4$.

Putting $x = 3$ in (i), we get $y = 1$.

Thus, we have the following table for $3x + 2y - 11 = 0$.

x	–1	1	3
y	7	4	1

Now, plot the points $A(-1, 7)$, $B(1, 4)$ and $C(3, 1)$ on the graph paper.

Join AB and BC to obtain the graph line ABC. Extend it on both ways.

Thus, the line ABC is the graph of the equation $3x + 2y - 11 = 0$.

Graph of $2x - 3y + 10 = 0$

$$2x - 3y + 10 = 0 \Rightarrow 3y = 2x + 10 \Rightarrow y = \frac{(2x + 10)}{3}. \qquad \ldots \text{(ii)}$$

Putting $x = -2$ in (ii), we get $y = 2$.

Putting $x = 1$ in (ii), we get $y = 4$.

Putting $x = 4$ in (ii), we get $y = 6$.

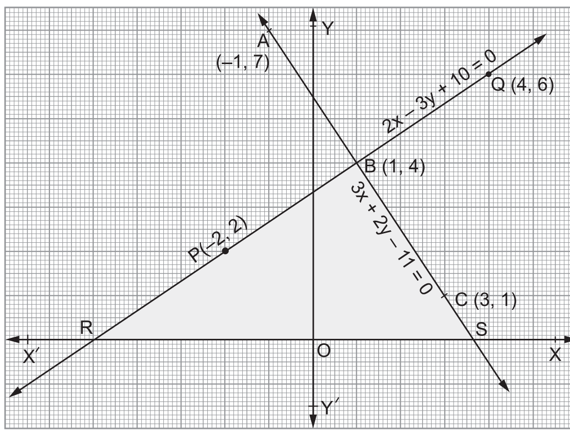
Thus, we have the following table for $2x - 3y + 10 = 0$.

x	–2	1	4
y	2	4	6

On the same graph paper as above, plot the points $P(-2, 2)$ and $Q(4, 6)$. The third point $B(1, 4)$ has already been plotted.

Now, join PB and BQ to obtain the graph line PBQ. Extend it on both ways.

Thus, the line PBQ is the graph of the equation $2x - 3y + 10 = 0$.



The two graph lines intersect at the point $B(1, 4)$.

$\therefore$ $x = 1, y = 4$ is the solution of the given system of equations.

These graph lines intersect the x-axis at R and S.

The region bounded by these lines and the x-axis has been shaded.

The shaded region is the $\triangle BRS$ with $B(1, 4)$, $R(-5, 0)$ and $S\left(\frac{11}{3}, 0\right)$.

EXAMPLE 5 *Solve the following system of linear equations graphically:*

$$3x + y - 11 = 0, x - y - 1 = 0.$$

Shade the region bounded by these lines and the y-axis.

Find the coordinates of the points where the graph lines cut the y-axis. [CBSE 2002C]

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $3x + y - 11 = 0$

$3x + y - 11 = 0 \Rightarrow y = (11 - 3x)$. … (i)

Putting $x = 2$ in (i), we get $y = 5$.

Putting $x = 3$ in (i), we get $y = 2$.

Putting $x = 5$ in (i), we get $y = -4$.

Thus, we have the following table for equation (i).

x	2	3	5
y	5	2	–4

On the graph paper, plot the points $A(2, 5)$, $B(3, 2)$ and $C(5, -4)$. Join AB and BC to get the graph line ABC.

Thus, the line ABC is the graph of the equation $3x + y - 11 = 0$.

Graph of $x - y - 1 = 0$

$x - y - 1 = 0 \Rightarrow y = (x - 1)$. … (ii)

Putting $x = -3$ in (ii), we get $y = -4$.

Putting $x = 0$ in (ii), we get $y = -1$.

Putting $x = 3$ in (ii), we get $y = 2$.

Thus, we have the following table for equation (ii).

x	–3	0	3
$-y$	–4	–1	2

On the same graph paper as above, plot the points $P(-3, -4)$ and $Q(0, -1)$. The third point $B(3, 2)$ is already plotted.

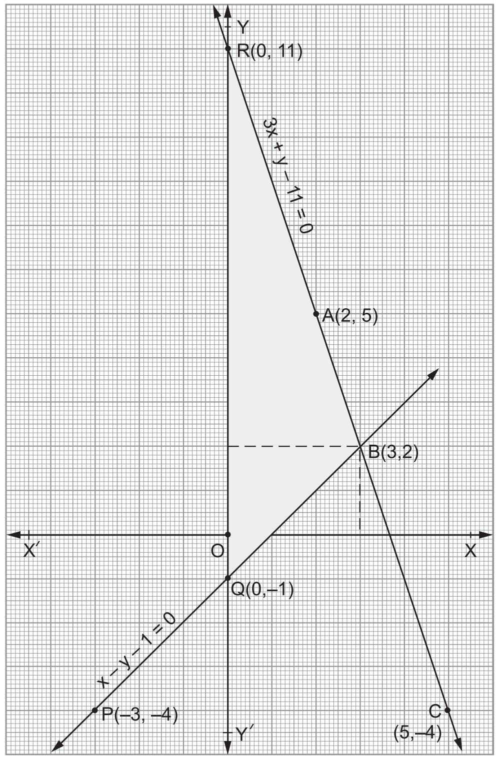
Join PQ and QB to get the graph line PQB.

Thus, line PQB is the graph of the equation $x - y - 1 = 0$.

The two graph lines intersect at the point $B(3, 2)$.

$\therefore$ $x = 3$, $y = 2$ is the solution of the given system of equations.

The region bounded by these lines and the y-axis has been shaded.



On extending the graph lines on both sides, we find that these graph lines intersect the y-axis at the points $Q(0, -1)$ and $R(0, 11)$.

GRAPH OF EQUATIONS HAVING INFINITELY MANY SOLUTIONS

Note Such graph lines just coincide.

EXAMPLE 6 *Show graphically that the system of equations*

$$3x - y = 2,\ 9x - 3y = 6$$

has an infinite number of solutions.

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $3x - y = 2$

$3x - y = 2 \Rightarrow y = (3x - 2)$. … (i)

Putting $x = -1$ in (i), we get $y = -5$.

Putting $x = 0$ in (i), we get $y = -2$.

Putting $x = 2$ in (i), we get $y = 4$.

Thus, we have the following table for $3x - y = 2$.

x	–1	0	2
y	–5	–2	4

Now, plot the points $A(-1, -5)$, $B(0, -2)$ and $C(2, 4)$ on the graph paper.

Join AB and BC to get the graph line ABC.

Extend the graph line ABC on both sides.

Thus, the line ABC is the graph of the equation $3x - y = 2$.

Graph of $9x - 3y = 6$

$9x - 3y = 6 \Rightarrow 3y = (9x - 6)$

$$\Rightarrow \quad y = \frac{(9x-6)}{3}. \qquad \ldots \text{(ii)}$$

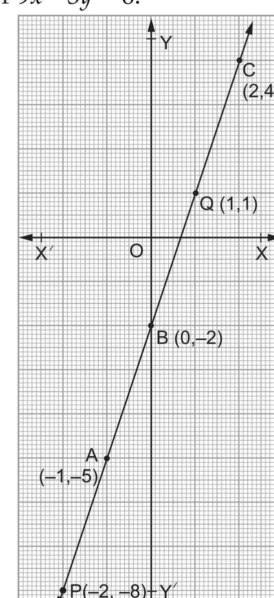
Putting $x = -2$ in (ii), we get $y = -8$.

Putting $x = 1$ in (ii), we get $y = 1$.

Putting $x = 2$ in (ii), we get $y = 4$.

Thus, we have the following table for $9x - 3y = 6$.

x	–2	1	2
y	–8	1	4



Now, plot the points $P(-2, -8)$ and $Q(1, 1)$ on the same graph paper. The point $C(2, 4)$ has already been plotted.

Join PQ and QC to obtain the line PQC.

Thus, the line PQC is the graph of $9x - 3y = 6$.

Thus, we find that the two graph lines coincide. Hence, the given system of equations has an infinite number of solutions.

GRAPH OF EQUATIONS HAVING NO SOLUTION

Note Such graph lines are parallel.

EXAMPLE 7 *Show graphically that the system of linear equations*

$$2x - 3y = 5, 6y - 4x = 3$$

is inconsistent, i.e., has no solution.

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and the y-axis respectively.

Graph of $2x - 3y = 5$

$$2x - 3y = 5 \Rightarrow 3y = (2x - 5)$$

$$\Rightarrow \quad y = \frac{(2x-5)}{3}. \qquad \ldots \text{(i)}$$

Putting $x = -2$ in (i), we get $y = -3$.

Putting $x = 1$ in (i), we get $y = -1$.

Putting $x = 4$ in (i), we get $y = 1$.

Thus, we have the following table for the equation $2x - 3y = 5$.

x	–2	1	4
y	–3	–1	1

Now, plot the points $A(-2, -3)$, $B(1, -1)$ and $C(4, 1)$ on the graph paper.

Join AB and BC to get the graph line ABC. Extend it on both ways.

Thus, the line ABC is the graph of the equation $2x - 3y = 5$.

Graph of $6y - 4x = 3$

$$6y - 4x = 3 \Rightarrow 6y = (3 + 4x)$$

$$\Rightarrow \quad y = \frac{(3+4x)}{6}. \qquad \ldots \text{(ii)}$$

Putting $x = -3$ in (ii), we get $y = \frac{-3}{2}$.

Putting $x = 0$ in (ii), we get $y = \frac{1}{2}$.

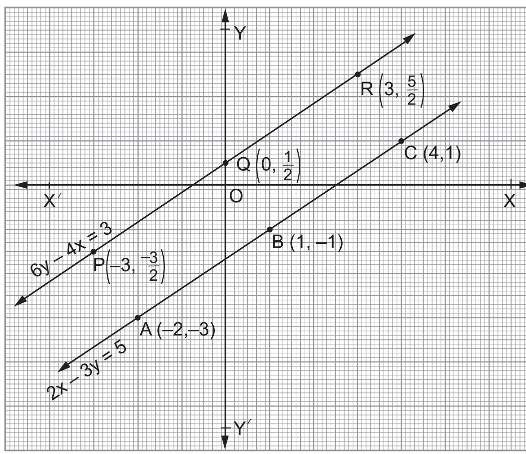
Putting $x = 3$ in (ii), we get $y = \frac{5}{2}$.

Thus, we have the following table for the equation $6y - 4x = 3$.

x	-3	0	3
y	$\frac{-3}{2}$	$\frac{1}{2}$	$\frac{5}{2}$

Now, plot the points $P\left(-3, \frac{-3}{2}\right)$, $Q\left(0, \frac{1}{2}\right)$ and $R\left(3, \frac{5}{2}\right)$ on the same graph paper as above. Join PQ and QR to get the graph line PQR. Extend it on both ways.

Then, the line PQR is the graph of the equation $6y - 4x = 3$.



It is clear from the graph that the two graph lines are parallel and do not intersect even when produced.

Hence, the given system of equations has no solution.

EXAMPLE 8 *Show graphically that the system of linear equations*

$$x - y = 8,\ 3x - 3y = 16$$

is inconsistent, i.e., has no solution.

SOLUTION On a graph paper, draw a horizontal line $X'OX$ and a vertical line YOY' as the x-axis and y-axis respectively.

Graph of $x - y = 8$

The first equation is $x - y = 8$.

Now, $x - y = 8 \Rightarrow y = x - 8$. … (i)

Putting $x = 3$ in (i), we get $y = -5$.

Putting $x = 4$ in (i), we get $y = -4$.

Putting $x = 5$ in (i), we get $y = -3$.

Thus, we have the following table for the equation $x - y = 8$.

x	3	4	5
y	–5	–4	–3

Now, we plot the points $A(3, -5)$, $B(4, -4)$ and $C(5, -3)$ on the graph paper. Join AB and BC to get the graph line ABC.

Graph of $3x - 3y = 16$

The second equation is $3x - 3y = 16$.

$$3x - 3y = 16 \Rightarrow 3y = 3x - 16 \Rightarrow y = \frac{(3x - 16)}{3}. \qquad \text{... (ii)}$$

Putting $x = 2$ in (ii), we get $y = \frac{-10}{3} = -3\frac{1}{3} = -3.3$.

Putting $x = 3$ in (ii), we get $y = \frac{-7}{3} = -2\frac{1}{3} = -2.3$.

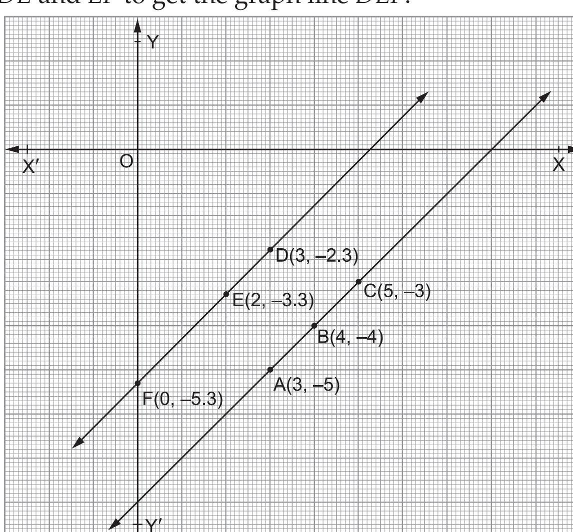
Putting $x = 0$ in (ii), we get $y = \frac{-16}{3} = -5\frac{1}{3} = -5.3$.

Thus, we have the following table for the equation $3x - 3y = 16$.

x	3	2	0
y	–2.3	–3.3	–5.3

Now, we plot $D(3, -2.3)$, $E(2, -3.3)$ and $F(0, -5.3)$ on the same graph paper as above.

Join DE and EF to get the graph line DEF.



It is clear from the graph that the lines *ABC* and *DEF* are parallel and do not meet when produced.

Hence, the given system of equations has no solution and therefore, it is inconsistent.

EXERCISE 3A

Solve each of the following systems of equations graphically:

1. $2x + 3y = 2$, $x - 2y = 8$. [CBSE 2007]
2. $3x + 2y = 4$, $2x - 3y = 7$. [CBSE 2006C]
3. $2x + 3y = 8$, $x - 2y + 3 = 0$. [CBSE 2005]
4. $2x - 5y + 4 = 0$, $2x + y - 8 = 0$. [CBSE 2005]
5. $3x + 2y = 12$, $5x - 2y = 4$. [CBSE 2006]
6. $3x + y + 1 = 0$, $2x - 3y + 8 = 0$. [CBSE 2007C]
7. $2x + 3y + 5 = 0$, $3x - 2y - 12 = 0$.
8. $2x - 3y + 13 = 0$, $3x - 2y + 12 = 0$.
9. $2x + 3y - 4 = 0$, $3x - y + 5 = 0$. [CBSE 2004C]
10. $x + 2y + 2 = 0$, $3x + 2y - 2 = 0$.

Solve each of the following given systems of equations graphically and find the vertices and area of the triangle formed by these lines and the x-axis:

11. $x - y + 3 = 0$, $2x + 3y - 4 = 0$.
12. $2x - 3y + 4 = 0$, $x + 2y - 5 = 0$. [CBSE 2005]
13. $4x - 3y + 4 = 0$, $4x + 3y - 20 = 0$. [CBSE 2008]
14. $x - y + 1 = 0$, $3x + 2y - 12 = 0$. [CBSE 2002]
15. $x - 2y + 2 = 0$, $2x + y - 6 = 0$.

Solve each of the following given systems of equations graphically and find the vertices and area of the triangle formed by these lines and the y-axis:

16. $2x - 3y + 6 = 0$, $2x + 3y - 18 = 0$. [CBSE 2004]
17. $4x - y - 4 = 0$, $3x + 2y - 14 = 0$. [CBSE 2006C]
18. $x - y - 5 = 0$, $3x + 5y - 15 = 0$. [CBSE 2009C]
19. $2x - 5y + 4 = 0$, $2x + y - 8 = 0$. [CBSE 2005]
20. $5x - y - 7 = 0$, $x - y + 1 = 0$.
21. $2x - 3y = 12$, $x + 3y = 6$. [CBSE 2008]

Show graphically that each of the following given systems of equations has infinitely many solutions:

22. $2x + 3y = 6$, $4x + 6y = 12$. [CBSE 2010]
23. $3x - y = 5$, $6x - 2y = 10$.
24. $2x + y = 6$, $6x + 3y = 18$.
25. $x - 2y = 5$, $3x - 6y = 15$.

Show graphically that each of the following given systems of equations is inconsistent, i.e., has no solution:

26. $x - 2y = 6$, $3x - 6y = 0$.
27. $2x + 3y = 4$, $4x + 6y = 12$.
28. $2x + y = 6$, $6x + 3y = 20$.
29. Draw the graphs of the following equations on the same graph paper:

$$2x + y = 2, \; 2x + y = 6.$$

Find the coordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium so formed. [HOTS]

HINT The line $2x + y = 2$ cuts the x-axis at $A(1, 0)$ and the y-axis at $B(0, 2)$.

The line $2x + y = 6$ cuts the x-axis at $C(3, 0)$ and the y-axis at $D(0, 6)$.

Area of trap. $ABDC$ = ar($\triangle OCD$) − ar($\triangle OAB$)

$$= \left(\frac{1}{2} \times 3 \times 6\right) - \left(\frac{1}{2} \times 1 \times 2\right) = 8 \text{ sq units.}$$

ANSWERS (EXERCISE 3A)

1. $x = 4, y = -2$ **2.** $x = 2, y = -1$ **3.** $x = 1, y = 2$ **4.** $x = 3, y = 2$
5. $x = 2, y = 3$ **6.** $x = -1, y = 2$ **7.** $x = 2, y = -3$ **8.** $x = -2, y = 3$
9. $x = -1, y = 2$ **10.** $x = 2, y = -2$
11. $(x = -1, y = 2)$; $A(-1, 2)$, $B(-3, 0)$, $C(2, 0)$; ar($\triangle ABC$) = 5 sq units
12. $(x = 1, y = 2)$; $A(1, 2)$, $B(-2, 0)$, $C(5, 0)$; ar($\triangle ABC$) = 7 sq units
13. $(x = 2, y = 4)$; $A(2, 4)$, $B(-1, 0)$, $C(5, 0)$; ar($\triangle ABC$) = 12 sq units
14. $(x = 2, y = 3)$; $A(2, 3)$, $B(-1, 0)$, $C(4, 0)$; ar($\triangle ABC$) = 7.5 sq units
15. $(x = 2, y = 2)$; $A(2, 2)$, $B(-2, 0)$, $C(3, 0)$; ar($\triangle ABC$) = 5 sq units
16. $(x = 3, y = 4)$; $A(3, 4)$, $B(0, 2)$, $C(0, 6)$; ar($\triangle ABC$) = 6 sq units
17. $(x = 2, y = 4)$; $A(2, 4)$, $B(0, -4)$, $C(0, 7)$; ar($\triangle ABC$) = 11 sq units
18. $(x = 5, y = 0)$; $A(5, 0)$, $B(0, -5)$, $C(0, 3)$; ar($\triangle ABC$) = 20 sq units
19. $(x = 3, y = 2)$; $A(3, 2)$, $B(0, 0.8)$, $C(0, 8)$; ar($\triangle ABC$) = 10.8 sq units
20. $(x = 2, y = 3)$; $A(2, 3)$, $B(0, -7)$, $C(0, 1)$; ar($\triangle ABC$) = 8 sq units

21. ($x = 6, y = 0$), $A(6, 0)$, $B(0, -4)$, $C(0, 2)$, ar($\triangle ABC$) = 18 sq units.

29. 8 sq units

SOLVING SIMULTANEOUS LINEAR EQUATIONS (BY ALGEBRAIC METHODS)

(I) SUBSTITUTION METHOD

METHOD Suppose we are given two linear equations in x and y. For solving these equations by the substitution method, we proceed according to the following steps.

Step 1. *Express y in terms of x in one of the given equations.*

Step 2. *Substitute this value of y in terms of x in the other equation. This gives a linear equation in x.*

Step 3. *Solve the linear equation in x obtained in Step 2.*

Step 4. *Substitute this value of x in the relation taken in Step 1 to obtain a linear equation in y.*

Step 5. *Solve the above linear equation in y to get the value of y.*

REMARK We may interchange the role of x and y in the above method.

SOLVED EXAMPLES

EXAMPLE 1 *Solve for x and y, using substitution method:*

$$2x + y = 7,\ 4x - 3y + 1 = 0.$$

SOLUTION The given system of equations is

$$2x + y = 7 \quad \ldots \text{(i)}$$

$$4x - 3y = -1. \quad \ldots \text{(ii)}$$

From (i), we get $y = (7 - 2x)$.

Substituting $y = (7 - 2x)$ in (ii), we get

$4x - 3(7 - 2x) = -1$

$\Rightarrow\ 4x - 21 + 6x = -1$

$\Rightarrow\ 10x = 20$

$\Rightarrow\ x = 2.$

Substituting $x = 2$ in (i), we get

$$2 \times 2 + y = 7 \Rightarrow y = 7 - 4 = 3.$$

Hence, the solution is $x = 2$, $y = 3$.

EXAMPLE 2 *Solve for x and y, using substitution method:*

$$\frac{3x}{2} - \frac{5y}{3} = -2, \ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}.$$

SOLUTION The given system of equations may be written as

$$9x - 10y + 12 = 0 \quad \ldots \text{(i)}$$

$$2x + 3y - 13 = 0. \quad \ldots \text{(ii)}$$

From (ii), we get $y = \dfrac{13 - 2x}{3}$.

Substituting $y = \dfrac{13 - 2x}{3}$ in (i), we get

$$9x - \frac{10(13 - 2x)}{3} + 12 = 0$$

$$\Rightarrow \quad 27x - 10(13 - 2x) + 36 = 0$$

$$\Rightarrow \quad 27x - 130 + 20x + 36 = 0$$

$$\Rightarrow \quad 47x - 94 = 0 \Rightarrow 47x = 94 \Rightarrow x = \frac{94}{47} = 2.$$

Substituting $x = 2$ in (i), we get

$$9 \times 2 - 10y + 12 = 0 \Rightarrow 10y = 30 \Rightarrow y = \frac{30}{10} = 3.$$

Hence, $x = 2$ and $y = 3$ is the required solution.

(II) ELIMINATION METHOD

METHOD In this method, we eliminate one of the unknown variables and proceed using the following steps.

Step 1. *Multiply the given equations by suitable numbers so as to make the coefficients of one of the unknown variables numerically equal.*

Step 2. *If the numerically equal coefficients are opposite in sign then add the new equations.*

Otherwise, subtract them.

Step 3. *The resulting equation is linear in one unknown variable.*

Solve it to get the value of one of the unknown quantities.

