4 Quadratic Equations

HISTORY

Babylonians were the first to solve the quadratic equations of the form $x^2 - px + q = 0$.

Brahmagupta (AD 598–665), an Indian mathematician gave an explicit formula to solve a quadratic equation of the form $ax^2 - bx = c$.

An Arab mathematician, Al-Khwarizmi in AD 800 also studied quadratic equations of various types.

An ancient Indian mathematician ***Shridharacharya*** derived the well-known ***Quadratic Formula*** for solving a quadratic equation $\mathbf{ax^2 + bx + c = 0}$ by the method of completing the square. This is being used as the standard formula for solving such an equation.

According to this formula, the roots of $ax^2 + bx + c = 0$ are given by $\alpha = \dfrac{-b + \sqrt{D}}{2a}$ and $\beta = \dfrac{-b - \sqrt{D}}{2a}$, where $D = (b^2 - 4ac)$ is called the ***discriminant*** of this equation.α

QUADRATIC EQUATION *A quadratic equation in the variable x is an equation of the form* $ax^2 + bx + c = 0$, *where a, b, c are real numbers and* $a \neq 0$.

ROOTS OF A QUADRATIC EQUATION *A real number* α *is called a root of the quadratic equation* $ax^2 + bx + c = 0$, $a \neq 0$ *if* $a\alpha^2 + b\alpha + c = 0$.

NOTE 1. If α is a root of $ax^2 + bx + c = 0$ then we say that

(i) $x = \alpha$ *satisfies the equation* $ax^2 + bx + c = 0$

or (ii) $x = \alpha$ *is a solution of the equation* $ax^2 + bx + c = 0$.

NOTE 2. The roots of a quadratic equation $ax^2 + bx + c = 0$ are called the ***zeros*** of the polynomial $ax^2 + bx + c$.

SOLVING A QUADRATIC EQUATION *Solving a quadratic equation means finding its roots.*

SOLVED EXAMPLES

EXAMPLE 1 *Which of the following are quadratic equations?*

(i) $x^2 - 5x + 3 = 0$ *(ii)* $2x^2 - 3\sqrt{2}x + 6 = 0$

(iii) $3x^2 - 2\sqrt{x} + 8 = 0$ *(iv)* $2x^2 - 3 = 0$

(v) $x + \frac{1}{x} = x^2$ *(vi)* $x^2 + \frac{1}{x^2} = 4\frac{1}{4}$

SOLUTION (i) Clearly, $(x^2 - 5x + 3)$ is a quadratic polynomial.

$\therefore$ $x^2 - 5x + 3 = 0$ is a quadratic equation.

(ii) Clearly, $(2x^2 - 3\sqrt{2}x + 6)$ is a quadratic polynomial.

$\therefore$ $2x^2 - 3\sqrt{2}x + 6 = 0$ is a quadratic equation.

(iii) $3x^2 - 2\sqrt{x} + 8$ is not of the form $ax^2 + bx + c = 0$.

$\therefore$ $3x^2 - 2\sqrt{x} + 8 = 0$ is not a quadratic equation.

(iv) $2x^2 - 3 = 0$ is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = 0$ and $c = -3$.

$\therefore$ $2x^2 - 3 = 0$ is a quadratic equation.

(v) $x + \frac{1}{x} = x^2 \Rightarrow x^2 + 1 = x^3 \Rightarrow x^3 - x^2 - 1 = 0.$

And, $(x^3 - x^2 - 1)$ being a polynomial of degree 3, it is not quadratic.

Hence, $x + \frac{1}{x} = x^2$ is not a quadratic equation.

(vi) $x^2 + \frac{1}{x^2} = \frac{17}{4} \Rightarrow 4x^4 + 4 = 17x^2 \Rightarrow 4x^4 - 17x^2 + 4 = 0.$

Clearly, $4x^4 - 17x^2 + 4$ is a polynomial of degree 4.

$\therefore$ $x^2 + \frac{1}{x^2} = \frac{17}{4}$ is not a quadratic equation.

EXAMPLE 2 *Check whether the following are quadratic equations:*

(i) $(2x - 1)(x - 3) = (x + 4)(x - 2)$ *(ii)* $(x + 2)^3 = 2x(x^2 - 1)$

(iii) $(x + 1)^3 = x^3 + x + 6$ *(iv)* $x(x + 3) + 6 = (x + 2)(x - 2)$

SOLUTION We have

(i) $(2x - 1)(x - 3) = (x + 4)(x - 2)$

$\Rightarrow$ $2x^2 - 7x + 3 = x^2 + 2x - 8 \Rightarrow x^2 - 9x + 11 = 0.$

This is of the form $ax^2 + bx + c = 0$, where $a = 1$, $b = -9$ and $c = 11$.

Hence, the given equation is a quadratic equation.

(ii) $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow \quad x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow \quad x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow \quad x^3 - 6x^2 - 14x - 8 = 0.$

This is not of the form $ax^2 + bx + c = 0.$

Hence, the given equation is not a quadratic equation.

(iii) $(x+1)^3 = x^3 + x + 6$

$\Rightarrow \quad x^3 + 1 + 3x(x+1) = x^3 + x + 6$

$\Rightarrow \quad 3x^2 + 2x - 5 = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = 3, b = 2$ and $c = -5.$

Hence, the given equation is a quadratic equation.

(iv) $x(x+3) + 6 = (x+2)(x-2)$

$\Rightarrow \quad x^2 + 3x + 6 = x^2 - 4$

$\Rightarrow \quad 3x + 10 = 0.$

This is not of the form $ax^2 + bx + c = 0.$

Hence, the given equation is not a quadratic equation.

EXAMPLE 3 *For the quadratic equation* $2x^2 - 5x - 3 = 0,$ *show that*

(i) $x = 3$ *is its solution.* *(ii)* $x = \frac{-1}{2}$ *is its solution.*

(iii) $x = 4$ *is not its solution.*

SOLUTION The given equation is $2x^2 - 5x - 3 = 0.$

(i) On substituting $x = 3$ in the given equation, we get

$$\text{LHS} = 2 \times 3^2 - 5 \times 3 - 3 = (18 - 15 - 3) = 0 = \text{RHS}.$$

$\therefore \quad x = 3$ is a solution of $2x^2 - 5x - 3 = 0.$

(ii) On substituting $x = \frac{-1}{2}$ in the given equation, we get

$$\text{LHS} = 2 \times \left(\frac{-1}{2}\right)^2 - 5 \times \left(\frac{-1}{2}\right) - 3$$

$$= \left\{2 \times \frac{1}{4} + 5 \times \frac{1}{2} - 3\right\}$$

$$= \left\{\frac{1}{2} + \frac{5}{2} - 3\right\} = 0 = \text{RHS}.$$

$\therefore \quad x = \frac{-1}{2}$ is a solution of $2x^2 - 5x - 3 = 0.$

(iii) On substituting $x = 4$ in the given equation, we get

$$\text{LHS} = 2 \times 4^2 - 5 \times 4 - 3 = (32 - 20 - 3) = 9 \neq 0.$$

Thus, LHS $\neq$ RHS.

$\therefore \quad x = 4$ is not a solution of $2x^2 - 5x - 3 = 0$.

EXAMPLE 4 *Show that* $\sqrt{2}$ *and* $-2\sqrt{2}$ *are the roots of the equation* $x^2 + \sqrt{2}x - 4 = 0$.

SOLUTION The given equation is $x^2 + \sqrt{2}x - 4 = 0$.

Putting $x = \sqrt{2}$ in the given equation, we get

$$\text{LHS} = (\sqrt{2})^2 + (\sqrt{2} \times \sqrt{2}) - 4 = (2 + 2 - 4) = 0 = \text{RHS}.$$

$\therefore \quad \sqrt{2}$ is a root of the given equation.

Putting $x = -2\sqrt{2}$ in the given equation, we get

$$\text{LHS} = (-2\sqrt{2})^2 + \sqrt{2} \times (-2\sqrt{2}) - 4 = (8 - 4 - 4) = 0 = \text{RHS}.$$

$\therefore \quad -2\sqrt{2}$ is a root of the given equation.

Hence, $\sqrt{2}$ and $-2\sqrt{2}$ are the roots of the given equation.

SOLVING A QUADRATIC EQUATION BY FACTORISATION

Let the given quadratic equation be $ax^2 + bx + c = 0$, where $a \neq 0$. Let $(ax^2 + bx + c)$ be expressible as the product of two linear expressions, say $(px + q)$ and $(rx + s)$, where p, q, r, s are real numbers such that $p \neq 0$ and $r \neq 0$. Then,

$$ax^2 + bx + c = 0 \Rightarrow (px + q)(rx + s) = 0$$
$$\Rightarrow (px + q) = 0 \quad \text{or} \quad (rx + s) = 0$$
$$\Rightarrow x = -\frac{q}{p} \quad \text{or} \quad x = -\frac{s}{r}.$$

EXAMPLE 5 *Solve:* $(x + 2)(3x - 5) = 0$.

SOLUTION We have

$$(x + 2)(3x - 5) = 0 \Rightarrow x + 2 = 0 \text{ or } 3x - 5 = 0$$
$$\Rightarrow x = -2 \quad \text{or} \quad x = \frac{5}{3}.$$

Hence, the roots of the given equation are –2 and $\frac{5}{3}$.

EXAMPLE 6 *Solve:* $5x^2 - 8x = 0$.

SOLUTION We have

$$5x^2 - 8x = 0 \Rightarrow x(5x - 8) = 0 \Rightarrow x = 0 \text{ or } 5x - 8 = 0$$
$$\Rightarrow x = 0 \text{ or } x = \frac{8}{5}.$$

Hence, 0 and $\frac{8}{5}$ are the roots of the given equation.

REMARK In Class IX, we have learnt how to factorise a quadratic polynomial by splitting the middle term. We shall use it for finding the roots of a quadratic equation.

EXAMPLE 7 *Solve:* $6x^2 - x - 2 = 0$ *by the factorisation method.*

SOLUTION We write, $-x = -4x + 3x$ as $(-4x) \times 3x = -12x^2 = 6x^2 \times (-2)$.

$\therefore \quad 6x^2 - x - 2 = 0$

$\Rightarrow \quad 6x^2 - 4x + 3x - 2 = 0 \Rightarrow 2x(3x-2) + (3x-2) = 0$

$\Rightarrow \quad (3x-2)(2x+1) = 0 \Rightarrow 3x - 2 = 0$ or $2x + 1 = 0$

$\Rightarrow \quad x = \frac{2}{3}$ or $x = \frac{-1}{2}$.

Hence, $\frac{2}{3}$ and $\frac{-1}{2}$ are the roots of the given equation.

EXAMPLE 8 *Solve:* $8x^2 - 22x - 21 = 0$ *by the factorisation method.*

SOLUTION We write,

$-22x = -28x + 6x$ as $8x^2 \times (-21) = -168x^2 = (-28x) \times 6x$.

$\therefore \quad 8x^2 - 22x - 21 = 0$

$\Rightarrow \quad 8x^2 - 28x + 6x - 21 = 0 \Rightarrow 4x(2x-7) + 3(2x-7) = 0$

$\Rightarrow \quad (2x-7)(4x+3) = 0 \Rightarrow 2x - 7 = 0$ or $4x + 3 = 0$

$\Rightarrow \quad x = \frac{7}{2}$ or $x = \frac{-3}{4}$.

Hence, $\frac{7}{2}$ and $\frac{-3}{4}$ are the roots of the given equation.

EXAMPLE 9 *Solve:* $6x^2 + 40 = 31x$.

SOLUTION The given equation may be written as $6x^2 - 31x + 40 = 0$.

We write,

$-31x = -16x - 15x$ as $6x^2 \times 40 = 240x^2 = (-16x) \times (-15x)$.

$\therefore \quad 6x^2 - 31x + 40 = 0$

$\Rightarrow \quad 6x^2 - 16x - 15x + 40 = 0 \Rightarrow 2x(3x-8) - 5(3x-8) = 0$

$\Rightarrow \quad (3x-8)(2x-5) = 0 \Rightarrow 3x - 8 = 0$ or $2x - 5 = 0$

$\Rightarrow \quad x = \frac{8}{3}$ or $x = \frac{5}{2}$.

Hence, $\frac{8}{3}$ and $\frac{5}{2}$ are the roots of the given equation.

EXAMPLE 10 *Solve:* $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0.$ [CBSE 2013]

SOLUTION Here, $4\sqrt{3} \times (-2\sqrt{3}) = -24$, $8 \times (-3) = -24$ and $8 + (-3) = 5$.

$\therefore \quad 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0 \Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow \quad (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0 \Rightarrow \sqrt{3}x + 2 = 0$ or $4x - \sqrt{3} = 0$

$\Rightarrow \quad x = \frac{-2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$

$\Rightarrow \quad x = \left(\frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-2\sqrt{3}}{3}$ or $x = \frac{\sqrt{3}}{4}\cdot$

EXAMPLE 11 *Solve:* $4x^2 - 12x + 9 = 0.$

SOLUTION Here, $4 \times 9 = 36$, $(-6) \times (-6) = 36$, and $(-6) + (-6) = -12$.

$\therefore \quad 4x^2 - 12x + 9 = 0$

$\Rightarrow \quad 4x^2 - 6x - 6x + 9 = 0 \Rightarrow 2x(2x - 3) - 3(2x - 3) = 0$

$\Rightarrow \quad (2x - 3)(2x - 3) = 0 \Rightarrow (2x - 3)^2 = 0$

$\Rightarrow \quad 2x - 3 = 0 \Rightarrow x = \frac{3}{2}\cdot$

Hence, $x = \frac{3}{2}$ is the repeated root of the given equation.

EXAMPLE 12 *Solve the following equation by using factorisation method:*

$4x^2 - 4ax + (a^2 - b^2) = 0.$ [CBSE 2012, '15]

SOLUTION We may write, $-4a = \{-2(a + b)\} + \{-2(a - b)\}$.

Also, $\{-2(a + b)\} \times \{-2(a - b)\} = 4(a^2 - b^2)$.

$\therefore \quad 4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow \quad 4x^2 - 2(a + b)x - 2(a - b)x + (a^2 - b^2) = 0$

$\Rightarrow \quad 2x\{2x - (a + b)\} - (a - b)\{2x - (a + b)\} = 0$

$\Rightarrow \quad \{2x - (a + b)\} \times \{2x - (a - b)\} = 0$

$\Rightarrow \quad 2x - (a + b) = 0$ or $2x - (a - b) = 0$

$\Rightarrow \quad x = \frac{(a + b)}{2}$ or $x = \frac{(a - b)}{2}\cdot$

Hence, $\frac{(a + b)}{2}$ and $\frac{(a - b)}{2}$ are the roots of the given equation.

EXAMPLE 13 *Solve the following equation by using factorisation method:*

$9x^2 - 6b^2x - (a^4 - b^4) = 0.$ [CBSE 2015]

SOLUTION We may write, $-6b^2 = 3(a^2 - b^2) - 3(a^2 + b^2)$.

Also, $\{3(a^2-b^2)\}\times\{-3(a^2+b^2)\} = -9(a^4-b^4)$

$\therefore \quad 9x^2-6b^2x-(a^4-b^4)=0$

$\Rightarrow \quad 9x^2+3(a^2-b^2)x-3(a^2+b^2)x-(a^4-b^4)=0$

$\Rightarrow \quad 3x\{3x+(a^2-b^2)\}-(a^2+b^2)\{3x+(a^2-b^2)\}=0$

$\Rightarrow \quad \{3x+(a^2-b^2)\}\times\{3x-(a^2+b^2)\}=0$

$\Rightarrow \quad 3x+(a^2-b^2)=0 \text{ or } 3x-(a^2+b^2)=0$

$\Rightarrow \quad x=\dfrac{(b^2-a^2)}{3} \text{ or } x=\dfrac{(a^2+b^2)}{3}.$

Hence, $\dfrac{(b^2-a^2)}{3}$ and $\dfrac{(a^2+b^2)}{3}$ are the required roots of the given equation.

EXAMPLE 14 *Solve:* $\dfrac{x+3}{x+2}=\dfrac{3x-7}{2x-3}$.

SOLUTION By cross multiplication, we get

$(x+3)(2x-3)=(x+2)(3x-7)$

$\Rightarrow \quad 2x^2+3x-9=3x^2-x-14 \Rightarrow x^2-4x-5=0$

$\Rightarrow \quad x^2-5x+x-5=0 \Rightarrow x(x-5)+(x-5)=0$

$\Rightarrow \quad (x-5)(x+1)=0 \Rightarrow x-5=0 \text{ or } x+1=0$

$\Rightarrow \quad x=5 \text{ or } x=-1.$

Hence, 5 and –1 are the roots of the given equation.

EXAMPLE 15 *Solve:* $\dfrac{14}{x+3}-1=\dfrac{5}{x+1},\ x\neq -3, -1.$ [CBSE 2014]

SOLUTION The given equation may be written as

$\dfrac{14}{x+3}-\dfrac{5}{x+1}=1$

$\Rightarrow \quad \dfrac{14(x+1)-5(x+3)}{(x+3)(x+1)}=1$

$\Rightarrow \quad \dfrac{(9x-1)}{x^2+4x+3}=1 \Rightarrow x^2+4x+3=9x-1$

$\Rightarrow \quad x^2-5x+4=0 \Rightarrow x^2-4x-x+4=0$

$\Rightarrow \quad x(x-4)-(x-4)=0 \Rightarrow (x-4)(x-1)=0$

$\Rightarrow \quad x-4=0 \text{ or } x-1=0$

$\Rightarrow \quad x=4 \text{ or } x=1.$

Hence, 4 and 1 are the roots of the given equation.

EXAMPLE 16 *Solve:* $\frac{1}{(x+4)} - \frac{1}{(x-7)} = \frac{11}{30}, x \neq -4, 7.$ [CBSE 2008, '10]

SOLUTION We have

$$\frac{1}{(x+4)} - \frac{1}{(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{-11}{(x^2-3x-28)} = \frac{11}{30}$$

$$\Rightarrow \frac{-1}{(x^2-3x-28)} = \frac{1}{30} \quad \text{[on dividing both sides by 11]}$$

$$\Rightarrow (x^2-3x-28) = -30 \quad \text{[by cross multiplication]}$$

$$\Rightarrow x^2-3x+2=0 \Rightarrow x^2-2x-x+2=0$$

$$\Rightarrow x(x-2)-(x-2)=0 \Rightarrow (x-2)(x-1)=0$$

$$\Rightarrow x-2=0 \text{ or } x-1=0$$

$$\Rightarrow x=2 \text{ or } x=1.$$

Hence, 2 and 1 are the roots of the given equation.

EXAMPLE 17 *Solve:* $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, [x \neq 0, x \neq -(a+b)].$ [CBSE 2013]

SOLUTION We have

$$\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{1}{(a+b+x)} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{b+a}{ab}$$

$$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{(a+b)}{ab}$$

$$\Rightarrow \frac{-1}{x(a+b+x)} = \frac{1}{ab} \quad \text{[on dividing both sides by } (a+b)\text{]}$$

$$\Rightarrow x(a+b+x) = -ab \quad \text{[by cross multiplication]}$$

$$\Rightarrow x^2+ax+bx+ab=0 \Rightarrow x(x+a)+b(x+a)=0$$

$$\Rightarrow (x+a)(x+b)=0 \Rightarrow x+a=0 \text{ or } x+b=0$$

$$\Rightarrow x=-a \text{ or } x=-b.$$

Hence, $-a$ and $-b$ are the roots of the given equation.

EXAMPLE 18 *Solve:* $\frac{x-2}{x-3} + \frac{x-4}{x-5} = 3\frac{1}{3}, x \neq 3, 5.$ [CBSE 2014]

SOLUTION We have

$$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$$

$$\Rightarrow \frac{(x-2)(x-5)+(x-4)(x-3)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(x^2-7x+10)+(x^2-7x+12)}{(x^2-8x+15)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(2x^2-14x+22)}{(x^2-8x+15)} = \frac{10}{3}$$

$$\Rightarrow \quad 3(2x^2-14x+22) = 10(x^2-8x+15)$$

[by cross multiplication]

$$\Rightarrow \quad 6x^2-42x+66 = 10x^2-80x+150$$

$$\Rightarrow \quad 4x^2-38x+84 = 0 \Rightarrow 2x^2-19x+42 = 0$$

$$\Rightarrow \quad 2x^2-12x-7x+42 = 0 \Rightarrow 2x(x-6)-7(x-6) = 0$$

$$\Rightarrow \quad (x-6)(2x-7) = 0 \Rightarrow x-6 = 0 \text{ or } 2x-7 = 0$$

$$x = 6 \text{ or } x = \frac{7}{2}.$$

Hence, 6 and $\frac{7}{2}$ are the roots of the given equation.

EXAMPLE 19 *Solve:* $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right) = 5,\ x \neq -3, \frac{1}{2}.$ [CBSE 2014]

SOLUTION On putting $\frac{2x-1}{x+3} = y,$ the given equation becomes

$$2y-\frac{3}{y} = 5 \Rightarrow 2y^2-3 = 5y$$

$$\Rightarrow 2y^2-5y-3 = 0 \Rightarrow 2y^2-6y+y-3 = 0$$

$$\Rightarrow 2y(y-3)+(y-3) = 0 \Rightarrow (y-3)(2y+1) = 0$$

$$\Rightarrow y-3 = 0 \text{ or } 2y+1 = 0$$

$$\Rightarrow y = 3 \text{ or } y = \frac{-1}{2}.$$

Case I $y = 3 \Rightarrow \frac{2x-1}{x+3} = 3$

$$\Rightarrow 2x-1 = 3(x+3) \text{ [by cross multiplication]}$$

$$\Rightarrow 2x-1 = 3x+9$$

$$\Rightarrow x = -10.$$

Case II $y = \frac{-1}{2} \Rightarrow \frac{2x-1}{x+3} = \frac{-1}{2}$

$$\Rightarrow 2(2x-1) = -(x+3)$$

$$\Rightarrow 5x = -1$$

$$\Rightarrow x = \frac{-1}{5}.$$

Hence, –10 and $\frac{-1}{5}$ are the roots of the given equation.