Step 4. *Substitute this value in any of the given equations.*

Step 5. *Solve it to get the value of the other unknown variable.*

EXAMPLE 3 *Solve for x and y using elimination method:*

$$10x + 3y = 75,\ 6x - 5y = 11.$$

SOLUTION The given equations are

$$10x + 3y = 75 \quad \ldots \text{(i)}$$

$$6x - 5y = 11. \quad \ldots \text{(ii)}$$

Multiplying (i) by 5 and (ii) by 3, we get

$$50x + 15y = 375 \quad \ldots \text{(iii)}$$

$$18x - 15y = 33. \quad \ldots \text{(iv)}$$

Adding (iii) and (iv), we get

$$68x = 408 \Rightarrow x = \frac{408}{68} \Rightarrow x = 6.$$

Putting $x = 6$ in (i), we get

$$(10 \times 6) + 3y = 75 \Rightarrow 60 + 3y = 75$$

$$\Rightarrow \quad 3y = 75 - 60 \Rightarrow 3y = 15 \Rightarrow y = 5.$$

$$\therefore \quad x = 6,\ y = 5.$$

EXAMPLE 4 *Solve for x and y:*

$$11x + 15y + 23 = 0,\ 7x - 2y - 20 = 0.$$

SOLUTION The given equations are

$$11x + 15y = -23 \quad \ldots \text{(i)}$$

$$7x - 2y = 20. \quad \ldots \text{(ii)}$$

Multiplying (i) by 2 and (ii) by 15 and adding the results, we get

$$22x + 105x = -46 + 300$$

$$\Rightarrow \quad 127x = 254$$

$$\Rightarrow \quad x = \frac{254}{127} = 2.$$

Putting $x = 2$ in (i), we get

$$22 + 15y = -23$$

$$\Rightarrow \quad 15y = -23 - 22$$

$$\Rightarrow \quad 15y = -45 \Rightarrow y = \frac{-45}{15} = -3.$$

Hence, $x = 2$ and $y = -3$.

<u>EXAMPLE 5</u> *Solve for x and y:* [CBSE 2000, '04C, '05]

$$0.4x - 1.5y = 6.5,$$
$$0.3x + 0.2y = 0.9.$$

SOLUTION Multiplying each of the equations by 10, we get

$$4x - 15y = 65 \quad \ldots \text{(i)}$$
$$3x + 2y = 9. \quad \ldots \text{(ii)}$$

Multiplying (i) by 2 and (ii) by 15 and adding, we get

$$8x + 45x = 130 + 135$$
$$\Rightarrow \quad 53x = 265$$
$$\Rightarrow \quad x = \frac{265}{53} = 5.$$

Putting $x = 5$ in (ii), we get

$$15 + 2y = 9 \Rightarrow 2y = 9 - 15 \Rightarrow 2y = -6 \Rightarrow y = -3.$$

Hence, $x = 5$ and $y = -3$.

<u>EXAMPLE 6</u> *Solve for x and y:*

$$\frac{4}{x} + 3y = 8, \frac{6}{x} - 4y = -5.$$ [CBSE 2010]

SOLUTION Putting $\frac{1}{x} = u$, the given equations become:

$$4u + 3y = 8 \quad \ldots \text{(i)}$$
$$6u - 4y = -5. \quad \ldots \text{(ii)}$$

Multiplying (i) by 4 and (ii) by 3 and adding the results, we get

$$16u + 18u = 32 - 15$$
$$\Rightarrow \quad 34u = 17$$
$$\Rightarrow \quad u = \frac{17}{34} = \frac{1}{2}. \quad \ldots \text{(iii)}$$

Putting $u = \frac{1}{2}$ in (i), we get

$$\left(4 \times \frac{1}{2}\right) + 3y = 8 \Rightarrow 2 + 3y = 8$$
$$\Rightarrow 3y = 8 - 2 \Rightarrow 3y = 6 \Rightarrow y = 2.$$

Now, $u = \frac{1}{2} \Rightarrow \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2.$

Hence, $x = 2$ and $y = 2$.

<u>EXAMPLE 7</u> *Solve for x and y:*

$$\frac{2}{x} + \frac{3}{y} = 13, \frac{5}{x} - \frac{4}{y} = -2 \ \ (x \neq 0 \text{ and } y \neq 0).$$ [CBSE 2008C]

SOLUTION Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become

$$2u + 3v = 13 \quad \ldots \text{(i)}$$

$$5u - 4v = -2. \quad \ldots \text{(ii)}$$

Multiplying (i) by 4 and (ii) by 3 and adding the results, we get

$$8u + 15u = 52 - 6$$

$$\Rightarrow \quad 23u = 46$$

$$\Rightarrow \quad u = \frac{46}{23} = 2.$$

Putting $u = 2$ in (i), we get

$$(2 \times 2) + 3v = 13 \Rightarrow 3v = 13 - 4 = 9 \Rightarrow v = 3.$$

Now, $u = 2 \Rightarrow \frac{1}{x} = 2 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}\cdot$

And, $v = 3 \Rightarrow \frac{1}{y} = 3 \Rightarrow 3y = 1 \Rightarrow y = \frac{1}{3}\cdot$

Hence, $x = \frac{1}{2}$ and $y = \frac{1}{3}\cdot$

EXAMPLE 8 *Solve for x and y:*

$$\frac{1}{2x} - \frac{1}{y} = -1, \frac{1}{x} + \frac{1}{2y} = 8 \ \ (x \neq 0, y \neq 0).$$ [CBSE 2004C]

SOLUTION Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become

$$\frac{u}{2} - v = -1 \Rightarrow u - 2v = -2 \quad \ldots \text{(i)}$$

$$u + \frac{v}{2} = 8 \Rightarrow 2u + v = 16. \quad \ldots \text{(ii)}$$

Multiplying (ii) by 2 and adding the result with (i), we get

$$u + 4u = -2 + 32$$

$$\Rightarrow \quad 5u = 30$$

$$\Rightarrow \quad u = \frac{30}{5} = 6.$$

Putting $u = 6$ in (i), we get

$$6 - 2v = -2 \Rightarrow 2v = 8 \Rightarrow v = 4.$$

Now, $u = 6 \Rightarrow \frac{1}{x} = 6 \Rightarrow 6x = 1 \Rightarrow x = \frac{1}{6}\cdot$

And, $v = 4 \Rightarrow \frac{1}{y} = 4 \Rightarrow 4y = 1 \Rightarrow y = \frac{1}{4}\cdot$

Hence, $x = \frac{1}{6}$ and $y = \frac{1}{4}\cdot$

EXAMPLE 9 *Solve for x and y:*

$$\frac{1}{7x}+\frac{1}{6y}=3,\ \frac{1}{2x}-\frac{1}{3y}=5\ \ (x\neq 0,\ y\neq 0).$$

SOLUTION Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, the given equations become

$$\frac{u}{7}+\frac{v}{6}=3 \Rightarrow 6u+7v=126 \qquad \ldots \text{(i)}$$

$$\frac{u}{2}-\frac{v}{3}=5 \Rightarrow 3u-2v=30. \qquad \ldots \text{(ii)}$$

Multiplying (i) by 2 and (ii) by 7 and adding the results, we get

$$12u+21u=252+210$$

$$\Rightarrow \quad 33u=462$$

$$\Rightarrow \quad u=\frac{462}{33}=14.$$

Putting $u=14$ in (i), we get

$$(6\times 14)+7v=126$$

$$\Rightarrow \quad 7v=126-84=42 \Rightarrow v=\frac{42}{7}=6.$$

Now, $u=14 \Rightarrow \frac{1}{x}=14 \Rightarrow 14x=1 \Rightarrow x=\frac{1}{14}\cdot$

And, $v=6 \Rightarrow \frac{1}{y}=6 \Rightarrow 6y=1 \Rightarrow y=\frac{1}{6}\cdot$

Hence, $x=\frac{1}{14}$ and $y=\frac{1}{6}\cdot$

EXAMPLE 10 *Solve for x and y:*

$$\frac{3a}{x}-\frac{2b}{y}+5=0,\ \frac{a}{x}+\frac{3b}{y}-2=0\ \ (x\neq 0,\ y\neq 0).$$ [CBSE 2005C]

SOLUTION Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, the given equations become

$$3au-2bv=-5 \qquad \ldots \text{(i)}$$

$$au+3bv=2. \qquad \ldots \text{(ii)}$$

Multiplying (ii) by 3 and subtracting (i) from the result, we get

$$9bv+2bv=6+5$$

$$\Rightarrow \quad 11bv=11 \Rightarrow v=\frac{11}{11b}=\frac{1}{b}\cdot$$

Putting $v=\frac{1}{b}$ in (ii), we get

$$au+3=2 \Rightarrow au=-1 \Rightarrow u=\frac{-1}{a}\cdot$$

Now, $u = \frac{-1}{a} \Rightarrow \frac{1}{x} = \frac{-1}{a} \Rightarrow -x = a \Rightarrow x = -a.$

And, $v = \frac{1}{b} \Rightarrow \frac{1}{y} = \frac{1}{b} \Rightarrow y = b.$

Hence, $x = -a$ and $y = b$.

EXAMPLE 11 *Solve for x and y:*

$$6x + 3y = 7xy,\ 3x + 9y = 11xy \quad (x \neq 0,\ y \neq 0).$$

SOLUTION On dividing each of the given equations by xy, we get

$$\frac{6}{y} + \frac{3}{x} = 7 \qquad \ldots \text{(i)}$$

$$\frac{3}{y} + \frac{9}{x} = 11. \qquad \ldots \text{(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in (i) and (ii), we get

$$6v + 3u = 7 \qquad \ldots \text{(iii)}$$

$$3v + 9u = 11. \qquad \ldots \text{(iv)}$$

Multiplying (iii) by 3 and subtracting (iv) from the result, we get

$$18v - 3v = 21 - 11$$

$$\Rightarrow \quad 15v = 10$$

$$\Rightarrow \quad v = \frac{10}{15} = \frac{2}{3}.$$

Putting $v = \frac{2}{3}$ in (iii), we get

$$\left(6 \times \frac{2}{3}\right) + 3u = 7 \Rightarrow 4 + 3u = 7 \Rightarrow 3u = 3 \Rightarrow u = 1.$$

Now, $u = 1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1.$

And, $v = \frac{2}{3} \Rightarrow \frac{1}{y} = \frac{2}{3} \Rightarrow 2y = 3 \Rightarrow y = \frac{3}{2}.$

Hence, $x = 1$ and $y = \frac{3}{2}.$

EXAMPLE 12 *Solve the following system of equations for x and y:*

$$\frac{5}{x-1} + \frac{1}{y-2} = 2,\ \frac{6}{x-1} - \frac{3}{y-2} = 1.$$ [CBSE 2008C, '09]

SOLUTION Putting $\frac{1}{x-1} = y$ and $\frac{1}{y-2} = v$, the given equations become

$$5u + v = 2 \qquad \ldots \text{(i)}$$

$$6u - 3v = 1. \qquad \ldots \text{(ii)}$$

Multiplying (i) by 3 and adding the result with (ii), we get

$$15u + 6u = 6 + 1$$

$$\Rightarrow \quad 21u = 7$$

$$\Rightarrow \quad u = \frac{7}{21} = \frac{1}{3}.$$

Putting $u = \frac{1}{3}$ in (i), we get

$$\left(5 \times \frac{1}{3}\right) + v = 2 \Rightarrow \frac{5}{3} + v = 2 \Rightarrow v = 2 - \frac{5}{3} \Rightarrow v = \frac{1}{3}.$$

Now, $u = \frac{1}{3} \Rightarrow \frac{1}{x-1} = \frac{1}{3} \Rightarrow x - 1 = 3 \Rightarrow x = 4$

[by cross multiplication].

Similarly, $v = \frac{1}{3} \Rightarrow \frac{1}{y-2} = \frac{1}{3} \Rightarrow y - 2 = 3 \Rightarrow y = 5$

[by cross multiplication].

Hence, $x = 4$ and $y = 5$.

EXAMPLE 13 *Solve for x and y:*

$$\frac{15}{x-y} + \frac{22}{x+y} = 5, \frac{40}{x-y} + \frac{55}{x+y} = 13, x \neq y \text{ and } x \neq -y.$$

[CBSE 2008C]

SOLUTION Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$, the given equations become

$$15u + 22v = 5 \qquad \dots \text{(i)}$$

$$40u + 55v = 13. \qquad \dots \text{(ii)}$$

Multiplying (ii) by 2 and (i) by 5 and subtracting the results, we get

$$80u - 75u = 26 - 25$$

$$\Rightarrow \quad 5u = 1$$

$$\Rightarrow \quad u = \frac{1}{5}.$$

Putting $u = \frac{1}{5}$ in (i), we get

$$\left(15 \times \frac{1}{5}\right) + 22v = 5$$

$$\Rightarrow \quad 3 + 22v = 5$$

$$\Rightarrow \quad 22v = 2 \Rightarrow v = \frac{2}{22} = \frac{1}{11}.$$

Now, $u = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5.$ ... (iii)

And, $v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11.$ ... (iv)

On adding (iii) and (iv), we get $2x = 16 \Rightarrow x = 8$.

On subtracting (iii) from (iv), we get $2y = 6 \Rightarrow y = 3$.

Hence, $x = 8$ and $y = 3$.

EXAMPLE 14 *Solve for x and y:*

$$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}, \frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2,$$

where $(2x+3y) \neq 0$ *and* $(3x-2y) \neq 0$. [CBSE 2004C]

SOLUTION Putting $\frac{1}{(2x+3y)} = u$ and $\frac{1}{3x-2y} = v$, the given equations become

$$\frac{u}{2} + \frac{12v}{7} = \frac{1}{2} \Rightarrow 7u + 24v = 7 \quad \text{... (i)}$$

and $7u + 4v = 2$. ... (ii)

Subtracting (ii) from (i), we get

$$20v = 5 \Rightarrow v = \frac{5}{20} = \frac{1}{4}.$$

Putting $v = \frac{1}{4}$ in (i), we get

$$7u + \left(24 \times \frac{1}{4}\right) = 7$$

$$\Rightarrow \quad 7u + 6 = 7 \Rightarrow 7u = 1 \Rightarrow u = \frac{1}{7}.$$

Now, $u = \frac{1}{7} \Rightarrow \frac{1}{(2x+3y)} = \frac{1}{7} \Rightarrow 2x + 3y = 7$ ... (iii)

and $v = \frac{1}{4} \Rightarrow \frac{1}{(3x-2y)} = \frac{1}{4} \Rightarrow 3x - 2y = 4$. ... (iv)

Multiplying (iii) by 2 and (iv) by 3 and adding the results, we get

$$4x + 9x = 14 + 12$$

$$\Rightarrow \quad 13x = 26 \Rightarrow x = 2.$$

Putting $x = 2$ in (iii), we get

$$(2 \times 2) + 3y = 7 \Rightarrow 3y = (7-4) = 3 \Rightarrow y = 1.$$

Hence, $x = 2$ and $y = 1$.

EXAMPLE 15 *Solve for x and y:*

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2, \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \quad (x \neq 0, y \neq 0).$$

SOLUTION Putting $\frac{1}{\sqrt{x}} = u$ and $\frac{1}{\sqrt{y}} = v$, the given equations become

$$2u + 3v = 2 \qquad \dots \text{(i)}$$

$$4u - 9v = -1. \qquad \dots \text{(ii)}$$

Multiplying (i) by 3 and adding the result with (ii), we get

$$6u + 4u = 6 - 1$$

$$\Rightarrow \quad 10u = 5$$

$$\Rightarrow \quad u = \frac{5}{10} \Rightarrow u = \frac{1}{2}.$$

Putting $u = \frac{1}{2}$ in (i), we get

$$\left(2 \times \frac{1}{2}\right) + 3v = 2$$

$$\Rightarrow \quad 1 + 3v = 2 \Rightarrow 3v = 1 \Rightarrow v = \frac{1}{3}.$$

Now, $u = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4.$

And, $v = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9.$

Hence, $x = 4$ and $x = 9$.

EXAMPLE 16 *Solve the following system of linear equations:*

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2,$$

$$(a+b)(x+y) = a^2 + b^2.$$

[CBSE 2004, '07C, '08]

SOLUTION The given equations may be written as

$$(a-b)x + (a+b)y = a^2 - 2ab - b^2 \qquad \dots \text{(i)}$$

$$(a+b)x + (a+b)y = a^2 + b^2. \qquad \dots \text{(ii)}$$

Subtracting (i) from (ii), we get

$$\{(a+b) - (a-b)\}x = 2ab + 2b^2$$

$$\Rightarrow \quad 2bx = 2b(a+b) \Rightarrow x = \frac{2b(a+b)}{2b}$$

$$\Rightarrow \quad x = (a+b).$$

Putting $x = (a+b)$ in (ii), we get

$$(a+b)^2 + (a+b)y = a^2 + b^2$$

$$\Rightarrow \quad (a+b)y = (a^2+b^2) - (a+b)^2$$

$$\Rightarrow \quad (a+b)y = (a^2+b^2) - (a^2+b^2+2ab)$$

$$\Rightarrow \quad (a+b)y = -2ab \Rightarrow y = \frac{-2ab}{(a+b)}.$$

$\therefore \quad x = (a+b)$ and $y = \frac{-2ab}{(a+b)}$ is the required solution.

EXAMPLE 17 *Solve for x and y:*

$$ax + by - a + b = 0, bx - ay - a - b = 0.$$ [CBSE 2000, '04C, '05]

SOLUTION The given equations may be written as

$$ax + by = a - b \quad \dots \text{(i)}$$

$$bx - ay = a + b. \quad \dots \text{(ii)}$$

Multiplying (i) by a and (ii) by b and adding the results, we get

$$(a^2 + b^2)x = (a^2 + b^2) \Rightarrow x = 1.$$

Putting $x = 1$ in (i), we get

$$(a \times 1) + by = a - b$$

$$\Rightarrow \quad a + by = a - b$$

$$\Rightarrow \quad by = -b \Rightarrow y = \frac{-b}{b} = -1.$$

Hence, $x = 1$ and $y = -1$.

EXAMPLE 18 *Solve the following system of linear equations:*

$$2(ax - by) + (a + 4b) = 0, 2(bx + ay) + (b - 4a) = 0.$$ [CBSE 2004]

SOLUTION The given equations may be written as

$$2ax - 2by = -a - 4b \quad \dots \text{(i)}$$

$$2bx + 2ay = 4a - b. \quad \dots \text{(ii)}$$

Multiplying (i) by a and (ii) by b and adding, we get

$$(2a^2 + 2b^2)x = (-a^2 - b^2)$$

$$\Rightarrow \quad 2(a^2 + b^2)x = -(a^2 + b^2) \Rightarrow x = \frac{-1}{2}.$$

Putting $x = \frac{-1}{2}$ in (i), we get

$$2a \times \left(\frac{-1}{2}\right) - 2by = -a - 4b$$

$$\Rightarrow \quad -a - 2by = -a - 4b$$

$$\Rightarrow \quad 2by = 4b \Rightarrow y = \frac{4b}{2b} = 2.$$

Hence, $x = \frac{-1}{2}$ and $y = 2$.

EXAMPLE 19 *Solve for x and y:*

$$\frac{ax}{b} - \frac{by}{a} = a + b, ax - by = 2ab.$$ [CBSE 2000, '04C, '05, '07C, '09]

SOLUTION The given equations may be written as

$$a^2x - b^2y = a^2b + ab^2 \quad \dots \text{(i)}$$

$$ax - by = 2ab. \quad \dots \text{(ii)}$$

Multiplying (ii) by b and subtracting the result from (i), we get

$$(a^2 - ab)x = a^2b - ab^2$$

$$\Rightarrow \quad (a^2 - ab)x = b(a^2 - ab) \Rightarrow x = b.$$

Putting $x = b$ in (ii), we get

$$ab - by = 2ab \Rightarrow by = -ab \Rightarrow y = \frac{-ab}{b} = -a.$$

Hence, $x = b$ and $y = -a$.

EXAMPLE 20 *Solve for x and y:*

$$\sqrt{2}x - \sqrt{3}y = 0, \sqrt{5}x + \sqrt{2}y = 0.$$ [CBSE 2000, '04C, '05]

SOLUTION The given system of equations is

$$\sqrt{2}x - \sqrt{3}y = 0 \quad \dots \text{(i)}$$

$$\sqrt{5}x + \sqrt{2}y = 0. \quad \dots \text{(ii)}$$

Multiplying (i) by $\sqrt{2}$ and (ii) by $\sqrt{3}$ and adding, we get

$$(2 + \sqrt{15})x = 0 \Rightarrow x = 0.$$

Putting $x = 0$ in (ii), we get

$$\sqrt{2}y = 0 \Rightarrow y = 0.$$

Hence, $x = 0$ and $y = 0$.

A SPECIAL CASE *When the coefficients of x and y in one equation are interchanged in the other.*

METHOD Step 1. Add the given equations and from it obtain an equation of the form $x + y = a$.

Step 2. Subtract the given equations and from it obtain an equation of the form $x - y = b$.

Now, $x + y = a$ and $x - y = b$ can be solved easily.

EXAMPLE 21 *Solve for x and y:*

$$37x + 43y = 123, 43x + 37y = 117.$$ [CBSE 2008]

SOLUTION The given equations are

$$37x + 43y = 123 \quad \dots \text{(i)}$$

$$43x + 37y = 117. \quad \dots \text{(ii)}$$

Clearly, the coefficients of x and y in one equation are interchanged in the other.

Adding (i) and (ii), we get

$$(37+43)x+(43+37)y=(123+117)$$

$\Rightarrow\quad 80x+80y=240$

$\Rightarrow\quad 80(x+y)=240$

$\Rightarrow\quad x+y=\dfrac{240}{80}\Rightarrow x+y=3.$ ... (iii)

Subtracting (i) from (ii), we get

$$6x-6y=-6$$

$\Rightarrow\quad 6(x-y)=-6\Rightarrow x-y=-1$ ... (iv)

Adding (iii) and (iv), we get $2x=2\Rightarrow x=1$.

Subtracting (iv) from (iii), we get $2y=4\Rightarrow y=2$.

Hence, $x=1$ and $y=2$.

EXAMPLE 22 *Solve for x and y:*

$$152x-378y=-74,\ -378x+152y=-604.$$

SOLUTION The given equations are

$152x-378y=-74$ ... (i)

$-378x+152y=-604.$ ... (ii)

Clearly, the coefficients of x and y in one equation are interchanged in the other.

Adding (i) and (ii), we get

$$\{152+(-378)\}x+\{(-378)+152\}y=-(74+604)$$

$\Rightarrow\quad (-226)x+(-226)y=-678$

$\Rightarrow\quad (-226)(x+y)=-678$

$\Rightarrow\quad (x+y)=\dfrac{-678}{-226}\Rightarrow x+y=3.$ ... (iii)

Subtracting (ii) from (i), we get

$$(152+378)x+(-378-152)y=(-74+604)$$

$\Rightarrow\quad 530x-530y=530$

$\Rightarrow\quad 530(x-y)=530\Rightarrow x-y=1.$ ... (iv)

Adding (iii) and (iv), we get $2x=4\Rightarrow x=2$.