EXAMPLE 20 *Solve:* $\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}$, $x \neq 0, -1, 2.$ [CBSE 2015]

SOLUTION We have

$$\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}$$

$$\Rightarrow \quad \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x} \Rightarrow \frac{7x-5}{2(x^2-x-2)} = \frac{23}{5x}$$

$\Rightarrow \quad 5x(7x-5) = 46(x^2-x-2)$ [by cross multiplication]

$$\Rightarrow \quad 35x^2 - 25x = 46x^2 - 46x - 92$$

$$\Rightarrow \quad 11x^2 - 21x - 92 = 0 \Rightarrow 11x^2 - 44x + 23x - 92 = 0$$

$$\Rightarrow \quad 11x(x-4) + 23(x-4) = 0 \Rightarrow (x-4)(11x+23) = 0$$

$$\Rightarrow \quad x - 4 = 0 \text{ or } 11x + 23 = 0$$

$$\Rightarrow \quad x = 4 \text{ or } x = \frac{-23}{11}.$$

Hence, 4 and $\frac{-23}{11}$ are the roots of the given equation.

EXAMPLE 21 *Solve:* $\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}$, $x \neq -3, \frac{1}{2}, \frac{-9}{7}.$ [CBSE 2009C[

SOLUTION We have

$$\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}$$

$$\Rightarrow \quad \frac{(2x-1)+(x+3)}{(x+3)(2x-1)} = \frac{11}{(7x+9)} \Rightarrow \frac{(3x+2)}{2x^2+5x-3} = \frac{11}{(7x+9)}$$

$\Rightarrow \quad (3x+2)(7x+9) = 11(2x^2+5x-3)$ [by cross multiplication]

$$\Rightarrow \quad 21x^2 + 41x + 18 = 22x^2 + 55x - 33$$

$$\Rightarrow \quad x^2 + 14x - 51 = 0 \Rightarrow x^2 + 17x - 3x - 51 = 0$$

$$\Rightarrow \quad x(x+17) - 3(x+17) = 0 \Rightarrow (x+17)(x-3) = 0$$

$$\Rightarrow \quad x + 17 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow \quad x = -17 \text{ or } x = 3.$$

Hence, –17 and 3 are the roots of the given equation.

EXAMPLE 22 *Solve:* $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1.$

SOLUTION We have

$$5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$$

$$\Rightarrow \quad 5^x \cdot 5 + 5^2 \cdot 5^{-x} = 126 \Rightarrow 5^x \cdot 5 + \frac{25}{5^x} = 126$$

$$\Rightarrow \quad 5y + \frac{25}{y} = 126, \text{ where } 5^x = y$$

$$\Rightarrow \quad 5y^2 - 126y + 25 = 0 \Rightarrow 5y^2 - 125y - y + 25 = 0$$

$\Rightarrow \quad 5y(y-25)-(y-25)=0 \Rightarrow (y-25)(5y-1)=0$

$\Rightarrow \quad y-25=0 \text{ or } 5y-1=0$

$\Rightarrow \quad y=25 \text{ or } y=\frac{1}{5}$

$\Rightarrow \quad 5^x=25=5^2 \text{ or } 5^x=5^{-1}$

$\Rightarrow \quad x=2 \text{ or } x=-1.$

Hence, 2 and –1 are the roots of the given equation.

EXERCISE 4A

1. Which of the following are quadratic equations in x?

(i) $x^2-x+3=0$

(ii) $2x^2+\frac{5}{2}x-\sqrt{3}=0$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

(iv) $\frac{1}{3}x^2+\frac{1}{5}x-2=0$

(v) $x^2-3x-\sqrt{x}+4=0$

(vi) $x-\frac{6}{x}=3$

(vii) $x+\frac{2}{x}=x^2$

(viii) $x^2-\frac{1}{x^2}=5$

(ix) $(x+2)^3=x^3-8$

(x) $(2x+3)(3x+2)=6(x-1)(x-2)$

(xi) $\left(x+\frac{1}{x}\right)^2=2\left(x+\frac{1}{x}\right)+3$

2. Which of the following are the roots of $3x^2+2x-1=0$?

(i) –1 (ii) $\frac{1}{3}$ (iii) $-\frac{1}{2}$

3. (i) Find the value of k for which $x=1$ is a root of the equation $x^2+kx+3=0$. Also, find the other root.

(ii) Find the values of a and b for which $x=\frac{3}{4}$ and $x=-2$ are the roots of the equation $ax^2+bx-6=0$.

4. Show that $x=-\frac{bc}{ad}$ is a solution of the quadratic equation $ad^2\left(\frac{ax}{b}+\frac{2c}{d}\right)x+bc^2=0.$ [CBSE 2017]

Solve each of the following quadratic equations:

5. $(2x-3)(3x+1)=0$

6. $4x^2+5x=0$

7. $3x^2-243=0$

8. $2x^2+x-6=0$

9. $x^2+6x+5=0$

10. $9x^2-3x-2=0$

11. $x^2+12x+35=0$

12. $x^2=18x-77$

13. $6x^2 + 11x + 3 = 0$
14. $6x^2 + x - 12 = 0$
15. $3x^2 - 2x - 1 = 0$
16. $4x^2 - 9x = 100$
17. $15x^2 - 28 = x$
18. $4 - 11x = 3x^2$
19. $48x^2 - 13x - 1 = 0$
20. $x^2 + 2\sqrt{2}x - 6 = 0$
21. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$ [CBSE 2017]
22. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$
23. $3\sqrt{7}x^2 + 4x - \sqrt{7} = 0$
24. $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$
25. $4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0$
26. $3x^2 - 2\sqrt{6}x + 2 = 0$ [CBSE 2010, '12]
27. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ [CBSE 2011]
28. $x^2 - 3\sqrt{5}x + 10 = 0$ [CBSE 2011]
29. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ [CBSE 2015]
30. $x^2 + 3\sqrt{3}x - 30 = 0$ [CBSE 2015]
31. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ [CBSE 2013]
32. $5x^2 + 13x + 8 = 0$ [CBSE 2013C]
33. $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$
34. $9x^2 + 6x + 1 = 0$
35. $100x^2 - 20x + 1 = 0$
36. $2x^2 - x + \frac{1}{8} = 0$
37. $10x - \frac{1}{x} = 3$
38. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$
39. $2x^2 + ax - a^2 = 0$ [CBSE 2015]
40. $4x^2 + 4bx - (a^2 - b^2) = 0$ [CBSE 2015, '17]
41. $4x^2 - 4a^2x + (a^4 - b^4) = 0$ [CBSE 2015]
42. $x^2 + 5x - (a^2 + a - 6) = 0$ [CBSE 2015]
43. $x^2 - 2ax - (4b^2 - a^2) = 0$ [CBSE 2015]
44. $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$ [CBSE 2015]
45. $x^2 + 6x - (a^2 + 2a - 8) = 0$ [CBSE 2015]
46. $abx^2 + (b^2 - ac)x - bc = 0$ [CBSE 2014]
47. $x^2 - 4ax - b^2 + 4a^2 = 0$ [CBSE 2012]
48. $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$
49. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$ [CBSE 2006]
50. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$ [CBSE 2005]
51. $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$ [CBSE 2009]
52. $\frac{16}{x} - 1 = \frac{15}{x+1}, \ x \neq 0, -1$ [CBSE 2014]
53. $\frac{4}{x} - 3 = \frac{5}{2x+3}, \ x \neq 0, \frac{-3}{2}$ [CBSE 2014]
54. $\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}, \ x \neq -1, \frac{1}{3}$ [CBSE 2014]

55. (i) $\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7},\ x\neq 1,-5$

(ii) $\frac{1}{2x-3}+\frac{1}{x-5}=1\frac{1}{9}, x\neq \frac{3}{2}, 5$ [CBSE 2017]

56. $\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$ [CBSE 2013]

57. $\frac{x+3}{x-2}-\frac{1-x}{x}=4\frac{1}{4},\ x\neq 2, 0$

58. $\frac{3x-4}{7}+\frac{7}{3x-4}=\frac{5}{2},\ x\neq \frac{4}{3}$ [CBSE 2010]

59. (i) $\frac{x}{x-1}+\frac{x-1}{x}=4\frac{1}{4},\ x\neq 0, 1$

(ii) $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x\neq -\frac{1}{2}, 1$ [CBSE 2017]

60. $\frac{x}{x+1}+\frac{x+1}{x}=2\frac{4}{15},\ x\neq 0,-1$ [CBSE 2014]

61. $\frac{x-4}{x-5}+\frac{x-6}{x-7}=3\frac{1}{3},\ x\neq 5, 7$ [CBSE 2014]

62. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3},\ x\neq 2, 4$ [CBSE 2017]

63. $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x},\ x\neq 0, 1, 2$ [CBSE 2013]

64. (i) $\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4},\ x\neq -1,-2,-4$ [CBSE 2013C]

(ii) $\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x\neq -1, -\frac{1}{5}, -4$ [CBSE 2017]

65. $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5,\ x\neq \frac{1}{3}, \frac{-3}{2}$ [CBSE 2014]

66. $3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11,\ x\neq \frac{3}{5}, \frac{-1}{7}$ [CBSE 2014]

67. $\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3,\ x\neq \frac{-1}{2}, \frac{3}{4}$

68. $\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0,\ x\neq -1$

69. $\frac{a}{(x-b)}+\frac{b}{(x-a)}=2,\ x\neq b, a$

70. $\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=(a+b),\ x\neq \frac{1}{a}, \frac{1}{b}$

71. $3^{(x+2)}+3^{-x}=10$

72. $4^{(x+1)}+4^{(1-x)}=10$

73. $2^{2x}-3\cdot 2^{(x+2)}+32=0$

ANSWERS (EXERCISE 4A)

1. (i), (ii), (iii), (iv), (vi), (ix) **2.** -1 and $\frac{1}{3}$

3. (i) $k = -4$, other root = 3 (ii) $a = 4, b = 5$ **5.** $x = \frac{3}{2}$ or $x = \frac{-1}{3}$

6. $x = 0$ or $x = \frac{-5}{4}$ **7.** $x = 9$ or $x = -9$ **8.** $x = -2$ or $x = \frac{3}{2}$

9. $x = -5$ or $x = -1$ **10.** $x = \frac{2}{3}$ or $x = \frac{-1}{3}$ **11.** $x = -7$ or $x = -5$

12. $x = 11$ or $x = 7$ **13.** $x = \frac{-3}{2}$ or $x = \frac{-1}{3}$ **14.** $x = \frac{-3}{2}$ or $x = \frac{4}{3}$

15. $x = 1$ or $x = \frac{-1}{3}$ **16.** $x = \frac{25}{4}$ or $x = -4$ **17.** $x = \frac{-4}{3}$ or $x = \frac{7}{5}$

18. $x = -4$ or $x = \frac{1}{3}$ **19.** $x = \frac{1}{3}$ or $x = \frac{-1}{16}$ **20.** $x = \sqrt{2}$ or $x = -3\sqrt{2}$

21. $x = -4\sqrt{3}$ or $x = \frac{2}{\sqrt{3}}$ **22.** $x = -3\sqrt{3}$ or $x = \frac{-2}{\sqrt{3}}$ **23.** $x = \frac{-\sqrt{7}}{3}$ or $x = \frac{1}{\sqrt{7}}$

24. $x = \frac{13}{\sqrt{7}}$ or $x = -\sqrt{7}$ **25.** $x = \frac{2\sqrt{2}}{\sqrt{3}}$ or $x = \frac{-\sqrt{3}}{4\sqrt{2}}$ **26.** $x = \frac{\sqrt{2}}{\sqrt{3}}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$

27. $x = \sqrt{6}$ or $x = \frac{-\sqrt{2}}{\sqrt{3}}$ **28.** $x = \sqrt{5}$ or $x = 2\sqrt{5}$ **29.** $x = \sqrt{3}$ or $x = 1$

30. $x = 2\sqrt{3}$ or $x = -5\sqrt{3}$ **31.** $x = -\sqrt{2}$ or $x = \frac{-5}{\sqrt{2}}$ **32.** $x = -1$ or $x = \frac{-8}{5}$

33. $x = 1$ or $x = \sqrt{2}$ **34.** $x = \frac{-1}{3}, x = \frac{-1}{3}$ **35.** $x = \frac{1}{10}, x = \frac{1}{10}$

36. $x = \frac{1}{4}, x = \frac{1}{4}$ **37.** $x = \frac{1}{2}$ or $x = \frac{-1}{5}$ **38.** $x = 2$ or $x = \frac{1}{2}$

39. $x = -a$ or $x = \frac{a}{2}$ **40.** $x = \frac{-(a+b)}{2}$ or $x = \frac{a-b}{2}$

41. $x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$ **42.** $x = -(a+3)$ or $x = (a-2)$

43. $x = (a-2b)$ or $x = (a+2b)$ **44.** $x = (b-5)$ or $x = (b+4)$

45. $x = -(a+4)$ or $x = (a-2)$ **46.** $x = \frac{-b}{a}$ or $x = \frac{c}{b}$

47. $x = (2a+b)$ or $x = (2a-b)$ **48.** $x = \frac{a^2}{2}$ or $x = \frac{b^2}{2}$

49. $x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$ **50.** $x = \frac{-1}{a^2}$ or $x = \frac{1}{b^2}$

51. $x = \frac{(a+2b)}{3}$ or $x = \frac{(2a+b)}{3}$ **52.** $x = 4$ or $x = -4$ **53.** $x = -2$ or $x = 1$

54. $x = 3$ or $x = 1$ **55.** (i) $x = -6$ or $x = 2$ (ii) $x = \frac{37}{20}$ or $x = 6$

56. $x = -a$ or $x = \frac{-b}{2}$ **57.** $x = 4$ or $x = \frac{-2}{9}$ **58.** $x = 6$ or $x = 2\frac{1}{2}$

59. (i) $x = \frac{4}{3}$ or $x = \frac{-1}{3}$ (ii) $x = -2$ **60.** $x = \frac{-5}{2}$ or $x = \frac{3}{2}$

61. $x = 8$ or $x = 5\frac{1}{2}$ **62.** $x = 5$ or $x = \frac{5}{2}$ **63.** $x = 3$ or $x = \frac{4}{3}$

64. (i) $x = 2$ or $x = \frac{-3}{2}$ (ii) $x = -\frac{11}{17}$ or $x = 1$ **65.** $x = -7$ or $x = 0$

66. $x = 1$ or $x = 0$ **67.** $x = \frac{-4}{3}$ or $x = \frac{1}{8}$ **68.** $x = \frac{-3}{2}$ or $x = -2$

69. $x = (a+b)$ or $x = \frac{(a+b)}{2}$ **70.** $x = \frac{(a+b)}{ab}$ or $x = \frac{2}{(a+b)}$

71. $x = -2$ or $x = 0$ **72.** $x = \frac{1}{2}$ or $x = \frac{-1}{2}$ **73.** $x = 2$ or $x = 3$

HINTS TO SOME SELECTED QUESTIONS

16. $4x^2 - 9x - 100 = 0 \Rightarrow 4x^2 - 25x + 16x - 100 = 0.$

17. $15x^2 - x - 28 = 0 \Rightarrow 15x^2 - 21x + 20x - 28 = 0.$

18. $3x^2 + 11x - 4 = 0 \Rightarrow 3x^2 + 12x - x - 4 = 0.$

19. $48x^2 - 13x - 1 = 0 \Rightarrow 48x^2 - 16x + 3x - 1 = 0.$

20. $x^2 + 2\sqrt{2}x - 6 = 0 \Rightarrow x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0 \Rightarrow x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0.$

21. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0 \Rightarrow \sqrt{3}x^2 + 12x - 2x - 8\sqrt{3} = 0 \Rightarrow \sqrt{3}x(x + 4\sqrt{3}) - 2(x + 4\sqrt{3}) = 0.$

22. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0 \Rightarrow \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0 \Rightarrow \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0.$

23. $3\sqrt{7}x^2 + 4x - \sqrt{7} = 0 \Rightarrow 3\sqrt{7}x^2 + 7x - 3x - \sqrt{7} = 0 \Rightarrow \sqrt{7}x(3x + \sqrt{7}) - (3x + \sqrt{7}) = 0.$

24. $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0 \Rightarrow \sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$
$\Rightarrow x(\sqrt{7}x - 13) + \sqrt{7}(\sqrt{7}x - 13) = 0.$

25. $4\sqrt{6}x^2 - 13x - 2\sqrt{6} = 0 \Rightarrow 4\sqrt{6}x^2 - 16x + 3x - 2\sqrt{6} = 0$
$\Rightarrow 4\sqrt{2}x(\sqrt{3}x - 2\sqrt{2}) + \sqrt{3}(\sqrt{3}x - 2\sqrt{2}) = 0.$

26. $3x^2 - 2\sqrt{6}x + 2 = 0 \Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0 \Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0.$

27. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0 \Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$
$\Rightarrow \sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0.$

28. $x^2 - 3\sqrt{5}x + 10 = 0 \Rightarrow x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0 \Rightarrow x(x - 2\sqrt{5}) - \sqrt{5}(x - 2\sqrt{5}) = 0.$

29. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0 \Rightarrow x^2 - \sqrt{3}x - x + \sqrt{3} = 0 \Rightarrow x(x - \sqrt{3}) - (x - \sqrt{3}) = 0.$

30. $x^2 + 3\sqrt{3}x - 30 = 0 \Rightarrow x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0 \Rightarrow x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = 0.$

31. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0 \Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0.$

32. $5x^2 + 13x + 8 = 0 \Rightarrow 5x^2 + 5x + 8x + 8 = 0.$

34. $9x^2+6x+1=0 \Rightarrow (3x+1)^2=0 \Rightarrow 3x+1=0.$

35. $100x^2-20x+1=0 \Rightarrow (10x-1)^2=0 \Rightarrow 10x-1=0.$

36. $16x^2-8x+1=0 \Rightarrow (4x-1)^2=0 \Rightarrow 4x-1=0.$

37. $10x^2-3x-1=0 \Rightarrow 10x^2-5x+2x-1=0 \Rightarrow 5x(2x-1)+(2x-1)=0.$

38. $2x^2-5x+2=0 \Rightarrow 2x^2-4x-x+2=0 \Rightarrow 2x(x-2)-(x-2)=0.$

39. $2x^2+ax-a^2=0 \Rightarrow 2x^2+2ax-ax-a^2=0 \Rightarrow 2x(x+a)-a(x+a)=0.$

40. $4x^2+4bx+(b^2-a^2)=0 \Rightarrow 4x^2+2(b+a)x+2(b-a)x+(b^2-a^2)=0.$

41. $4x^2-4a^2x+(a^4-b^4)=0 \Rightarrow 4x^2-2(a^2+b^2)x-2(a^2-b^2)x+(a^4-b^4)=0$

$$\Rightarrow 2x[2x-(a^2+b^2)]-(a^2-b^2)[2x-(a^2+b^2)]=0.$$

42. $x^2+5x-(a+3)(a-2)=0 \Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0.$

43. $x^2-2ax+(a^2-4b^2)=0 \Rightarrow x^2-2ax+(a+2b)(a-2b)=0$

$$\Rightarrow x^2-(a+2b)x-(a-2b)x+(a+2b)(a-2b)=0.$$

44. $x^2-(2b-1)x+(b-5)(b+4)=0 \Rightarrow x^2-(b-5)x-(b+4)x+(b-5)(b+4)=0$

$$\Rightarrow x[x-(b-5)]-(b+4)[x-(b-5)]=0.$$

45. $x^2+6x-(a+4)(a-2)=0 \Rightarrow x^2+(a+4)x-(a-2)x-(a+4)(a-2)=0.$

46. $abx^2+b^2x-acx-bc=0 \Rightarrow bx(ax+b)-c(ax+b)=0.$

47. $x^2-4ax+(2a+b)(2a-b)=0 \Rightarrow x^2-(2a+b)x-(2a-b)x+(4a^2-b^2)=0.$

48. $4x^2-2a^2x-2b^2x+a^2b^2=0 \Rightarrow 2x(2x-a^2)-b^2(2x-a^2)=0.$

49. $12abx^2-9a^2x+8b^2x-6ab=0 \Rightarrow 3ax(4bx-3a)+2b(4bx-3a)=0.$

50. $b^2x(a^2x+1)-(a^2x+1)=0.$

51. $9x^2-9(a+b)x+(2a+b)(a+2b)=0$

$$\Rightarrow \quad 9x^2-3\{(2a+b)+(a+2b)\}x+(2a+b)(a+2b)=0$$

$$\Rightarrow \quad 9x^2-3(2a+b)x-3(a+2b)x+(2a+b)(a+2b)=0.$$

56. $\dfrac{1}{2a+b+2x}-\dfrac{1}{2x}=\dfrac{1}{2a}+\dfrac{1}{b}.$

57. $\dfrac{x(x+3)-(1-x)(x-2)}{x(x-2)}=\dfrac{17}{4} \Rightarrow \dfrac{(x^2+3x)(3x-x^2-2)}{(x^2-2x)}=\dfrac{17}{4}$

$$\Rightarrow \quad 4(2x^2+2)=17(x^2-2x) \Rightarrow 9x^2-34x-8=0 \Rightarrow 9x^2-36x+2x-8=0.$$

58. Put $\dfrac{3x-4}{7}=y$. Then, $y+\dfrac{1}{y}=\dfrac{5}{2} \Rightarrow 2y^2-5y+2=0.$

65. Putting $\dfrac{3x-1}{2x+3}=y$, the given equation becomes

$$3y-\frac{2}{y}=5 \Rightarrow 3y^2-5y-2=0 \Rightarrow 3y^2-6y+y-2=0.$$

69. The given equation is

$$\left(\frac{a}{x-b}-1\right)+\left(\frac{b}{x-a}-1\right)=0$$

$$\Rightarrow \quad \frac{(a-x+b)}{(x-b)}+\frac{(a-x+b)}{(x-a)}=0$$

$$\Rightarrow \quad (a-x+b)\cdot\left[\frac{1}{(x-b)}+\frac{1}{(x-a)}\right]=0$$

$$\Rightarrow \quad (a-x+b)\left[\frac{2x-(a+b)}{(x-a)(x-b)}\right]=0$$

$$\Rightarrow \quad (a-x+b)[2x-(a+b)]=0$$

$$\Rightarrow \quad x=(a+b) \quad \text{or} \quad x=\frac{(a+b)}{2}.$$

70. The given equation is

$$\left\{\frac{a}{(ax-1)}-b\right\}+\left\{\frac{b}{(bx-1)}-a\right\}=0$$

$$\Rightarrow \quad \frac{(a-abx+b)}{(ax-1)}+\frac{(a-abx+b)}{(bx-1)}=0$$

$$\Rightarrow \quad (a-abx+b)\cdot\left(\frac{1}{ax-1}+\frac{1}{bx-1}\right)=0.$$

71. $3^x\times 3^2+\frac{1}{3^x}=10 \Rightarrow 9y+\frac{1}{y}=10$, where $y=3^x$.