Subtracting (iv) from (iii), we get $2y = 2 \Rightarrow y = 1$.

Hence, $x = 2$ and $y = 1$.

SOME MORE EXAMPLES

EXAMPLE 23 *Solve for x and y:*

$$\frac{x+1}{2} + \frac{y-1}{3} = 8, \frac{x-1}{3} + \frac{y+1}{2} = 9.$$ [CBSE 2007]

SOLUTION The given equations may be written as

$$3(x+1) + 2(y-1) = 48 \Rightarrow 3x + 2y = 47 \quad \ldots \text{(i)}$$

and $2(x-1) + 3(y+1) = 54 \Rightarrow 2x + 3y = 53$. … (ii)

Multiplying (i) by 2 and (ii) by 3 and subtracting, we get

$$(4-9)y = 94 - 159$$

$$\Rightarrow \quad -5y = -65 \Rightarrow y = \frac{-65}{-5} \Rightarrow y = 13.$$

Putting $y = 13$ in (i), we get

$$3x + (2 \times 13) = 47 \Rightarrow 3x = (47 - 26) = 21$$

$$\Rightarrow \quad x = \frac{21}{3} = 7.$$

Hence, $x = 7$ and $y = 13$.

EXAMPLE 24 *Solve for x and y:*

$$\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}, \frac{4}{x} + \frac{9}{y} = \frac{21}{xy} \quad (x \neq 0, y \neq 0).$$

SOLUTION Multiplying each equation throughout by xy, we get

$$2y + 3x = 9 \quad \ldots \text{(i)}$$

$$4y + 9x = 21. \quad \ldots \text{(ii)}$$

Multiplying (i) by 3 and subtracting (ii) from the result, we get

$$(6-4)y = (27-21) \Rightarrow 2y = 6 \Rightarrow y = 3.$$

Putting $y = 3$ in (i), we get

$$(2 \times 3) + 3x = 9 \Rightarrow 6 + 3x = 9 \Rightarrow 3x = 3 \Rightarrow x = 1.$$

Hence, $x = 1$ and $y = 3$.

EXAMPLE 25 *Solve for x and y:*

$$\frac{xy}{x+y} = \frac{6}{5}, \frac{xy}{y-x} = 6 \quad (x \neq 0, y \neq 0 \text{ and } x \neq y).$$

SOLUTION The given equations may be written as

$$\frac{x+y}{xy} = \frac{5}{6} \Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{5}{6} \qquad \ldots \text{(i)}$$

$$\frac{y-x}{xy} = \frac{1}{6} \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{6} \qquad \ldots \text{(ii)}$$

Adding (i) and (ii), we get

$$\frac{2}{x} = \left(\frac{5}{6} + \frac{1}{6}\right) = \frac{6}{6} = 1 \Rightarrow x = 2.$$

Subtracting (ii) from (i), we get

$$\frac{2}{y} = \left(\frac{5}{6} - \frac{1}{6}\right) = \frac{4}{6} = \frac{2}{3} \Rightarrow y = 3.$$

Hence, $x = 2$ and $y = 3$.

EXAMPLE 26 *Solve for x and y:*

$$\frac{7x-2y}{xy} = 5, \frac{8x+7y}{xy} = 15 \quad (x \neq 0, y \neq 0).$$

SOLUTION We have

$$\frac{7x-2y}{xy} = 5 \Rightarrow \frac{7}{y} - \frac{2}{x} = 5 \qquad \ldots \text{(i)}$$

$$\frac{8x+7y}{xy} = 15 \Rightarrow \frac{8}{y} + \frac{7}{x} = 15. \qquad \ldots \text{(ii)}$$

Multiplying (i) by 7 and (ii) by 2 and adding the results, we get

$$\left(\frac{49}{y} + \frac{16}{y}\right) = (35 + 30)$$

$$\Rightarrow \quad \frac{65}{y} = 65 \Rightarrow 65y = 65 \Rightarrow y = 1.$$

Putting $y = 1$ in (ii), we get

$$\frac{8}{1} + \frac{7}{x} = 15 \Rightarrow \frac{7}{x} = (15 - 8) = 7 \Rightarrow 7x = 7 \Rightarrow x = 1.$$

Hence, $x = 1$ and $y = 1$.

EXERCISE 3B

Solve for x and y:

1. $x + y = 3$,

   $4x - 3y = 26$.

2. $x - y = 3$,

   $\frac{x}{3} + \frac{y}{2} = 6$.

3. $2x+3y=0,$
$3x+4y=5.$

4. $2x-3y=13,$
$7x-2y=20.$

5. $3x-5y-19=0,$
$-7x+3y+1=0.$

6. $2x-y+3=0,$
$3x-7y+10=0.$

7. $\frac{x}{2}-\frac{y}{9}=6,$
$\frac{x}{7}+\frac{y}{3}=5.$

8. $\frac{x}{3}+\frac{y}{4}=11,$
$\frac{5x}{6}-\frac{y}{3}=-7.$

9. $4x-3y=8,$
$6x-y=\frac{29}{3}.$

10. $2x-\frac{3y}{4}=3,$
$5x=2y+7.$

11. $2x+5y=\frac{8}{3},$
$3x-2y=\frac{5}{6}.$

12. $2x+3y+1=0,$
$\frac{7-4x}{3}=y.$

13. $0.4x+0.3y=1.7,$
$0.7x-0.2y=0.8.$

14. $0.3x+0.5y=0.5,$
$0.5x+0.7y=0.74$

15. $7(y+3)-2(x+2)=14,$
$4(y-2)+3(x-3)=2.$

16. $6x+5y=7x+3y+1=2(x+6y-1)$

17. $\frac{x+y-8}{2}=\frac{x+2y-14}{3}=\frac{3x+y-12}{11}$

HINT $a=b=c \Rightarrow a=b$ and $b=c$.

18. $\frac{5}{x}+6y=13,$
$\frac{3}{x}+4y=7 \quad (x\neq 0).$

19. $x+\frac{6}{y}=6,$
$3x-\frac{8}{y}=5 \quad (y\neq 0).$ [CBSE 2007]

20. $2x-\frac{3}{y}=9,$
$3x+\frac{7}{y}=2 \quad (y\neq 0).$

21. $\frac{3}{x}-\frac{1}{y}+9=0,$
$\frac{2}{x}+\frac{3}{y}=5 \quad (x\neq 0, y\neq 0).$

22. $\frac{9}{x}-\frac{4}{y}=8,$
$\frac{13}{x}+\frac{7}{y}=101 \quad (x\neq 0, y\neq 0).$

23. $\frac{5}{x}-\frac{3}{y}=1,$
$\frac{3}{2x}+\frac{2}{3y}=5 \quad (x\neq 0, y\neq 0).$

24. $\frac{1}{2x}+\frac{1}{3y}=2,$
$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6} \quad (x\neq 0, y\neq 0).$

25. $4x+6y=3xy,$
$8x+9y=5xy \quad (x\neq 0, y\neq 0).$

26. $x+y=5xy,$
$3x+2y=13xy \quad (x\neq 0, y\neq 0).$

27. $\frac{5}{x+y}-\frac{2}{x-y}=-1,$
$\frac{15}{x+y}+\frac{7}{x-y}=10.$

28. $\frac{3}{x+y}+\frac{2}{x-y}=2,$
$\frac{9}{x+y}-\frac{4}{x-y}=1.$

29. $\frac{5}{x+1}-\frac{2}{y-1}=\frac{1}{2},$
$\frac{10}{x+1}+\frac{2}{y-1}=\frac{5}{2},\quad x\neq-1$ and $y\neq 1.$

30. $\frac{44}{x+y}+\frac{30}{x-y}=10,$
$\frac{55}{x+y}+\frac{40}{x-y}=13.$

31. $\frac{10}{x+y}+\frac{2}{x-y}=4,$
$\frac{15}{x+y}-\frac{9}{x-y}=-2.$

32. $71x+37y=253,$
$37x+71y=287.$ [CBSE 2007C]

33. $217x+131y=913,$
$131x+217y=827.$

34. $23x-29y=98,$
$29x-23y=110.$

35. $\frac{2x+5y}{xy}=6,$
$\frac{4x-5y}{xy}=-3.$

36. $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4},$
$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}.$

37. $\frac{1}{2(x+2y)}+\frac{5}{3(3x-2y)}=\frac{-3}{2},$
$\frac{5}{4(x+2y)}-\frac{3}{5(3x-2y)}=\frac{61}{60}.$

38. $\frac{2}{(3x+2y)}+\frac{3}{(3x-2y)}=\frac{17}{5},$
$\frac{5}{(3x+2y)}+\frac{1}{(3x-2y)}=2.$

39. $3(2x+y)=7xy,$
$3(x+3y)=11xy\quad(x\neq 0$ and $y\neq 0).$

40. $x+y=a+b,$
$ax-by=a^2-b^2.$

41. $\frac{x}{a}+\frac{y}{b}=2,$
$ax-by=a^2-b^2.$ [CBSE 2005]

42. $px+qy=p-q,$
$qx-py=p+q.$

43. $\frac{x}{a}-\frac{y}{b}=0,$
$ax+by=(a^2+b^2).$

44. $6(ax+by)=3a+2b,$
$6(bx-ay)=3b-2a.$

45. $ax-by=a^2+b^2,$
$x+y=2a.$ [CBSE 2006C]

46. $\frac{bx}{a}-\frac{ay}{b}+a+b=0,$
$bx-ay+2ab=0.$ [CBSE 2006]

47. $\frac{bx}{a}+\frac{ay}{b}=a^2+b^2,$
$x+y=2ab.$ [CBSE 2010]

48. $x+y=a+b,$
$ax-by=a^2-b^2.$

49. $a^2x+b^2y=c^2,$
$b^2x+a^2y=d^2.$

50. $\frac{x}{a}+\frac{y}{b}=a+b,\ \frac{x}{a^2}+\frac{y}{b^2}=2.$

ANSWERS (EXERCISE 3B)

1. $x=5, y=-2$ **2.** $x=9, y=6$ **3.** $x=15, y=-10$ 4. $x=2, y=-3$

5. $x = -2, y = -5$ **6.** $x = -1, y = 1$ **7.** $x = 14, y = 9$ **8.** $x = 6, y = 36$

9. $x = \frac{3}{2}, y = \frac{-2}{3}$ **10.** $x = 3, y = 4$ **11.** $x = \frac{1}{2}, y = \frac{1}{3}$ **12.** $x = 4, y = -3$

13. $x = 2, y = 3$ **14.** $x = 0.5, y = 0.7$ **15.** $x = 5, y = 1$ **16.** $x = 3, y = 2$

17. $x = 2, y = 6$ **18.** $x = \frac{1}{5}, y = -2$ **19.** $x = 3, y = 2$ **20.** $x = 3, y = -1$

21. $x = \frac{-1}{2}, y = \frac{1}{3}$ **22.** $x = \frac{1}{4}, y = \frac{1}{7}$ **23.** $x = \frac{1}{2}, y = \frac{1}{3}$ **24.** $x = \frac{1}{2}, y = \frac{1}{3}$

25. $x = 3, y = 4$ **26.** $x = \frac{1}{2}, y = \frac{1}{3}$ **27.** $x = 3, y = 2$ **28.** $x = \frac{5}{2}, y = \frac{1}{2}$

29. $x = 4, y = 5$ **30.** $x = 8, y = 3$ **31.** $x = \frac{21}{8}, y = \frac{9}{8}$ **32.** $x = 2, y = 3$

33. $x = 3, y = 2$ **34.** $x = 3, y = -1$ **35.** $x = 1, y = 2$ **36.** $x = 1, y = 1$

37. $x = \frac{1}{2}, y = \frac{5}{4}$ **38.** $x = 1, y = 1$ **39.** $x = 1, y = \frac{3}{2}$ **40.** $x = a, y = b$

41. $x = a, y = b$ **42.** $x = 1, y = -1$ **43.** $x = a, y = b$ **44.** $x = \frac{1}{2}, y = \frac{1}{3}$

45. $x = a + b, y = a - b$ **46.** $x = -a, y = b$ **47.** $x = ab, y = ab$

48. $x = a, y = b$ **49.** $x = \frac{(a^2c^2 - b^2d^2)}{(a^4 - b^4)}, y = \frac{(a^2d^2 - b^2c^2)}{(a^4 - b^4)}$ **50.** $x = a^2, y = b^2$

HINTS TO SOME SELECTED QUESTIONS

9. The second equation is $18x - 3y = 29$.

11. Given equations are $6x + 15y = 8$, $18x - 12y = 5$.

12. $\frac{7 - 4x}{3} = y \Rightarrow 7 - 4x = 3y \Rightarrow 4x + 3y = 7$.

13. Multiply each of the given equations throughout by 10.

14. Multiply each of the given equations throughout by 10.

15. The given equations are

$$2x - 7y = 3, 3x + 4y = 19.$$

16. $6x + 5y = 7x + 3y + 1 \Rightarrow x - 2y = -1$.

$7x + 3y + 1 = 2x + 12y - 2 \Rightarrow 5x - 9y = -3$.

17. $\frac{x + y - 8}{2} = \frac{x + 2y - 14}{3} \Rightarrow 3(x + y - 8) = 2(x + 2y - 14)$

$\Rightarrow x - y = -4.$ … (i)

$\frac{x + 2y - 14}{3} = \frac{3x + y - 12}{11} \Rightarrow 11(x + 2y - 14) = 3(3x + y - 12)$

$\Rightarrow 2x + 19y = 118.$ … (ii)

Now, solve (i) and (ii).

18. Multiply (i) by 3 and (ii) by 5 and subtract.

19. Multiply (i) by 4 and (ii) by 3 and add.

23. Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ to get

$$5u - 3v = 1 \text{ and } \frac{3u}{2} + \frac{2v}{3} = 5 \text{ or } 9u + 4v = 30.$$

Now, solve $5u - 3v = 1$ and $9u + 4v = 30$.

24. Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get

$$\frac{u}{2} + \frac{v}{3} = 2 \Rightarrow 3u + 2v = 12 \quad \dots \text{(i)}$$

$$\frac{u}{3} + \frac{v}{2} = \frac{13}{6} \Rightarrow 2u + 3v = 13. \quad \dots \text{(ii)}$$

Add (i) and (ii). Subtract (ii) from (i).

25. Divide each equation throughout by xy to get $\frac{4}{y} + \frac{6}{x} = 3$, $\frac{8}{y} + \frac{9}{x} = 5$.

27. Put $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$.

29. Put $\frac{1}{x+1} = u$ and $\frac{1}{y-1} = v$.

32. Add (i) and (ii) to get $108(x+y) = 540 \Rightarrow x + y = 5$.

Subtract (ii) from (i) to get $34(x-y) = -34 \Rightarrow x - y = -1$.

35. $\frac{2}{y} + \frac{5}{x} = 6$, $\frac{4}{y} - \frac{5}{x} = -3$. Now, add.

36. Put $\frac{1}{(3x+y)} = u$ and $\frac{1}{(3x-y)} = v$.

37. Put $\frac{1}{(x+2y)} = u$ and $\frac{1}{(3x-2y)} = v$.

39. $6x + 3y = 7xy \Rightarrow \frac{6}{y} + \frac{3}{x} = 7.$ ... (i)

$3x + 9y = 11xy \Rightarrow \frac{3}{y} + \frac{9}{x} = 11.$ ... (ii)

40. Multiply (i) by b and add (ii) to the result.

41. $bx + ay = 2ab$ ... (i) and $ax - by = a^2 - b^2$. ... (ii)

Multiply (i) by b and (ii) by a and add.

42. Multiply (i) by p and (ii) by q and add.

43. $bx - ay = 0$ ... (i) and $ax + by = (a^2 + b^2)$. ... (ii)

Multiply (i) by b and (ii) by a and add.

44. $6ax + 6by = 3a + 2b$ ... (i) and $6bx - 6ay = 3b - 2a$. ... (ii)

Multiply (i) by a and (ii) by b and add.

45. Multiply (ii) by b and add the result with (i).

46. $b^2x - a^2y + a^2b + ab^2 = 0$ ... (i) and $b^2x - aby + 2ab^2 = 0$. ... (ii)

Now, subtract (ii) from (i).

47. $b^2x + a^2y = a^3b + ab^3$ ... (i) and $x + y = 2ab$. ... (ii)

Multiply (ii) by b^2 and subtract the result from (i).

48. Multiply (i) by b and add the result with (ii).

49. Multiply (i) by b^2 and (ii) by a^2 and subtract the results.

50. $bx + ay = a^2b + ab^2$ … (i) and $b^2x + a^2y = 2a^2b^2$. … (ii)

…………………………………………………………………………

METHOD OF CROSS MULTIPLICATION

THEOREM *The system of two linear equations*

$$a_1x + b_1y + c_1 = 0, \; a_2x + b_2y + c_2 = 0,$$

where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, *has a unique solution, given by*

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)}, \; y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}.$$

PROOF The given equations are

$$a_1x + b_1y + c_1 = 0 \quad \ldots \text{(i)}$$

$$a_2x + b_2y + c_2 = 0. \quad \ldots \text{(ii)}$$

Multiplying (i) by b_2, (ii) by b_1 and subtracting, we get

$$(a_1b_2 - a_2b_1)x = (b_1c_2 - b_2c_1)$$

$$\Rightarrow \quad x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \quad \left[\because \; \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow (a_1b_2 - a_2b_1) \neq 0\right].$$

Multiplying (ii) by a_1, (i) by a_2 and subtracting, we get

$$(a_1b_2 - a_2b_1)y = (c_1a_2 - c_2a_1)$$

$$\Rightarrow \quad y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)} \quad [\because \; (a_1b_2 - a_2b_1) \neq 0].$$

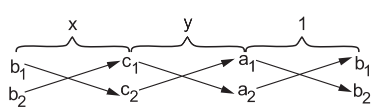
Hence, a unique solution exists, which is given by

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)}, \; y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}.$$

This is customarily written as

$$\frac{x}{(b_1c_2 - b_2c_1)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}.$$

REMARK The following diagram helps in remembering the above solution.



Rule Numbers with downward arrows are multiplied first; and from this product, the product of numbers with upward arrows is subtracted.

SOLVED EXAMPLES

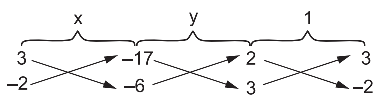
EXAMPLE 1 *Solve the system of equations* $2x+3y=17, 3x-2y=6$ *by the method of cross multiplication.*

SOLUTION The given equations may be written as

$$2x+3y-17=0 \quad \ldots \text{(i)}$$

$$3x-2y-6=0. \quad \ldots \text{(ii)}$$

By cross multiplication, we have



$$\therefore \quad \frac{x}{\{3\times(-6)-(-2)\times(-17)\}}=\frac{y}{\{(-17)\times 3-(-6)\times 2\}}$$

$$=\frac{1}{\{2\times(-2)-3\times 3\}}$$

$$\Rightarrow \quad \frac{x}{(-18-34)}=\frac{y}{(-51+12)}=\frac{1}{(-4-9)}$$

$$\Rightarrow \quad \frac{x}{-52}=\frac{y}{-39}=\frac{1}{-13}$$

$$\Rightarrow \quad x=\frac{-52}{-13}=4,\ y=\frac{-39}{-13}=3.$$

Hence, $x=4$ and $y=3$ is the required solution.

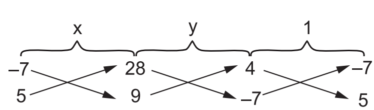
EXAMPLE 2 *Solve* $4x-7y+28=0, 5y-7x+9=0.$

SOLUTION The given equations are

$$4x-7y+28=0 \quad \ldots \text{(i)}$$

$$-7x+5y+9=0. \quad \ldots \text{(ii)}$$

By cross multiplication, we have



$$\therefore \quad \frac{x}{\{(-7)\times 9-5\times 28\}}=\frac{y}{\{28\times(-7)-9\times 4\}}=\frac{1}{\{4\times 5-(-7)\times(-7)\}}$$

$$\Rightarrow \quad \frac{x}{(-63-140)} = \frac{y}{(-196-36)} = \frac{1}{(20-49)}$$

$$\Rightarrow \quad \frac{x}{-203} = \frac{y}{-232} = \frac{1}{-29}$$

$$\Rightarrow \quad x = \left(\frac{-203}{-29}\right) = 7 \text{ and } y = \left(\frac{-232}{-29}\right) = 8.$$

Hence, $x = 7$ and $y = 8$ is the required solution.

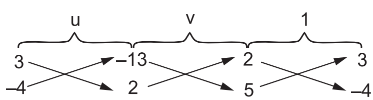
EXAMPLE 3 *Solve* $\frac{2}{x} + \frac{3}{y} = 13, \frac{5}{x} - \frac{4}{y} = -2$, *where* $x \neq 0$ *and* $y \neq 0$.

SOLUTION Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given equations become

$$2u + 3v - 13 = 0 \quad \ldots \text{(i)}$$

$$5u - 4v + 2 = 0. \quad \ldots \text{(ii)}$$

By cross multiplication, we have



$$\therefore \quad \frac{u}{[3 \times 2 - (-4) \times (-13)]} = \frac{v}{[(-13) \times 5 - 2 \times 2]} = \frac{1}{[2 \times (-4) - 5 \times 3]}$$

$$\Rightarrow \quad \frac{u}{-46} = \frac{v}{-69} = \frac{1}{-23}$$

$$\Rightarrow \quad u = \left(\frac{-46}{-23}\right) = 2, v = \left(\frac{-69}{-23}\right) = 3$$

$$\Rightarrow \quad \frac{1}{x} = 2, \frac{1}{y} = 3$$

$$\Rightarrow \quad x = \frac{1}{2}, y = \frac{1}{3}.$$

Hence, $x = \frac{1}{2}$ and $y = \frac{1}{3}$ is the required solution.

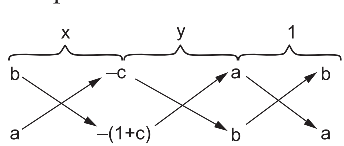
EXAMPLE 4 *Solve* $ax + by = c, bx + ay = 1 + c$.

SOLUTION The given equations may be written as

$$ax + by - c = 0 \quad \ldots \text{(i)}$$

$$bx + ay - (1 + c) = 0. \quad \ldots \text{(ii)}$$

By cross multiplication, we have



$$\therefore \quad \frac{x}{-b(1+c)+ac} = \frac{y}{-cb+a(1+c)} = \frac{1}{(a^2-b^2)}$$

$$\Rightarrow \quad x = \frac{c(a-b)-b}{(a^2-b^2)} \text{ and } y = \frac{c(a-b)+a}{(a^2-b^2)}.$$

Hence, $x = \dfrac{c(a-b)-b}{(a^2-b^2)}$ and $y = \dfrac{c(a-b)+a}{(a^2-b^2)}$.