72. $4^x\times 4+\frac{4}{4^x}=10 \Rightarrow 2y+\frac{2}{y}=5$, where $y=4^x$.

73. $2^{2x}-3\times 2^2\times 2^x+32=0 \Rightarrow y^2-12y+32=0$, where $2^x=y$

$$\Rightarrow y^2-8y-4y+32=0.$$

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

METHOD In fact, we can convert any quadratic equation to the form $(ax+b)^2-c^2=0$ and then we can easily find its roots.

SOLVED EXAMPLES

EXAMPLE 1 *Solve the equation $x^2-10x-2=0$ by the method of completing the square.*

SOLUTION We have

$$x^2-10x-2=0$$

$$\Rightarrow \quad x^2-10x=2$$

$$\Rightarrow \quad x^2-2\times x\times 5+5^2=2+5^2 \quad [\text{adding } 5^2 \text{ on both sides}]$$

$$\Rightarrow \quad (x-5)^2=(2+25)=27$$

$$\Rightarrow \quad x-5=\pm\sqrt{27}=\pm 3\sqrt{3} \quad [\text{taking square root on both sides}]$$

$$\Rightarrow \quad x-5=3\sqrt{3} \quad \text{or} \quad x-5=-3\sqrt{3}$$

$$\Rightarrow \quad x=(5+3\sqrt{3}) \quad \text{or} \quad x=(5-3\sqrt{3}).$$

Hence, $(5+3\sqrt{3})$ and $(5-3\sqrt{3})$ are the roots of the given equation.

EXAMPLE 2 *Solve the equation* $3x^2 - 5x + 2 = 0$ *by the method of completing the square.*

SOLUTION We have

$$3x^2 - 5x + 2 = 0$$

$\Rightarrow$ $9x^2 - 15x + 6 = 0$ [multiplying each term by 3]

$\Rightarrow$ $9x^2 - 15x = -6$

$\Rightarrow$ $(3x)^2 - 2 \times 3x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$

[adding $\left(\frac{5}{2}\right)^2$ on both sides]

$\Rightarrow$ $\left(3x - \frac{5}{2}\right)^2 = \left(-6 + \frac{25}{4}\right) = \frac{(-24 + 25)}{4} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$\Rightarrow$ $\left(3x - \frac{5}{2}\right) = \pm\frac{1}{2}$ [taking square root on both sides]

$\Rightarrow$ $3x - \frac{5}{2} = \frac{1}{2}$ or $3x - \frac{5}{2} = \frac{-1}{2}$

$\Rightarrow$ $3x = \left(\frac{1}{2} + \frac{5}{2}\right) = \frac{6}{2} = 3$ or $3x = \left(\frac{-1}{2} + \frac{5}{2}\right) = \frac{4}{2} = 2$

$\Rightarrow$ $3x = 3$ or $3x = 2 \Rightarrow x = 1$ or $x = \frac{2}{3}$.

Hence, 1 and $\frac{2}{3}$ are the roots of the given equation.

EXAMPLE 3 *Solve the equation* $2x^2 + x - 4 = 0$ *by the method of completing the square.*

SOLUTION We have

$$2x^2 + x - 4 = 0$$

$\Rightarrow$ $4x^2 + 2x - 8 = 0$ [multiplying both sides by 2]

$\Rightarrow$ $4x^2 + 2x = 8$

$\Rightarrow$ $(2x)^2 + 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = 8 + \left(\frac{1}{2}\right)^2$

[adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\Rightarrow$ $\left(2x + \frac{1}{2}\right)^2 = \left(8 + \frac{1}{4}\right) = \frac{33}{4} = \left(\frac{\sqrt{33}}{2}\right)^2$

$\Rightarrow$ $2x + \frac{1}{2} = \pm\left(\frac{\sqrt{33}}{2}\right)$ [taking square root on both sides]

$\Rightarrow$ $2x + \frac{1}{2} = \frac{\sqrt{33}}{2}$ or $2x + \frac{1}{2} = \frac{-\sqrt{33}}{2}$

$$\Rightarrow \quad 2x = \left(\frac{\sqrt{33}}{2} - \frac{1}{2}\right) = \frac{(\sqrt{33} - 1)}{2}$$

$$\text{or} \quad 2x = \frac{-\sqrt{33}}{2} - \frac{1}{2} = \frac{-(\sqrt{33} + 1)}{2}$$

$$\Rightarrow \quad x = \frac{(\sqrt{33} - 1)}{4} \text{ or } x = \frac{-(\sqrt{33} + 1)}{4}.$$

Hence, $\dfrac{-1 + \sqrt{33}}{4}$ and $\dfrac{-1 - \sqrt{33}}{4}$ are the roots of the given equation.

EXAMPLE 4 *By using the method of completing the square, show that the equation* $4x^2 + 3x + 5 = 0$ *has no real roots.*

SOLUTION We have

$$4x^2 + 3x + 5 = 0$$

$$\Rightarrow \quad 4x^2 + 3x = -5$$

$$\Rightarrow \quad (2x)^2 + 2 \times 2x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2 = -5 + \left(\frac{3}{4}\right)^2$$

[adding $\left(\frac{3}{4}\right)^2$ on both sides]

$$\Rightarrow \quad \left(2x + \frac{3}{4}\right)^2 = \left(-5 + \frac{9}{16}\right) = \frac{(-80 + 9)}{16} = \frac{-71}{16} < 0.$$

But, $\left(2x + \frac{3}{4}\right)^2$ cannot be negative for any real value of x.

So, there is no real value of x satisfying the given equation.

Hence, the given equation has no real roots.

EXAMPLE 5 *Solve the equation* $10x - \frac{1}{x} = 3$ *by the method of completing the square.*

SOLUTION We have

$$10x - \frac{1}{x} = 3$$

$$\Rightarrow \quad 10x^2 - 1 = 3x$$

$$\Rightarrow \quad 10x^2 - 3x = 1$$

$$\Rightarrow \quad 100x^2 - 30x = 10 \quad \text{[multiplying each side by 10]}$$

$$\Rightarrow \quad (10x)^2 - 2 \times 10x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$$

[adding $\left(\frac{3}{2}\right)^2$ on both sides]

$$\Rightarrow \quad \left(10x - \frac{3}{2}\right)^2 = \left(10 + \frac{9}{4}\right) = \frac{49}{4} = \left(\frac{7}{2}\right)^2$$

$$\Rightarrow \quad 10x - \frac{3}{2} = \pm\frac{7}{2} \qquad \text{[taking square root on both sides]}$$

$$\Rightarrow \quad 10x - \frac{3}{2} = \frac{7}{2} \text{ or } 10x - \frac{3}{2} = \frac{-7}{2}$$

$$\Rightarrow \quad 10x = \left(\frac{7}{2} + \frac{3}{2}\right) = \frac{10}{2} = 5 \text{ or } 10x = \left(\frac{-7}{2} + \frac{3}{2}\right) = \frac{-4}{2} = -2$$

$$\Rightarrow \quad 10x = 5 \text{ or } 10x = -2 \Rightarrow x = \frac{5}{10} = \frac{1}{2} \text{ or } x = \frac{-2}{10} = \frac{-1}{5}.$$

Hence, $\frac{1}{2}$ and $\frac{-1}{5}$ are the roots of the given equation.

EXAMPLE 6 *Solve the equation $a^2x^2 - 3abx + 2b^2 = 0$ by the method of completing the square.*

SOLUTION We have

$$a^2x^2 - 3abx + 2b^2 = 0$$

$$\Rightarrow \quad a^2x^2 - 3abx = -2b^2$$

$$\Rightarrow \quad (ax)^2 - 2 \times (ax) \times \frac{3b}{2} + \left(\frac{3b}{2}\right)^2 = -2b^2 + \left(\frac{3b}{2}\right)^2$$

$$\text{[adding } \left(\frac{3b}{2}\right)^2 \text{ on both sides]}$$

$$\Rightarrow \quad \left(ax - \frac{3b}{2}\right)^2 = \left(-2b^2 + \frac{9b^2}{4}\right) = \frac{(-8b^2 + 9b^2)}{4} = \frac{b^2}{4} = \left(\frac{b}{2}\right)^2$$

$$\Rightarrow \quad \left(ax - \frac{3b}{2}\right) = \pm\frac{b}{2} \qquad \text{[taking square root on both sides]}$$

$$\Rightarrow \quad \left(ax - \frac{3b}{2}\right) = \frac{b}{2} \text{ or } \left(ax - \frac{3b}{2}\right) = \frac{-b}{2}$$

$$\Rightarrow \quad ax = \left(\frac{b}{2} + \frac{3b}{2}\right) = \frac{4b}{2} = 2b \text{ or } ax = \left(\frac{-b}{2} + \frac{3b}{2}\right) = \frac{2b}{2} = b$$

$$\Rightarrow \quad x = \frac{2b}{a} \text{ or } x = \frac{b}{a}.$$

Hence, $\frac{2b}{a}$ and $\frac{b}{a}$ are the roots of the given equation.

EXAMPLE 7 *Solve the equation $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ by the method of completing the square.*

SOLUTION We have

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$\Rightarrow \quad x^2 - (\sqrt{3} + 1)x = -\sqrt{3}$$

$$\Rightarrow \quad x^2 - 2 \times x \times \left(\frac{\sqrt{3} + 1}{2}\right) + \left(\frac{\sqrt{3} + 1}{2}\right)^2 = -\sqrt{3} + \left(\frac{\sqrt{3} + 1}{2}\right)^2$$

[adding $\left(\frac{\sqrt{3}+1}{2}\right)^2$ on both sides]

$$\Rightarrow \quad \left\{x - \frac{(\sqrt{3}+1)}{2}\right\}^2 = \left\{\frac{(\sqrt{3}+1)^2}{4} - \sqrt{3}\right\}$$

$$\Rightarrow \quad \frac{(\sqrt{3}+1)^2 - 4\sqrt{3}}{4} = \left(\frac{\sqrt{3}-1}{2}\right)^2$$

$$\Rightarrow \quad \left\{x - \frac{(\sqrt{3}+1)}{2}\right\} = \pm\frac{(\sqrt{3}-1)}{2}$$

[taking square root on both sides]

$$\Rightarrow \quad x - \frac{(\sqrt{3}+1)}{2} = \frac{(\sqrt{3}-1)}{2} \text{ or } x - \frac{(\sqrt{3}+1)}{2} = \frac{-(\sqrt{3}-1)}{2}$$

$$\Rightarrow \quad x = \frac{(\sqrt{3}-1)}{2} + \frac{(\sqrt{3}+1)}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\text{or } x = \frac{(-\sqrt{3}+1)}{2} + \frac{(\sqrt{3}+1)}{2} = \frac{2}{2} = 1$$

$$\Rightarrow \quad x = \sqrt{3} \text{ or } x = 1.$$

Hence, $\sqrt{3}$ and 1 are the roots of the given equation.

EXERCISE 4B

Solve each of the following equations by using the method of completing the square:

1. $x^2 - 6x + 3 = 0$
2. $x^2 - 4x + 1 = 0$
3. $x^2 + 8x - 2 = 0$
4. $4x^2 + 4\sqrt{3}x + 3 = 0$
5. $2x^2 + 5x - 3 = 0$
6. $3x^2 - x - 2 = 0$
7. $8x^2 - 14x - 15 = 0$
8. $7x^2 + 3x - 4 = 0$
9. $3x^2 - 2x - 1 = 0$
10. $5x^2 - 6x - 2 = 0$
11. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$
12. $4x^2 + 4bx - (a^2 - b^2) = 0$
13. $x^2 - (\sqrt{2}+1)x + \sqrt{2} = 0$
14. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$
15. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
16. By using the method of completing the square, show that the equation $2x^2 + x + 4 = 0$ has no real roots.

ANSWERS (EXERCISE 4B)

1. $x = (3+\sqrt{6})$ or $x = (3-\sqrt{6})$ **2.** $x = (2+\sqrt{3})$ or $x = (2-\sqrt{3})$

3. $x = (-4+3\sqrt{2})$ or $x = (-4-3\sqrt{2})$ **4.** $x = \frac{-\sqrt{3}}{2}$ or $x = \frac{-\sqrt{3}}{2}$

5. $x = \frac{1}{2}$ or $x = -3$ **6.** $x = 1$ or $x = \frac{-2}{3}$ **7.** $x = \frac{5}{2}$ or $x = \frac{-3}{4}$

8. $x=-1$ or $x=\frac{4}{7}$ **9.** $x=1$ or $x=\frac{-1}{3}$

10. $x=\frac{3+\sqrt{19}}{5}$ or $x=\frac{3-\sqrt{19}}{5}$ **11.** $x=2$ or $x=\frac{1}{2}$

12. $x=\frac{-(a+b)}{2}$ or $x=\frac{(a-b)}{2}$ **13.** $x=\sqrt{2}$ or $x=1$

14. $x=\frac{-1}{\sqrt{2}}$ or $x=2\sqrt{2}$ **15.** $x=-\sqrt{3}$ or $x=\frac{-7}{\sqrt{3}}$

....

QUADRATIC FORMULA [SHRIDHARACHARYA'S RULE]

Consider the quadratic equation $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$. Then,

$$ax^2+bx+c=0$$

$$\Rightarrow \quad ax^2+bx=-c$$

$$\Rightarrow \quad x^2+\frac{b}{a}\cdot x=\frac{-c}{a} \qquad \text{[dividing throughout by } a\text{]}$$

$$\Rightarrow \quad x^2+\frac{b}{a}\cdot x+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2 \qquad \left[\text{adding } \left(\frac{b}{2a}\right)^2 \text{ on both sides}\right]$$

$$\Rightarrow \quad \left(x+\frac{b}{2a}\right)^2=\left(\frac{-c}{a}+\frac{b^2}{4a^2}\right)$$

$$\Rightarrow \quad \left(x+\frac{b}{2a}\right)^2=\frac{(b^2-4ac)}{4a^2}$$

$$\Rightarrow \quad \left(x+\frac{b}{2a}\right)=\frac{\pm\sqrt{b^2-4ac}}{2a}, \text{ when } (b^2-4ac) \geq 0$$

$$\Rightarrow \quad x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

This is called the *quadratic formula* or *Shridharacharya's rule*.

Thus, $ax^2+bx+c=0$ has two roots α and β, given by

$$\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \beta=\frac{-b-\sqrt{b^2-4ac}}{2a}.$$

DISCRIMINANT For the equation $ax^2+bx+c=0$, the expression $D=(b^2-4ac)$ is called the *discriminant*.

AN IMPORTANT NOTE The roots of $ax^2+bx+c=0$ are real only when $(b^2-4ac) \geq 0$.

Taking $(b^2 - 4ac) = D$, the roots of $ax^2 + bx + c = 0$ are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}.$$

SOLVED EXAMPLES

EXAMPLE 1 *Show that the equation* $9x^2 + 7x - 2 = 0$ *has real roots and solve it.*

SOLUTION The given equation is $9x^2 + 7x - 2 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 9, b = 7$ and $c = -2$.

$\therefore \quad D = (b^2 - 4ac) = (7^2 - 4 \times 9 \times (-2)] = 121 > 0.$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{121} = 11$.

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(-7 + 11)}{2 \times 9} = \frac{4}{18} = \frac{2}{9},$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{(-7 - 11)}{2 \times 9} = \frac{-18}{18} = -1.$$

Hence, the required roots are $\frac{2}{9}$ and -1.

EXAMPLE 2 *Show that the equation* $x^2 + 6x + 6 = 0$ *has real roots and solve it.*

SOLUTION The given equation is $x^2 + 6x + 6 = 0$.

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = 6$ and $c = 6$.

$\therefore \quad D = (b^2 - 4ac) = (36 - 4 \times 1 \times 6) = 12 > 0.$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{12} = 2\sqrt{3}$.

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(-6 + 2\sqrt{3})}{2 \times 1} = \frac{(-6 + 2\sqrt{3})}{2} = (-3 + \sqrt{3}),$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{(-6 - 2\sqrt{3})}{2 \times 1} = \frac{(-6 - 2\sqrt{3})}{2} = (-3 - \sqrt{3}).$$

Hence, $(-3 + \sqrt{3})$ and $(-3 - \sqrt{3})$ are the roots of the given equation.

EXAMPLE 3 *Show that the equation* $2x^2 + 5\sqrt{3}x + 6 = 0$ *has real roots and solve it.*

SOLUTION The given equation is $2x^2 + 5\sqrt{3}x + 6 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = 5\sqrt{3}$ and $c = 6.$

$\therefore \quad D = (b^2 - 4ac) = [(5\sqrt{3})^2 - 4 \times 2 \times 6] = (75 - 48) = 27 > 0.$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{27} = 3\sqrt{3}.$

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(-5\sqrt{3} + 3\sqrt{3})}{2 \times 2} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2},$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{(-5\sqrt{3} - 3\sqrt{3})}{2 \times 2} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}.$$

Hence, $\frac{-\sqrt{3}}{2}$ and $-2\sqrt{3}$ are the roots of the given equation.

EXAMPLE 4 *Using quadratic formula, solve for x:*

$$p^2x^2 + (p^2 - q^2)x - q^2 = 0.$$ [CBSE 2014]

SOLUTION The given equation is $p^2x^2 + (p^2 - q^2)x - q^2 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = p^2, b = (p^2 - q^2)$ and $c = -q^2.$

$$\therefore \quad D = (b^2 - 4ac) = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2)$$
$$= (p^2 - q^2)^2 + 4p^2q^2 = (p^2 + q^2)^2 > 0.$$

So, the given equation has real roots.

Now, $\sqrt{D} = (p^2 + q^2).$

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2},$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1.$$

Hence, $\frac{q^2}{p^2}$ and -1 are the roots of the given equation.

EXAMPLE 5 *Using quadratic formula, solve for x:*

$$9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0.$$ [CBSE 2009]

SOLUTION The given equation is $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0.$

This is of the form $Ax^2 + Bx + C = 0,$ where

$A = 9, B = -9(a + b)$ and $C = (2a^2 + 5ab + 2b^2).$

$$\therefore \quad D = (B^2 - 4AC) = 81(a+b)^2 - 36(2a^2 + 5ab + 2b^2)$$
$$= 81(a^2 + b^2 + 2ab) - 36(2a^2 + 5ab + 2b^2)$$
$$= 9a^2 + 9b^2 - 18ab = 9(a^2 + b^2 - 2ab)$$
$$= 9(a-b)^2 \geq 0.$$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{9(a-b)^2} = 3(a-b)$.

$$\therefore \quad \alpha = \frac{-B + \sqrt{D}}{2A} = \frac{9(a+b) + 3(a-b)}{2 \times 9} = \frac{6(2a+b)}{18} = \frac{(2a+b)}{3},$$
$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{9(a+b) - 3(a-b)}{2 \times 9} = \frac{6(a+2b)}{18} = \frac{(a+2b)}{3}.$$

Hence, $\frac{(2a+b)}{3}$ and $\frac{(a+2b)}{3}$ are the roots of the given equation.

EXAMPLE 6 *Using quadratic formula, solve for x:*

$$abx^2 + (b^2 - ac)x - bc = 0.$$ [CBSE 2014]

SOLUTION The given equation is $abx^2 + (b^2 - ac)x - bc = 0$.

This is of the form $Ax^2 + Bx + C = 0$, where

$A = ab$, $B = (b^2 - ac)$ and $C = -bc$.

$$\therefore \quad D = (B^2 - 4AC) = (b^2 - ac)^2 + 4ab^2c$$
$$= b^4 + a^2c^2 - 2ab^2c + 4ab^2c$$
$$= b^4 + a^2c^2 + 2ab^2c = (b^2 + ac)^2 > 0.$$

So, the given equation has real roots.

Now, $\sqrt{D} = (b^2 + ac)$.

$$\therefore \quad \alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} = \frac{2ac}{2ab} = \frac{c}{b},$$
$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} = \frac{-2b^2}{2ab} = \frac{-b}{a}.$$

Hence, $\frac{c}{b}$ and $\frac{-b}{a}$ are the roots of the given equation.

EXAMPLE 7 *Solve for x:* $\frac{1}{x} - \frac{1}{(x-2)} = 3,\ x \neq 0, 2.$ [CBSE 2010]

SOLUTION The given equation may be written as

$$\frac{(x-2) - x}{x(x-2)} = 3 \Rightarrow 3x(x-2) = -2$$
$$\Rightarrow 3x^2 - 6x + 2 = 0. \qquad \ldots \text{(i)}$$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 3$, $b = -6$ and $c = 2$.

$\therefore \quad D = (b^2 - 4ac) = (-6)^2 - 4 \times 3 \times 2 = 36 - 24 = 12 > 0.$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{12} = 2\sqrt{3}$.

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{6 + 2\sqrt{3}}{2 \times 3} = \frac{6 + 2\sqrt{3}}{6} = \frac{3 + \sqrt{3}}{3},$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{6 - 2\sqrt{3}}{2 \times 3} = \frac{6 - 2\sqrt{3}}{6} = \frac{3 - \sqrt{3}}{3}.$$

Hence, the required values of x are $\frac{(3 + \sqrt{3})}{3}$ and $\frac{(3 - \sqrt{3})}{3}$.

EXAMPLE 8 *Solve for x:* $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4.$ [CBSE 2014]

SOLUTION The given equation is

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(x^2 - 5x + 4) + (x^2 - 5x + 6)}{(x^2 - 6x + 8)} = \frac{10}{3} \Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$$

$$\Rightarrow \quad 3(2x^2 - 10x + 10) = 10(x^2 - 6x + 8)$$

$$\Rightarrow \quad 6x^2 - 30x + 30 = 10x^2 - 60x + 80$$

$$\Rightarrow \quad 4x^2 - 30x + 50 = 0 \Rightarrow 2x^2 - 15x + 25 = 0. \qquad \dots \text{(i)}$$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -15$ and $c = 25$.