EXERCISE 3C

Solve each of the following systems of equations by using the method of cross multiplication:

1. $x+2y+1=0$, $2x-3y-12=0$.
2. $3x-2y+3=0$, $4x+3y-47=0$.
3. $6x-5y-16=0$, $7x-13y+10=0$.
4. $3x+2y+25=0$, $2x+y+10=0$.
5. $2x+5y=1$, $2x+3y=3$.
6. $2x+y=35$, $3x+4y=65$.
7. $7x-2y=3$, $22x-3y=16$.
8. $\dfrac{x}{6}+\dfrac{y}{15}=4$, $\dfrac{x}{3}-\dfrac{y}{12}=\dfrac{19}{4}$.
9. $\dfrac{1}{x}+\dfrac{1}{y}=7$, $\dfrac{2}{x}+\dfrac{3}{y}=17$ $(x \neq 0, y \neq 0)$.
10. $\dfrac{5}{(x+y)}-\dfrac{2}{(x-y)}+1=0$, $\dfrac{15}{(x+y)}+\dfrac{7}{(x-y)}-10=0$ $(x \neq y, x \neq -y)$.
11. $\dfrac{ax}{b}-\dfrac{by}{a}=a+b$, $ax-by=2ab$. [CBSE 2007C]
12. $2ax+3by=(a+2b)$, $3ax+2by=(2a+b)$.
13. $\dfrac{a}{x}-\dfrac{b}{y}=0$, $\dfrac{ab^2}{x}+\dfrac{a^2b}{y}=(a^2+b^2)$, where $x \neq 0$ and $y \neq 0$.

ANSWERS (EXERCISE 3C)

1. $x=3, y=-2$ **2.** $x=5, y=9$ **3.** $x=6, y=4$ **4.** $x=5, y=-20$
5. $x=3, y=-1$ **6.** $x=15, y=5$ **7.** $x=1, y=2$ **8.** $x=18, y=15$
9. $x=\dfrac{1}{4}, y=\dfrac{1}{3}$ **10.** $x=3, y=2$ **11.** $x=b, y=-a$
12. $x=\dfrac{(4a-b)}{5a}, y=\dfrac{(4b-a)}{5b}$ **13.** $x=a, y=b$

HINTS TO SOME SELECTED QUESTIONS

8. The given equations are $5x+2y-120=0$ and $4x-y-57=0$.
9. Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, we get

   $u+v-7=0$ and $2u+3v-17=0$.

   Now, solve for u and v.
10. Putting $\frac{1}{(x+y)}=u$ and $\frac{1}{(x-y)}=v$, we get

    $5u-2v+1=0$ and $15u+7v-10=0$.

    Now, solve for u and v.
11. The given equations are $a^2x-b^2y-(a^2b+ab^2)=0$ and $ax-by-2ab=0$.
12. $2ax+3by-(a+2b)=0$ and $3ax+2by-(2a+b)=0$.
13. Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Then, $au-bv=0$, $ab^2u+a^2bv-(a^2+b^2)=0$.

CONDITIONS FOR SOLVABILITY OF LINEAR EQUATIONS

CONSISTENT AND INCONSISTENT SYSTEMS OF LINEAR EQUATIONS

A system of equations $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ is said to be *consistent* if it has at least one solution. On the other hand, the above system is said to be *inconsistent* if it has no solution at all.

CONDITIONS FOR SOLVABILITY OF LINEAR EQUATIONS

The system of a pair of linear equations

$$a_1x+b_1y+c_1=0,\ a_2x+b_2y+c_2=0$$

(i) *has a unique solution,*

if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

and the solution is $x=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right)$, $y=\left(\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$;

(ii) *has an infinite number of solutions,*

if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$;

(iii) *has no solution* (i.e., *inconsistent*),

if $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

SOLVED EXAMPLES

EXAMPLE 1 *Show that the system of equations*

$$2x + 5y = 17, 5x + 3y = 14$$

has a unique solution. Find the solution.

SOLUTION The given system of equations is

$$2x + 5y - 17 = 0, 5x + 3y - 14 = 0.$$

These equations are of the form

$$a_1 x + b_1 y + c_1 = 0, a_2 x + b_2 y + c_2 = 0,$$

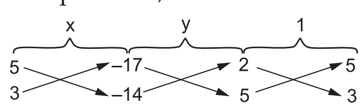
where $a_1 = 2, b_1 = 5, c_1 = -17$ and $a_2 = 5, b_2 = 3, c_2 = -14$.

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{5}, \frac{b_1}{b_2} = \frac{5}{3}, \frac{c_1}{c_2} = \frac{-17}{-14} = \frac{17}{14}.$$

Thus, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Hence, the given system of equations has a unique solution.

By cross multiplication, we have



$$\therefore \quad \frac{x}{(-70+51)} = \frac{y}{(-85+28)} = \frac{1}{(6-25)}$$

$$\Rightarrow \quad \frac{x}{-19} = \frac{y}{-57} = \frac{1}{-19}$$

$$\Rightarrow \quad x = \frac{-19}{-19} = 1 \text{ and } y = \frac{-57}{-19} = 3.$$

Hence, $x = 1$ and $y = 3$ is the required solution.

EXAMPLE 2 *Find the values of k for which the system of equations*

$$x - 2y = 3, 3x + ky = 1$$

has a unique solution.

SOLUTION The given system of equations is

$$x - 2y - 3 = 0, 3x + ky - 1 = 0.$$

These equations are of the form

$$a_1 x + b_1 y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0,$$

where $a_1 = 1, b_1 = -2, c_1 = -3$ and $a_2 = 3, b_2 = k, c_2 = -1$.

For a unique solution, we must have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

$\therefore \quad \frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6.$

Hence, the given system of equations will have a unique solution for all real values of k, other than –6.

EXAMPLE 3 *Show that the system of equations*

$$4x + 6y = 7, 12x + 18y = 21$$

has infinitely many solutions.

SOLUTION The given system of equations is

$$4x + 6y - 7 = 0, 12x + 18y - 21 = 0.$$

These equations are of the form

$$a_1 x + b_1 y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0,$$

where $a_1 = 4$, $b_1 = 6$, $c_1 = -7$ and $a_2 = 12$, $b_2 = 18$, $c_2 = -21$.

$$\therefore \quad \frac{a_1}{a_2} = \frac{4}{12} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{6}{18} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \left(\frac{-7}{-21}\right) = \frac{1}{3}.$$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Hence, the given system of equations has infinitely many solutions.

EXAMPLE 4 *Find the value of k for which the following pair of linear equations has infinitely many solutions:*

$$2x - 3y = 7, (k+1)x + (1-2k)y = (5k-4).$$ [CBSE 2008C]

SOLUTION The given equations are

$$2x - 3y - 7 = 0,$$

$$(k+1)x + (1-2k)y + (4-5k) = 0.$$

These equations are of the form

$$a_1 x + b_1 y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0,$$

where $a_1 = 2$, $b_1 = -3$, $c_1 = -7$

and $\quad a_2 = (k+1)$, $b_2 = (1-2k)$, $c_2 = (4-5k)$.

Let the given system of equations have infinitely many solutions.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \quad \frac{2}{(k+1)} = \frac{-3}{(1-2k)} = \frac{-7}{(4-5k)}$$

$\Rightarrow \quad \frac{2}{(k+1)} = \frac{3}{(2k-1)} = \frac{7}{(5k-4)}$

$\Rightarrow \quad \frac{2}{(k+1)} = \frac{3}{(2k-1)}$ and $\frac{3}{(2k-1)} = \frac{7}{(5k-4)}$

$\Rightarrow \quad 4k-2 = 3k+3$ and $15k-12 = 14k-7$

$\Rightarrow \quad k = 5$ and $k = 5$.

Hence, $k = 5$.

EXAMPLE 5 *Find the value of k for which the given system of equations has infinitely many solutions:*

$$kx + 3y = k - 3,\ 12x + ky = k.$$

SOLUTION The given system of equations is

$$kx + 3y + (3-k) = 0,$$

$$12x + ky - k = 0.$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = k$, $b_1 = 3$, $c_1 = (3-k)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$.

Let the given system of equations have infinitely many solutions.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} = \frac{(3-k)}{-k}$

$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$

$\Rightarrow \quad \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} = \frac{k-3}{k}$

$\Rightarrow \quad k^2 = 36$ and $k^2 - 6k = 0$

$\Rightarrow \quad (k = 6$ or $k = -6)$ and $k(k-6) = 0$

$\Rightarrow \quad (k = 6$ or $k = -6)$ and $(k = 0$ or $k = 6)$

$\Rightarrow \quad k = 6$.

Hence, the given system of equations has infinitely many solutions when $k = 6$.

EXAMPLE 6 *Find the value of k for which the given system of equations has infinitely many solutions:*

$$x + (k+1)y = 5,$$

$$(k+1)x + 9y + (1-8k) = 0.$$ [CBSE 2003]

SOLUTION The given system of equations is

$$x + (k+1)y - 5 = 0 \qquad \ldots \text{(i)}$$

$$(k+1)x + 9y + (1-8k) = 0. \qquad \ldots \text{(ii)}$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = 1, b_1 = (k+1), c_1 = -5$

and $a_2 = (k+1), b_2 = 9, c_2 = (1-8k)$.

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{(k+1)}, \frac{b_1}{b_2} = \frac{(k+1)}{9} \text{ and } \frac{c_1}{c_2} = \frac{-5}{(1-8k)} = \frac{5}{(8k-1)}.$$

Let the given system of equations have infinitely many solutions.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \quad \frac{1}{(k+1)} = \frac{(k+1)}{9} = \frac{5}{(8k-1)}$$

$$\Rightarrow \quad \frac{1}{(k+1)} = \frac{(k+1)}{9} \text{ and } \frac{(k+1)}{9} = \frac{5}{(8k-1)}$$

$$\Rightarrow \quad (k+1)^2 = 9 \text{ and } (k+1)(8k-1) = 45$$

$$\Rightarrow \quad (k+1 = 3 \text{ or } k+1 = -3) \text{ and } 8k^2 + 7k - 46 = 0$$

$$\Rightarrow \quad (k = 2 \text{ or } k = -4) \text{ and } (k-2)(8k+23) = 0$$

$$\Rightarrow \quad (k = 2 \text{ or } k = -4) \text{ and } \left(k = 2 \text{ or } k = \frac{-23}{8}\right)$$

$$\Rightarrow \quad k = 2.$$

Hence, the given system of equations will have infinitely many solutions when $k = 2$.

EXAMPLE 7 *Find the values of a and b for which the following pair of linear equations have an infinite number of solutions:*

$$2x + 3y = 7, (a-b)x + (a+b)y = 3a + b - 2.$$ [CBSE 2008C]

SOLUTION The given equations are

$$2x + 3y - 7 = 0, \qquad \ldots \text{(i)}$$

$$(a-b)x + (a+b)y + (2 - 3a - b) = 0. \qquad \ldots \text{(ii)}$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = (a-b), b_2 = (a+b), c_2 = (2-3a-b)$.

$\therefore \quad \frac{a_1}{a_2} = \frac{2}{(a-b)}, \frac{b_1}{b_2} = \frac{3}{(a+b)}$ and $\frac{c_1}{c_2} = \frac{-7}{(2-3a-b)} = \frac{7}{(3a+b-2)}$.

Let the given system of equations have infinitely many solutions.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{(a-b)} = \frac{3}{(a+b)} = \frac{7}{(3a+b-2)}$

$\Rightarrow \quad \frac{2}{(a-b)} = \frac{3}{(a+b)}$ and $\frac{3}{(a+b)} = \frac{7}{(3a+b-2)}$

$\Rightarrow \quad 2a+2b = 3a-3b$ and $9a+3b-6 = 7a+7b$

$\Rightarrow \quad a-5b = 0$ and $2a-4b = 6$

$\Rightarrow \quad a-5b = 0$ and $a-2b = 3$.

On solving $a-5b=0$ and $a-2b=3$, we get $a=5$ and $b=1$.

Hence, the required values are $a=5$ and $b=1$.

EXAMPLE 8 *Find the values of m and n for which the following system of linear equations has infinitely many solutions:*

$3x+4y=12,$

$(m+n)x+2(m-n)y=(5m-1).$

SOLUTION The given system of equations is

$3x+4y-12=0,$

$(m+n)x+2(m-n)y+(1-5m)=0.$

These equations are of the form

$a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0,$

where $a_1=3, b_1=4, c_1=-12$

and $\quad a_2=(m+n), b_2=2(m-n), c_2=(1-5m).$

$\therefore \quad \frac{a_1}{a_2} = \frac{3}{(m+n)}, \frac{b_1}{b_2} = \frac{2}{(m-n)}$ and $\frac{c_1}{c_2} = \frac{12}{(5m-1)}$.

Let the given system of equations have infinitely many solutions.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{(m+n)} = \frac{2}{(m-n)} = \frac{12}{(5m-1)}$

$\Rightarrow \quad \frac{3}{(m+n)} = \frac{2}{(m-n)}$ and $\frac{2}{(m-n)} = \frac{12}{(5m-1)}$

$\Rightarrow \quad 3m-3n = 2m+2n$ and $10m-2 = 12m-12n$

$\Rightarrow \quad m - 5n = 0 \qquad \ldots \text{(i)} \quad \text{and} \quad m - 6n + 1 = 0. \qquad \ldots \text{(ii)}$

On solving (i) and (ii), we get $m = 5$ and $n = 1$.
Hence, the required values are $m = 5$ and $n = 1$.

EXAMPLE 9 *Show that the system of equations*

$$3x - 5y = 7, 6x - 10y = 3$$

has no solution.

SOLUTION The given system of equations is

$$3x - 5y - 7 = 0 \text{ and } 6x - 10y - 3 = 0.$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = 3, b_1 = -5, c_1 = -7$ and $a_2 = 6, b_2 = -10, c_2 = -3$.

$\therefore \quad \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-7}{-3} = \frac{7}{3}.$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Hence, the given system of equations has no solution.

EXAMPLE 10 *Find the value of k for which the system of equations*

$$x + 2y = 5, 3x + ky - 15 = 0$$

has no solution.

SOLUTION The given system of equations is

$$x + 2y - 5 = 0 \qquad \ldots \text{(i)}$$

$$3x + ky - 15 = 0. \qquad \ldots \text{(ii)}$$

These equations are of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = 1, b_1 = 2, c_1 = -5$ and $a_2 = 3, b_2 = k, c_2 = -15$.

$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{2}{k} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}.$

Let the given system of equations have no solution.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{1}{3} = \frac{2}{k} \neq \frac{1}{3}$

$\Rightarrow \quad \frac{1}{3} = \frac{2}{k} \text{ and } \frac{2}{k} \neq \frac{1}{3}$

$\Rightarrow \quad k = 6$ and $k \neq 6$, which is impossible.

Hence, there is no value of k for which the given system of equations has no solution.

EXAMPLE 11 *Find the values of k for which the pair of linear equations*

$$kx + 3y = k - 2 \text{ and } 12x + ky = k$$

has no solution. [CBSE 2009]

SOLUTION The given equations are

$$kx + 3y + (2 - k) = 0 \text{ and } 12x + ky - k = 0.$$

These equations are of the form

$$a_1 x + b_1 y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0,$$

where $a_1 = k$, $b_1 = 3$, $c_1 = (2 - k)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$.

$\therefore \quad \frac{a_1}{a_2} = \frac{k}{12}; \frac{b_1}{b_2} = \frac{3}{k}$ and $\frac{c_1}{c_2} = \frac{(2-k)}{-k} = \frac{(k-2)}{k}$.

Let the given system of equations have no solution.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} \neq \frac{(k-2)}{k}$

$\Rightarrow \quad \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{(k-2)}{k}$

$\Rightarrow \quad k^2 = 36$ and $k^2 - 2k \neq 3k$

$\Rightarrow \quad k^2 = 36$ and $k^2 - 5k \neq 0$

$\Rightarrow \quad (k = 6$ or $k = -6)$ and $k(k - 5) \neq 0$.

Case 1. *When* $k = 6$.

In this case, $k(k - 5) = 6(6 - 5) = 6 \times 1 = 6 \neq 0$.

Case 2. *When* $k = -6$.

In this case, $k(k - 5) = (-6)(-6 - 5) = (-6) \times (-11) = 66 \neq 0$.

Thus, in each case, the given system has no solution.

Hence, $k = 6$ or $k = -6$.

EXAMPLE 12 *Find the value of k for which the pair of linear equations*

$$(3k + 1)x + 3y - 2 = 0, (k^2 + 1)x + (k - 2)y - 5 = 0$$

has no solution.

SOLUTION The given linear equations are of the form

$$a_1 x + b_1 y + c_1 = 0 \text{ and } a_2 x + b_2 y + c_2 = 0,$$

where $a_1 = (3k + 1)$, $b_1 = 3$, $c_1 = -2$

and $\quad a_2 = (k^2 + 1)$, $b_2 = (k - 2)$, $c_2 = -5$.

$\therefore \quad \frac{a_1}{a_2} = \frac{(3k+1)}{(k^2+1)}, \frac{b_1}{b_2} = \frac{3}{(k-2)}$ and $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$.

Let the given system of equations have no solution.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{(3k+1)}{(k^2+1)} = \frac{3}{(k-2)} \neq \frac{2}{5}$

$\Rightarrow \quad \frac{(3k+1)}{(k^2+1)} = \frac{3}{(k-2)}$ ... (i) and $\frac{3}{(k-2)} \neq \frac{2}{5}$. ... (ii)

Now, $\frac{(3k+1)}{(k^2+1)} = \frac{3}{(k-2)} \Rightarrow (3k+1)(k-2) = 3(k^2+1)$

$\Rightarrow 3k^2 - 6k + k - 2 = 3k^2 + 3$

$\Rightarrow 5k = -5 \Rightarrow k = -1.$

When $k = -1$, then clearly, $\frac{3}{(k-2)} \neq \frac{2}{5}$.

Hence, the given system of equations has no solution when $k = -1$.

EXAMPLE 13 *Find the value of k for which the following pair of linear equations has no solution:*

$3x + y = 1, (2k-1)x + (k-1)y = 2k+1.$

SOLUTION The given linear equations are

$3x + y - 1 = 0$ ... (i)

$(2k-1)x + (k-1)y - (2k+1) = 0.$ ... (ii)

These equations are of the form

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 3, b_1 = 1, c_1 = -1$

and $a_2 = (2k-1), b_2 = (k-1), c_2 = -(2k+1)$.

$\therefore \quad \frac{a_1}{a_2} = \frac{3}{(2k-1)}, \frac{b_1}{b_2} = \frac{1}{(k-1)}, \frac{c_1}{c_2} = \frac{-1}{-(2k+1)} = \frac{1}{(2k+1)}$.

Let the given system of equations have no solution.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{(2k-1)} = \frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$

$\Rightarrow \quad \frac{3}{(2k-1)} = \frac{1}{(k-1)}$ and $\frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$

$\Rightarrow \quad 3k - 3 = 2k - 1$ and $\frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$

$\Rightarrow \quad k = 2$ and $\frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$.

Clearly, when $k = 2$, then $\frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$

$$\left[\text{as } \frac{1}{(2-1)} \neq \frac{1}{(4+1)}\right].$$

Hence, the given system of equations will have no solution, when $k = 2$.

EXAMPLE 14 *Find the values of k for which the system of equations*

$$kx - y = 2, 6x - 2y = 3$$

has (i) a unique solution, (ii) no solution.

(iii) Is there a value of k for which the given system has infinitely many solutions?

SOLUTION The given system of equations is

$$kx - y - 2 = 0, 6x - 2y - 3 = 0.$$

This is of the form

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0,$$

where $a_1 = k$, $b_1 = -1$, $c_1 = -2$ and $a_2 = 6$, $b_2 = -2$, $c_2 = -3$.

(i) For a unique solution, we must have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

$\therefore \quad \frac{k}{6} \neq \frac{-1}{-2} \Rightarrow \frac{k}{6} \neq \frac{1}{2} \Rightarrow k \neq 3.$

Hence, the given system of equations will have a unique solution when $k \neq 3$.

(ii) For no solution, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

$\therefore \quad \frac{k}{6} = \frac{-1}{-2} \neq \frac{-2}{-3}$

$\Rightarrow \quad \frac{k}{6} = \frac{1}{2} \neq \frac{2}{3}$

$\Rightarrow \quad \frac{k}{6} = \frac{1}{2}$ and $\frac{k}{6} \neq \frac{2}{3} \Rightarrow k = 3$ and $k \neq 4$.

Clearly, $k = 3$ also satisfies the condition $k \neq 4$.

Hence, the given system of equations will have no solution when $k = 3$.

(iii) For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

i.e., $\frac{k}{6} = \frac{1}{2} = \frac{1}{3}$, which is never possible, as $\frac{1}{2} \neq \frac{1}{3}$.

Hence, there is no real value of k for which the given system of equations has infinitely many solutions.

HOMOGENEOUS SYSTEM OF EQUATIONS

The system of equations $a_1x + b_1y = 0$ *and* $a_2x + b_2y = 0$ *has*

(i) *the zero solution,* $x = 0$ *and* $y = 0$ *when* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) *an infinite number of nonzero solutions when* $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

Note The homogeneous system of equations is always consistent.

EXAMPLE 15 *Find the value of k for which the system of equations*

$$3x + 5y = 0,\ kx + 10y = 0$$

has a nonzero solution.

SOLUTION The given equations are of the form

$$a_1x + b_1y = 0 \text{ and } a_2x + b_2y = 0,$$

where $a_1 = 3$, $b_1 = 5$ and $a_2 = k$, $b_2 = 10$.

Then, $\frac{a_1}{a_2} = \frac{3}{k}$ and $\frac{b_1}{b_2} = \frac{5}{10} = \frac{1}{2}$.

Let the given system of equations have a nonzero solution.

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = \frac{1}{2} \Rightarrow k = 6$.

Hence, the required value of k is 6.

EXERCISE 3D

Show that each of the following systems of equations has a unique solution and solve it:

1. $3x + 5y = 12,\ 5x + 3y = 4.$
2. $2x - 3y = 17,\ 4x + y = 13.$
3. $\frac{x}{3} + \frac{y}{2} = 3,\ x - 2y = 2.$

Find the value of k for which each of the following systems of equations has a unique solution:

4. $2x + 3y - 5 = 0,\ kx - 6y - 8 = 0.$
5. $x - ky = 2,\ 3x + 2y + 5 = 0.$
6. $5x - 7y - 5 = 0,\ 2x + ky - 1 = 0.$
7. $4x + ky + 8 = 0,\ x + y + 1 = 0.$

8. $4x - 5y = k,\ 2x - 3y = 12.$

9. $kx + 3y = (k - 3),\ 12x + ky = k.$

10. Show that the system of equations

$$2x - 3y = 5,\ 6x - 9y = 15$$

has an infinite number of solutions.

11. Show that the system of equations

$$6x + 5y = 11,\ 9x + \frac{15}{2}y = 21$$

has no solution.

12. For what value of k does the system of equations

$$kx + 2y = 5,\ 3x - 4y = 10$$

have (i) a unique solution, (ii) no solution?

13. For what value of k does the system of equations

$$x + 2y = 5,\ 3x + ky + 15 = 0$$

have (i) a unique solution, (ii) no solution?

14. For what value of k does the system of equations

$$x + 2y = 3,\ 5x + ky + 7 = 0$$

have (i) a unique solution, (ii) no solution?

Also, show that there is no value of k for which the given system of equations has infinitely many solutions.

Find the value of k for which each of the following systems of linear equations has an infinite number of solutions:

15. $2x + 3y = 7,$
$(k - 1)x + (k + 2)y = 3k.$ [CBSE 2010]

16. $2x + (k - 2)y = k,$
$6x + (2k - 1)y = (2k + 5).$ [CBSE 2000C]

17. $kx + 3y = (2k + 1),$
$2(k + 1)x + 9y = (7k + 1).$ [CBSE 2000C]

18. $5x + 2y = 2k,$
$2(k + 1)x + ky = (3k + 4).$ [CBSE 2003C]

19. $(k - 1)x - y = 5,$
$(k + 1)x + (1 - k)y = (3k + 1).$ [CBSE 2003]

20. $(k - 3)x + 3y = k,$
$kx + ky = 12.$

Find the values of a and b for which each of the following systems of linear equations has an infinite number of solutions:

21. $(a - 1)x + 3y = 2,$
$6x + (1 - 2b)y = 6.$ [CBSE 2002C]

22. $(2a-1)x+3y=5,$
$3x+(b-1)y=2.$ [CBSE 2001C]

23. $2x-3y=7,$
$(a+b)x-(a+b-3)y=4a+b.$ [CBSE 2002]

24. $2x+3y=7,$
$(a+b+1)x+(a+2b+2)y=4(a+b)+1.$ [CBSE 2003]

25. $2x+3y=7,$
$(a+b)x+(2a-b)y=21.$ [CBSE 2001]

26. $2x+3y=7,$
$2ax+(a+b)y=28.$ [CBSE 2001]

Find the value of k for which each of the following systems of equations has no solution:

27. $8x+5y=9,\ kx+10y=15.$

28. $kx+3y=3,\ 12x+ky=6.$

29. $3x-y-5=0,\ 6x-2y+k=0 \quad (k\neq 0).$ [CBSE 2008]

30. $kx+3y=k-3,\ 12x+ky=k.$ [CBSE 2009]

31. Find the value of k for which the system of equations
$$5x-3y=0,\ 2x+ky=0$$
has a nonzero solution.

ANSWERS (EXERCISE 3D)

1. $x=-1, y=3$ **2.** $x=4, y=-3$ **3.** $x=6, y=2$ **4.** $k\neq -4$
5. $k\neq \frac{-2}{3}$ **6.** $k\neq \frac{-14}{5}$ **7.** $k\neq 4$
8. k is any real number **9.** k is any real number other than 6 and –6
12. (i) $k\neq \frac{-3}{2}$ (ii) $k=\frac{-3}{2}$ **13.** (i) $k\neq 6$ (ii) $k=6$ **14.** (i) $k\neq 10$ (ii) $k=10$
15. $k=7$ **16.** $k=5$ **17.** $k=2$ **18.** $k=4$ **19.** $k=3$ **20.** $k=6$
21. $a=3, b=-4$ **22.** $a=\frac{17}{4}, b=\frac{11}{5}$ **23.** $a=-5, b=-1$ **24.** $a=3, b=2$
25. $a=5, b=1$ **26.** $a=4, b=8$ **27.** $k=16$ **28.** $k=-6$
29. $k\neq -10$ **30.** $k=-6$ **31.** $k=\frac{-6}{5}$

HINTS TO SOME SELECTED QUESTIONS

9. $\frac{k}{12}\neq\frac{3}{k}\Rightarrow k^2\neq 36\Rightarrow k\neq 6$ and $k\neq -6$.

So, for a unique solution, we may take any real value of k, other than 6 and –6.

12. (i) $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow \frac{k}{3} \neq \frac{-1}{2} \Rightarrow k \neq \frac{-3}{2}$.

(ii) $\frac{k}{3} = \frac{2}{-4} \neq \frac{5}{10} \Rightarrow \frac{k}{3} = \frac{-1}{2}$ and $\frac{k}{3} \neq \frac{1}{2} \Rightarrow k = \frac{-3}{2}$.

13. (i) $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$.

(ii) $\frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15} \Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-1}{3} \Rightarrow k = 6$.

14. (i) $\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$.

(ii) $\frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = 10$.

Clearly, $\frac{1}{5} = \frac{2}{k} = \frac{-3}{7}$ is never true, as $\frac{1}{5} = \frac{-3}{7}$ is false.

20. $\frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12} \Rightarrow \frac{k-3}{k} = \frac{3}{k}$ and $\frac{3}{k} = \frac{k}{12}$.

$\therefore$ $k^2 - 6k = 0$ and $k^2 = 36 \Rightarrow k(k-6) = 0$ and $k^2 = 36$

$\Rightarrow$ $(k = 0$ or $k = 6)$ and $(k = 6$ or $k = -6)$

$\Rightarrow$ $k = 6$.

21. $\frac{a-1}{6} = \frac{3}{1-2b} = \frac{-2}{-6} = \frac{1}{3}$

$\Rightarrow$ $\frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{1-2b} = \frac{1}{3}$

$\Rightarrow$ $3a - 3 = 6$ and $1 - 2b = 9$

$\Rightarrow$ $3a = 9$ and $2b = -8 \Rightarrow a = 3$ and $b = -4$.

28. $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6} \Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$

$\Rightarrow k^2 = 36$ and $k \neq 6 \Rightarrow k = -6$.

30. $\frac{k}{12} = \frac{3}{k} \neq \frac{k-3}{k} \Rightarrow k^2 = 36$ and $k(k-6) \neq 0 \Rightarrow k = -6$.

WORD PROBLEMS

SOLVED EXAMPLES

PROBLEMS ON MONEY MATTERS

EXAMPLE 1 8 *chairs and* 5 *tables for a classroom cost* ₹ 10500, *while* 5 *chairs and* 3 *tables cost* ₹ 6450. *Find the cost of each chair and that of each table.*

SOLUTION Let the cost of each chair be ₹ x and that of each table be ₹ y.

Then, $8x + 5y = 10500$ ... (i)

and $5x + 3y = 6450$. ... (ii)

On multiplying (ii) by 5 and (i) by 3 and subtracting the results, we get

$$25x - 24x = 32250 - 31500 \Rightarrow x = 750.$$

Putting $x = 750$ in (i), we get

$$8 \times 750 + 5y = 10500 \Rightarrow 6000 + 5y = 10500$$

$\Rightarrow$ $5y = (10500 - 6000) = 4500$

$\Rightarrow$ $y = 900.$

$\therefore$ cost of each chair = ₹ 750 and cost of each table = ₹ 900.

EXAMPLE 2 *The coach of a cricket team buys 7 bats and 6 balls for* ₹ 13200. *Later, he buys* 3 *bats and* 5 *balls for* ₹ 5900. *Find the cost of each bat and each ball.*

SOLUTION Let the cost of each bat be ₹ x and the cost of each ball be ₹ y.

Then, $7x + 6y = 13200.$ … (i)

And, $3x + 5y = 5900.$ … (ii)

On multiplying (i) by 5, (ii) by 6 and subtracting the results, we get

$$35x - 18x = 66000 - 35400 \Rightarrow 17x = 30600$$

$\Rightarrow$ $x = \dfrac{30600}{17} = 1800.$

Putting $x = 1800$ in (ii), we get

$$5400 + 5y = 5900 \Rightarrow 5y = 5900 - 5400$$

$\Rightarrow$ $5y = 500$

$\Rightarrow$ $y = 100.$

$\therefore$ cost of each bat = ₹ 1800 and cost of each ball = ₹ 100.

EXAMPLE 3 37 *pens and* 53 *pencils together cost* ₹ 955, *while* 53 *pens and* 37 *pencils together cost* ₹ 1115. *Find the cost of a pen and that of a pencil.*

SOLUTION Let the cost of each pen be ₹ x and that of a pencil be ₹ y.

Then, $37x + 53y = 955.$ … (i)

And, $53x + 37y = 1115.$ … (ii)

Adding (i) and (ii), we get

$$90x + 90y = 2070 \Rightarrow 90(x + y) = 2070$$

$\Rightarrow$ $x + y = \dfrac{2070}{90}$

$\Rightarrow$ $x + y = 23.$ … (iii)

On subtracting (i) from (ii), we get

$$16x - 16y = 160 \Rightarrow 16(x - y) = 160$$

$\Rightarrow \quad (x - y) = 10.$ ... (iv)

Adding (iii) and (iv), we get $2x = 33 \Rightarrow x = \frac{33}{2} = 16.50.$

Subtracting (iv) from (iii), we get $2y = 13 \Rightarrow y = \frac{13}{2} = 6.50.$

$\therefore$ cost of each pen = ₹ 16.50 and cost of each pencil = ₹ 6.50.

EXAMPLE 4 *Taxi charges in a city consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a person travels* 60 km, *he pays* ₹ 960, *and for travelling* 80 km, *he pays* ₹ 1260. *Find the fixed charges and the rate per kilometre.*

SOLUTION Let the fixed charges be ₹ x and the other charges be ₹ y per km.

Then, $x + 60y = 960.$ ... (i)

And, $x + 80y = 1260.$ ... (ii)

On subtracting (i) from (ii), we get

$$20y = 300 \Rightarrow y = \frac{300}{20} \Rightarrow y = 15.$$

Putting $y = 15$ in (i), we get

$$x + (60 \times 15) = 960 \Rightarrow x = 960 - 900 \Rightarrow x = 60.$$

$\therefore$ fixed charges = ₹ 60 and the rate per km = ₹ 15 per km.

EXAMPLE 5 *A part of monthly hostel charges in a school is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for* 22 *days, he has to pay* ₹ 4250 *as hostel charges, whereas a student B, who takes food for* 28 *days, pays* ₹ 5150 *as hostel charges. Find the fixed charges and the cost of food per day.*

SOLUTION Let the fixed charges be ₹ x per month and the cost of meals per day be ₹ y. Then, we have

$x + 22y = 4250$ ... (i)

and $x + 28y = 5150.$ ... (ii)

On subtracting (i) from (ii), we get

$$6y = 900 \Rightarrow y = 150.$$

Putting $y = 150$ in (i), we get

$$x + (22 \times 150) = 4250$$

$\Rightarrow \quad x + 3300 = 4250$

$\Rightarrow \quad x = 4250 - 3300 = 950.$

$\therefore \quad x = 950$ and $y = 150$.

Hence, the fixed charges are ₹ 950 per month and the cost of food is ₹ 150 per day.

EXAMPLE 6 *The monthly incomes of A and B are in the ratio* 8 : 7 *and their expenditures are in the ratio* 19 : 16. *If each saves* ₹ 5000 *per month, find the monthly income of each.*

SOLUTION Let the monthly incomes of A and B be ₹ $8x$ and ₹ $7x$ respectively, and let their expenditures be ₹ $19y$ and ₹ $16y$ respectively.

Then, A's monthly savings = ₹ $(8x - 19y)$.

And, B's monthly savings = ₹ $(7x - 16y)$.

But, the monthly saving of each is ₹ 5000.

$\therefore \quad 8x - 19y = 5000$ ... (i)

and $7x - 16y = 5000.$ ... (ii)

Multiplying (ii) by 19, (i) by 16 and subtracting the results, we get

$(19 \times 7 - 16 \times 8)x = (19 \times 5000 - 16 \times 5000)$

$\Rightarrow \quad (133 - 128)x = 5000 \times (19 - 16)$

$\Rightarrow \quad 5x = 15000 \Rightarrow x = 3000.$

$\therefore$ A's monthly income = ₹ $(8x)$ = ₹ (8×3000) = ₹ 24000.

And, B's monthly income = ₹ $(7x)$ = ₹ (7×3000) = ₹ 21000.

EXAMPLE 7 *On selling a TV at* 5% *gain and a fridge at* 10% *gain, a shopkeeper gains* ₹ 3250. *But, if he sells the TV at* 10% *gain and the fridge at* 5% *loss, he gains* ₹ 1500. *Find the actual cost price of TV and that of the fridge.*

SOLUTION Let the cost price of the TV set be ₹ x and that of the fridge be ₹ y.

Then, total CP of TV and fridge = ₹ $(x + y)$.

SP in first case = ₹ $\left(\frac{105x}{100} + \frac{110y}{100}\right)$ = ₹ $\left(\frac{21x}{20} + \frac{11y}{10}\right)$.

$\therefore$ gain in this case = ₹ $\left\{\left(\frac{21x}{20} + \frac{11y}{10}\right) - (x + y)\right\}$ = ₹ $\left(\frac{x}{20} + \frac{y}{10}\right)$.

So, $\frac{x}{20}+\frac{y}{10}=3250 \Rightarrow x+2y=65000.$ … (i)

SP in second case = ₹ $\left(\frac{110x}{100}+\frac{95y}{100}\right)$ = ₹ $\left(\frac{11x}{10}+\frac{19y}{20}\right)$.

∴ gain in this case = ₹ $\left\{\left(\frac{11x}{10}+\frac{19y}{20}\right)-(x+y)\right\}$ = ₹ $\left(\frac{x}{10}-\frac{y}{20}\right)$.

So, $\frac{x}{10}-\frac{y}{20}=1500 \Rightarrow 2x-y=30000.$ … (ii)

Multiplying (ii) by 2 and adding the result with (i), we get

$5x=60000+65000 \Rightarrow 5x=125000 \Rightarrow x=25000.$

Putting $x=25000$ in (i), we get

$25000+2y=65000 \Rightarrow 2y=65000-25000=40000$

$\Rightarrow$ $y=20000.$

∴ $x=25000$ and $y=20000.$

Hence, the CP of the TV set is ₹ 25000 and that of the fridge is ₹ 20000.

EXAMPLE 8 *A man invested an amount at 12% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of ₹ 2600. But, if he had interchanged the amounts invested, he would have received ₹ 140 less. What amounts did he invest at the different rates?*

SOLUTION Let the amount invested at 12% be ₹ x and that invested at 10% be ₹ y. Then,

total annual interest

= ₹ $\left(\frac{x\times 12\times 1}{100}+\frac{y\times 10\times 1}{100}\right)$ = ₹ $\left(\frac{6x+5y}{50}\right)$.

∴ $\frac{6x+5y}{50}=2600 \Rightarrow 6x+5y=130000.$ … (i)

Again, the amount invested at 12% is ₹ y and that invested at 10% is ₹ x.

Total annual interest at the new rates

= ₹ $\left(\frac{y\times 12\times 1}{100}+\frac{x\times 10\times 1}{100}\right)$ = ₹ $\left(\frac{6y+5x}{50}\right)$.

But, interest received at the new rates = ₹ (2600 − 140) = ₹ 2460.

∴ $\frac{6y+5x}{50}=2460 \Rightarrow 5x+6y=123000.$ … (ii)

Adding (i) and (ii), we get

$$11x + 11y = 253000 \Rightarrow 11(x + y) = 253000$$

$$\Rightarrow \quad x + y = 23000. \qquad \ldots \text{(iii)}$$

Subtracting (ii) from (i), we get

$$x - y = 7000. \qquad \ldots \text{(iv)}$$

Adding (iii) and (iv), we get $2x = 30000 \Rightarrow x = 15000$.

Putting $x = 15000$ in (i), we get

$$15000 + y = 23000 \Rightarrow y = 23000 - 15000 = 8000.$$

$\therefore \quad x = 15000$ and $y = 8000$.

Hence, the amount at 12% is ₹ 15000 and that at 10% is ₹ 8000.

EXAMPLE 9 *Each one of A and B has some money. If A gives ₹ 30 to B then B will have twice the money left with A. But, if B gives ₹ 10 to A then A will have thrice as much as is left with B. How much money does each have?*

SOLUTION Suppose A and B have ₹ x and ₹ y respectively.

Case I *When A gives ₹ 30 to B.*

Then, money with A = ₹ $(x - 30)$

and money with B = ₹ $(y + 30)$.

$$\therefore \quad (y + 30) = 2(x - 30) \Rightarrow 2x - y = 90. \qquad \ldots \text{(i)}$$

Case II *When B gives ₹ 10 to A.*

Then, money with A = ₹ $(x + 10)$

and money with B = ₹ $(y - 10)$.

$$\therefore \quad (x + 10) = 3(y - 10) \Rightarrow x - 3y = -40. \qquad \ldots \text{(ii)}$$

On multiplying (ii) by 2 and subtracting the result from (i), we get

$$5y = 170 \Rightarrow y = \frac{170}{5} = 34.$$

Putting $y = 34$ in (i), we get

$$2x - 34 = 90 \Rightarrow 2x = (90 + 34) = 124 \Rightarrow x = \frac{124}{2} = 62.$$

$\therefore \quad x = 62$ and $y = 34$.

Hence, A has ₹ 62 and B has ₹ 34.

PROBLEMS ON NUMBERS

EXAMPLE 10 *The students of a class are made to stand in rows. If 4 students are extra in each row, there would be 2 rows less. If 4 students are less in each row, there would be 4 rows more. Find the number of students in the class.*

SOLUTION Let the number of rows be x and the number of students in each row be y.

Then, the total number of students = xy.

Case I *When there are 4 more students in each row.*

Then, number of students in each row = $(y+4)$.

And, number of rows = $(x-2)$.

Total number of students = $(x-2)(y+4)$.

$\therefore \quad (x-2)(y+4) = xy \Rightarrow 4x-2y=8$

$\Rightarrow 2x-y=4.$ ... (i)

Case II *When 4 students are removed from each row.*

Then, number of students in each row = $(y-4)$.

And, number of rows = $(x+4)$.

Total number of students = $(x+4)(y-4)$.

$\therefore \quad (x+4)(y-4) = xy \Rightarrow 4y-4x=16$

$\Rightarrow 4(y-x)=16$

$\Rightarrow (y-x)=4.$ ... (ii)

Adding (i) and (ii), we get $x=8$.

Putting $x=8$ in (ii), we get

$y-8=4 \Rightarrow y=12.$

Thus, $x=8$ and $y=12$.

This shows that there are 8 rows and there are 12 students in each row.

Hence, the number of students in the class = $xy = 8\times 12 = 96$.

EXAMPLE 11 *The sum of two numbers is* 1000 *and the difference between their squares is* 256000. *Find the numbers.*

SOLUTION Let the larger number be x and the smaller number be y.

Then, $x+y=1000$ ... (i)

and $x^2-y^2=256000.$ ... (ii)

On dividing (ii) by (i), we get

$$\frac{x^2-y^2}{x+y}=\frac{256000}{1000} \Rightarrow x-y=256. \qquad \ldots \text{(iii)}$$

Adding (i) and (iii), we get

$$2x=1256 \Rightarrow x=628.$$

Substituting $x=628$ in (i), we get $y=372$.

Hence, the required numbers are 628 and 372.

EXAMPLE 12 *If three times the larger of two numbers is divided by the smaller one, we get 4 as the quotient and 3 as the remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as the quotient and 1 as the remainder. Find the numbers.*

SOLUTION Let the larger number be x and the smaller one be y.

We know that

dividend = (divisor × quotient) + remainder.

Using the above result and the given conditions, we have

$$3x=4y+3 \Rightarrow 3x-4y=3 \qquad \ldots \text{(i)}$$

and $7y=5x+1 \Rightarrow 5x-7y=-1$. … (ii)

Multiplying (i) by 5, (ii) by 3 and subtracting, we get $y=18$.

Putting $y=18$ in (i), we get

$$3x-(4\times 18)=3 \Rightarrow 3x-72=3$$

$$\Rightarrow \quad 3x=75 \Rightarrow x=25.$$

Hence, the required numbers are 25 and 18.

EXAMPLE 13 *The sum of two numbers is 8 and the sum of their reciprocals is* $\frac{8}{15}$. *Find the numbers.* [CBSE 2009]

SOLUTION Let the required numbers be x and y.

Then, $x+y=8$. … (i)

And, $\frac{1}{x}+\frac{1}{y}=\frac{8}{15} \Rightarrow \frac{x+y}{xy}=\frac{8}{15}$

$$\Rightarrow \frac{8}{xy}=\frac{8}{15} \qquad \text{[using (i)]}$$

$$\Rightarrow xy=15.$$

$$\therefore \quad (x-y)=\sqrt{(x+y)^2-4xy}$$

$$=\sqrt{8^2-4\times 15}=\sqrt{64-60}=\sqrt{4}=\pm 2.$$

Thus, we have

$$\left.\begin{array}{ll} x+y=8 & \ldots\text{(i)} \\ x-y=2 & \ldots\text{(ii)} \end{array}\right\} \text{ or } \left\{\begin{array}{ll} x+y=8 & \ldots\text{(iii)} \\ x-y=-2 & \ldots\text{(iv)} \end{array}\right.$$

On solving (i) and (ii), we get $x = 5$ and $y = 3$.

On solving (iii) and (iv), we get $x = 3$ and $y = 5$.

Hence, the required numbers are 5 and 3.

EXAMPLE 14 *The difference of two numbers is* 4 *and the difference of their reciprocals is* $\frac{4}{21}\cdot$ *Find the numbers.* [CBSE 2008]

SOLUTION Let the larger number be x and the smaller number be y.

Then, $x - y = 4$. … (i)

And, $\frac{1}{y} - \frac{1}{x} = \frac{4}{21}$ $\quad\left[\because\ x > y \Rightarrow \frac{1}{y} > \frac{1}{x}\right]$

$$\Rightarrow \frac{x-y}{xy} = \frac{4}{21}$$

$$\Rightarrow \frac{4}{xy} = \frac{4}{21} \Rightarrow xy = 21 \quad \text{[using (i)].}$$

$$\therefore \quad (x+y) = \sqrt{(x-y)^2 + 4xy}$$

$$= \sqrt{4^2 + 4 \times 21} = \sqrt{16+84} = \sqrt{100} = \pm 10.$$

Thus, we have

$$\left.\begin{array}{ll} x-y=4 & \ldots\text{(i)} \\ x+y=10 & \ldots\text{(ii)} \end{array}\right\} \text{ or } \left\{\begin{array}{ll} x-y=4 & \ldots\text{(iii)} \\ x+y=-10 & \ldots\text{(iv)} \end{array}\right.$$

On solving (i) and (ii), we get $x = 7$ and $y = -3$.

On solving (iii) and (iv), we get $x = -3$ and $y = -7$.

Hence, the required numbers are (7 and 3) or (–3 and –7).