$\therefore \quad D = (b^2 - 4ac) = \{(-15)^2 - 4 \times 2 \times 25\} = (225 - 200) = 25 > 0.$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{25} = 5$.

$$\therefore \quad \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{(15 + 5)}{2 \times 2} = \frac{20}{4} = 5,$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{(15 - 5)}{2 \times 2} = \frac{10}{4} = \frac{5}{2}.$$

Hence, the required values of x are 5 and $\frac{5}{2}$.

EXERCISE 4C

Find the discriminant of each of the following equations:

1. (i) $2x^2 - 7x + 6 = 0$ (ii) $3x^2 - 2x + 8 = 0$
   (iii) $2x^2 - 5\sqrt{2}x + 4 = 0$ (iv) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$
   (v) $(x-1)(2x-1) = 0$ (vi) $1 - x = 2x^2$

Find the roots of each of the following equations, if they exist, by applying the quadratic formula:

2. $x^2 - 4x - 1 = 0$
3. $x^2 - 6x + 4 = 0$
4. $2x^2 + x - 4 = 0$
5. $25x^2 + 30x + 7 = 0$
6. $16x^2 = 24x + 1$
7. $15x^2 - 28 = x$
8. $2x^2 - 2\sqrt{2}x + 1 = 0$
9. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ [CBSE 2013, '17]
10. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$ [CBSE 2011]
11. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ [CBSE 2015]
12. $2x^2 + 6\sqrt{3}x - 60 = 0$ [CBSE 2011, '15]
13. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ [CBSE 2013]
14. $3x^2 - 2\sqrt{6}x + 2 = 0$ [CBSE 2012]
15. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$ [CBSE 2011]
16. $x^2 + x + 2 = 0$
17. $2x^2 + ax - a^2 = 0$ [CBSE 2015]
18. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ [CBSE 2015]

   HINT $D = (\sqrt{3} + 1)^2 - 4\sqrt{3} = (\sqrt{3} - 1)^2$.

19. $2x^2 + 5\sqrt{3}x + 6 = 0$
20. $3x^2 - 2x + 2 = 0$
21. $x + \frac{1}{x} = 3, x \neq 0$
22. $\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$ [CBSE 2010]
23. $x - \frac{1}{x} = 3, x \neq 0$ [CBSE 2010]
24. $\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$

   HINT $\frac{m}{n}x^2 + 2x + \left(\frac{n}{m} - 1\right) = 0 \Rightarrow m^2x^2 + 2mnx + (n^2 - mn) = 0$.

25. $36x^2 - 12ax + (a^2 - b^2) = 0$
26. $x^2 - 2ax + (a^2 - b^2) = 0$
27. $x^2 - 2ax - (4b^2 - a^2) = 0$ [CBSE 2015]
28. $x^2 + 6x - (a^2 + 2a - 8) = 0$ [CBSE 2015]
29. $x^2 + 5x - (a^2 + a - 6) = 0$ [CBSE 2015]
30. $x^2 - 4ax - b^2 + 4a^2 = 0$ [CBSE 2012]
31. $4x^2 - 4a^2x + (a^4 - b^4) = 0$ [CBSE 2015]

32. $4x^2 + 4bx - (a^2 - b^2) = 0$ [CBSE 2015]

33. $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$ [CBSE 2015]

34. $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

35. $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0, a \neq 0$ and $b \neq 0$ [CBSE 2006]

36. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0,$ where $a \neq 0$ and $b \neq 0$ [CBSE 2009]

ANSWERS (EXERCISE 4C)

1. (i) 1 (ii) –92 (iii) 18 (iv) 32 (v) 1 (vi) 9

2. $x = (2 + \sqrt{5})$ or $x = (2 - \sqrt{5})$ **3.** $x = (3 + \sqrt{5})$ or $x = (3 - \sqrt{5})$

4. $x = \frac{(-1 + \sqrt{33})}{4}$ or $x = \frac{(-1 - \sqrt{33})}{4}$ **5.** $x = \frac{(-3 + \sqrt{2})}{5}$ or $x = \frac{(-3 - \sqrt{2})}{5}$

6. $x = \frac{(3 + \sqrt{10})}{4}$ or $x = \frac{(3 - \sqrt{10})}{4}$ **7.** $x = \frac{7}{5}$ or $x = \frac{-4}{3}$

8. $x = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **9.** $x = -\sqrt{2}$ or $x = \frac{-5}{\sqrt{2}}$ **10.** $x = \frac{2\sqrt{3}}{3}$ or $x = -4\sqrt{3}$

11. $x = \sqrt{6}$ or $x = \frac{-\sqrt{2}}{\sqrt{3}}$ **12.** $x = 2\sqrt{3}$ or $x = -5\sqrt{3}$ **13.** $x = \frac{\sqrt{3}}{4}$ or $x = \frac{-2}{\sqrt{3}}$

14. $x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ **15.** $x = \frac{\sqrt{3}}{2}$ or $x = \frac{1}{\sqrt{3}}$ **16.** Do not exist

17. $x = \frac{a}{2}$ or $x = -a$ **18.** $x = 1$ or $x = \sqrt{3}$ **19.** $x = \frac{-\sqrt{3}}{2}$ or $x = -2\sqrt{3}$

20. Do not exist **21.** $x = \frac{3 + \sqrt{5}}{2}$ or $x = \frac{3 - \sqrt{5}}{2}$

22. $x = \frac{3 + \sqrt{3}}{3}$ or $x = \frac{3 - \sqrt{3}}{3}$ **23.** $x = \frac{3 + \sqrt{13}}{2}$ or $x = \frac{3 - \sqrt{13}}{2}$

24. $x = \frac{-n + \sqrt{mn}}{m}$ or $x = \frac{n - \sqrt{mn}}{m}$ **25.** $x = \frac{(a + b)}{6}$ or $x = \frac{(a - b)}{6}$

26. $x = (a + b)$ or $x = (a - b)$ **27.** $x = (a + 2b)$ or $x = (a - 2b)$

28. $x = (a - 2)$ or $x = -(4 + a)$ **29.** $x = (a - 2)$ or $x = -(a + 3)$

30. $x = (2a - b)$ or $x = (2a + b)$ **31.** $x = \frac{1}{2}(a^2 + b^2)$ or $x = \frac{1}{2}(a^2 - b^2)$

32. $x = \frac{1}{2}(a - b)$ or $x = -\frac{1}{2}(a + b)$ **33.** $x = (b + 4)$ or $x = (b - 5)$

34. $x = \frac{-2b}{3a}$ or $x = \frac{-2b}{a}$ **35.** $x = \frac{4b^2}{a^2}$ or $x = \frac{-3a^2}{b^2}$ **36.** $x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$

....

NATURE OF THE ROOTS OF A QUADRATIC EQUATION

Let the given equation be $ax^2 + bx + c = 0$, where $a \neq 0$. Then, the *discriminant* is given by $D = (b^2 - 4ac)$. And, the roots of the given equation are

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}.$$

Case I *When* $D > 0$

In this case, the roots are *real* and *distinct*. These roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}.$$

Case II *When* $D = 0$

In this case, the roots are *real* and *equal*.

Each root $= \frac{(-b)}{2a}$.

Case III *When* $D < 0$

In this case, the roots are *imaginary*, and we say that the given equation has no real roots.

SUMMARY

Value	Nature of roots	Roots
$D > 0$	Real and unequal	$\frac{-b \pm \sqrt{D}}{2a}$
$D = 0$	Real and equal	Each root $= \frac{-b}{2a}$
$D < 0$	No real roots	None

SOLVED EXAMPLES

EXAMPLE 1 *Find the nature of the roots of the quadratic equation* $4x^2 - 5x + 3 = 0$.

SOLUTION The given equation is $4x^2 - 5x + 3 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 4, b = -5$ and $c = 3$.

$\therefore \quad D = (b^2 - 4ac) = \{(-5)^2 - 4 \times 4 \times 3\} = (25 - 48) = -23 < 0.$

Hence, the given equation has no real roots.

EXAMPLE 2 *Show that the equation* $2x^2 - 6x + 3 = 0$ *has real roots and find these roots.*

SOLUTION The given equation is $2x^2 - 6x + 3 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -6$ and $c = 3$.

$\therefore \quad D = (b^2 - 4ac) = [(-6)^2 - 4 \times 2 \times 3] = (36 - 24) = 12 > 0.$

So, the given equation has real unequal roots.

Solving $2x^2 - 6x + 3 = 0$ by quadratic formula, we have

$$x = \frac{6 \pm \sqrt{36 - 4 \times 2 \times 3}}{(2 \times 2)} = \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm \sqrt{12}}{4}$$

$$\Rightarrow \quad x = \frac{6 \pm 2\sqrt{3}}{4} \Rightarrow x = \frac{3 \pm \sqrt{3}}{2}.$$

So, $\frac{(3 + \sqrt{3})}{2}$ and $\frac{(3 - \sqrt{3})}{2}$ are the roots of the given equation.

EXAMPLE 3 *Show that the equation* $x^2 + ax - 4 = 0$ *has real and distinct roots for all real values of a.*

SOLUTION The given equation is $x^2 + ax - 4 = 0$.

This is of the form $Ax^2 + Bx + C = 0$, where $A = 1$, $B = a$ and $C = -4$.

$\therefore \quad D = (B^2 - 4AC) = \{a^2 - 4 \times 1 \times (-4)\} = (a^2 + 16) > 0$ for all real values of a.

Thus, $D > 0$ for all real values of a.

Hence, the given equation has real and distinct roots for all real values of a.

EXAMPLE 4 *Find the nature of the roots of the quadratic equation* $3x^2 - 4\sqrt{3}x + 4 = 0$ *and hence solve it.*

SOLUTION The given equation is $3x^2 - 4\sqrt{3}x + 4 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 3$, $b = -4\sqrt{3}$ and $c = 4$.

$\therefore \quad D = (b^2 - 4ac) = \{(-4\sqrt{3})^2 - 4 \times 3 \times 4\} = (48 - 48) = 0.$

This shows that the given quadratic equation has real and equal roots.

Each root $= \frac{-b}{2a} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}.$

Hence, $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$ are the roots of the given equation.

EXAMPLE 5 *Find the values of k for which the quadratic equation* $2x^2 + kx + 3 = 0$ *has two real equal roots.*

SOLUTION The given equation is $2x^2 + kx + 3 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = k$ and $c = 3$.

$\therefore \quad D = (b^2 - 4ac) = (k^2 - 4 \times 2 \times 3) = (k^2 - 24)$.

For real and equal roots, we much have

$$D = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k^2 = 24 \Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}.$$

Hence, $2\sqrt{6}$ and $-2\sqrt{6}$ are the required values of k.

EXAMPLE 6 *Find the value of k for which the roots of the quadratic equation* $kx(x-2) + 6 = 0$ *are equal.* [CBSE 2017]

SOLUTION The given equation is $kx^2 - 2kx + 6 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = k$, $b = -2k$ and $c = 6$.

$\therefore \quad D = (b^2 - 4ac) = (4k^2 - 4 \times k \times 6) = (4k^2 - 24k)$.

For equal roots, we must have

$$D = 0 \Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k-6) = 0 \Rightarrow k = 0 \text{ or } k = 6.$$

Now, $k = 0$, we get $6 = 0$, which is absurd.

$\therefore \quad k \neq 0$ and hence $k = 6$.

EXAMPLE 7 *Find the value of k for which the quadratic equation* $(k+4)x^2 + (k+1)x + 1 = 0$ *has two real equal roots.* [CBSE 2014]

SOLUTION The given equation is $(k+4)x^2 + (k+1)x + 1 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = (k+4)$, $b = (k+1)$ and $c = 1$.

$$\therefore \quad D = (b^2 - 4ac) = (k+1)^2 - 4 \times (k+4) \times 1 = (k+1)^2 - 4(k+4)$$
$$= (k^2 + 1 + 2k - 4k - 16) = (k^2 - 2k - 15).$$

For equal roots, we must have

$$D = 0 \Rightarrow k^2 - 2k - 15 = 0$$
$$\Rightarrow k^2 - 5k + 3k - 15 = 0 \Rightarrow k(k-5) + 3(k-5) = 0$$
$$\Rightarrow (k-5)(k+3) = 0 \Rightarrow k - 5 = 0 \text{ or } k + 3 = 0$$
$$\Rightarrow k = 5 \text{ or } k = -3.$$

Hence, the required value of k is 5 or –3.

EXAMPLE 8 *Find the nonzero value of k for which the quadratic equation* $kx^2 + 1 - 2(k-1)x + x^2 = 0$ *has equal roots. Hence, find the roots of the equation.* [CBSE 2015]

SOLUTION The given equation is $(k+1)x^2 - 2(k-1)x + 1 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = (k+1)$, $b = -2(k-1)$ and $c = 1$.

$\therefore \quad D = (b^2 - 4ac) = \{4(k-1)^2 - 4 \times (k+1) \times 1\} = 4(k^2 - 3k)$.

For equal roots, we must have

$$D = 0 \Rightarrow 4(k^2 - 3k) = 0 \Rightarrow 4k(k-3) = 0 \Rightarrow k = 0 \text{ or } k = 3.$$

$\therefore$ the required nonzero value of k is 3.

Putting $k = 3$, the given equation becomes

$$4x^2 - 4x + 1 = 0 \Rightarrow (2x-1)^2 = 0 \Rightarrow (2x-1) = 0 \Rightarrow x = \frac{1}{2}.$$

Hence, the required roots are $\frac{1}{2}$ and $\frac{1}{2}$.

EXAMPLE 9 *If –4 is a root of the equation $x^2 + px - 4 = 0$ and the equation $x^2 + px + q = 0$ has equal roots, find the values of p and q.*

SOLUTION Since –4 is a root of the equation $x^2 + px - 4 = 0$, we have

$$(-4)^2 + p(-4) - 4 = 0 \Rightarrow 4p = (16-4) = 12 \Rightarrow p = 3.$$

Now, the roots of $x^2 + px + q = 0$ being equal, we have

$$p^2 - 4q = 0 \Rightarrow 3^2 - 4q = 0 \Rightarrow 4q = 9 \Rightarrow q = \frac{9}{4}.$$

Hence, $p = 3$ and $q = \frac{9}{4}$.

EXAMPLE 10 *If –2 is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.*

[CBSE 2015]

SOLUTION Since –2 is a root of the equation $3x^2 + 7x + p = 0$, we have

$$3 \times (-2)^2 + 7 \times (-2) + p = 0 \Rightarrow 12 - 14 + p = 0 \Rightarrow p = 2.$$

For $p = 2$, the other given equation becomes

$$x^2 + 4kx + k(k-1) + 2 = 0.$$

This is of the form $ax^2 + bx + c = 0$, where

$$a = 1, b = 4k \text{ and } c = (k^2 - k + 2).$$

$\therefore \quad D = (b^2 - 4ac) = \{16k^2 - 4 \times 1 \times (k^2 - k + 2)\} = (12k^2 + 4k - 8)$.

For equal roots we must have

$$D = 0 \Rightarrow 12k^2 + 4k - 8 = 0 \Rightarrow 4(3k^2 + k - 2) = 0$$
$$\Rightarrow 3k^2 + k - 2 = 0 \Rightarrow 3k^2 + 3k - 2k - 2 = 0$$
$$\Rightarrow 3k(k+1) - 2(k+1) = 0 \Rightarrow (k+1)(3k-2) = 0$$

$$\Rightarrow k+1=0 \text{ or } 3k-2=0 \Rightarrow k=-1 \text{ or } k=\frac{2}{3}.$$

Hence, the required value of k is -1 or $\frac{2}{3}$.

EXAMPLE 11 *Prove that both the roots of the equation*

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

are real but they are equal only when $a=b=c$.

SOLUTION The given equation may be written as

$$3x^2-2x(a+b+c)+(ab+bc+ca)=0.$$

$$\begin{aligned}\therefore\quad D &= 4(a+b+c)^2-12(ab+bc+ca)\\ &= 4[(a+b+c)^2-3(ab+bc+ca)]\\ &= 4(a^2+b^2+c^2-ab-bc-ca)\\ &= 2(2a^2+2b^2+2c^2-2ab-2bc-2ca)\\ &= 2[(a-b)^2+(b-c)^2+(c-a)^2]\ge 0\end{aligned}$$

$$[\because\ (a-b)^2\ge 0, (b-c)^2\ge 0 \text{ and } (c-a)^2\ge 0].$$

This shows that both the roots of the given equation are real.

For equal roots, we must have $D=0$.

$$\begin{aligned}\text{Now, } D=0 &\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0\\ &\Rightarrow (a-b)=0, (b-c)=0 \text{ and } (c-a)=0\\ &\Rightarrow a=b=c.\end{aligned}$$

Hence, the roots are equal only when $a=b=c$.

EXAMPLE 12 *If the roots of the equation* $(b-c)x^2+(c-a)x+(a-b)=0$ *are equal, prove that* $2b=a+c$. [CBSE 2002C, '06, '17]

SOLUTION Clearly, $x=1$ satisfies the given equation.

Since its roots are equal, so 1 and 1 are its roots.

$\therefore$ product of roots of the given equation $=(1\times 1)=1$.

But, product of roots $=\frac{a-b}{b-c}$. $\left[\because\ \text{product of roots } = \frac{C}{A}\right]$

$$\therefore\quad \frac{a-b}{b-c}=1 \Rightarrow a-b=b-c \Rightarrow 2b=a+c.$$

Hence, $2b=a+c$.

EXAMPLE 13 *Show that the equation* $3x^2+7x+8=0$ *is not true for any real value of* x.

SOLUTION The given equation is $3x^2+7x+8=0$.

$$\therefore\quad D=(7^2-4\times 3\times 8)=(49-96)=-47<0.$$

So, the given equation has no real roots.

Hence, the given equation is not true for any real value of x.

EXAMPLE 14 *Show that the equation* $2(a^2+b^2)x^2+2(a+b)x+1=0$ *has no real roots, when* $a \neq b$.

SOLUTION The given equation is $2(a^2+b^2)x^2+2(a+b)x+1=0$.

$\therefore \quad D = 4(a+b)^2 - 8(a^2+b^2)$
$= -4(a^2+b^2-2ab) = -4(a-b)^2 < 0$, when $a-b \neq 0$.

So, the given equation has no real roots, when $a \neq b$.

EXAMPLE 15 *Find the values of k for which the equation* $x^2+5kx+16=0$ *has no real roots.* [CBSE 2013C]

SOLUTION The given equation is $x^2+5kx+16=0$.

This is of the form $ax^2+bx+c=0$, where $a=1, b=5k$ and $c=16$.

$\therefore \quad D = (b^2-4ac) = (25k^2 - 4\times 1\times 16) = (25k^2-64)$.

Since, the given equation has no real root, we have

$$D<0 \Rightarrow 25k^2-64<0 \Rightarrow 25k^2<64$$
$$\Rightarrow k^2 < \frac{64}{25} \Rightarrow k^2 < \left(\frac{8}{5}\right)^2$$
$$\Rightarrow \frac{-8}{5} < k < \frac{8}{5}.$$

Hence, the required real values of k are such that $\frac{-8}{5} < k < \frac{8}{5}$.

EXAMPLE 16 *Find the values of k for which the given equation has real roots:*
(i) $kx^2-6x-2=0$ *(ii)* $3x^2+2x+k=0$ *(iii)* $2x^2+kx+2=0$

SOLUTION (i) The given equation is $kx^2-6x-2=0$.

$\therefore \quad D = [(-6)^2 - 4\times k\times(-2)] = (36+8k)$.

The given equation will have real roots if $D \geq 0$.

Now, $D \geq 0 \Rightarrow 36+8k \geq 0 \Rightarrow k \geq \frac{-36}{8} \Rightarrow k \geq \frac{-9}{2}$.

(ii) The given equation is $3x^2+2x+k=0$.

$\therefore \quad D = (2^2 - 4\times 3\times k) = (4-12k)$.

The given equation will have real roots if $D \geq 0$.

Now, $D \geq 0 \Rightarrow 4-12k \geq 0 \Rightarrow 12k \leq 4 \Rightarrow k \leq \frac{1}{3}$.

(iii) The given equation is $2x^2 + kx + 2 = 0$.

$\therefore \quad D = (k^2 - 4 \times 2 \times 2) = (k^2 - 16)$.

The given equation will have real roots if $D \geq 0$.

Now, $D \geq 0 \Rightarrow (k^2 - 16) \geq 0 \Rightarrow k^2 \geq 16 \Rightarrow k \geq 4$ or $k \leq -4$.

EXAMPLE 17 *Determine the positive value of p for which the equations* $x^2 + 2px + 64 = 0$ *and* $x^2 - 8x + 2p = 0$ *will both have real roots.* [CBSE 2013C]

SOLUTION Let D_1 and D_2 be the discriminants of the first and second given equations respectively.

For real roots, we must have $D_1 \geq 0$ and $D_2 \geq 0$.

Now, $D_1 \geq 0$ and $D_2 \geq 0$

$\Rightarrow \quad (4p^2 - 4 \times 64) \geq 0 \text{ and } (64 - 8p) \geq 0$

$\Rightarrow \quad p^2 - 64 \geq 0 \text{ and } 64 - 8p \geq 0$

$\Rightarrow \quad p^2 \geq 64 \text{ and } 8p \leq 64$

$\Rightarrow \quad p \geq 8 \text{ and } p \leq 8 \quad [\because \ p \text{ is positive}]$

$\Rightarrow \quad p = 8$.

Hence, $p = 8$.

EXERCISE 4D

1. Find the nature of the roots of the following quadratic equations:

   (i) $2x^2 - 8x + 5 = 0$ (ii) $3x^2 - 2\sqrt{6}x + 2 = 0$

   (iii) $5x^2 - 4x + 1 = 0$ (iv) $5x(x - 2) + 6 = 0$

   (v) $12x^2 - 4\sqrt{15}x + 5 = 0$ (vi) $x^2 - x + 2 = 0$

2. If a and b are distinct real numbers, show that the quadratic equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$ has no real roots.

3. Show that the roots of the equation $x^2 + px - q^2 = 0$ are real for all real values of p and q.

4. For what values of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal? [CBSE 2008C]

5. For what value of k are the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ real and equal? [CBSE 2013]

6. For what values of p are the roots of the equation $4x^2 + px + 3 = 0$ real and equal? [CBSE 2014]

7. Find the nonzero value of k for which the roots of the quadratic equation $9x^2 - 3kx + k = 0$ are real and equal. [CBSE 2014]
8. (i) Find the values of k for which the quadratic equation $(3k+1)x^2 + 2(k+1)x + 1 = 0$ has real and equal roots. [CBSE 2014]

   (ii) Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots. [CBSE 2017]
9. Find the values of p for which the quadratic equation $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ has real and equal roots. [CBSE 2014]
10. Find the values of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0, p \neq -1$ has equal roots. Hence, find the roots of the equation. [CBSE 2015]
11. If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k. [CBSE 2014]
12. If 3 is a root of the quadratic equation $x^2 - x + k = 0$, find the value of p so that the roots of the equation $x^2 + k(2x + k + 2) + p = 0$ are equal. [CBSE 2015]
13. If –4 is a root of the equation $x^2 + 2x + 4p = 0$, find the value of k for which the quadratic equation $x^2 + px(1 + 3k) + 7(3 + 2k) = 0$ has equal roots. [CBSE 2015]
14. If the quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$. [CBSE 2014, '17]
15. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are real and equal, show that either $a = 0$ or $(a^3 + b^3 + c^3) = 3abc$. [CBSE 2017]
16. Find the values of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real roots.
17. Find the value of α for which the equation $(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$ has equal roots. [CBSE 2013]
18. Find the value of k for which the roots of $9x^2 + 8kx + 16 = 0$ are real and equal.
19. Find the values of k for which the given quadratic equation has real and distinct roots:

    (i) $kx^2 + 6x + 1 = 0$ (ii) $x^2 - kx + 9 = 0$

    (iii) $9x^2 + 3kx + 4 = 0$ (iv) $5x^2 - kx + 1 = 0$
20. If a and b are real and $a \neq b$ then show that the roots of the equation $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0$ are real and unequal.

21. (i) If the roots of the equation $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$ are equal, prove that $\frac{a}{b}=\frac{c}{d}\cdot$ [CBSE 2017]

(ii) If $ad \neq bc$ then prove that the equation

$(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$ has no real roots. [CBSE 2017]

22. If the roots of the equations $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ac}\,x+b=0$ are simultaneously real then prove that $b^2=ac$.

ANSWERS (EXERCISE 4D)

1. (i) Real and unequal (ii) Real and equal (iii) Not real (iv) Not real (v) Real and equal (vi) Not real

4. $k=9$ or $k=-9$ **5.** $k=0$ or $k=2$ **6.** $p=4\sqrt{3}$ or $p=-4\sqrt{3}$

7. $k=4$ **8.** (i) $k=0$ or $k=1$ (ii) $k=2$ **9.** $p=4$ or $p=\frac{-4}{7}$

10. $p=3$ or $p=-1$ **11.** $k=\frac{49}{28}$ **12.** $p=12$ **13.** $k=2$ or $k=\frac{-10}{9}$

16. $p \geq 8$ or $p \leq -8$ **17.** $\alpha=14$ **18.** $k=3$ or $k=-3$

19. (i) $k<9$ (ii) $k>6$ or $k<-6$ (iii) $k>4$ or $k<-4$ (iv) $k>2\sqrt{5}$ or $k<-2\sqrt{5}$

HINTS TO SOME SELECTED QUESTIONS

2. $D=4(a+b)^2-8(a^2+b^2)$

$=4[(a+b)^2-2(a^2+b^2)]=4(-a^2-b^2+2ab)$

$=-4(a^2+b^2-2ab)=-4(a-b)^2<0.$

3. $D=(p^2+4q^2)\geq 0.$

4. $D=4k^2-324$. For real and equal roots, we must have, $D=0$.

5. Given equation is $kx^2-2\sqrt{5}kx+10=0$.

$\therefore \quad D=20k^2-40k=20k(k-2)$.

For real and equal roots, we must have $D=0$.

6. For real and equal roots, we must have $D=0$.

$\therefore \quad D=0 \Rightarrow p^2-48=0 \Rightarrow p^2=48 \Rightarrow p=\pm\sqrt{48} \Rightarrow p=4\sqrt{3}$ or $-4\sqrt{3}$.

9. $D=(7p+2)^2-4(2p+1)(7p-3)=-7p^2+24p+16$.

$\therefore \quad D=0 \Rightarrow 7p^2-24p-16=0 \Rightarrow 7p^2-28p+4p-16=0$

$\Rightarrow 7p(p-4)+4(p-4)=0 \Rightarrow (p-4)(7p+4)=0.$

15. $D=4(a^2-bc)^2-4(c^2-ab)(b^2-ac)$

Now, $D=0 \Rightarrow (a^2-bc)^2-(c^2-ab)(b^2-ac)=0$

$\Rightarrow a^4-3a^2bc+ac^3+ab^3=0$

$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

16. $D = (p^2 - 64)$. So, $D \geq 0 \Rightarrow p^2 \geq 64 \Rightarrow p \geq 8$ or $p \leq -8$.

19. (i) $36 - 4k > 0 \Rightarrow 36 > 4k \Rightarrow 4k < 36 \Rightarrow k < 9$.

(ii) $k^2 - 36 > 0 \Rightarrow k^2 > 36 \Rightarrow k > 6$ *or* $k < -6$.

(iii) $9k^2 - 144 > 0 \Rightarrow k^2 > 16 \Rightarrow k > 4$ or $k < -4$.

(iv) $k^2 - 20 > 0 \Rightarrow k^2 > 20 \Rightarrow k > 2\sqrt{5}$ or $k < -2\sqrt{5}$.

20. $D = 25(a+b)^2 + 8(a-b)^2$

$= 17(a+b)^2 + 8[(a+b)^2 + (a-b)^2] = 17(a+b)^2 + 16(a^2 + b^2) > 0$.

21. (i) $4(ac+bd)^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow \quad (a^2+b^2)(c^2+d^2) - (ac+bd)^2 = 0$

$\Rightarrow \quad a^2d^2 + b^2c^2 - 2abcd = 0 \Rightarrow (ad-bc)^2 = 0 \Rightarrow ad - bc = 0$

$\Rightarrow \quad ad = bc \Rightarrow \frac{a}{b} = \frac{c}{d}$.

22. $4b^2 - 4ac \geq 0 \Rightarrow b^2 - ac \geq 0 \Rightarrow b^2 \geq ac$.

$4ac - 4b^2 \geq 0 \Rightarrow ac - b^2 \geq 0 \Rightarrow b^2 \leq ac$

Hence, $b^2 = ac$.

WORD PROBLEMS ON QUADRATIC EQUATIONS

SOLVED EXAMPLES

PROBLEMS ON NUMBERS

EXAMPLE 1 *If the sum of two natural numbers is 27 and their product is 182, find the numbers.*

SOLUTION Let the required numbers be x and $(27 - x)$. Then,

$x(27 - x) = 182$

$\Rightarrow \quad 27x - x^2 = 182 \Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow \quad x^2 - 13x - 14x + 182 = 0$

$\Rightarrow \quad x(x-13) - 14(x-13) = 0$

$\Rightarrow \quad (x-13)(x-14) = 0$

$\Rightarrow \quad x - 13 = 0$ or $x - 14 = 0$

$\Rightarrow \quad x = 13$ or $x = 14$.

Hence, the required numbers are 13 and 14.

EXAMPLE 2 *The sum of the squares of two consecutive odd numbers is* 394. *Find the numbers.* [CBSE 2014]

SOLUTION Let the required consecutive odd numbers be x and $(x + 2)$. Then,

$x^2 + (x+2)^2 = 394$

$\Rightarrow \quad 2x^2 + 4x - 390 = 0 \Rightarrow x^2 + 2x - 195 = 0$

$\Rightarrow \quad x^2 + 15x - 13x - 195 = 0$

$\Rightarrow \quad x(x+15) - 13(x+15) = 0$

$\Rightarrow \quad (x+15)(x-13) = 0$

$\Rightarrow \quad x + 15 = 0 \text{ or } x - 13 = 0$

$\Rightarrow \quad x = -15 \text{ or } x = 13$

$\Rightarrow \quad x = 13. \quad$ [rejecting $x = -15$]

Hence, the required numbers are 13 and 15.

EXAMPLE 3 *The sum of the squares of two consecutive even numbers is* 340. *Find the numbers.* [CBSE 2014]

SOLUTION Let the required consecutive even numbers be x and $(x + 2)$.

Then,

$x^2 + (x+2)^2 = 340$

$\Rightarrow \quad 2x^2 + 4x - 336 = 0 \Rightarrow x^2 + 2x - 168 = 0$

$\Rightarrow \quad x^2 + 14x - 12x - 168 = 0$

$\Rightarrow \quad x(x+14) - 12(x+14) = 0$

$\Rightarrow \quad (x+14)(x-12) = 0$

$\Rightarrow \quad x + 14 = 0 \text{ or } x - 12 = 0$

$\Rightarrow \quad x = -14 \text{ or } x = 12$

$\Rightarrow \quad x = 12 \quad$ [rejecting $x = -14$]

Hence, the required numbers are 12 and 14.

EXAMPLE 4 *The sum of the squares of two consecutive multiples of* 7 *is* 637. *Find the multiples.* [CBSE 2014]

SOLUTION Let the required consecutive multiples of 7 be $7x$ and $7(x+1)$.

Then, $(7x)^2 + \{7(x+1)\}^2 = 637$

$\Rightarrow \quad 49x^2 + (7x+7)^2 = 637 \Rightarrow 98x^2 + 98x - 588 = 0$

$\Rightarrow \quad x^2 + x - 6 = 0 \Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow \quad x(x+3)-2(x+3)=0 \Rightarrow (x+3)(x-2)=0$

$\Rightarrow \quad x+3=0$ or $x-2=0$

$\Rightarrow \quad x=-3$ or $x=2$

$\Rightarrow \quad x=2$ [neglecting $x=-3$]

Hence, the required numbers are (7×2) and (7×3), i.e., 14 and 21.

EXAMPLE 5 *The sum of two natural numbers is* 9 *and the sum of their reciprocals is* $\frac{1}{2}\cdot$ *Find the numbers.* [CBSE 2012]

SOLUTION Let the required natural numbers be x and $(9-x)$. Then,

$$\frac{1}{x}+\frac{1}{(9-x)}=\frac{1}{2}$$

$\Rightarrow \quad \dfrac{(9-x)+x}{x(9-x)}=\dfrac{1}{2}$

$\Rightarrow \quad x(9-x)=18$ [by cross multiplication]

$\Rightarrow \quad x^2-9x+18=0 \Rightarrow x^2-6x-3x+18=0$

$\Rightarrow \quad x(x-6)-3(x-6)=0 \Rightarrow (x-6)(x-3)=0$

$\Rightarrow \quad x-6=0$ or $x-3=0 \Rightarrow x=6$ or $x=3$.

Hence, the required natural numbers are 6 and 3.

EXAMPLE 6 *The difference of two natural numbers is* 5 *and the difference of their reciprocals is* $\frac{1}{10}\cdot$ *Find the numbers.* [CBSE 2014]

SOLUTION Let the required natural numbers be x and $(x-5)$. Then,

$$x>x-5 \Rightarrow \frac{1}{x}<\frac{1}{x-5} \Rightarrow \frac{1}{x-5}>\frac{1}{x}\cdot$$

$\therefore \quad \dfrac{1}{x-5}-\dfrac{1}{x}=\dfrac{1}{10}$

$\Rightarrow \quad \dfrac{x-(x-5)}{(x-5)x}=\dfrac{1}{10} \Rightarrow \dfrac{5}{(x-5)x}=\dfrac{1}{10}$ [by cross multiplication]

$\Rightarrow \quad (x-5)x=50 \Rightarrow x^2-5x-50=0$

$\Rightarrow \quad x^2-10x+5x-50=0 \Rightarrow x(x-10)+5(x-10)=0$

$\Rightarrow \quad (x-10)(x+5)=0$

$\Rightarrow \quad x-10=0$ or $x+5=0$

$\Rightarrow \quad x=10$ or $x=-5$

$\Rightarrow \quad x=10$ [$\because$ -5 is not a natural number]

Hence, the required natural numbers are 10 and 5.

EXAMPLE 7 *The difference of squares of two numbers is* 180. *The square of the smaller number is* 8 *times the larger number. Find the two numbers.* [CBSE 2012]

SOLUTION Let the required numbers be x and y such that $x > y$.

Then, $x^2 - y^2 = 180.$ … (i)

And, $y^2 = 8x.$ … (ii)

From (i) and (ii), we get

$x^2 - 8x - 180 = 0$

$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0 \Rightarrow x(x-18) + 10(x-18) = 0$

$\Rightarrow \quad (x-18)(x+10) = 0 \Rightarrow x - 18 = 0 \text{ or } x + 10 = 0$

$\Rightarrow \quad x = 18 \text{ or } x = -10.$

Now, $x = 18 \Rightarrow y^2 = (8 \times 18) = 144$

$\Rightarrow y = 12 \text{ or } y = -12.$

Also, $x = -10 \Rightarrow y^2 = [8 \times (-10)] = -80,$ which is not possible.

Hence, the numbers are (18 and 12) or (18 and –12).

EXAMPLE 8 *The numerator of a fraction is* 3 *less than its denominator. If* 2 *is added to both of its numerator and denominator then the sum of the new fraction and original fraction is* $\frac{29}{20}\cdot$ *Find the original fraction.* [CBSE 2015]

SOLUTION Let the denominator of the required fraction be x.

Then, its numerator $= (x-3)$.

So, the original fraction is $\frac{(x-3)}{x}$.

$\therefore \quad \frac{(x-3)+2}{x+2} + \frac{(x-3)}{x} = \frac{29}{20}$

$\Rightarrow \quad \frac{(x-1)}{(x+2)} + \frac{(x-3)}{x} = \frac{29}{20}$

$\Rightarrow \quad \frac{x(x-1) + (x-3)(x+2)}{(x+2)x} = \frac{29}{20}$

$\Rightarrow \quad \frac{(x^2-x) + (x^2-x-6)}{x^2+2x} = \frac{29}{20}$

$\Rightarrow \quad 20(2x^2 - 2x - 6) = 29(x^2 + 2x)$

$\Rightarrow \quad 40x^2 - 40x - 120 = 29x^2 + 58x$

$\Rightarrow \quad 11x^2 - 98x - 120 = 0$

$\Rightarrow \quad 11x^2 - 110x + 12x - 120 = 0$

$\Rightarrow \quad 11x(x-10)+12(x-10)=0$

$\Rightarrow \quad (x-10)(11x+12)=0$

$\Rightarrow \quad x-10=0 \text{ or } 11x+12=0$

$\Rightarrow \quad x=10 \text{ or } x=\frac{-12}{11}$

$\Rightarrow \quad x=10$ [$\because$ denominator of a fraction is never negative]

$\therefore$ denominator = 10 and numerator $=(10-3)=7$.

Hence, the required fraction is $\frac{7}{10}$.

EXAMPLE 9 *The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is* $2\frac{16}{21}$, *find the fraction.*

SOLUTION Let the numerator of the required fraction be x.

Then, its denominator $=(2x+1)$.

$\therefore$ fraction $=\frac{x}{(2x+1)}$ and its reciprocal $=\frac{(2x+1)}{x}$.

$\therefore \quad \frac{x}{(2x+1)}+\frac{(2x+1)}{x}=\frac{58}{21}$

$\Rightarrow \quad 21\times[x^2+(2x+1)^2]=58x(2x+1)$

$\Rightarrow \quad 21\times[5x^2+4x+1]=116x^2+58x$

$\Rightarrow \quad 11x^2-26x-21=0$

$\Rightarrow \quad 11x^2-33x+7x-21=0 \Rightarrow 11x(x-3)+7(x-3)=0$

$\Rightarrow \quad (x-3)(11x+7)=0 \Rightarrow x-3=0 \text{ or } 11x+7=0$

$\Rightarrow \quad x=3 \text{ or } x=\frac{-7}{11}$

$\therefore \quad x=3$ [$\because$ numerator cannot be a negative fraction].

$\therefore$ required fraction $=\frac{x}{(2x+1)}=\frac{3}{7}$.

EXAMPLE 10 *A two-digit number is 5 times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number.*

SOLUTION Let the tens and units digits of the required number be x and y respectively. Then,

$$10x+y=5(x+y) \Rightarrow 4y=5x \Rightarrow y=\frac{5x}{4} \qquad \dots \text{(i)}$$

$$10x+y=2xy+5 \Rightarrow 10x+\frac{5x}{4}=2x\times\frac{5x}{4}+5 \qquad \text{[using (i)]}$$

$$\Rightarrow \frac{45x}{4}=\frac{10x^2}{4}+5 \Rightarrow 10x^2-45x+20=0$$

$$\Rightarrow 2x^2-9x+4=0 \Rightarrow 2x^2-8x-x+4=0$$

$$\Rightarrow 2x(x-4)-(x-4)=0$$
$$\Rightarrow (x-4)(2x-1)=0$$
$$\Rightarrow x-4=0 \text{ or } 2x-1=0$$
$$\Rightarrow x=4 \text{ or } x=\frac{1}{2}$$
$$\Rightarrow x=4 \quad [\because \text{ a digit cannot be a fraction}].$$

Putting $x=4$ in (i), we get $y=5$.

$\therefore \quad x=4$ and $y=5$.

Hence, the required number is 45.

EXAMPLE 11 *A two-digit number is such that the product of its digits is* 18. *When* 63 *is subtracted from the number, the digits interchange their places. Find the number.* [CBSE 2006C]

SOLUTION Let the tens and units digits of the required number be x and y respectively. Then,

$$xy=18 \Rightarrow y=\frac{18}{x} \quad \ldots \text{(i)}$$

And, $(10x+y)-63=10y+x$

$$\Rightarrow \quad 9x-9y=63 \Rightarrow x-y=7 \quad \ldots \text{(ii)}$$

Putting $y=\frac{18}{x}$ from (i) into (ii), we get

$$x-\frac{18}{x}=7$$
$$\Rightarrow \quad x^2-18=7x \Rightarrow x^2-7x-18=0$$
$$\Rightarrow \quad x^2-9x+2x-18=0 \Rightarrow x(x-9)+2(x-9)=0$$
$$\Rightarrow \quad (x-9)(x+2)=0 \Rightarrow x-9=0 \text{ or } x+2=0$$
$$\Rightarrow \quad x=9 \text{ or } x=-2$$
$$\Rightarrow \quad x=9 \quad [\because \text{ a digit cannot be negative}].$$

Putting $x=9$ in (i), we get $y=2$.

Thus, the tens digit is 9 and the units digit is 2.

Hence, the required number is 92.

GENERAL PROBLEMS ON MONEY MATTERS

EXAMPLE 12 *A person on tour has* ₹ 4200 *for his expenses. If he extends his tour for* 3 *days, he has to cut down his daily expenses by* ₹ 70. *Find the original duration of the tour.* [CBSE 2008C]

SOLUTION Let the original duration of the tour be x days. Then,

$$\frac{4200}{x} - \frac{4200}{(x+3)} = 70$$

$$\Rightarrow \quad 4200 \times \left[\frac{1}{x} - \frac{1}{(x+3)}\right] = 70 \Rightarrow \frac{(x+3)-x}{x(x+3)} = \frac{70}{4200}$$

$$\Rightarrow \quad x(x+3) = 180 \Rightarrow x^2 + 3x - 180 = 0$$

$$\Rightarrow \quad x^2 + 15x - 12x - 180 = 0 \Rightarrow x(x+15) - 12(x+15) = 0$$

$$\Rightarrow \quad (x+15)(x-12) = 0 \Rightarrow x + 15 = 0 \text{ or } x - 12 = 0$$

$$\Rightarrow \quad x = -15 \text{ or } x = 12$$

$$\Rightarrow \quad x = 12 \quad [\because \text{ number of days cannot be negative}].$$

$\therefore$ original duration of the tour is 12 days.

EXAMPLE 13 *A bookseller buys a number of books for ₹ 1760. If he had bought 4 more books for the same amount, each book would have cost ₹ 22 less. How many books did he buy?*

SOLUTION Let the bookseller buy x books for ₹ 1760.

Then, cost of each book = ₹ $\frac{1760}{x}$.

Again, cost of $(x + 4)$ books = ₹ 1760

$\therefore$ cost of each book, now = ₹ $\frac{1760}{(x+4)}$.

$$\therefore \quad \frac{1760}{x} - \frac{1760}{(x+4)} = 22$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{(x+4)} = \frac{22}{1760} \Rightarrow \frac{(x+4)-x}{x(x+4)} = \frac{1}{80}$$

$$\Rightarrow \quad \frac{4}{(x^2+4x)} = \frac{1}{80} \Rightarrow x^2 + 4x = 320$$

$$\Rightarrow \quad x^2 + 4x - 320 = 0 \Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow \quad x(x+20) - 16(x+20) = 0 \Rightarrow (x+20)(x-16) = 0$$

$$\Rightarrow \quad x + 20 = 0 \text{ or } x - 16 = 0 \Rightarrow x = -20 \text{ or } x = 16$$

$$\Rightarrow \quad x = 16 \quad [\because \text{ number of books cannot be negative}]$$

Hence, the bookseller bought 16 books.

EXAMPLE 14 *Some students planned a picnic. The total budget for hiring a bus was ₹ 1440. Later on, eight of these refused to go and instead paid their total share of money towards the fee of one economically weaker student of their class, and thus, the cost for each member who went for picnic, increased by ₹ 30.*

(i) How many students attended the picnic?

(ii) How much money in total was paid towards the fee? Which value is reflected in this question? [CBSE 2013C]

SOLUTION Let x students planned the picnic.

Then, $(x-8)$ students attended the picnic.

Total bus charges = ₹ 1440.

$$\therefore \quad \frac{1440}{(x-8)} - \frac{1440}{x} = 30$$

$$\Rightarrow \quad \frac{1}{(x-8)} - \frac{1}{x} = \frac{30}{1440} \Rightarrow \frac{x-(x-8)}{(x-8)x} = \frac{1}{48}$$

$$\Rightarrow \quad \frac{8}{(x^2-8x)} = \frac{1}{48} \Rightarrow x^2 - 8x = 384 \quad \text{[by cross multiplication]}$$

$$\Rightarrow \quad x^2 - 8x - 384 = 0 \Rightarrow x^2 - 24x + 16x - 384 = 0$$

$$\Rightarrow \quad x(x-24) + 16(x-24) = 0 \Rightarrow (x-24)(x+16) = 0$$

$$\Rightarrow \quad x - 24 = 0 \text{ or } x + 16 = 0 \Rightarrow x = 24 \text{ or } x = -16$$

$$\Rightarrow \quad x = 24 \quad [\because \text{ number of students cannot be negative}]$$

Thus, 24 students planned the picnic.