EXAMPLE 15 *The sum of the digits of a two-digit number is* 12. *The number obtained by interchanging its digits exceeds the given number by* 18. *Find the number.* [CBSE 2006]

SOLUTION Let the ten's digit of the required number be x and the unit's digit be y. Then,

$$x + y = 12. \quad \ldots\text{(i)}$$

Required number = $(10x + y)$.

Number obtained on reversing the digits = $(10y + x)$.

$\therefore \quad (10y + x) - (10x + y) = 18 \Rightarrow 9y - 9x = 18$

$$\Rightarrow y - x = 2. \qquad \ldots \text{(ii)}$$

On adding (i) and (ii), we get

$$2y = 14 \Rightarrow y = 7.$$

Putting $y = 7$ in (i), we get

$$x + 7 = 12 \Rightarrow x = 12 - 7 = 5.$$

$\therefore \quad x = 5$ and $y = 7$.

Hence, the required number is 57.

EXAMPLE 16 *The sum of a two-digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.*

[CBSE 2002]

SOLUTION Let the ten's and unit's digits of the required number be x and y respectively.

Then, the number $= (10x + y)$.

The number obtained on reversing the digits $= (10y + x)$.

$\therefore \quad (10y + x) + (10x + y) = 99 \Rightarrow 11(x + y) = 99 \Rightarrow x + y = 9.$

Also, $(x - y) = \pm 3$.

Thus, we have

$$\left.\begin{array}{ll} x + y = 9 & \ldots \text{(i)} \\ x - y = 3 & \ldots \text{(ii)} \end{array}\right\} \quad \text{or} \quad \left\{\begin{array}{ll} x + y = 9 & \ldots \text{(iii)} \\ x - y = -3 & \ldots \text{(iv)} \end{array}\right.$$

From (i) and (ii), we get $x = 6, y = 3$.

From (iii) and (iv), we get $x = 3, y = 6$.

Hence, the required number is 63 or 36.

EXAMPLE 17 *Seven times a two-digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digits is 3, find the number.*

SOLUTION Let the ten's and unit's digits of the required number be x and y respectively.

Then, the number $= (10x + y)$.

The number obtained by reversing the digits $= (10y + x)$.

$\therefore \quad 7(10x + y) = 4(10y + x) \Rightarrow 33(2x - y) = 0$

$\Rightarrow \quad 2x - y = 0 \Rightarrow y = 2x \qquad \ldots \text{(i)}$

Thus, unit's digit = 2 times the ten's digit.

$\therefore$ (unit's digit) > (ten's digit) and so $y > x$.

$\therefore$ $y - x = 3$. ... (ii)

Using (i) in (ii), we get $(2x - x) = 3 \Rightarrow x = 3$.

On substituting $x = 3$ in (i), we get $y = 2 \times 3 = 6$.

Hence, the required number is 36.

EXAMPLE 18 *A two-digit number is such that the product of its digits is* 14. *If* 45 *is added to the number, the digits interchange their places. Find the number.* [CBSE 2005]

SOLUTION Let the ten's and unit's digits of the required number be x and y respectively. Then, $xy = 14$.

Required number = $(10x + y)$.

Number obtained on reversing its digits = $(10y + x)$.

$\therefore$ $(10x + y) + 45 = (10y + x)$

$\Rightarrow$ $9(y - x) = 45 \Rightarrow y - x = 5$. ... (i)

Now, $(y + x)^2 - (y - x)^2 = 4xy$

$\Rightarrow$ $(y + x) = \sqrt{(y - x)^2 + 4xy} = \sqrt{25 + 4 \times 14} = \sqrt{81}$

$\Rightarrow$ $y + x = 9$ ... (ii) [$\because$ digits are never negative].

On adding (i) and (ii), we get

$2y = 14 \Rightarrow y = 7$.

Putting $y = 7$ in (ii), we get

$7 + x = 9 \Rightarrow x = 9 - 7 = 2$.

$\therefore$ $x = 2$ and $y = 7$.

Hence, the required number is 27.

EXAMPLE 19 *A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.* [CBSE 2005]

SOLUTION Let the ten's digit of the required number be x and its unit's digit be y.

Then, $10x + y = 4(x + y) \Rightarrow 6x - 3y = 0 \Rightarrow 2x - y = 0$. ... (i)

Also, $10x + y = 2xy$. ... (ii)

Putting $y = 2x$ from (i) in (ii), we get

$$10x + 2x = 4x^2 \Rightarrow 4x^2 - 12x = 0$$
$$\Rightarrow 4x(x-3) = 0$$
$$\Rightarrow x - 3 = 0 \Rightarrow x = 3 \quad [\because \text{ ten's digit}, x \neq 0].$$

Putting $x = 3$ in (i), we get $y = 6$.

Thus, ten's digit = 3 and unit's digit = 6.

Hence, the required number is 36.

EXAMPLE 20 *A fraction becomes* $\frac{1}{3}$, *if 2 is added to both of its numerator and denominator. If 3 is added to both of its numerator and denominator then it becomes* $\frac{2}{5}\cdot$ *Find the fraction.* [CBSE 2009C]

SOLUTION Let the required fraction be $\frac{x}{y}\cdot$ Then,

$$\frac{x+2}{y+2} = \frac{1}{3} \Rightarrow 3x + 6 = y + 2$$
$$\Rightarrow 3x - y = -4. \qquad \ldots \text{(i)}$$

Also, $\frac{x+3}{y+3} = \frac{2}{5} \Rightarrow 5x + 15 = 2y + 6$

$$\Rightarrow 5x - 2y = -9. \qquad \ldots \text{(ii)}$$

Multiplying (i) by 2 and subtracting (ii) from the result, we get

$$6x - 5x = -8 + 9 \Rightarrow x = 1.$$

Putting $x = 1$ in (i), we get

$$3 - y = -4 \Rightarrow y = 7.$$

Thus, $x = 1$ and $y = 7$.

Hence, the required fraction is $\frac{1}{7}\cdot$

EXAMPLE 21 *The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes* $\frac{1}{2}\cdot$ *Find the fraction.* [CBSE 2010]

SOLUTION Let the required fraction be $\frac{x}{y}\cdot$ Then,

$$(x + y) = 2y - 3 \Rightarrow x - y = -3. \qquad \ldots \text{(i)}$$

And, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - 2 = y - 1$

$$\Rightarrow 2x - y = 1. \qquad \ldots \text{(ii)}$$

On subtracting (i) from (ii), we get $x = 4$.

Putting $x = 4$ in (i), we get $y = x + 3 = 4 + 3 = 7$.

$\therefore \quad x = 4$ and $y = 7$.

Hence, the required fraction is $\frac{4}{7}$.

EXAMPLE 22 *In a given fraction, if the numerator is multiplied by 2 and the denominator is reduced by 5, we get* $\left(\frac{6}{5}\right)$. *But if the numerator of the given fraction is increased by 8 and the denominator is doubled, we get* $\left(\frac{2}{5}\right)$. *Find the fraction.*

SOLUTION Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{2x}{y-5} = \frac{6}{5} \Rightarrow 10x = 6(y-5)$$

$$\Rightarrow 10x - 6y = -30$$

$$\Rightarrow 5x - 3y = -15 \qquad \ldots \text{(i)}$$

and $$\frac{x+8}{2y} = \frac{2}{5} \Rightarrow 5(x+8) = 4y$$

$$\Rightarrow 5x - 4y = -40. \qquad \ldots \text{(ii)}$$

On subtracting (ii) from (i), we get $y = 25$.

Putting $y = 25$ in (i), we get

$$5x - (3 \times 25) = -15 \Rightarrow 5x - 75 = -15 \Rightarrow 5x = 60 \Rightarrow x = 12.$$

$\therefore \quad x = 12$ and $y = 25$.

Hence, the required fraction is $\frac{12}{25}$.

PROBLEMS ON AGES

EXAMPLE 23 *Five years ago, A was thrice as old as B and ten years later A shall be twice as old as B. What are the present ages of A and B?* [CBSE 2002C]

SOLUTION Let the present ages of B and A be x years and y years respectively. Then,

B's age 5 years ago = $(x - 5)$ years

and A's age 5 years ago = $(y - 5)$ years.

$\therefore \quad (y - 5) = 3(x - 5) \Rightarrow 3x - y = 10. \qquad \ldots$ (i)

B's age 10 years hence = $(x + 10)$ years.

A's age 10 years hence = $(y + 10)$ years.

$\therefore \quad (y + 10) = 2(x + 10) \Rightarrow 2x - y = -10. \qquad \ldots$ (ii)

On subtracting (ii) from (i), we get $x = 20$.

Putting $x = 20$ in (i), we get

$$(3 \times 20) - y = 10 \Rightarrow y = 60 - 10 = 50.$$

$\therefore \quad x = 20$ and $y = 50$.

Hence, B's present age = 20 years

and A's present age = 50 years.

EXAMPLE 24 *A man's age is three times the sum of the ages of his two sons. After* 5 *years, his age will be twice the sum of the ages of his two sons. Find the age of the man.* [CBSE 2003]

SOLUTION Let the present age of the man be x years and the sum of the present ages of his two sons be y years. Then,

$$x = 3y \Rightarrow x - 3y = 0. \quad \ldots \text{(i)}$$

Man's age after 5 years = $(x + 5)$ years.

Sum of the ages of his two sons after 5 years = $(y + 5 + 5)$ years = $(y + 10)$ years.

$$\therefore \quad (x + 5) = 2(y + 10) \Rightarrow x - 2y = 15. \quad \ldots \text{(ii)}$$

On subtracting (i) from (ii), we get $y = 15$.

Putting $y = 15$ in (i), we get $x = 45$.

$\therefore \quad x = 45$ and $y = 15$.

Hence, the present age of the man is 45 years.

PROBLEMS ON TIME AND DISTANCE

EXAMPLE 25 *A man travels* 370 km, *partly by train and partly by car. If he covers* 250 km *by train and the rest by car, it takes him* 4 *hours. But, if he travels* 130 km *by train and the rest by car, he takes* 18 *minutes longer. Find the speed of the train and that of the car.*

SOLUTION Let the speed of the train be x km/hr and that of the car be y km/hr.

Case I Distance covered by train = 250 km.

Distance covered by car = (370 − 250) km = 120 km.

Time taken to cover 250 km by train = $\frac{250}{x}$ hours.

Time taken to cover 120 km by car = $\frac{120}{y}$ hours.

Total time taken = 4 hours.

$$\therefore \quad \frac{250}{x}+\frac{120}{y}=4 \Rightarrow \frac{125}{x}+\frac{60}{y}=2. \qquad \ldots \text{(i)}$$

Case II Distance covered by train = 130 km.

Distance covered by car = (370 − 130) km = 240 km.

Time taken to cover 130 km by train = $\frac{130}{x}$ hours.

Time taken to cover 240 km by car = $\frac{240}{y}$ hours.

Total time taken = $4\frac{18}{60}$ hours = $4\frac{3}{10}$ hours = $\frac{43}{10}$ hours.

$$\therefore \quad \frac{130}{x}+\frac{240}{y}=\frac{43}{10} \Rightarrow \frac{1300}{x}+\frac{2400}{y}=43. \qquad \ldots \text{(ii)}$$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, equations (i) and (ii) become

$$125u+60v=2 \quad \ldots \text{(iii)} \quad \text{and} \quad 1300u+2400v=43. \quad \ldots \text{(iv)}$$

On multiplying (iii) by 40 and subtracting (iv) from the result, we get

$$5000u-1300u=80-43 \Rightarrow 3700u=37$$

$$\Rightarrow \quad u=\frac{37}{3700}=\frac{1}{100} \Rightarrow \frac{1}{x}=\frac{1}{100} \Rightarrow x=100.$$

Putting $u=\frac{1}{100}$ in (iv), we get

$$\left(1300\times\frac{1}{100}\right)+2400v=43 \Rightarrow 2400v=43-13=30$$

$$\Rightarrow \quad v=\frac{30}{2400}=\frac{1}{80} \Rightarrow \frac{1}{y}=\frac{1}{80} \Rightarrow y=80.$$

$\therefore \quad x=100$ and $y=80$.

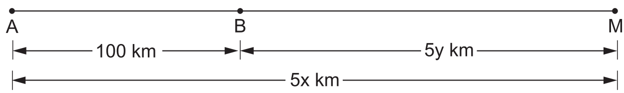
Hence, the speed of the train is 100 km/hr and that of the car is 80 km/hr.

EXAMPLE 26 *Places A and B are* 100 km *apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in* 5 *hours. If they travel towards each other, they meet in* 1 *hour. What are the speeds of the two cars?* [CBSE 2009]

SOLUTION Let the speeds of the cars from A and B be x km/hr and y km/hr respectively.

Case I *When the two cars move in the same direction:*

In this case, let them meet at M after 5 hours.



Then, $AM = 5x$ km.

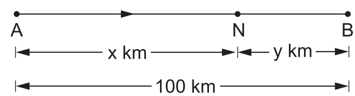
And, $BM = 5y$ km.

$\therefore \quad AM - BM = AB \Rightarrow 5x - 5y = 100$

$\Rightarrow \quad 5(x - y) = 100 \Rightarrow x - y = 20.$ ... (i)

Case II *When the two cars move in the opposite directions:*

Let one car move from A to B and let the other move from B to A. Let them meet at N after 1 hour.



Then, $AN = x$ km and $BN = y$ km.

$\therefore \quad AN + BN = AB \Rightarrow x + y = 100.$ ... (ii)

Adding (i) and (ii), we get $2x = 120 \Rightarrow x = 60$.

Putting $x = 60$ in (ii), we get $60 + y = 100 \Rightarrow y = 40$.

$\therefore$ speed of the car from $A = 60$ km/hr,

and speed of the car from $B = 40$ km/hr.

EXAMPLE 27 *A train covered a certain distance at a uniform speed. If the train had been* 6 kmph *faster, it would have taken* 4 *hours less than the scheduled time. And, if the train were slower by* 6 kmph, *it would have taken* 6 *hours more than the scheduled time. Find the length of the journey.*

SOLUTION Let the original speed of the train be x kmph and let the time taken to complete the journey be y hours.

Length of whole journey $= (xy)$ km.

Case I *When speed* $= (x + 6)$ kmph *and time taken* $= (y - 4)$ hours.

Length of total journey $= (x + 6)(y - 4)$ km.

$\therefore \quad xy = (x + 6)(y - 4) \Rightarrow xy = xy - 4x + 6y - 24$

$\Rightarrow \quad 4x - 6y = -24 \Rightarrow 2x - 3y = -12.$ ... (i)

Case II *When speed* $= (x - 6)$ kmph *and time taken* $= (y + 6)$ hours.

Length of total journey $= (x - 6)(y + 6)$ km.

$\therefore \quad xy = (x-6)(y+6) \Rightarrow xy = xy + 6x - 6y - 36$

$\Rightarrow \quad 6x - 6y = 36 \Rightarrow x - y = 6.$ … (ii)

On multiplying (ii) by 3 and subtracting (i) from the result, we get

$3x - 2x = 18 + 12 \Rightarrow x = 30.$

On putting $x = 30$ in (ii), we get $y = 30 - 6 = 24$.

$\therefore \quad x = 30$ and $y = 24$.

So, length of journey = (xy) km = (30×24) km = 720 km.

PROBLEMS ON BOATS AND STREAM

EXAMPLE 28 *A man can row downstream* 20 km *in* 2 *hours, and upstream* 4 km *in* 2 *hours. Find his speed of rowing in still water. Also, find the speed of the current.*

SOLUTION Let the speed of the man in still water be x km/hr and let the speed of the current be y km/hr.

Speed downstream = $(x + y)$ km/hr.

Speed upstream = $(x - y)$ km/hr.

But, speed downstream = $\frac{\text{distance}}{\text{time}} = \frac{20}{2}$ km/hr = 10 km/hr.

And, speed upstream = $\frac{\text{distance}}{\text{time}} = \frac{4}{2}$ km/hr = 2 km/hr.

$\therefore \quad x + y = 10$ … (i) and $x - y = 2$ … (ii).

Adding (i) and (ii), we get $2x = 12 \Rightarrow x = 6$.

Putting $x = 6$ in (i), we get $6 + y = 10 \Rightarrow y = 4$.

Thus, $x = 6$ and $y = 4$.

Hence, the speed of the man in still water is 6 km/hr and the speed of the current is 4 km/hr.

EXAMPLE 29 *A boat goes* 16 km *upstream and* 24 km *downstream in* 6 *hours. Also, it covers* 12 km *upstream and* 36 km *downstream in the same time. Find the speed of the boat in still water and that of the stream.*

SOLUTION Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr. Then, speed upstream = $(x-y)$ km/hr

and speed downstream = $(x+y)$ km/hr.

Time taken to cover 16 km upstream = $\frac{16}{(x-y)}$ hours.

Time taken to cover 24 km downstream = $\frac{24}{(x+y)}$ hours.

Total time taken = 6 hours.

$\therefore \quad \frac{16}{x-y}+\frac{24}{x+y}=6.$ ... (i)

Again, time taken to cover 12 km upstream = $\frac{12}{(x-y)}$ hours.

Time taken to cover 36 km downstream = $\frac{36}{(x+y)}$ hours.

Total time taken = 6 hours.

$\therefore \quad \frac{12}{x-y}+\frac{36}{x+y}=6.$ ... (ii)

Putting $\frac{1}{(x-y)}=u$ and $\frac{1}{(x+y)}=v$ in (i) and (ii), we get

$16u+24v=6 \Rightarrow 8u+12v=3,$ ... (iii)

$12u+36v=6 \Rightarrow 2u+6v=1.$ ... (iv)

On multiplying (iv) by 4 and subtracting (iii) from the result, we get

$12v=1 \Rightarrow v=\frac{1}{12}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{12} \Rightarrow x+y=12.$ ... (v)

On multiplying (iv) by 2 and subtracting the result from (iii), we get

$4u=1 \Rightarrow u=\frac{1}{4}$

$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{4} \Rightarrow x-y=4.$ ... (vi)

On adding (v) and (vi), we get $2x=16 \Rightarrow x=8.$

On subtracting (vi) from (v), we get $2y=8 \Rightarrow y=4.$

$\therefore$ speed of the boat in still water = 8 km/hr,

and speed of the stream = 4 km/hr.

PROBLEMS ON TIME AND WORK

EXAMPLE 30 *8 men and 12 boys can finish a piece of work in 5 days, while 6 men and 8 boys can finish it in 7 days. Find the time taken by 1 man alone and that by 1 boy alone to finish the work.*

SOLUTION Suppose 1 man alone can finish the work in x days and 1 boy alone can finish it in y days.

Then, 1 man's 1 day's work $= \frac{1}{x}$.

And, 1 boy's 1 day's work $= \frac{1}{y}$.

8 men and 12 boys can finish the work in 5 days

$\Rightarrow$ (8 men's 1 day's work) + (12 boys' 1 day's work) $= \frac{1}{5}$

$\Rightarrow \frac{8}{x} + \frac{12}{y} = \frac{1}{5}$

$\Rightarrow 8u + 12v = \frac{1}{5}$ ... (i), where $\frac{1}{x} = u$ and $\frac{1}{y} = v$.

Again, 6 men and 8 boys can finish the work in 7 days

$\Rightarrow$ (6 men's 1 day's work) + (8 boys' 1 day's work) $= \frac{1}{7}$

$\Rightarrow \frac{6}{x} + \frac{8}{y} = \frac{1}{7}$

$\Rightarrow 6u + 8v = \frac{1}{7}$. ... (ii)

On multiplying (i) by 3, (ii) by 4 and subtracting the results, we get

$$4v = \left(\frac{3}{5} - \frac{4}{7}\right) = \frac{1}{35} \Rightarrow v = \frac{1}{140} \Rightarrow \frac{1}{y} = \frac{1}{140} \Rightarrow y = 140.$$

On putting $v = \frac{1}{140}$ in (ii), we get

$$6u = \left(\frac{1}{7} - \frac{8}{140}\right) = \frac{12}{140} \Rightarrow u = \left(\frac{12}{140} \times \frac{1}{6}\right) = \frac{1}{70}$$

$\Rightarrow \frac{1}{x} = \frac{1}{70} \Rightarrow x = 70.$

$\therefore$ one man alone can finish the work in 70 days,

and one boy alone can finish the work in 140 days.

PROBLEMS ON AREA

EXAMPLE 31 *If the length of a rectangle is reduced by 5 units and its breadth is increased by 2 units then the area of the rectangle is reduced by 80 sq units. However, if we increase its length by 10 units and decrease* *the breadth by 5 units, its area is increased by 50 sq units. Find the length and breadth of the rectangle.*

SOLUTION Let the length and breadth of the rectangle be x units and y units respectively.

Then, area of the rectangle = xy sq units.

Case I *When the length is reduced by 5 units and the breadth is increased by 2 units.*

Then, new length = $(x-5)$ units

and new breadth = $(y+2)$ units.

$\therefore$ new area = $(x-5)(y+2)$ sq units.

$\therefore$ $xy-(x-5)(y+2)=80 \Rightarrow 5y-2x=70.$ … (i)

Case II *When the length is increased by 10 units and the breadth is decreased by 5 units.*

Then, new length = $(x+10)$ units

and new breadth = $(y-5)$ units.

$\therefore$ new area = $(x+10)(y-5)$ sq units.

$\therefore$ $(x+10)(y-5)-xy=50$

$\Rightarrow$ $10y-5x=100 \Rightarrow 2y-x=20.$ … (ii)

On multiplying (ii) by 2 and subtracting the result from (i), we get $y=30$.

Putting $y=30$ in (ii), we get

$$(2\times 30)-x=20 \Rightarrow 60-x=20 \Rightarrow x=(60-20)=40.$$

$\therefore$ $x=40$ and $y=30$.

Hence, length = 40 units and breadth = 30 units.

PROBLEMS ON GEOMETRY

EXAMPLE 32 *In a* $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. *Find the angles.*

SOLUTION Let $\angle A = x^\circ$ and $\angle B = y^\circ$.

Then, $\angle C = 3\angle B = (3y)^\circ$.

Now, $\angle C = 2(\angle A + \angle B)$

$\Rightarrow$ $3y=2(x+y) \Rightarrow 2x-y=0.$ … (i)

We know that the sum of the angles of a triangle is 180°.

$\therefore \quad \angle A + \angle B + \angle C = 180^\circ \Rightarrow x + y + 3y = 180$

$\Rightarrow x + 4y = 180.$ ... (ii)

On multiplying (i) by 4 and adding the result with (ii), we get

$8x + x = 180 \Rightarrow 9x = 180 \Rightarrow x = 20.$

Putting $x = 20$ in (i), we get $y = (2 \times 20) = 40.$

Thus, $x = 20$ and $y = 40.$

$\therefore \quad \angle A = 20^\circ, \angle B = 40^\circ$ and $\angle C = (3 \times 40^\circ) = 120^\circ.$

EXAMPLE 33 *Find the four angles of a cyclic quadrilateral ABCD in which* $\angle A = (2x - 1)^\circ, \angle B = (y + 5)^\circ, \angle C = (2y + 15)^\circ$ *and* $\angle D = (4x - 7)^\circ$.