(i) Number of students who attended the picnic = (24 – 8) = 16.

(ii) Share of 24 students = ₹ 1440

Share of 8 students = ₹ $\left(\frac{1440}{24} \times 8\right)$ = ₹ 480.

$\therefore$ money paid towards the fee = ₹ 480.

The value reflected in the given question is '*charity*'.

EXAMPLE 15 *The total cost of a certain length of a piece of wire is ₹ 200. If the piece was 5 metres longer and each metre of wire costs ₹ 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?* [CBSE 2015]

SOLUTION Let the original length of the piece of wire be x metres.

Cost of this piece = ₹ 200.

$\therefore$ its original rate per metre = ₹ $\frac{200}{x}$.

New length = $(x+5)$ metres.

New cost per metre = ₹ $\frac{200}{(x+5)}$.

$$\therefore \quad \frac{200}{x} - \frac{200}{(x+5)} = 2$$

$\Rightarrow \quad \frac{1}{x}-\frac{1}{(x+5)}=\frac{2}{200} \Rightarrow \frac{(x+5)-x}{x(x+5)}=\frac{1}{100}$

$\Rightarrow \quad \frac{5}{(x^2+5x)}=\frac{1}{100} \Rightarrow x^2+5x=500$

$\Rightarrow \quad x^2+5x-500=0 \Rightarrow x^2+25x-20x-500=0$

$\Rightarrow \quad x(x+25)-20(x+25)=0 \Rightarrow (x+25)(x-20)=0$

$\Rightarrow \quad x+25=0$ or $x-20=0 \Rightarrow x=-25$ or $x=20$

$\Rightarrow \quad x=20$ [$\because$ length cannot be negative]

$\therefore$ original length of wire = 20 m.

Original rate = ₹ $\frac{200}{20}$ per m = ₹ 10 per m.

EXAMPLE 16 *₹ 6500 were divided equally among a certain number of persons. Had there been 15 more persons, each would have got ₹ 30 less. Find the original number of persons.*

SOLUTION Let the original number of persons be x.

Total amount to be divided = ₹ 6500.

Share of each = ₹ $\frac{6500}{x}$.

New number of persons = $(x + 15)$.

Now, share of each = ₹ $\frac{6500}{(x+15)}$.

$\therefore \quad \frac{6500}{x}-\frac{6500}{(x+15)}=30$

$\Rightarrow \quad \frac{1}{x}-\frac{1}{(x+15)}=\frac{30}{6500} \Rightarrow \frac{(x+15)-x}{x(x+15)}=\frac{3}{650}$

$\Rightarrow \quad \frac{15}{(x^2+15x)}=\frac{3}{650} \Rightarrow 3(x^2+15x)=9750$

$\Rightarrow \quad x^2+15x=3250 \Rightarrow x^2+15x-3250=0$

$\Rightarrow \quad x^2+65x-50x-3250=0 \Rightarrow x(x+65)-50(x+65)=0$

$\Rightarrow \quad (x+65)(x-50)=0 \Rightarrow x+65=0$ or $x-50=0$

$\Rightarrow \quad x=-65$ or $x=50$

$\Rightarrow \quad x=50$ [$\because$ number of persons cannot be negative]

Hence, the original number of persons is 50.

PROBLEMS ON AGES

EXAMPLE 17 *A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.* [CBSE 2010]

SOLUTION Let the present age of her sister be x years.

Then, girl's present age = $2x$ years.

Product of their ages 4 years hence $= (x+4)(2x+4)$.

$\therefore \quad (x+4)(2x+4) = 160$

$\Rightarrow \quad 2x^2 + 12x - 144 = 0 \Rightarrow x^2 + 6x - 72 = 0$

$\Rightarrow \quad x^2 + 12x - 6x - 72 = 0 \Rightarrow x(x+12) - 6(x+12) = 0$

$\Rightarrow \quad (x+12)(x-6) = 0 \Rightarrow x+12 = 0$ or $x - 6 = 0$

$\Rightarrow \quad x = -12$ or $x = 6$

$\Rightarrow \quad x = 6$ [$\because$ the age cannot be negative]

$\therefore$ sister's present age = 6 years and girl's present age = 12 years.

EXAMPLE 18 *The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be* 4 *years more than three times the age of his son. Find their present ages.* [CBSE 2009C]

SOLUTION Let the present age of the son be x years.

Then, the present age of the man is $(2x^2)$ years.

Age of the son 8 years hence = $(x + 8)$ years.

Age of the man 8 years hence $(2x^2 + 8)$ years.

$\therefore \quad (2x^2 + 8) = 3(x+8) + 4$

$\Rightarrow \quad 2x^2 - 3x - 20 = 0 \Rightarrow 2x^2 - 8x + 5x - 20 = 0$

$\Rightarrow \quad 2x(x-4) + 5(x-4) = 0 \Rightarrow (x-4)(2x+5) = 0$

$\Rightarrow \quad x - 4 = 0$ or $2x + 5 = 0$

$\Rightarrow \quad x = 4$ or $x = \frac{-5}{2}$

$\Rightarrow \quad x = 4$ [$\because$ age cannot be negative].

$\therefore$ son's present age = 4 years, and

man's present age $= (2 \times 4^2)$ years = 32 years.

EXAMPLE 19 *The sum of the reciprocals of Arun's ages (in years)* 3 *years ago and five years from now is* $\frac{1}{3}\cdot$ *Find his present age.*

SOLUTION Let Arun's present age be x years.

Arun's age 3 years ago = $(x - 3)$ years.

Arun's age 5 years hence = $(x + 5)$ years.

$$\therefore \quad \frac{1}{(x-3)} + \frac{1}{(x+5)} = \frac{1}{3}$$

$\Rightarrow \quad \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow (x-3)(x+5) = 6(x+1)$

$\Rightarrow \quad x^2 + 2x - 15 = 6x + 6 \Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0 \Rightarrow x(x-7) + 3(x-7) = 0$

$\Rightarrow \quad (x-7)(x+3) = 0 \Rightarrow x - 7 = 0$ or $x + 3 = 0$

$\Rightarrow \quad x = 7$ or $x = -3$

$\Rightarrow \quad x = 7$ [$\because$ age cannot be negative]

Hence, Arun's present age is 7 years.

EXAMPLE 20 *The sum of the ages of a man and his son is* 45 *years. Five years ago, the product of their ages (in years) was* 124. *Find their present ages.*

SOLUTION Let the present age of the man be x years.

Then, the present age of the son = $(45 - x)$ years.

Age of the man 5 years ago = $(x - 5)$ years.

Age of the son 5 years ago = $(45 - x - 5)$ years = $(40 - x)$ years.

$\therefore \quad (x-5)(40-x) = 124$

$\Rightarrow \quad -x^2 + 45x - 200 = 124 \Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow \quad x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x-36) - 9(x-36) = 0$

$\Rightarrow \quad (x-36)(x-9) = 0 \Rightarrow x - 36 = 0$ or $x - 9 = 0$

$\Rightarrow \quad x = 36$ or $x = 9$.

But, the man's age cannot be 9 years. So, we reject $x = 9$.

Hence, the man's present age = 36 years.

And, the son's present age = $(45 - 36)$ years = 9 years.

EXAMPLE 21 7 *years ago Varun's age was five times the square of Swati's age.* 3 *years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.* [CBSE 2006C]

SOLUTION Let Swati's age 7 years ago be x years.

Then, Varun's age 7 years ago $= 5x^2$ years.

Swati's present age = $(x + 7)$ years.

Varun's present age $= (5x^2 + 7)$ years.

Swati's age 3 years hence = $(x + 7 + 3)$ years = $(x + 10)$ years.

Varun's age 3 years hence $= (5x^2 + 7 + 3)$ years

$= (5x^2 + 10)$ years.

$\therefore \quad (x+10) = \frac{2}{5}(5x^2+10)$

$\Rightarrow \quad 5x+50 = 10x^2+20 \Rightarrow 10x^2-5x-30=0$

$\Rightarrow \quad 2x^2-x-6=0 \Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow \quad 2x(x-2)+3(x-2)=0 \Rightarrow (x-2)(2x+3)=0$

$\Rightarrow \quad x-2=0 \text{ or } 2x+3=0 \Rightarrow x=2 \text{ or } x=\frac{-3}{2}$

$\Rightarrow \quad x=2$ [$\because$ age cannot be negative]

$\therefore$ Swati's present age = (2 + 7) years = 9 years.

Varun's present age $= (5 \times 2^2 + 7)$ years = 27 years.

PROBLEMS ON SPEED AND TIME

EXAMPLE 22 *A bus travels at a certain average speed for a distance of* 75 km *and then travels a distance of* 90 km *at an average speed of* 10 km/hr *more than the first speed. If it takes* 3 *hours to complete the total journey, find its original speed.* [CBSE 2015]

SOLUTION Let the original speed of the bus be x km/hr.

Time taken to cover 75 km $= \frac{75}{x}$ hours.

New speed = $(x + 10)$ km/hr.

Time taken to cover 90 km with new speed $= \frac{90}{(x+10)}$ hours.

Total time taken to cover the whole journey = 3 hours.

$\therefore \quad \frac{75}{x} + \frac{90}{(x+10)} = 3$

$\Rightarrow \quad \frac{75(x+10)+90x}{x(x+10)} = 3 \Rightarrow 165x+750 = 3x^2+30x$

$\Rightarrow \quad 3x^2-135x-750=0 \Rightarrow x^2-45x-250=0$

$\Rightarrow \quad x^2-50x+5x-250=0 \Rightarrow x(x-50)+5(x-50)=0$

$\Rightarrow \quad (x-50)(x+5)=0 \Rightarrow x-50=0 \text{ or } x+5=0$

$\Rightarrow \quad x=50 \text{ or } x=-5 \Rightarrow x=50$ [$\because$ speed cannot be negative]

Hence, the original speed of the bus was 50 km/hr.

EXAMPLE 23 *In a flight of* 2800 km, *an aircraft was slowed down due to bad weather. Its average speed is reduced by* 100 km/hr *and time of flight increased by* 30 *minutes. Find the original duration of the flight.* [CBSE 2012]

SOLUTION Let the original speed of the aircraft be x km/hr.

Time taken to cover 2800 km $= \frac{2800}{x}$ hours.

Reduced speed $= (x - 100)$ km/hr.

Time taken to cover 2800 km at this speed $= \frac{2800}{(x-100)}$ hours.

$\therefore \quad \frac{2800}{(x-100)} - \frac{2800}{x} = \frac{30}{60}$

$\Rightarrow \quad \frac{1}{(x-100)} - \frac{1}{x} = \frac{1}{2 \times 2800} \Rightarrow \frac{x-(x-100)}{(x-100)x} = \frac{1}{5600}$

$\Rightarrow \quad \frac{100}{(x^2-100x)} = \frac{1}{5600} \Rightarrow x^2 - 100x - 560000 = 0$

$\Rightarrow \quad x^2 - 800x + 700x - 560000 = 0$

$\Rightarrow \quad x(x-800) + 700(x-800) = 0$

$\Rightarrow \quad (x-800)(x+700) = 0$

$\Rightarrow \quad x - 800 = 0$ or $x + 700 = 0$

$\Rightarrow \quad x = 800$ or $x = -700$

$\Rightarrow \quad x = 800$ [$\because$ speed cannot be negative]

$\therefore$ original speed of the aircraft = 800 km/hr.

original duration of the flight $= \frac{2800}{800}$ hours

= 3 hours 30 minutes.

EXAMPLE 24 *An aeroplane left* 30 *minutes later than its scheduled time and in order to reach its destination* 1500 km *away in time, it had to increase its speed by* 250 km/hr *from its usual speed. Determine its usual speed.* [CBSE 2005C]

SOLUTION Let the usual speed be x km/hr.

Actual speed = $(x + 250)$ km/hr.

Time taken at usual speed $= \left(\frac{1500}{x}\right)$ hr.

Time taken at actual speed $= \left(\frac{1500}{x+250}\right)$ hr.

Difference between the two times taken $= \frac{1}{2}$ hr.

$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$\Rightarrow \quad \frac{1}{x} - \frac{1}{(x+250)} = \frac{1}{3000} \Rightarrow \frac{(x+250)-x}{x(x+250)} = \frac{1}{3000}$

$\Rightarrow \quad \frac{250}{(x^2+250x)} = \frac{1}{3000} \Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow \quad x(x+1000) - 750(x+1000) = 0 \Rightarrow (x+1000)(x-750) = 0$

$\Rightarrow \quad x + 1000 = 0 \text{ or } x - 750 = 0 \Rightarrow x = -1000 \text{ or } x = 750$

$\Rightarrow \quad x = 750$ [$\because$ speed cannot be negative]

Hence, the usual speed of the aeroplane was 750 km/hr.

EXAMPLE 25 *If a man walks* 1 km/hr *faster than his usual speed then he covers a distance of* 3 km *in* 15 *minutes less time. Find his usual speed.* [CBSE 2014]

SOLUTION Let the usual speed of the man be x km/hr.

Time taken to cover 3 km at usual speed $= \frac{3}{x}$ hours.

Actual speed of the man = $(x + 1)$ km/hr.

Time taken to cover 3 km at actual speed $= \frac{3}{(x+1)}$ hours.

$\therefore \quad \frac{3}{x} - \frac{3}{(x+1)} = \frac{15}{60} \Rightarrow \frac{3}{x} - \frac{3}{(x+1)} = \frac{1}{4}$

$\Rightarrow \quad \frac{1}{x} - \frac{1}{(x+1)} = \frac{1}{12} \Rightarrow \frac{(x+1)-x}{x(x+1)} = \frac{1}{12}$

$\Rightarrow \quad \frac{1}{(x^2+x)} = \frac{1}{12} \Rightarrow x^2 + x - 12 = 0$

$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0 \Rightarrow x(x+4) - 3(x+4) = 0$

$\Rightarrow \quad (x+4)(x-3) = 0 \Rightarrow x + 4 = 0 \text{ or } x - 3 = 0$

$\Rightarrow \quad x = -4 \text{ or } x = 3 \Rightarrow x = 3$ [$\because$ speed cannot be negative]

Hence, the usual speed of the man is 3 km/hr.

EXAMPLE 26 *A motorboat whose speed in still water is* 24 km/hr*, takes* 1 *hour more to go* 32 km *upstream than to return downstream to the same spot. Find the speed of the stream.* [CBSE 2014]

SOLUTION Speed of the motorboat in still water = 24 km/hr.

Let the speed of the stream be x km/hr.

Then, speed upstream = $(24 - x)$ km/hr.

Speed downstream = $(24 + x)$ km/hr.

Time taken to go 32 km upstream $= \frac{32}{(24-x)}$ hours.

Time taken to return 32 km downstream $= \frac{32}{(24+x)}$ hours.

$\therefore \quad \frac{32}{(24-x)} - \frac{32}{(24+x)} = 1$

$\Rightarrow \quad \dfrac{1}{(24-x)} - \dfrac{1}{(24+x)} = \dfrac{1}{32} \Rightarrow \dfrac{(24+x)-(24-x)}{(24-x)(24+x)} = \dfrac{1}{32}$

$\Rightarrow \quad \dfrac{2x}{(576-x^2)} = \dfrac{1}{32} \Rightarrow 576 - x^2 = 64x$

$\Rightarrow \quad x^2 + 64x - 576 = 0 \Rightarrow x^2 + 72x - 8x - 576 = 0$

$\Rightarrow \quad x(x+72) - 8(x+72) = 0 \Rightarrow (x+72)(x-8) = 0$

$\Rightarrow \quad x + 72 = 0$ or $x - 8 = 0$

$\Rightarrow \quad x = -72$ or $x = 8$

$\Rightarrow \quad x = 8$ [$\because$ speed of the stream cannot be negative]

Hence, the speed of the stream is 8 km/hr.

EXAMPLE 27 *A sailor can row a boat* 8 km *downstream and return back to the starting point in* 1 *hour* 40 *minutes. If the speed of the stream is* 2 km/hr, *find the speed of the boat in still water.*

SOLUTION Let the speed of the boat in still water be x km/hr.

Speed of the stream = 2 km/hr.

$\therefore$ speed downstream = $(x + 2)$ km/hr,

speed upstream = $(x - 2)$ km/hr.

Time taken to cover 8 km downstream and return back to the starting point $= \dfrac{8}{(x+2)} + \dfrac{8}{(x-2)}\cdot$ But, this time is given as $1\dfrac{40}{60}$ hours $= 1\dfrac{2}{3}$ hours $= \dfrac{5}{3}$ hours.

$\therefore \quad \dfrac{8}{x+2} + \dfrac{8}{x-2} = \dfrac{5}{3}$

$\Rightarrow \quad \dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{5}{24} \Rightarrow \dfrac{(x-2)+(x+2)}{(x+2)(x-2)} = \dfrac{5}{24}$

$\Rightarrow \quad \dfrac{2x}{(x^2-4)} = \dfrac{5}{24} \Rightarrow 5x^2 - 20 = 48x$

$\Rightarrow \quad 5x^2 - 48x - 20 = 0 \Rightarrow 5x^2 - 50x + 2x - 20 = 0$

$\Rightarrow \quad 5x(x-10) + 2(x-10) = 0 \Rightarrow (x-10)(5x+2) = 0$

$\Rightarrow \quad x - 10 = 0$ or $5x + 2 = 0$

$\Rightarrow \quad x = 10$ or $x = \dfrac{-2}{5}$

$\Rightarrow \quad x = 10$ [$\because$ speed of the boat cannot be negative]

Hence, the speed of the boat in still water is 10 km/hr.

PROBLEMS ON TIME AND WORK

EXAMPLE 28 *A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.* [CBSE 2017]

SOLUTION Suppose B alone takes x days to finish the work and A alone can finish it in $(x - 6)$ days.

B's 1 day's work $= \frac{1}{x}$.

A's 1 day's work $= \frac{1}{(x-6)}$.

(A + B)'s 1 day's work $= \frac{1}{4}$.

$\therefore \quad \frac{1}{x} + \frac{1}{(x-6)} = \frac{1}{4}$

$\Rightarrow \quad \frac{(x-6)+x}{x(x-6)} = \frac{1}{4} \Rightarrow \frac{(2x-6)}{(x^2-6x)} = \frac{1}{4}$

$\Rightarrow \quad x^2 - 6x = 8x - 24 \Rightarrow x^2 - 14x + 24 = 0$

$\Rightarrow \quad x^2 - 12x - 2x + 24 = 0 \Rightarrow x(x-12) - 2(x-12) = 0$

$\Rightarrow \quad (x-12)(x-2) = 0 \Rightarrow x - 12 = 0 \text{ or } x - 2 = 0$

$\Rightarrow \quad x = 12 \text{ or } x = 2$

$\Rightarrow \quad x = 12 \quad [\because \ x = 2 \Rightarrow (x-6) < 0]$

Hence, B alone can finish the work in 12 days.

EXAMPLE 29 *Two water taps together can fill a tank in* $9\frac{3}{8}$ *hours. The tap of larger diameter takes* 10 *hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.* [CBSE 2008]

SOLUTION Let the smaller tap fill the tank in x hours.

Then, the larger tap fills it in $(x - 10)$ hours.

Time taken by both together to fill the tank $= \frac{75}{8}$ hours.

Part filled by the smaller tap in 1 hr $= \frac{1}{x}$.

Part filled by the larger tap in 1 hr $= \frac{1}{(x-10)}$.

Part filled by both the taps in 1 hr $= \frac{8}{75}$.

$\therefore \quad \frac{1}{x} + \frac{1}{(x-10)} = \frac{8}{75}$

$\Rightarrow \quad \dfrac{(x-10)+x}{x(x-10)} = \dfrac{8}{75} \Rightarrow \dfrac{(2x-10)}{x(x-10)} = \dfrac{8}{75}$

$\Rightarrow \quad 75(2x-10) = 8x(x-10)$ [by cross multiplication]

$\Rightarrow \quad 150x - 750 = 8x^2 - 80x$

$\Rightarrow \quad 8x^2 - 230x + 750 = 0 \Rightarrow 4x^2 - 115x + 375 = 0$

$\Rightarrow \quad 4x^2 - 100x - 15x + 375 = 0 \Rightarrow 4x(x-25) - 15(x-25) = 0$

$\Rightarrow \quad (x-25)(4x-15) = 0 \Rightarrow x - 25 = 0 \text{ or } 4x - 15 = 0$

$\Rightarrow \quad x = 25 \text{ or } x = \dfrac{15}{4}$

$\Rightarrow \quad x = 25 \quad \left[\because \; x = \dfrac{15}{4} \Rightarrow (x-10) < 0\right].$

Hence, the time taken by the smaller tap to fill the tank = 25 hours.

And, the time taken by the larger tap to fill the tank

= (25 –10) hours = 15 hours.

PROBLEMS ON AREAS

EXAMPLE 30 *The perimeter of a rectangular field is* 82 m *and its area is* 400 m^2. *Find the dimensions of the field.*

SOLUTION We know that 2(length + breadth) = perimeter.

$\therefore$ (length + breadth) $= \dfrac{1}{2} \times$ perimeter $= \dfrac{1}{2} \times 82$ m = 41 m.

Let the length of the field be x metres.

Then, its breadth = $(41 - x)$ m.

$\therefore$ area of the field $= x(41-x)\text{ m}^2 = (41x - x^2)\text{ m}^2$.

But, area $= 400\text{ m}^2$ (given)

$\therefore \quad 41x - x^2 = 400$

$\Rightarrow \quad x^2 - 41x + 400 = 0 \Rightarrow x^2 - 25x - 16x + 400 = 0$

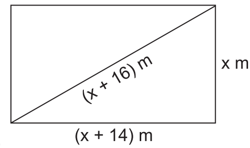
$\Rightarrow \quad x(x-25) - 16(x-25) = 0 \Rightarrow (x-25)(x-16) = 0$

$\Rightarrow \quad x - 25 = 0 \text{ or } x - 16 = 0 \Rightarrow x = 25 \text{ or } x = 16.$

$\therefore$ length of the field = 25 m and breadth of the field = 16 m.

EXAMPLE 31 *The diagonal of a rectangular field is* 16 m *more than the shorter side. If the longer side is* 14 m *more than the shorter side then find the lengths of the sides of the field.* [CBSE 2015]

SOLUTION Let the shorter side of the field be x metres.



Then, longer side $= (x+14)$ m.

And, diagonal $= (x+16)$ m.

$\therefore \quad (x+16)^2 - (x+14)^2 = x^2$

$\Rightarrow \quad (2x+30)\times 2 = x^2 \Rightarrow x^2 - 4x - 60 = 0$

$\Rightarrow \quad x^2 - 10x + 6x - 60 = 0 \Rightarrow x(x-10) + 6(x-10) = 0$

$\Rightarrow \quad (x-10)(x+6) = 0 \Rightarrow x - 10 = 0$ or $x + 6 = 0$

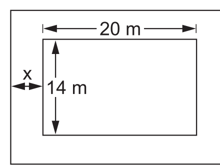
$\Rightarrow \quad x = 10$ or $x = -6$

$\Rightarrow \quad x = 10$ [$\because$ breadth cannot be negative]

$\therefore$ breadth = 10 m and length = (10 + 14) m = 24 m.

EXAMPLE 32 *A rectangular field is* 20 m *long and* 14 m *wide. There is a path of equal width all around it, having an area of* 111 sq m. *Find the width of the path.*

SOLUTION Let the width of the path be x metres.



Length of the field including the path

$= (20 + 2x)$ m.

Breadth of the field including the path

$= (14 + 2x)$ m.

Area of the field including the path $= (20+2x)(14+2x)\ \text{m}^2$.

Area of the field excluding the path $= (20\times 14)\ \text{m}^2 = 280\ \text{m}^2$.

$\therefore$ the area of the path $= [(20+2x)(14+2x) - 280]\ \text{m}^2$.

$\therefore \quad (20+2x)(14+2x) - 280 = 111$

$\Rightarrow \quad 4x^2 + 68x - 111 = 0 \Rightarrow 4x^2 + 74x - 6x - 111 = 0$

$\Rightarrow \quad 2x(2x+37) - 3(2x+37) = 0 \Rightarrow (2x+37)(2x-3) = 0$

$\Rightarrow \quad x = -\dfrac{37}{2}$ or $x = \dfrac{3}{2}$

$\Rightarrow \quad x = \dfrac{3}{2} = 1.5$ [$\because$ width can never be negative].

Hence, the width of the path is 1.5 m.

EXAMPLE 33 *Sum of the areas of two squares is* $260\ \text{m}^2$. *If the difference of their perimeters is* 24 m *then find the sides of the two squares.*

[CBSE 2009C]

SOLUTION Let the sides of the two squares be a metres and b metres.

Then, their areas are $(a^2)\ \text{m}^2$ and $(b^2)\ \text{m}^2$ respectively.

And, their perimeters are $(4a)$ m and $(4b)$ m respectively.

$\therefore \quad 4a - 4b = 24 \Rightarrow 4(a-b) = 24$

$\Rightarrow a - b = 6 \Rightarrow b = (a-6) \qquad \ldots \text{(i)}$

Sum of their areas $= 260\ \text{m}^2$.

$\therefore \quad a^2 + b^2 = 260$

$\Rightarrow \quad a^2 + (a-6)^2 = 260 \quad$ [using (i)]

$\Rightarrow \quad 2a^2 - 12a - 224 = 0 \Rightarrow a^2 - 6a - 112 = 0$

$\Rightarrow \quad a^2 - 14a + 8a - 112 = 0$

$\Rightarrow \quad a(a-14) + 8(a-14) = 0 \Rightarrow (a-14)(a+8) = 0$

$\Rightarrow \quad a - 14 = 0$ or $a + 8 = 0 \Rightarrow a = 14$ or $a = -8$

$\Rightarrow \quad a = 14 \quad$ [$\because$ side of a square cannot be negative].

$\therefore \quad a = 14$ and $b = (14-6) = 8$.

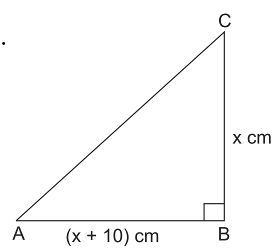
Hence, the sides of the square are 14 m and 8 m respectively.

EXAMPLE 34 *The area of a right-angled triangle is* 600 sq cm. *If the base of the triangle exceeds the altitude by* 10 cm, *find the dimensions of the triangle.*

SOLUTION Let the altitude of the triangle be x cm.

Then, its base $= (x+10)$ cm.

Area of the triangle $= \frac{1}{2}x(x+10)\ \text{cm}^2$.



$\therefore \quad \frac{1}{2}x(x+10) = 600$

$\Rightarrow \quad x(x+10) = 1200$

$\Rightarrow \quad x^2 + 10x - 1200 = 0$

$\Rightarrow \quad x^2 + 40x - 30x - 1200 = 0$

$\Rightarrow \quad x(x+40) - 30(x+40) = 0$

$\Rightarrow \quad (x+40)(x-30) = 0$

$\Rightarrow \quad x = -40$ or $x = 30$

$\Rightarrow \quad x = 30 \quad$ [$\because$ altitude cannot be negative].

Thus, the altitude of the triangle = 30 cm.

And, the base of the triangle = (30 + 10) cm = 40 cm.

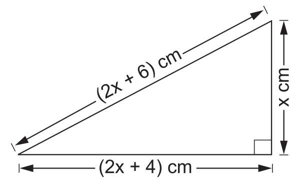
EXAMPLE 35 *The hypotenuse of a right-angled triangle is* 6 cm *more than twice the shortest side. If the third side is* 2 cm *less than the hypotenuse, find the sides of the triangle.* [CBSE 2007]

SOLUTION Let the shortest side of the triangle be x cm.

Then, its hypotenuse = $(2x + 6)$ cm.

And, its third side = $(2x + 6 - 2)$ cm = $(2x + 4)$ cm.

By Pythagoras' theorem, we have



$$(2x + 6)^2 = x^2 + (2x + 4)^2$$

$\Rightarrow \quad 4x^2 + 24x + 36 = 5x^2 + 16x + 16$

$\Rightarrow \quad x^2 - 8x - 20 = 0$

$\Rightarrow \quad x^2 - 10x + 2x - 20 = 0$

$\Rightarrow \quad x(x - 10) + 2(x - 10) = 0 \Rightarrow (x - 10)(x + 2) = 0$

$\Rightarrow \quad x - 10 = 0 \text{ or } x + 2 = 0 \Rightarrow x = 10 \text{ or } x = -2$

$\Rightarrow \quad x = 10$ [$\because$ the side of a triangle cannot be negative]

$\therefore$ shorter side = 10 cm, longer side $= (2 \times 10 + 4)$ cm = 24 cm

and hypotenuse $= (2 \times 10 + 6)$ cm = 26 cm.

Hence, the sides of the given triangle are 10 cm, 24 cm and 26 cm.

EXERCISE 4E

PROBLEMS ON NUMBERS

1. The sum of a natural number and its square is 156. Find the number.
2. The sum of a natural number and its positive square root is 132. Find the number.
3. The sum of two natural numbers is 28 and their product is 192. Find the numbers.
4. The sum of the squares of two consecutive positive integers is 365. Find the integers.
5. The sum of the squares of two consecutive positive odd numbers is 514. Find the numbers.
6. The sum of the squares of two consecutive positive even numbers is 452. Find the numbers.
7. The product of two consecutive positive integers is 306. Find the integers.
8. Two natural numbers differ by 3 and their product is 504. Find the numbers.
9. Find two consecutive multiples of 3 whose product is 648.
10. Find two consecutive positive odd integers whose product is 483.
11. Find two consecutive positive even integers whose product is 288.

12. The sum of two natural numbers is 9 and the sum of their reciprocals is $\frac{1}{2}\cdot$ Find the numbers. [CBSE 2012]
13. The sum of two natural numbers is 15 and the sum of their reciprocals is $\frac{3}{10}\cdot$ Find the numbers. [CBSE 2005]
14. The difference of two natural numbers is 3 and the difference of their reciprocals is $\frac{3}{28}\cdot$ Find the numbers. [CBSE 2014]
15. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{5}{14}\cdot$ Find the numbers. [CBSE 2014]
16. The sum of the squares of two consecutive multiples of 7 is 1225. Find the multiples.
17. The sum of a natural number and its reciprocal is $\frac{65}{8}\cdot$ Find the number.
18. Divide 57 into two parts whose product is 680.
19. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}\cdot$
20. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.
21. Find two natural numbers, the sum of whose squares is 25 times their sum and also equal to 50 times their difference.
22. The difference of the squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers. [CBSE 2007]
23. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46. Find the integers. [CBSE 2010]
24. A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.
25. A two-digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number. [CBSE 2012]
26. The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\frac{9}{10}\cdot$ Find the fraction.
27. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}\cdot$ Find the fraction. [CBSE 2012]

28. The sum of a number and its reciprocal is $2\frac{1}{30}\cdot$ Find the number.

SOME GENERAL PROBLEMS

29. A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were left. When he increased the size of the square by one student, he found that he was short of 25 students. Find the number of students.
30. 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.
31. In a class test, the sum of Kamal's marks in mathematics and English is 40. Had he got 3 marks more in mathematics and 4 marks less in English, the product of the marks would have been 360. Find his marks in two subjects separately. [CBSE 2008C]
32. Some students planned a picnic. The total budget for food was ₹ 2000. But, 5 students failed to attend the picnic and thus the cost for food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food? [CBSE 2010]
33. If the price of a book is reduced by ₹ 5, a person can buy 4 more books for ₹ 600. Find the original price of the book.
34. A person on tour has ₹ 10800 for his expenses. If he extends his tour by 4 days, he has to cut down his daily expenses by ₹ 90. Find the original duration of the tour.
35. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately. [CBSE 2008]
36. A man buys a number of pens for ₹ 180. If he had bought 3 more pens for the same amount, each pen would have cost him ₹ 3 less. How many pens did he buy?
37. A dealer sells an article for ₹ 75 and gains as much per cent as the cost price of the article. Find the cost price of the article. [CBSE 2011]

PROBLEMS ON AGES

38. (i) One year ago, a man was 8 times as old as his son. Now, his age is equal to the square of his son's age. Find their present ages.

(ii) A man is $3\frac{1}{2}$ times as old as his son. If the sum of the squares of their ages is 1325, find the ages of the father and the son. [CBSE 2017]

39. The sum of the reciprocals of Meena's ages (in years) 3 years ago and 5 years hence is $\frac{1}{3}\cdot$ Find her present age.

40. The sum of the ages of a boy and his brother is 25 years, and the product of their ages in years is 126. Find their ages.

41. The product of Tanvy's age (in years) 5 years ago and her age 8 years later is 30. Find her present age.

42. Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.

PROBLEMS ON TIME AND DISTANCE

43. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck. [CBSE 2015]

44. While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalise the injured and so the plane started late by 30 minutes. To reach the destination, 1500 km away, in time, the pilot increased the speed by 100 km/hour. Find the original speed of the plane.

    Do you appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time? [CBSE 2013]

45. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less then it would have taken 3 hours more to cover the same distance. Find the usual speed of the train. [CBSE 2013C]

46. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed? [CBSE 2015]

47. A train travels 180 km at a uniform speed. If the speed had been 9 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train. [CBSE 2013]

48. A train covers a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km/hour, it takes 2 hours less in the journey. Find the original speed of the train. [CBSE 2017]
49. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find its usual speed. [CBSE 2007]
50. The distance between Mumbai and Pune is 192 km. Travelling by the Deccan Queen, it takes 48 minutes less than another train. Calculate the speed of the Deccan Queen if the speeds of the two trains differ by 20 km/hr. [CBSE 2003]
51. A motor boat whose speed in still water is 18 km/hr, takes 1 hour more to go 24 km upstream than to return to the same spot. Find the speed of the stream. [CBSE 2014]
52. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream. [CBSE 2017]
53. A motorboat whose speed is 9 km/hr in still water, goes 15 km downstream and comes back in a total time of 3 hours 45 minutes. Find the speed of the stream. [CBSE 2007C]

PROBLEMS ON TIME AND WORK AND PIPES AND CISTERN

54. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.
55. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank then how much time will each tap take to fill the tank? [CBSE 2017]
56. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. [CBSE 2010]
57. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. [CBSE 2011]

PROBLEMS ON AREA AND GEOMETRY

58. The length of a rectangle is twice its breadth and its area is 288 cm^2. Find the dimensions of the rectangle.

59. The length of a rectangular field is three times its breadth. If the area of the field be 147 sq metres, find the length of the field.
60. The length of a hall is 3 metres more than its breadth. If the area of the hall is 238 sq metres, calculate its length and breadth.
61. The perimeter of a rectangular plot is 62 m and its area is 228 sq metres. Find the dimensions of the plot.
62. A rectangular field is 16 m long and 10 m wide. There is a path of uniform width all around it, having an area of 120 m^2. Find the width of the path.
63. The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares. [CBSE 2008]
64. The length of a rectangle is thrice as long as the side of a square. The side of the square is 4 cm more than the width of the rectangle. Their areas being equal, find their dimensions.
65. A farmer prepares a rectangular vegetable garden of area 180 sq metres. With 39 metres of barbed wire, he can fence the three sides of the garden, leaving one of the longer sides unfenced. Find the dimensions of the garden.
66. The area of a right triangle is 600 cm^2. If the base of the triangle exceeds the altitude by 10 cm, find the dimensions of the triangle.
67. The area of a right-angled triangle is 96 sq metres. If the base is three times the altitude, find the base.
68. The area of a right-angled triangle is 165 sq metres. Determine its base and altitude if the latter exceeds the former by 7 metres.
69. The hypotenuse of a right-angled triangle is 20 metres. If the difference between the lengths of the other sides be 4 metres, find the other sides.
70. The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.
71. The hypotenuse of a right-angled triangle is 1 metre less than twice the shortest side. If the third side is 1 metre more than the shortest side, find the sides of the triangle.

ANSWERS (EXERCISE 4E)

1. 12	**2.** 121	**3.** 16 and 12	**4.** 13 and 14	**5.** 15 and 17
6. 14 and 16	**7.** 17 and 18	**8.** 24 and 21	**9.** 24 and 27	**10.** 21 and 23
11. 16 and 18	**12.** 3 and 6	**13.** 10 and 5	**14.** 7 and 4	**15.** 7 and 2

16. 21 and 28 **17.** 8 **18.** 17 and 40 **19.** 12 and 15

20. 10 and 6 **21.** 30 and 10 **22.** 9 and 6 **23.** 4, 5, 6

24. 36 **25.** 27 **26.** $\frac{2}{5}$ **27.** $\frac{2}{5}$ **28.** $\frac{5}{6}$ or $\frac{6}{5}$ **29.** 600 **30.** 50

31. (21 and 19) or (12 and 28) **32.** 20, ₹ 100 **33.** ₹ 30 **34.** 20 days

35. Either he got 12 marks in maths and 16 in science or he got 9 marks in maths and 19 in science

36. 12 pens **37.** ₹ 50

38. (i) 49 years, 7 years (ii) 35 years, 10 years **39.** 7 years

40. 18 years and 7 years **41.** 7 years **42.** 29 years, 5 years

43. 60 km/hr **44.** 500 km/hr **45.** 40 km/hr **46.** 36 km/hr

47. 36 km/hr **48.** 25 km/hr **49.** 25 km/hr **50.** 80 km/hr

51. 6 km/hr **52.** 5 km/hr **53.** 3 km/hr **54.** 30 days

55. 5 hours, 8 hours **56.** 20 min, 25 min **57.** 9 hours, 18 hours

58. length = 24 cm, breadth = 12 cm **59.** 21 m

60. breadth = 14 m, length = 17 m **61.** length = 19 m, breadth = 12 m

62. 2 m **63.** 24 m, 8 m **64.** (l = 18 cm, b = 2 cm), side = 6 cm

65. Either (l = 24 m, b = 7.5 m) or (l = 15 m, b = 12 m)

66. 30 cm, 40 cm, 50 cm **67.** 24 m **68.** base = 15 m, altitude = 22 m

69. 16 m, 12 m **70.** base = 15 cm, altitude = 8 cm, hypotenuse = 17 cm

71. 3 m, 4 m, 5 m

HINTS TO SOME SELECTED QUESTIONS

1. $x + x^2 = 156 \Rightarrow x^2 + x - 156 = 0 \Rightarrow x^2 + 13x - 12x - 156 = 0.$

2. $x + \sqrt{x} = 132 \Rightarrow y^2 + y = 132$, where $\sqrt{x} = y$

   $\therefore \quad y^2 + y - 132 = 0 \Rightarrow y^2 + 12y - 11y - 132 = 0.$

3. Let the required numbers be x and $(28 - x)$.

   Then, $x(28 - x) = 192.$

4. $x^2 + (x+1)^2 = 365 \Rightarrow 2x^2 + 2x - 364 = 0 \Rightarrow x^2 + x - 182 = 0$

   $\therefore \quad x^2 + 14x - 13x - 182 = 0.$

5. $x^2 + (x+2)^2 = 514 \Rightarrow 2x^2 + 4x - 510 = 0 \Rightarrow x^2 + 2x - 255 = 0$

   $\Rightarrow x^2 + 17x - 15x - 255 = 0.$

6. $x^2 + (x+2)^2 = 452 \Rightarrow 2x^2 + 4x - 448 = 0 \Rightarrow x^2 + 2x - 224 = 0$

   $\Rightarrow x^2 + 16x - 14x - 224 = 0.$

7. $x(x+1)=306 \Rightarrow x^2+x-306=0 \Rightarrow x^2+18x-17x-306=0.$

8. $x(x-3)=504 \Rightarrow x^2-3x-504=0 \Rightarrow x^2-24x+21x-504=0.$

9. $(3x)\{3(x+1)\}=648 \Rightarrow x(x+1)=72 \Rightarrow x^2+9x-8x-72=0.$

10. $x(x+2)=483 \Rightarrow x^2+2x-483=0 \Rightarrow x^2+23x-21x-483=0.$

11. $x(x+2)=288 \Rightarrow x^2+2x-288=0 \Rightarrow x^2+18x-16x-288=0.$

14. Let the required numbers be x and $(x-3)$. Then,

$$\frac{1}{(x-3)}-\frac{1}{x}=\frac{3}{28}.$$

18. $x(57-x)=680 \Rightarrow x^2-57x+680=0 \Rightarrow x^2-40x-17x+680=0.$

19. $\frac{1}{x}+\frac{1}{(27-x)}=\frac{3}{20} \Rightarrow \frac{27-x+x}{x(27-x)}=\frac{3}{20} \Rightarrow x^2-27x+180=0$

$$\Rightarrow x^2-15x-12x+180=0.$$

20. Let the two parts be x and $(16-x)$ such that x is larger. Then,

$$2x^2-(16-x)^2=164 \Rightarrow x^2+32x-420=0 \Rightarrow x^2+42x-10x-420=0.$$

21. Let the required natural numbers be a and b. Then,

$a^2+b^2=25(a+b)$ and $(a^2+b^2)=50(a-b)$.

$\therefore$ $25(a+b)=50(a-b) \Rightarrow a+b=2(a-b) \Rightarrow a=3b.$

$\therefore$ $(3b)^2+b^2=25\times 4b \Rightarrow 10b^2-100b=0$

$$\Rightarrow 10b(b-10)=0 \Rightarrow b=10.$$

22. Let the required natural numbers be a and b. Then,

$$(a^2-b^2=45 \text{ and } b^2=4a) \Rightarrow (a^2-4a-45)=0.$$

23. Let the required numbers be x, $(x+1)$ and $(x+2)$. Then,

$$x^2+(x+1)(x+2)=46 \Rightarrow 2x^2+3x-44=0 \Rightarrow 2x^2+11x-8x-44=0$$

$$\Rightarrow (2x+11)(x-4).$$

$x\neq\frac{-11}{2}$, since x is an integer.

24. Let the tens digit be x and units digit be y. Then,

$(10x+y)=4(x+y) \Rightarrow 6x=3y \Rightarrow y=2x.$

$(10x+y)=2xy \Rightarrow (10x+2x)=4x^2 \Rightarrow 4x^2-12x=0$

$$\Rightarrow 4x(x-3)=0 \Rightarrow x=3 \quad [\because\ x\neq 0].$$

25. Let the tens digit be x and units digit be y. Then,

$xy=14$ and $10x+y+45=10y+x$

$\Rightarrow$ $xy=14$ and $9(y-x)=45 \Rightarrow xy=14$ and $y-x=5$

$\Rightarrow$ $x(x+5)=14$ and $y=(5+x)$.