SOLUTION We know that the sum of the opposite angles of a cyclic quadrilateral is 180°.

$\therefore \quad \angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ.$

Now, $\angle A + \angle C = 180^\circ \Rightarrow (2x - 1) + (2y + 15) = 180$

$\Rightarrow 2(x + y) = 166$

$\Rightarrow x + y = 83.$ ... (i)

And, $\angle B + \angle D = 180^\circ \Rightarrow (y + 5) + (4x - 7) = 180$

$\Rightarrow 4x + y = 182.$ ... (ii)

On subtracting (i) from (ii), we get

$3x = 182 - 83 = 99 \Rightarrow x = 33.$

Putting $x = 33$ in (i), we get

$33 + y = 83 \Rightarrow y = (83 - 33) \Rightarrow y = 50.$

Thus, $x = 33$ and $y = 50.$

$\therefore \quad \angle A = \{(2 \times 33) - 1\}^\circ = 65^\circ, \angle B = (50 + 5)^\circ = 55^\circ.$

$\angle C = \{(2 \times 50) + 15\}^\circ = 115^\circ$ and $\angle D = \{(4 \times 33) - 7\}^\circ = 125^\circ.$

Hence, $\angle A = 65^\circ, \angle B = 55^\circ, \angle C = 115^\circ$ and $\angle D = 125^\circ.$

EXERCISE 3E

1. 5 chairs and 4 tables together cost ₹ 5600, while 4 chairs and 3 tables together cost ₹ 4340. Find the cost of a chair and that of a table.
2. 23 spoons and 17 forks together cost ₹ 1770, while 17 spoons and 23 forks together cost ₹ 1830. Find the cost of a spoon and that of a fork.

3. A lady has only 25-paisa and 50-paisa coins in her purse. If she has 50 coins in all totalling ₹ 19.50, how many coins of each kind does she have?
4. The sum of two numbers is 137 and their difference is 43. Find the numbers.
5. Find two numbers such that the sum of twice the first and thrice the second is 92, and four times the first exceeds seven times the second by 2.
6. Find two numbers such that the sum of thrice the first and the second is 142, and four times the first exceeds the second by 138.
7. If 45 is subtracted from twice the greater of two numbers, it results in the other number. If 21 is subtracted from twice the smaller number, it results in the greater number. Find the numbers.
8. If three times the larger of two numbers is divided by the smaller, we get 4 as the quotient and 8 as the remainder. If five times the smaller is divided by the larger, we get 3 as the quotient and 5 as the remainder. Find the numbers.
9. If 2 is added to each of two given numbers, their ratio becomes 1 : 2. However, if 4 is subtracted from each of the given numbers, the ratio becomes 5 : 11. Find the numbers.
10. The difference between two numbers is 14 and the difference between their squares is 448. Find the numbers.
11. The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number. [CBSE 2006]
12. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.
13. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number. [CBSE 2004]
14. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.
15. A number consists of two digits. When it is divided by the sum of its digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number. [CBSE 1999C]

16. A two-digit number is such that the product of its digits is 35. If 18 is added to the number, the digits interchange their places. Find the number. [CBSE 2006]

17. A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number. [CBSE 2006C]

18. The sum of a two-digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

19. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both of the numerator and the denominator, the fraction becomes $\frac{3}{4}\cdot$ Find the fraction. [CBSE 2003]

20. If 2 is added to the numerator of a fraction, it reduces to $\left(\frac{1}{2}\right)$ and if 1 is subtracted from the denominator, it reduces to $\left(\frac{1}{3}\right)\cdot$ Find the fraction.

21. The denominator of a fraction is greater than its numerator by 11. If 8 is added to both its numerator and denominator, it becomes $\frac{3}{4}\cdot$ Find the fraction.

22. Find a fraction which becomes $\left(\frac{1}{2}\right)$ when 1 is subtracted from the numerator and 2 is added to the denominator, and the fraction becomes $\left(\frac{1}{3}\right)$ when 7 is subtracted from the numerator and 2 is subtracted from the denominator.

23. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. [CBSE 2010]

24. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}\cdot$ Find the numbers. [CBSE 2005]

25. There are two classrooms A and B. If 10 students are sent from A to B, the number of students in each room becomes the same. If 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in each room.

26. Taxi charges in a city consist of fixed charges and the remaining depending upon the distance travelled in kilometres. If a man travels

80 km, he pays ₹ 1330, and travelling 90 km, he pays ₹ 1490. Find the fixed charges and rate per km.

27. A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹ 4500, whereas a student B who takes food for 30 days, has to pay ₹ 5200. Find the fixed charges per month and the cost of the food per day.
28. A man invested an amount at 10% per annum and another amount at 8% per annum simple interest. Thus, he received ₹ 1350 as annual interest. Had he interchanged the amounts invested, he would have received ₹ 45 less as interest. What amounts did he invest at different rates?
29. The monthly incomes of A and B are in the ratio 5 : 4 and their monthly expenditures are in the ratio 7 : 5. If each saves ₹ 9000 per month, find the monthly income of each.
30. A man sold a chair and a table together for ₹ 1520, thereby making a profit of 25% on chair and 10% on table. By selling them together for ₹ 1535, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each.
31. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours. But, if they travel towards each other, they meet in 1 hour. Find the speed of each car. [CBSE 2007C]
32. A train covered a certain distance at a uniform speed. If the train had been 5 kmph faster, it would have taken 3 hours less than the scheduled time. And, if the train were slower by 4 kmph, it would have taken 3 hours more than the scheduled time. Find the length of the journey.
33. Abdul travelled 300 km by train and 200 km by taxi taking 5 hours 30 minutes. But, if he travels 260 km by train and 240 km by taxi, he takes 6 minutes longer. Find the speed of the train and that of the taxi. [CBSE 2006C]
34. Places A and B are 160 km apart on a highway. One car starts from A and another from B at the same time. If they travel in the same direction, they meet in 8 hours. But, if they travel towards each other, they meet in 2 hours. Find the speed of each car. [CBSE 2009C]
35. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Find the speed of the sailor in still water and the speed of the current.

36. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.
37. 2 men and 5 boys can finish a piece of work in 4 days, while 3 men and 6 boys can finish it in 3 days. Find the time taken by one man alone to finish the work and that taken by one boy alone to finish the work.
38. The length of a room exceeds its breadth by 3 metres. If the length is increased by 3 metres and the breadth is decreased by 2 metres, the area remains the same. Find the length and the breadth of the room.
39. The area of a rectangle gets reduced by 8 m^2, when its length is reduced by 5 m and its breadth is increased by 3 m. If we increase the length by 3 m and breadth by 2 m, the area is increased by 74 m^2 . Find the length and the breadth of the rectangle.
40. The area of a rectangle gets reduced by 67 square metres, when its length is increased by 3 m and breadth is decreased by 4 m. If the length is reduced by 1 m and breadth is increased by 4 m, the area is increased by 89 square metres. Find the dimensions of the rectangle.
41. A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Delhi costs ₹ 4150 while one full and one half reserved first class tickets cost ₹ 6255. What is the basic first class full fare and what is the reservation charge? [HOTS]
42. Five years hence, a man's age will be three times the age of his son. Five years ago, the man was seven times as old as his son. Find their present ages.
43. Two years ago, a man was five times as old as his son. Two years later, his age will be 8 more than three times the age of his son. Find their present ages. [CBSE 2008]
44. If twice the son's age in years is added to the father's age, the sum is 70. But, if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.
45. The present age of a woman is 3 years more than three times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.
46. On selling a tea set at 5% loss and a lemon set at 15% gain, a crockery seller gains ₹ 7. If he sells the tea set at 5% gain and the lemon set at 10% gain, he gains ₹ 13. Find the actual price of each of the tea set and the lemon set. [HOTS]

47. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Mona paid ₹ 27 for a book kept for 7 days, while Tanvy paid ₹ 21 for the book she kept for 5 days. Find the fixed charge and the charge for each extra day.

48. A chemist has one solution containing 50% acid and a second one containing 25% acid. How much of each should be used to make 10 litres of a 40% acid solution? [HOTS]

49. A jeweller has bars of 18-carat gold and 12-carat gold. How much of each must be melted together to obtain a bar of 16-carat gold, weighing 120 g? (Given: Pure gold is 24-carat). [HOTS]

50. 90% and 97% pure acid solutions are mixed to obtain 21 litres of 95% pure acid solution. Find the quantity of each type of acids to be mixed to form the mixture. [HOTS]

51. The larger of the two supplementary angles exceeds the smaller by 18°. Find them.

52. In a $\triangle ABC$, $\angle A = x^\circ$, $\angle B = (3x-2)^\circ$, $\angle C = y^\circ$ and $\angle C - \angle B = 9^\circ$. Find the three angles.

53. In a cyclic quadrilateral $ABCD$, it is given that $\angle A = (2x+4)^\circ$, $\angle B = (y+3)^\circ$, $\angle C = (2y+10)^\circ$ and $\angle D = (4x-5)^\circ$. Find the four angles.

ANSWERS (EXERCISE 3E)

1. ₹ 560, ₹ 700 **2.** ₹ 40, ₹ 50 **3.** 22, 28 **4.** 90, 47 **5.** 25, 14
6. 40, 22 **7.** 37, 29 **8.** 20, 13 **9.** 34, 70 **10.** 23, 9
11. 57 **12.** 63 **13.** 78 **14.** 35 **15.** 54 **16.** 57 **17.** 92
18. 47 or 74 **19.** $\frac{3}{5}$ **20.** $\frac{3}{10}$ **21.** $\frac{25}{36}$ **22.** $\frac{15}{26}$ **23.** $\frac{5}{9}$ **24.** 12 and 4
25. 100 in A and 80 in B **26.** fixed charges = ₹ 50, rate per km = ₹ 16
27. fixed charges = ₹ 1000, cost of food per day = ₹ 140
28. ₹ 8500 at 10% p.a. and ₹ 6250 at 8% p.a. **29.** ₹ 30000, ₹ 24000
30. ₹ 600, ₹ 700 **31.** 40 km/hr, 30 km/hr **32.** 1080 km
33. 100 kmph, 80 kmph **34.** 50 kmph, 30 kmph **35.** 10 kmph, 2 kmph
36. 6 kmph, 2 kmph **37.** 18 days, 36 days
38. length = 15 m, breadth = 12 m **39.** length = 19 m, breadth = 10 m
40. length = 28 m, breadth = 19 m
41. first class full fare = ₹ 4090, reservation charges = ₹ 60
42. 40 years, 10 years **43.** 42 years, 10 years **44.** 40 years, 15 years

45. 33 years, 10 years **46.** tea set = ₹ 100, lemon set = ₹ 80

47. ₹ 15, ₹ 3 per day **48.** 50% solution = 6 litres, 25% solution = 4 litres

49. 80 g, 40 g **50.** 6 litres, 15 lires **51.** 99°, 81°

52. $\angle A = 25^\circ, \angle B = 73^\circ, \angle C = 82^\circ$

53. $\angle A = 70^\circ, \angle B = 53^\circ, \angle C = 110^\circ, \angle D = 127^\circ$

HINTS TO SOME SELECTED QUESTIONS

3. Let the number of 25-p and 50-p coins be x and y respectively.

Then, $x + y = 50$ and $\frac{x}{4} + \frac{y}{2} = \frac{1950}{100}$.

8. Let the larger number be x and the smaller number be y.

Then, $3x = 4y + 8$ and $5y = 3x + 5$.

10. $x - y = 14$ and $x^2 - y^2 = 448$.

$\therefore \quad x + y = \frac{(x^2 - y^2)}{(x - y)} = \frac{448}{14} = 32.$

15. Let the ten's digit be x and the unit's digit be y. Then,

$\frac{10x + y}{x + y} = 6$ and $10x + y - 9 = 10y + x$.

16. Let the ten's digit be x and the unit's digit be y. Then,

$xy = 35$ and $10x + y + 18 = 10y + x \Rightarrow y - x = 2$.

$\therefore \quad (y + x)^2 - (y - x)^2 = 4xy \Rightarrow (y + x) = \sqrt{(y - x)^2 + 4xy} = \sqrt{144} = 12.$

Now, solve $y - x = 2$ and $y + x = 12$.

18. Let the ten's digit be x and the unit's digit be y.

Then, $(10x + y) + (10y + x) = 121 \Rightarrow 11(x + y) = 121 \Rightarrow x + y = 11$.

$(x + y = 11, x - y = 3)$ or $(x + y = 11, y - x = 3)$.

21. Let the required fraction be $\frac{x}{y}$.

Then, $x + 11 = y \Rightarrow x - y = -11$.

And, $\frac{x + 8}{y + 8} = \frac{3}{4} \Rightarrow 4x + 32 = 3y + 24 \Rightarrow 4x - 3y = -8$.

23. Let the required fraction be $\frac{x}{y}$.

Then, $x + y - 4 = 2x \Rightarrow y - x = 4$.

$\frac{x + 3}{y + 3} = \frac{2}{3} \Rightarrow 3x + 9 = 2y + 6 \Rightarrow 2y - 3x = 3$.

24. $x + y = 16$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{3} \Rightarrow \frac{y + x}{xy} = \frac{1}{3} \Rightarrow xy = 16 \times 3 = 48$.

$(x - y)^2 = (x + y)^2 - 4xy = (16)^2 - (4 \times 48) = 256 - 192 = 64$

$\Rightarrow \quad x - y = 8$ or $x - y = -8$.

$\therefore \quad (x + y = 16, x - y = 8)$ or $(x + y = 16, x - y = -8)$.

25. Let the number of students in room A and room B be x and y respectively. Then,

$x - 10 = y + 10 \Rightarrow x - y = 20.$ … (i)

And, $(x+20) = 2(y-20) \Rightarrow x - 2y = -60.$ ... (ii)

Now, solve (i) and (ii) to get x and y.

26. Let the fixed charges be ₹ x and rate per km be ₹ y. Then,

$$x + 80y = 1330 \text{ and } x + 90y = 1490.$$

27. Let the fixed charges be ₹ x and the charges for food be ₹ y per day. Then,

$$x + 25y = 4500 \text{ and } x + 30y = 5200.$$

28. Suppose he invested ₹ x at 10% p.a. and ₹ y at 8% p.a.

$$\frac{(x \times 10 \times 1)}{100} + \frac{(y \times 8 \times 1)}{100} = 1350 \Rightarrow 5x + 4y = 67500. \quad \text{... (i)}$$

$$\frac{(x \times 8 \times 1)}{100} + \frac{(y \times 10 \times 1)}{100} = 1350 - 45 \Rightarrow 4x + 5y = 65250. \quad \text{... (ii)}$$

Add (i) and (ii). Then, subtract (ii) from (i).

29. $5x - 7y = 9000$ and $4x - 5y = 9000$. Find x and y.

30. Let the CP of a chair be ₹ x and that of a table be ₹ y. Then,

$$\frac{125x}{100} + \frac{110y}{100} = 1520 \Rightarrow 25x + 22y = 30400. \quad \text{... (i)}$$

$$\frac{110x}{100} + \frac{125y}{100} = 1535 \Rightarrow 22x + 25y = 30700. \quad \text{... (ii)}$$

On adding (i) and (ii), we get $47(x+y) = 61100 \Rightarrow x + y = 1300.$

On subtracting (i) from (ii), we get $3(y-x) = 300 \Rightarrow y - x = 100.$

$\therefore \quad x = 600$ and $y = 700$.

31. Let the speed of the car from A be x kmph and that of the car from B be y kmph.

Then, $7x - 7y = 70 \Rightarrow x - y = 10.$ ... (i)

And, $x + y = 70.$ ... (ii)

32. Let the original speed of the train be x kmph and let the time taken to complete the journey be y hours.

Then, length of the journey = xy km.

Case I Speed = $(x+5)$ kmph and time taken = $(y-3)$ hours.

$\therefore \quad xy = (x+5)(y-3) \Rightarrow 5y - 3x = 15.$ ... (i)

Case II Speed = $(x-4)$ kmph and time taken = $(y+3)$ hours.

$\therefore \quad xy = (x-4)(y+3) \Rightarrow 3x - 4y = 12.$ ... (ii)

From (i) and (ii), we get $x = 40$ and $y = 27$.

$\therefore \quad$ length of the journey = $xy = 40 \times 27$ km = 1080 km.

33. Let the speed of the train be x kmph and that of the taxi be y kmph.

Then, $\frac{300}{x} + \frac{200}{y} = \frac{11}{2} \Rightarrow 600u + 400v = 11$ ... (i), where $\frac{1}{x} = u$ and $\frac{1}{y} = v$.

And, $\frac{260}{x} + \frac{240}{y} = \frac{28}{5} \Rightarrow 325u + 300v = 7.$ ... (ii)

35. Let the speed of the sailor in still water be x kmph and the speed of the current be y kmph.

Then, speed downstream = $(x + y)$ km/hr.

And, speed upstream = $(x - y)$ km/hr.

$\therefore \quad \frac{8}{x+y} = \frac{40}{60} = \frac{2}{3} \Rightarrow x + y = 12$ … (i)

and $\frac{8}{x-y} = 1 \Rightarrow x - y = 8.$ … (ii)

Solve (i) and (ii).

37. Let 1 man's 1 day's work be $\frac{1}{x}$ and 1 boy's 1 day's work be $\frac{1}{y}$.

Then, $\frac{2}{x} + \frac{5}{y} = \frac{1}{4} \Rightarrow 2u + 5v = \frac{1}{4}$

and $\frac{3}{x} + \frac{6}{y} = \frac{1}{3} \Rightarrow 3u + 6v = \frac{1}{3}$.

38. Let length = x metres and breadth = y metres.

Then, $x = y + 3$ and $(x + 3)(y - 2) = xy$.

$\therefore \quad x - y = 3$ and $3y - 2x = 6$.

39. Let length = x metres and breadth = y metres. Then,

$xy - (x - 5)(y + 3) = 8 \Rightarrow 3x - 5y = 7.$

And, $(x + 3)(y + 2) - xy = 74 \Rightarrow 2x + 3y = 68.$

40. Let length = x metres and breadth = y metres. Then,

$xy - (x + 3)(y - 4) = 67 \Rightarrow 4x - 3y = 55$

and $(x - 1)(y + 4) - xy = 89 \Rightarrow 4x - y = 93.$

41. Let the basic first class full fare be ₹ x and the reservation charge be ₹ y. Then,

$x + y = 4150.$ … (i)

And, $(x + y) + \left(\frac{1}{2}x + y\right) = 6255 \Rightarrow 3x + 4y = 12510.$ … (ii)

42. Let the present ages of the man and his son be x years and y years respectively. Then,

$x + 5 = 3(y + 5) \Rightarrow x - 3y = 10$ … (i)

and $x - 5 = 7(y - 5) \Rightarrow x - 7y = -30.$ … (ii)

43. Let the present ages of the man and his son be x years and y years respectively. Then,

$(x - 2) = 5(y - 2) \Rightarrow x - 5y = -8$ … (i)

and $(x + 2) = 3(y + 2) + 8 \Rightarrow x - 3y = 12.$ … (ii)

44. Let father's age be x years and son's age be y years. Then,

$x + 2y = 70$ and $2x + y = 95$.

45. Let woman's present age be x years and daughter's present age be y years. Then,

$x = 3y + 3 \Rightarrow x - 3y = 3$

and, $x + 3 = 2(y + 3) + 10 \Rightarrow x - 2y = 13.$

46. Let the cost price of the tea set be ₹ x and that of the lemon set be ₹ y. Then,

$$\left(\frac{95x}{100} + \frac{115y}{100}\right) - (x + y) = 7 \Rightarrow \frac{(19x + 23y)}{20} - (x + y) = 7$$

$\Rightarrow 3y - x = 140.$ … (i)

And, $\left(\frac{105x}{100}+\frac{110y}{100}\right)-(x+y)=13 \Rightarrow \frac{(21x+22y)}{20}-(x+y)=13$

$\Rightarrow x+2y=260.$ ... (ii)

Now, solve (i) and (ii).

47. Let the fixed charge be ₹ x and charge for each extra day be ₹ y.

Then, $x+(7-3)y=27 \Rightarrow x+4y=27$ ... (i)

and, $x+(5-3)y=21 \Rightarrow x+2y=21.$ ... (ii)

48. Let x litres of 50% solution be mixed with y litres of 25% solution.

Then, $x+y=10$ ... (i)

and, $\frac{50x}{100}+\frac{25y}{100}=\frac{40\times 10}{100} \Rightarrow 2x+y=16.$ ... (ii)

49. Let x g of 18-carat gold be mixed with y g of 12-carat gold to get 120 g of 16-carat gold.

Then, $x+y=120.$ ... (i)

Gold % in 18-carat gold $=\left(\frac{18}{24}\times 100\right)\%=75\%$

Gold % in 12-carat gold $=\left(\frac{12}{24}\times 100\right)\%=50\%$

Gold % in 16-carat gold $=\left(\frac{16}{24}\times 100\right)\%=\frac{200}{3}\%$

$\therefore$ 75% of x + 50% of $y=\frac{200}{3}\%$ of 120

$\Rightarrow \frac{75x}{100}+\frac{50y}{100}=\frac{200\times 120}{3\times 100} \Rightarrow 3x+2y=320.$ ... (ii)

Now, solve (i) and (ii).

50. Let x litres of 90% pure solution be mixed with y litres of 97% pure solution to get 21 litres of 95% pure solution.

Then, $x+y=21$ ... (i)

and $\frac{90x}{100}+\frac{97y}{100}=\frac{95\times 21}{100} \Rightarrow 90x+97y=1995.$ ... (ii)

Now, solve (i) and (ii).

51. $x-y=18$ and $x+y=180$. Find x and y.

52. $\angle A+\angle B+\angle C=180^\circ \Rightarrow x+(3x-2)+y=180 \Rightarrow 4x+y=182.$ ... (i)

$\angle C-\angle B=9^\circ \Rightarrow y-3x+2=9 \Rightarrow y-3x=7.$ ... (ii)

From (i) and (ii), we get $7x=175 \Rightarrow x=25.$

Putting $x=25$ in (i), we get $y=82.$

$\therefore \angle A=25^\circ, \angle B=(3\times 25-2)^\circ=73^\circ, \angle C=82^\circ.$

53. We have, $\angle A+\angle C=180^\circ$ and $\angle B+\angle D=180^\circ$.

$\therefore (2x+4)+(2y+10)=180 \Rightarrow x+y=83$ ... (i)

and $(y+3)+(4x-5)=180 \Rightarrow 4x+y=182.$ ... (ii)

From (i) and (ii), we get

$\therefore 3x=99 \Rightarrow x=33.$ And, $33+y=83 \Rightarrow y=50.$

EXERCISE 3F

Very-Short and Short-Answer Questions

1. Write the number of solutions of the following pair of linear equations:

   $x+2y-8=0,\ 2x+4y=16.$ [CBSE 2009]

2. Find the value of k for which the following pair of linear equations have infinitely many solutions:

   $2x+3y=7,\ (k-1)x+(k+2)y=3k.$ [CBSE 2010]

3. For what value of k does the following pair of linear equations have infinitely many solutions?

   $10x+5y-(k-5)=0$ and $20x+10y-k=0.$

4. For what value of k will the following pair of linear equations have no solution?

   $2x+3y=9,\ 6x+(k-2)y=(3k-2).$ [CBSE 2010]

5. Write the number of solutions of the following pair of linear equations:

   $x+3y-4=0$ and $2x+6y-7=0.$

6. Write the value of k for which the system of equations $3x+ky=0$, $2x-y=0$ has a unique solution.
7. The difference between two numbers is 5 and the difference between their squares is 65. Find the numbers.
8. The cost of 5 pens and 8 pencils is ₹ 120, while the cost of 8 pens and 5 pencils is ₹ 153. Find the cost of 1 pen and that of 1 pencil.
9. The sum of two numbers is 80. The larger number exceeds four times the smaller one by 5. Find the numbers.
10. A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digits are reversed. Find the number.
11. A man purchased 47 stamps of 20 p and 25 p for ₹ 10. Find the number of each type of stamps.
12. A man has some hens and cows. If the number of heads be 48 and number of feet be 140, how many cows are there?
13. If $\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$ and $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, find the values of x and y.
14. If $\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$ and $\frac{x}{2}+y=1$ then find the value of $(x+y)$.
15. If $12x+17y=53$ and $17x+12y=63$ then find the value of $(x+y)$.

16. Find the value of k for which the system $3x + 5y = 0$, $kx + 10y = 0$ has a nonzero solution.
17. Find k for which the system $kx - y = 2$ and $6x - 2y = 3$ has a unique solution.
18. Find k for which the system $2x + 3y - 5 = 0$, $4x + ky - 10 = 0$ has an infinite number of solutions.
19. Show that the system $2x + 3y - 1 = 0$, $4x + 6y - 4 = 0$ has no solution.
20. Find k for which the system $x + 2y = 3$ and $5x + ky + 7 = 0$ is inconsistent.
21. Solve: $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$.

ANSWERS (EXERCISE 3F)

1. infinitely many **2.** $k = 7$ **3.** $k = 10$ **4.** $k = 11$ **5.** 0
6. $k \neq \frac{-3}{2}$ **7.** 9 and 4 **8.** ₹ 16, ₹ 5 **9.** 65 and 15 **10.** 64
11. 35 and 12 **12.** 22 **13.** $x = 1, y = 3$ **14.** $\frac{3}{2}$ **15.** 4
16. $k = 6$ **17.** $k \neq 3$ **18.** $k = 6$ **20.** $k = \frac{2}{5}$ **21.** $x = \frac{5}{2}, y = \frac{1}{2}$

HINTS TO SOME SELECTED QUESTIONS

1. Given equations are $x + 2y - 8 = 0$ and $2x + 4y - 16 = 0$.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$

So, there are infinitely many solutions.

2. $2x + 3y - 7 = 0$ and $(k-1)x + (k+2)y - 3k = 0$.

For infinitely many solutions, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$

$\therefore \quad \frac{2}{(k-1)} = \frac{3}{(k+2)}$ and $\frac{3}{(k+2)} = \frac{7}{3k}$

$\Rightarrow \quad 2k + 4 = 3k - 3$ and $9k = 7k + 14 \Rightarrow k = 7$.

3. For infinitely many solutions, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$

$\therefore \quad \frac{10}{20} = \frac{5}{10} = \frac{k-5}{k} \Rightarrow \frac{k-5}{k} = \frac{1}{2} \Rightarrow 2k - 10 = k \Rightarrow k = 10.$

4. For no solution, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$

$\therefore \quad \frac{2}{6} = \frac{3}{k-2}$ and $\frac{2}{6} \neq \frac{9}{3k-2} \Rightarrow \frac{3}{k-2} = \frac{1}{3}$ and $\frac{9}{3k-2} \neq \frac{1}{3}$

$\Rightarrow \quad k - 2 = 9$ and $3k - 2 \neq 27 \Rightarrow k = 11$ and $k \neq \frac{29}{3}.$

Hence, $k = 11$.

5. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Hence, the given system has no solution.

6. For unique solution, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

$\therefore \quad \frac{3}{2} \neq \frac{k}{-1} \Rightarrow k \neq \frac{-3}{2}$.

7. Let the required numbers be x and y. Then,

$$x - y = 5 \text{ and } x^2 - y^2 = 65 \Rightarrow \frac{x^2 - y^2}{x - y} = \frac{65}{5} = 13 \Rightarrow x + y = 13.$$

Solving $x - y = 5$ and $x + y = 13$, we get $x = 9$ and $y = 4$.

8. Let the cost of 1 pen be ₹ x and that of 1 pencil be ₹ y. Then,

$5x + 8y = 120$ … (i) and $8x + 5y = 153$. … (ii)

Adding (i) and (ii), we get $13(x + y) = 273 \Rightarrow x + y = 21$. … (iii)

Subtracting (i) from (ii), we get $3(x - y) = 33 \Rightarrow x - y = 11$. … (iv)

From (iii) and (iv), we get $x = 16$ and $y = 5$.

9. Let the required numbers be x and y such that $x > y$. Then,

$x + y = 80$ … (i) and $x - 4y = 5$. … (ii)

Solve (i) and (ii).

10. Let the ten's digit be x and unit's digit be y. Then,

$x + y = 10$ … (i) and $(10x + y) - 18 = (10y + x) \Rightarrow x - y = 2$. … (ii)

Solve (i) and (ii).

11. Let x stamps of 20 p and y stamps of 25 p be purchased. Then,

$x + y = 47$ … (i) and $20x + 25y = 1000 \Rightarrow 4x + 5y = 200$. … (ii)

Solve (i) and (ii).

12. Let there be x cows and y hens. Then,

$x + y = 48$ … (i) and $4x + 2y = 140 \Rightarrow 2x + y = 70$. … (ii)

Subtracting (i) from (ii), we get $x = 22$.

13. The given equations are

$2y + 3x = 9$ … (i) and $4y + 9x = 21$. … (ii)

Solve (i) and (ii).

14. Given equations are $3x + 4y = 5$ … (i) and $x + 2y = 2$. … (ii)

Solve (i) and (ii) for x and y and find $(x + y)$.

15. Adding the given equations, we get

$$29(x + y) = 116 \Rightarrow x + y = \frac{116}{29} = 4.$$

16. The given system has a nonzero solution when $\frac{3}{k} = \frac{5}{10} \Rightarrow k = 6$.

17. For a unique solution, we have $\frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq 3$.

18. For an infinite number of solutions, we have

$$\frac{2}{4} = \frac{3}{k} = \frac{-5}{10} \Rightarrow k = 6.$$

19. Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Hence, the given system has no solution.

20. The system will be inconsistent when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

$\therefore \quad \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = \frac{2}{5}$.

21. Put $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ to get $3u + 2v = 2$ and $9u - 4v = 1$.

This gives $u = \frac{1}{3}$ and $v = \frac{1}{2}$. So, $x + y = 3$ and $x - y = 2$.
Solve for x and y.

....

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. If $2x + 3y = 12$ and $3x - 2y = 5$ then
   (a) $x = 2, y = 3$ (b) $x = 2, y = -3$ (c) $x = 3, y = 2$ (d) $x = 3, y = -2$
2. If $x - y = 2$ and $\frac{2}{x+y} = \frac{1}{5}$ then
   (a) $x = 4, y = 2$ (b) $x = 5, y = 3$ (c) $x = 6, y = 4$ (d) $x = 7, y = 5$
3. If $\frac{2x}{3} - \frac{y}{2} + \frac{1}{6} = 0$ and $\frac{x}{2} + \frac{2y}{3} = 3$ then
   (a) $x = 2, y = 3$ (b) $x = -2, y = 3$ (c) $x = 2, y = -3$ (d) $x = -2, y = -3$
4. If $\frac{1}{x} + \frac{2}{y} = 4$ and $\frac{3}{y} - \frac{1}{x} = 11$ then
   (a) $x = 2, y = 3$ (b) $x = -2, y = 3$ (c) $x = \frac{-1}{2}, y = 3$ (d) $x = \frac{-1}{2}, y = \frac{1}{3}$
5. If $\frac{2x+y+2}{5} = \frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$ then
   (a) $x = 1, y = 1$ (b) $x = -1, y = -1$ (c) $x = 1, y = 2$ (d) $x = 2, y = 1$
6. If $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$ then
   (a) $x = \frac{1}{2}, y = \frac{3}{2}$ (b) $x = \frac{5}{2}, y = \frac{1}{2}$ (c) $x = \frac{3}{2}, y = \frac{1}{2}$ (d) $x = \frac{1}{2}, y = \frac{5}{2}$
7. If $4x + 6y = 3xy$ and $8x + 9y = 5xy$ then
   (a) $x = 2, y = 3$ (b) $x = 1, y = 2$ (c) $x = 3, y = 4$ (d) $x = 1, y = -1$
8. If $29x + 37y = 103$ and $37x + 29y = 95$ then
   (a) $x = 1, y = 2$ (b) $x = 2, y = 1$ (c) $x = 3, y = 2$ (d) $x = 2, y = 3$
9. If $2^{x+y} = 2^{x-y} = \sqrt{8}$ then the value of y is
   (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) none of these

10. If $\frac{2}{x}+\frac{3}{y}=6$ and $\frac{1}{x}+\frac{1}{2y}=2$ then
    (a) $x=1, y=\frac{2}{3}$ (b) $x=\frac{2}{3}, y=1$ (c) $x=1, y=\frac{3}{2}$ (d) $x=\frac{3}{2}, y=1$
11. The system $kx-y=2$ and $6x-2y=3$ has a unique solution only when
    (a) $k=0$ (b) $k\neq 0$ (c) $k=3$ (d) $k\neq 3$
12. The system $x-2y=3$ and $3x+ky=1$ has a unique solution only when
    (a) $k=-6$ (b) $k\neq -6$ (c) $k=0$ (d) $k\neq 0$
13. The system $x+2y=3$ and $5x+ky+7=0$ has no solution, when
    (a) $k=10$ (b) $k\neq 10$ (c) $k=\frac{-7}{3}$ (d) $k=-21$
14. If the lines given by $3x+2ky=2$ and $2x+5y+1=0$ are parallel then the value of k is
    (a) $\frac{-5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{2}$ (d) $\frac{15}{4}$
15. For what value of k do the equations $kx-2y=3$ and $3x+y=5$ represent two lines intersecting at a unique point?
    (a) $k=3$ (b) $k=-3$
    (c) $k=6$ (d) all real values except -6
16. The pair of equations $x+2y+5=0$ and $-3x-6y+1=0$ has
    (a) a unique solution (b) exactly two solutions
    (c) infinitely many solutions (d) no solution
17. The pair of equations $2x+3y=5$ and $4x+6y=15$ has
    (a) a unique solution (b) exactly two solutions
    (c) infinitely many solutions (d) no solution
18. If a pair of linear equations is consistent then their graph lines will be
    (a) parallel (b) always coincident
    (c) always intersecting (d) intersecting or coincident
19. If a pair of linear equations is inconsistent then their graph lines will be
    (a) parallel (b) always coincident
    (c) always intersecting (d) intersecting or coincident
20. In a $\triangle ABC$, $\angle C=3\angle B=2(\angle A+\angle B)$, then $\angle B=?$
    (a) 20° (b) 40° (c) 60° (d) 80°
21. In a cyclic quadrilateral $ABCD$, it is being given that $\angle A=(x+y+10)^\circ$, $\angle B=(y+20)^\circ$, $\angle C=(x+y-30)^\circ$ and $\angle D=(x+y)^\circ$. Then, $\angle B=?$
    (a) 70° (b) 80° (c) 100° (d) 110°

22. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. The number is
   (a) 96 (b) 69 (c) 87 (d) 78

23. In a given fraction, if 1 is subtracted from the numerator and 2 is added to the denominator, it becomes $\frac{1}{2}\cdot$ If 7 is subtracted from the numerator and 2 is subtracted from the denominator, it becomes $\frac{1}{3}\cdot$ The fraction is
   (a) $\frac{13}{24}$ (b) $\frac{15}{26}$ (c) $\frac{16}{27}$ (d) $\frac{16}{21}$

24. 5 years hence, the age of a man shall be 3 times the age of his son while 5 years earlier the age of the man was 7 times the age of his son. The present age of the man is
   (a) 45 years (b) 50 years (c) 47 years (d) 40 years

25. The graphs of the equations $6x-2y+9=0$ and $3x-y+12=0$ are two lines which are
   (a) coincident
   (b) parallel
   (c) intersecting exactly at one point
   (d) perpendicular to each other

26. The graphs of the equations $2x+3y-2=0$ and $x-2y-8=0$ are two lines which are
   (a) coincident
   (b) parallel
   (c) intersecting exactly at one point
   (d) perpendicular to each other

27. The graphs of the equations $5x-15y=8$ and $3x-9y=\frac{24}{5}$ are two lines which are
   (a) coincident
   (b) parallel
   (c) intersecting exactly at one point
   (d) perpendicular to each other

ANSWERS (MCQ)

1. (c) **2.** (c) **3.** (a) **4.** (d) **5.** (a) **6.** (b) **7.** (c) **8.** (a) **9.** (c)
10. (b) **11.** (d) **12.** (b) **13.** (a) **14.** (d) **15.** (d) **16.** (d) **17.** (d) **18.** (d)
19. (a) **20.** (b) **21.** (b) **22.** (d) **23.** (b) **24.** (d) **25.** (b) **26.** (c) **27.** (a)

HINTS TO SOME SELECTED QUESTIONS

3. $4x - 3y = -1$, $3x + 4y = 18$. Solve.

4. Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get $u + 2v = 4$ and $-u + 3v = 11$.

5. $\frac{2x + y + 2}{5} = \frac{3x - y + 1}{3} \Rightarrow 6x + 3y + 6 = 15x - 5y + 5 \Rightarrow 9x - 8y = 1.$ ... (i)

   $\frac{3x - y + 1}{3} = \frac{3x + 2y + 1}{6} \Rightarrow 18x - 6y + 6 = 9x + 6y + 3 \Rightarrow 9x - 12y = -3.$ ... (ii)

   Solve (i) and (ii) to get $x = 1$ and $y = 1$.

6. Put $\frac{1}{x + y} = u$ and $\frac{1}{x - y} = v$ to get $3u + 2v = 2$ and $9u - 4v = 1$.

   Solve for u and v to get $u = \frac{1}{3}$ and $v = \frac{1}{2}\cdot$

   $\therefore \quad x + y = 3$ and $x - y = 2$.

7. Divide throughout by xy and put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ to get

   $4v + 6u = 3$ ... (i) and $8v + 9u = 5.$ ... (ii)

   This gives $u = \frac{1}{3}$ and $v = \frac{1}{4}\cdot$ Hence, $x = 3$ and $y = 4$.

8. Adding (i) and (ii), we get $66(x + y) = 198 \Rightarrow x + y = 3$.

   Subtracting (ii) from (i), we get $8(y - x) = 8 \Rightarrow y - x = 1$.

9. $2^{x+y} = 2^{x-y} = 2^{3/2} \Rightarrow x + y = \frac{3}{2}$ and $x - y = \frac{3}{2}\cdot$ So, $y = 0$.

10. Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$. Then, $2u + 3v = 6$ ... (i) and $u + \frac{1}{2}v = 2 \Rightarrow 2u + v = 4.$ ... (ii)

11. For a unique solution, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\cdot$

    $\therefore \quad \frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq 3.$

12. For a unique solution, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\cdot$ So, $\frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6.$

13. For no solution, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\cdot$

    $\therefore \quad \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = 10$ and $k \neq \frac{-14}{3}\cdot$ Hence, $k = 10$.

14. For parallel lines, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\cdot$

    $\therefore \quad \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \Rightarrow k = \frac{15}{4}$ and $k \neq -5 \Rightarrow k = \frac{15}{4}\cdot$

15. For a unique intersecting point, we have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\cdot$

    $\therefore \quad \frac{k}{3} \neq \frac{-2}{1} \Rightarrow k \neq -6.$ Hence, correct answer is (d).

16. Here, $\frac{a_1}{a_2} = \frac{1}{-3} = \frac{-1}{3}$, $\frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3}$ and $\frac{c_1}{c_2} = \frac{5}{1}\cdot$

    $\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\cdot$ So, the given system has no solution.

17. Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. So, the given system has no solution.

18. If a pair of linear equations is consistent then their graph lines will be intersecting or coincident.

20. Let $C = 3B = 2(A+B) = x°$.

Then, $C = x°$, $B = \left(\frac{x}{3}\right)^{\circ}$ and $(A+B) = \left(\frac{x}{2}\right)^{\circ}$.

$(A+B) + C = 180° \Rightarrow \frac{x}{2} + x = 180 \Rightarrow 3x = 360 \Rightarrow x = 120$.

$\therefore \quad \angle B = \left(\frac{120}{3}\right)^{\circ} = 40°$.

21. $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

This gives $x + y = 100$ and $x + 2y = 160$.

22. Let the ten's digit be x and the unit's digit be y. Then,

$x + y = 15 \quad \ldots \text{(i)} \quad$ and $(10y + x) - (10x + y) = 9 \Rightarrow y - x = 1. \quad \ldots \text{(ii)}$

23. Let the required fraction be $\frac{x}{y}$. Then,

$\frac{x-1}{y+2} = \frac{1}{2} \Rightarrow 2x - 2 = y + 2 \Rightarrow 2x - y = 4 \quad \ldots \text{(i)}$

$\frac{x-7}{y-2} = \frac{1}{3} \Rightarrow 3x - 21 = y - 2 \Rightarrow 3x - y = 19. \quad \ldots \text{(ii)}$

24. Let the present ages of the man and his son be x years and y years respectively. Then,

$(x+5) = 3(y+5) \Rightarrow x - 3y = 10 \quad \ldots \text{(i)}$

$(x-5) = 7(y-5) \Rightarrow x - 7y = -30. \quad \ldots \text{(ii)}$

25. $\frac{a_1}{a_2} = \frac{6}{3} = \frac{2}{1}$, $\frac{b_1}{b_2} = \frac{-2}{-1} = \frac{2}{1}$ and $\frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

So, the system is inconsistent and hence the lines are parallel.

26. $\frac{a_1}{a_2} = \frac{2}{1}$, $\frac{b_1}{b_2} = \frac{3}{-2}$. So, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Thus, the system has a unique solution and therefore the lines intersect exactly at one point.

27. The equations are $5x - 15y - 8 = 0$ and $15x - 45y - 24 = 0$.

$\therefore \quad \frac{a_1}{a_2} = \frac{5}{15} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-15}{-45} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and therefore the system has infinitely many solutions.

Hence, the graph lines are coincident.

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TEST YOURSELF

MCQ

1. The graphic representation of the equations $x + 2y = 3$ and $2x + 4y + 7 = 0$ gives a pair of
   (a) parallel lines (b) intersecting lines
   (c) coincident lines (d) none of these
2. If $2x - 3y = 7$ and $(a + b)x - (a + b - 3)y = 4a + b$ have an infinite number of solutions then
   (a) $a = 5, b = 1$ (b) $a = -5, b = 1$
   (c) $a = 5, b = -1$ (d) $a = -5, b = -1$
3. The pair of equations $2x + y = 5$, $3x + 2y = 8$ has
   (a) a unique solution (b) two solutions
   (c) no solution (d) infinitely many solutions
4. If $x = -y$ and $y > 0$, which of the following is wrong?
   (a) $x^2y > 0$ (b) $x + y = 0$ (c) $xy < 0$ (d) $\frac{1}{x} - \frac{1}{y} = 0$

Short-Answer Questions

5. Show that the system of equations $-x + 2y + 2 = 0$ and $\frac{1}{2}x - \frac{1}{2}y - 1 = 0$ has a unique solution.
6. For what values of k is the system of equations $kx + 3y = k - 2$, $12x + ky = k$ inconsistent?
7. Show that the equations $9x - 10y = 21$, $\frac{3x}{2} - \frac{5y}{3} = \frac{7}{2}$ have infinitely many solutions.
8. Solve the system of equations $x - 2y = 0$, $3x + 4y = 20$.
9. Show that the paths represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$ are parallel.
10. The difference between two numbers is 26 and one number is three times the other. Find the numbers.
11. Solve: $23x + 29y = 98$, $29x + 23y = 110$.
12. Solve: $6x + 3y = 7xy$ and $3x + 9y = 11xy$.
13. Find the value of k for which the system of equations $3x + y = 1$ and $kx + 2y = 5$ has (i) a unique solution, (ii) no solution.
14. In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the measure of each one of $\angle A$, $\angle B$ and $\angle C$.

15. 5 pencils and 7 pens together cost ₹ 195 while 7 pencils and 5 pens together cost ₹ 153. Find the cost of each one of the pencil and the pen.

16. Solve the following system of equations graphically:

$$2x - 3y = 1,\ 4x - 3y + 1 = 0.$$

Long-Answer Questions

17. Find the angles of a cyclic quadrilateral $ABCD$ in which $\angle A = (4x + 20)^\circ$, $\angle B = (3x - 5)^\circ$, $\angle C = (4y)^\circ$ and $\angle D = (7y + 5)^\circ$.

18. Solve for x and y: $\frac{35}{x+y} + \frac{14}{x-y} = 19,\ \frac{14}{x+y} + \frac{35}{x-y} = 37.$

19. If 1 is added to both the numerator and the denominator of a fraction, it becomes $\frac{4}{5}\cdot$ If, however, 5 is subtracted from both the numerator and the denominator, the fraction becomes $\frac{1}{2}\cdot$ Find the fraction.

20. Solve: $\frac{ax}{b} - \frac{by}{a} = a + b,\ ax - by = 2ab.$

ANSWERS (TEST YOURSELF)

1. (a) **2.** (d) **3.** (a) **4.** (d) **6.** $k = \pm 6$ **8.** $x = 4, y = 2$

10. 39, 13 **11.** $x = 3, y = 1$ **12.** $x = 1, y = \frac{3}{2}$ **13.** (i) $k \neq 6$ (ii) $k = 6$

14. $\angle A = 20^\circ, \angle B = 40^\circ, \angle C = 120^\circ$ **15.** ₹ 4 per pencil, ₹ 25 per pen

16. $x = -1, y = -1$ **17.** $\angle A = 120^\circ, \angle B = 70^\circ, \angle C = 60^\circ, \angle D = 110^\circ$

18. $x = 4, y = 3$ **19.** $\frac{7}{9}$ **20.** $x = b, y = -a$

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