26. Let the required fraction be $\frac{a}{(a+3)}\cdot$ Then,

$$\frac{a}{(a+3)}+\frac{(a+3)}{a}=\frac{29}{10} \Rightarrow 10[a^2+(a+3)^2]=29a(a+3)$$

$$\Rightarrow 9a^2+27a-90=0 \Rightarrow a^2+3a-10=0.$$

27. Let the required fraction in simplest form be $\frac{x-3}{x}\cdot$ Then,

$$\frac{x-3}{x}-\frac{x-3}{x+1}=\frac{1}{15} \Rightarrow (x-3)\left[\frac{1}{x}-\frac{1}{x+1}\right]=\frac{1}{15}$$

$\therefore \quad 15(x-3)=x(x+1) \Rightarrow x^2-14x+45=0 \Rightarrow x=5 \text{ or } x=9.$

So, the required fraction is $\frac{2}{5}\cdot$ $\left[\text{neglecting } \frac{6}{9}\right]$

28. Let the required number be x. Then, $x+\frac{1}{x}=\frac{61}{30}\cdot$

$\therefore \quad 30x^2-61x+30=0 \Rightarrow 30x^2-36x-25x+30=0.$

29. Let there be x rows and the number of students in each row be x.

Then, total number of students $=(x^2+24)$.

$$x^2+24=(x+1)^2-25.$$

30. Let the total number of students be x. Then,

$$\frac{300}{x}-\frac{300}{(x+10)}=1 \Rightarrow x^2+10x-3000=0$$

$$\Rightarrow (x-50)(x+60)=0 \Rightarrow x=50.$$

31. Suppose he gets x marks in mathematics and y marks in English. Then,

$$x+y=40 \text{ and } (x+3)(y-4)=360.$$

$$(x+3)(40-x-4)=360 \Rightarrow (x+3)(36-x)=360$$

$\Rightarrow \quad x^2-33x+252=0 \Rightarrow (x-21)(x-12)=0.$

32. Let x students attended the picnic. Then, $(x+5)$ planned it.

$\therefore \quad \frac{2000}{x}-\frac{2000}{x+5}=20 \Rightarrow \frac{1}{x}-\frac{1}{x+5}=\frac{20}{2000}=\frac{1}{100} \Rightarrow x^2+5x-500=0.$

$\therefore \quad x^2+25x-20x-500=0.$

33. Let the original price per book be ₹ x. Then, reduced price = ₹ $(x-5)$.

$\therefore \quad \frac{600}{x-5}-\frac{600}{x}=4 \Rightarrow \frac{1}{x-5}-\frac{1}{x}=\frac{4}{600}=\frac{1}{150}\cdot$

34. Let the original duration of the tour be x days. Then,

$$\frac{10800}{x}-\frac{10800}{(x+4)}=90 \Rightarrow \frac{1}{x}-\frac{1}{x+4}=\frac{1}{120} \Rightarrow x^2+4x-480=0$$

$\therefore \quad x^2+24x-20x-480=0.$

35. Suppose P gets x marks in mathematics and y marks in science. Then,

$$x+y=28 \Rightarrow y=28-x.$$

And $(x+3)(y-4)=180 \Rightarrow (x+3)(28-x-4)=180$

$\therefore \quad (x+3)(24-x)=180 \Rightarrow x^2-21x+108=0.$

36. Suppose he bought x pens.

$\therefore \quad \frac{180}{x}-\frac{180}{(x+3)}=3 \Rightarrow \frac{1}{x}-\frac{1}{x+3}=\frac{1}{60} \Rightarrow x(x+3)-180=0$

$\therefore \quad x^2+15x-12x-180=0.$

37. Let the CP of the article be ₹ x. Then, gain = x%.

$\therefore \quad \text{SP} = ₹ \left\{\frac{(100+x)x}{100}\right\} = ₹ \left\{\frac{x^2+100x}{100}\right\}$

$\Rightarrow \quad \dfrac{x^2+100x}{100} = 75 \Rightarrow x^2+100x-7500=0 \Rightarrow x^2+150x-50x-7500=0.$

38. (i) Let the son's present age be x years.

Son's age 1 year ago = $(x - 1)$ years.

Man's age 1 year ago = $8(x - 1)$ years = $(8x - 8)$ years.

Man's present age = $(8x - 8 + 1)$ years = $(8x - 7)$ years.

$\therefore \quad (8x-7) = x^2 \Rightarrow x^2 - 8x + 7 = 0.$

39. Let Meena's present age be x years. Then,

$$\frac{1}{(x-3)} + \frac{1}{(x+5)} = \frac{1}{3} \Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow x^2+2x-15 = 3(2x+2)$$

$\therefore \quad x^2 - 4x - 21 = 0.$

40. $x(25-x) = 126 \Rightarrow x^2 - 25x + 126 = 0 \Rightarrow x^2 - 18x - 7x + 126 = 0.$

41. Let Tanvy's present age be x years.

Then, $(x-5)(x+8) = 30 \Rightarrow x^2+3x-70=0 \Rightarrow x^2+10x-7x-70=0.$

42. Let son's age 2 years ago be x years.

Then, man's age 2 years ago $= 3x^2$.

$\therefore \quad 3x^2+5 = 4(x+5) \Rightarrow 3x^2-4x-15=0 \Rightarrow 3x^2-9x+5x-15=0 \Rightarrow x=3.$

Son's age $= (x+2)$ years = 5 years, man's age $= (3x^2+2)$ years = 29 years.

43. Let the first speed be x km/hr. Then,

$$\frac{150}{x} + \frac{200}{(x+20)} = 5 \Rightarrow 50\left[\frac{3}{x} + \frac{4}{(x+20)}\right] = 5$$

$\Rightarrow \quad 10[3(x+20)+4x] = x(x+20) \Rightarrow x^2-50x-600=0.$

44. Let the original speed of the plane be x km/hr. Then,

$$\frac{1500}{x} - \frac{1500}{(x+100)} = \frac{30}{60} \Rightarrow \frac{1}{x} - \frac{1}{(x+100)} = \frac{1}{3000} \Rightarrow x^2+100x-300000=0$$

$\Rightarrow \quad x^2+600x-500x-300000=0.$

His promptness is appreciable.

45. Let the usual speed of the train be x km/hr. Then,

$$\frac{480}{(x-8)} - \frac{480}{x} = 3 \Rightarrow \frac{1}{x-8} - \frac{1}{x} = \frac{3}{480} = \frac{1}{160}.$$

$\Rightarrow \quad 160[x-(x-8)] = x(x-8) \Rightarrow x^2-8x-1280=0$

$\Rightarrow \quad x^2-40x+32x-1280=0.$

46. Let the first speed be x km/hr. Then,

$$\frac{54}{x} + \frac{63}{(x+6)} = 3 \Rightarrow 9\left[\frac{6}{x} + \frac{7}{(x+6)}\right] = 3$$

$\Rightarrow \quad 3[6(x+6)+7x] = x(x+6) \Rightarrow x(x+6) = 3(13x+36)$

$\Rightarrow \quad x^2-33x-108=0.$

47. Let the speed be x km/hr. Then,

$$\frac{180}{x} - \frac{180}{(x+9)} = 1$$

$\Rightarrow \quad x^2+9x-1620=0 \Rightarrow x^2+45x-36x-1620=0.$

49. $\frac{300}{x}-\frac{300}{x+5}=2 \Rightarrow \frac{1}{x}-\frac{1}{x+5}=\frac{1}{150} \Rightarrow x^2+5x-750=0.$

$\Rightarrow \quad x^2+30x-25x-750=0.$

50. Let the speed of the Deccan queen be x km/hr. Then,

$$\frac{192}{(x-20)}-\frac{192}{x}=\frac{48}{60} \Rightarrow \frac{1}{x-20}-\frac{1}{x}=\frac{1}{240}.$$

$\Rightarrow \quad x^2-20x-4800=0 \Rightarrow x^2-80x+60x-4800=0.$

51. Let the speed of the stream be x km/hr.

Then, speed downstream = $(18 + x)$ km/hr

and speed upstream = $(18 - x)$ km/hr.

$\therefore \quad \frac{24}{(18-x)}-\frac{24}{(18+x)}=1 \Rightarrow \frac{1}{(18-x)}-\frac{1}{(18+x)}=\frac{1}{24}$

$\Rightarrow \quad x^2+48x-324=0 \Rightarrow x^2+54x-6x-324=0.$

52. Let the speed of the stream be x km/hr.

Time to go 30 km upstream $=\frac{30 \text{ km}}{(15-x) \text{ km}}$ hr $=\frac{30}{(15-x)}$ hr.

Time to go 30 km downstream $=\frac{30 \text{ km}}{(15+x) \text{ km}}$ hr $=\frac{30}{(15+x)}$ hr.

$\therefore \quad \frac{30}{(15-x)}+\frac{30}{(15+x)}=4\frac{1}{2}=\frac{9}{2}$

$\Rightarrow \quad 30(15+x)+30(15-x)=\frac{9}{2}(225-x^2)$

$\Rightarrow \quad 450+30x+450-30x=\frac{9}{2}(225-x^2)$

$\Rightarrow \quad 200=225-x^2 \Rightarrow x^2=25 \Rightarrow x=5.$

53. Let the speed of the stream be x km/hr.

Then, speed downstream = $(9 + x)$ km/hr

and speed upstream = $(9 - x)$ km/hr.

$\therefore \quad \frac{15}{9+x}+\frac{15}{9-x}=3\frac{45}{60}=3\frac{3}{4}=\frac{15}{4}$

$\Rightarrow \quad \frac{1}{9+x}+\frac{1}{9-x}=\frac{1}{4} \Rightarrow 4[(9-x)+(9+x)]=(9+x)(9-x)$

$\Rightarrow \quad 81-x^2=72 \Rightarrow x^2=9 \Rightarrow x=3.$

54. Suppose B takes x days to finish the work. Then, A takes $(x - 10)$ days.

$\therefore \quad \frac{1}{x}+\frac{1}{(x-10)}=\frac{1}{12} \Rightarrow x^2-34x+120=0 \Rightarrow x^2-30x-4x+120=0.$

55. Let the faster pipe take x hr to fill it. Then, the other takes $(x + 3)$ hr.

$\therefore \quad \frac{1}{x}+\frac{1}{(x+3)}=\frac{13}{40} \Rightarrow \frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$

$\Rightarrow \quad 13x^2-41x-120=0$

$\Rightarrow \quad x=\frac{41 \pm \sqrt{1681+6240}}{26}=\frac{41 \pm \sqrt{7921}}{26}=\frac{(41 \pm 89)}{26} \Rightarrow x=5.$

56. Let the faster pipe take x min to fill it. Then, the other takes $(x + 5)$ min.

$\therefore \quad \frac{1}{x}+\frac{1}{(x+5)}=\frac{9}{100} \Rightarrow 100(2x+5)=9(x^2+5x)$

$\Rightarrow \quad 9x^2-155x-500=0 \Rightarrow 9x^2-180x+25x-500=0.$

HINT Quadratic formula may be used.

57. Let the faster tap take x hours to fill the tank.

Then, the slower tap takes $(x + 9)$ hours to fill it.

$\therefore \quad \frac{1}{x}+\frac{1}{(x+9)}=\frac{1}{6} \Rightarrow 6(2x+9)=x(x+9)$

$\Rightarrow \quad x^2-3x-54=0 \Rightarrow x^2-9x+6x-54=0.$

58. $2x \times x = 288 \Rightarrow x^2 = 144 \Rightarrow x = 12$ cm.

59. $3x \times x = 147 \Rightarrow x^2 = 49 \Rightarrow x = 7$ m. So, length = 21 m.

60. $x(x+3)=238 \Rightarrow x^2+3x-238=0 \Rightarrow x^2+17x-14x-238=0.$

$\therefore$ breadth = 14 m and length = 17 m.

61. $2(l+b)=62 \Rightarrow l+b=31.$ Let $b = x$ m. Then, $l = (31-x)$ m

$\therefore \quad (31-x)x=228 \Rightarrow x^2-31x+228=0 \Rightarrow x^2-19x-12x+228=0.$

62. Let the width of the path be x m.

So, the length of the field with path = $(16+2x)$ m

and breadth of the field with path = $(10+2x)$ m.

$\therefore \quad (16+2x)(10+2x)-16\times 10=120$

$\Rightarrow \quad 160+52x+4x^2-160=120$

$\Rightarrow \quad 4x^2+52x-120=0 \Rightarrow x^2+13x-30=0$

$\Rightarrow \quad x^2+15x-2x-30=0 \Rightarrow x(x+15)-2(x+15)=0$

$\Rightarrow \quad (x+15)(x-2)=0 \Rightarrow x=-15$ or 2.

So, width of the path = 2 m.

63. Let the sides of the squares be a metres and b metres.

Then, $4a-4b=64 \Rightarrow 4(a-b)=64 \Rightarrow (a-b)=16.$

And, $a^2+b^2=640 \Rightarrow a^2+(a-16)^2=640$

$\Rightarrow 2a^2-32a-384=0 \Rightarrow a^2-16a-192=0$

$\Rightarrow a^2-24a+8a-192=0 \Rightarrow (a-24)(a+8)=0 \Rightarrow a=24.$

$\therefore \quad a = 24$ m, $b = 8$ m.

64. Let each side of the square be x cm.

Then, length of the rectangle = $3x$ cm.

Width of the rectangle = $(x - 4)$ cm.

$\therefore \quad 3x(x-4)=x^2 \Rightarrow 2x^2-12x=0 \Rightarrow 2x(x-6)=0 \Rightarrow x=6.$

65. Let the length be x metres. Then, breadth $=\frac{180}{x}$ m.

$\therefore \quad x+\frac{180}{x}+\frac{180}{x}=39 \Rightarrow x+\frac{360}{x}=39$

$\Rightarrow \quad x^2-39x+360=0 \Rightarrow x^2-24x-15x+360=0$

$\Rightarrow \quad (x-24)(x-15)=0 \Rightarrow x=24$ or $x=15$.

Either $(l=24 \text{ m}, b=7.5 \text{ m})$ or $(l=15 \text{ m}, b=12 \text{ m})$.

66. Let the altitude be x cm. Then, base = $(x + 10)$ cm.

$\therefore \quad \frac{1}{2}(x+10)x=600 \Rightarrow x^2+10x-1200=0$

$\Rightarrow \quad (x+40)(x-30)=0 \Rightarrow x=30.$

$\therefore$ base = 40 cm, altitude = 30 cm.

$\therefore$ hypotenuse $=\sqrt{(40)^2+(30)^2}=\sqrt{1600+900}=\sqrt{2500}=50$ cm.

67. Let the altitude be x metres. Then, base = $3x$ metres.

$\therefore \quad \frac{1}{2}\times 3x\times x=96 \Rightarrow x^2=64 \Rightarrow x=8$ m. Hence, base = 24 m.

68. Let the base be x metres. Then, altitude = $(x + 7)$ m.

$\therefore \quad \frac{1}{2}\times x\times(x+7)=165 \Rightarrow x^2+7x-330=0$

$\Rightarrow \quad x^2+22x-15x-330=0 \Rightarrow (x-15)(x+22)=0 \Rightarrow x=15.$

69. Let the other sides be x metres and $(x - 4)$ metres.

$\therefore \quad x^2+(x-4)^2=(20)^2 \Rightarrow 2x^2-8x-384=0$

$\Rightarrow \quad x^2-4x-192=0 \Rightarrow (x-16)(x+12)=0 \Rightarrow x=16.$

70. Let the base be x cm. Then, hypotenuse = $(x + 2)$ cm.

$(x+2)-(2\times \text{altitude})=1 \Rightarrow 2\times \text{altitude}=(x+1)$

$\Rightarrow \text{altitude}=\frac{1}{2}(x+1)$ cm.

$\therefore \quad (x+2)^2=x^2+\frac{1}{4}(x+1)^2 \Rightarrow 4(x+2)^2=4x^2+(x+1)^2$

$\Rightarrow \quad 4(x^2+4x+4)=5x^2+2x+1 \Rightarrow x^2-14x-15=0$

$\Rightarrow \quad (x-15)(x+1)=0 \Rightarrow x=15.$

$\therefore$ base = 15 cm, hypotenuse = 17 cm, altitude = 8 cm.

71. Let the shortest side be x metres.

Then, its hypotenuse $=(2x-1)$ m and third side $=(x+1)$ m.

$\therefore \quad (2x-1)^2=x^2+(x+1)^2 \Rightarrow 2x^2-6x=0 \Rightarrow 2x(x-3)=0.$

SUMMARY OF RESULTS

1. If α and β are the roots of a quadratic equation $ax^2+bx+c=0$ then

   (i) $\alpha+\beta=\frac{-b}{a}$, (ii) $\alpha\beta=\frac{c}{a}$.

2. The roots of the equation $ax^2+bx+c=0$ are given by

$$\alpha=\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

3. For the equation $ax^2+bx+c=0, a\neq 0$ the *discriminant* is given by $D=(b^2-4ac)$.

4. Nature of Roots of quadratic equation $ax^2 + bx + c = 0, a \neq 0$.

Value of D	Nature of Roots	
(i) $D > 0$ and D is a perfect square.	Real, unequal and rational	$\frac{-b \pm \sqrt{D}}{2a}$
(ii) $D > 0$ and D is not a perfect square.	Real, unequal and irrational	$\frac{-b \pm \sqrt{D}}{2a}$
(iii) $D = 0$	Real and equal	Each $= \frac{-b}{2a}$
(iv) $D < 0$	Imaginary	None

....

TEST YOURSELF

MCQ

1. Which of the following is a quadratic equation?

(a) $x^2 - 3\sqrt{x} + 2 = 0$ (b) $x + \frac{1}{x} = x^2$

(c) $x^2 + \frac{1}{x^2} = 5$ (d) $2x^2 - 5x = (x-1)^2$

2. Which of the following is a quadratic equation?

(a) $(x^2 + 1) = (2 - x)^2 + 3$ (b) $x^3 - x^2 = (x-1)^3$

(c) $2x^2 + 3 = (5 + x)(2x - 3)$ (d) None of these

3. Which of the following is not a quadratic equation?

(a) $3x - x^2 = x^2 + 5$ (b) $(x+2)^2 = 2(x^2 - 5)$

(c) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$ (d) $(x-1)^2 = 3x^2 + x - 2$

4. If $x = 3$ is a solution of the equation $3x^2 + (k-1)x + 9 = 0$ then $k = ?$

(a) 11 (b) –11 (c) 13 (d) –13

5. If one root of the equation $2x^2 + ax + 6 = 0$ is 2 then $a = ?$

(a) 7 (b) –7 (c) $\frac{7}{2}$ (d) $\frac{-7}{2}$

6. The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

(a) 2 (b) –2 (c) 6 (d) –6

7. If the product of the roots of the equation $x^2 - 3x + k = 10$ is –2 then the value of k is

(a) –2 (b) –8 (c) 8 (d) 12

8. The ratio of the sum and product of the roots of the equation $7x^2 - 12x + 18 = 0$ is

(a) 7 : 12 (b) 7 : 18 (c) 3 : 2 (d) 2 : 3

9. If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$ then the other root is

(a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) –3 (d) 3

10. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root then the value of k is

(a) 0 (b) 1 (c) 2 (d) 5

11. If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product then the value of k is

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

12. The roots of a quadratic equation are 5 and –2. Then, the equation is

(a) $x^2 - 3x + 10 = 0$ (b) $x^2 - 3x - 10 = 0$

(c) $x^2 + 3x - 10 = 0$ (d) $x^2 + 3x + 10 = 0$

13. If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is

(a) $x^2 - 6x + 6 = 0$ (b) $x^2 + 6x - 6 = 0$

(c) $x^2 - 6x - 6 = 0$ (d) $x^2 + 6x + 6 = 0$

14. If α and β are the roots of the equation $3x^2 + 8x + 2 = 0$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = ?$

(a) $\frac{-3}{8}$ (b) $\frac{2}{3}$ (c) –4 (d) 4

15. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal of each other if

(a) $a = b$ (b) $b = c$ (c) $c = a$ (d) none of these

16. If the roots of the equation $ax^2 + bx + c = 0$ are equal then $c = ?$

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$ (c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

17. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then $k = ?$

(a) 2 or 0 (b) –2 or 0 (c) 2 or –2 (d) 0 only

18. If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots then $k = ?$

(a) 1 or 4 (b) –1 or 4 (c) 1 or –4 (d) –1 or –4

19. If the equation $4x^2 - 3kx + 1 = 0$ has equal roots then $k = ?$

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{1}{3}$ (c) $\pm\frac{3}{4}$ (d) $\pm\frac{4}{3}$

20. The roots of $ax^2 + bx + c = 0, a \neq 0$ are real and unequal, if $(b^2 - 4ac)$ is
(a) > 0 (b) $= 0$ (c) < 0 (d) none of these

21. In the equation $ax^2 + bx + c = 0,$ it is given that $D = (b^2 - 4ac) > 0.$ Then, the roots of the equation are
(a) real and equal (b) real and unequal
(c) imaginary (d) none of these

22. The roots of the equation $2x^2 - 6x + 7 = 0$ are
(a) real, unequal and rational (b) real, unequal and irrational
(c) real and equal (d) imaginary

23. The roots of the equation $2x^2 - 6x + 3 = 0$ are
(a) real, unequal and rational (b) real, unequal and irrational
(c) real and equal (d) imaginary

24. If the roots of $5x^2 - kx + 1 = 0$ are real and distinct then
(a) $-2\sqrt{5} < k < 2\sqrt{5}$ (b) $k > 2\sqrt{5}$ only
(c) $k < -2\sqrt{5}$ only (d) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

25. If the equation $x^2 + 5kx + 16 = 0$ has no real roots then
(a) $k > \frac{8}{5}$ (b) $k < \frac{-8}{5}$
(c) $\frac{-8}{5} < k < \frac{8}{5}$ (d) none of these

26. If the equation $x^2 - kx + 1 = 0$ has no real roots then
(a) $k < -2$ (b) $k > 2$
(c) $-2 < k < 2$ (d) none of these

27. For what values of k, the equation $kx^2 - 6x - 2 = 0$ has real roots?
(a) $k \leq \frac{-9}{2}$ (b) $k \geq \frac{-9}{2}$
(c) $k \leq -2$ (d) None of these

28. The sum of a number and its reciprocal is $2\frac{1}{20}\cdot$ The number is
(a) $\frac{5}{4}$ or $\frac{4}{5}$ (b) $\frac{4}{3}$ or $\frac{3}{4}$ (c) $\frac{5}{6}$ or $\frac{6}{5}$ (d) $\frac{1}{6}$ or 6

29. The perimeter of a rectangle is 82 m and its area is 400 m^2. The breadth of the rectangle is
(a) 25 m (b) 20 m (c) 16 m (d) 9 m

30. The length of a rectangular field exceeds its breadth by 8 m and the area of the field is 240 m^2. The breadth of the field is [CBSE 2014]
(a) 20 m (b) 30 m (c) 12 m (d) 16 m

31. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are [CBSE 2012]

(a) $-2, \frac{3}{2}$ (b) $2, \frac{-3}{2}$ (c) $-2, \frac{-3}{2}$ (d) $2, \frac{3}{2}$

32. The sum of two natural numbers is 8 and their product is 15. Find the numbers. [CBSE 2012]

Very-Short-Answer Questions

33. Show that $x = -3$ is a solution of $x^2 + 6x + 9 = 0$. [CBSE 2008]

34. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$. [CBSE 2008]

35. If $x = \frac{-1}{2}$ is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k. [CBSE 2015]

36. Find the roots of the quadratic equation $2x^2 - x - 6 = 0$. [CBSE 2012]

37. Find the solution of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$. [CBSE 2009]

38. If the roots of the quadratic equation $2x^2 + 8x + k = 0$ are equal then find the value of k. [CBSE 2014]

39. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots then find the value of p. [CBSE 2015]

40. If 1 is a root of the equation $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$ then find the value of ab. [CBSE 2012]

41. If one zero of the polynomial $x^2 - 4x + 1$ is $(2 + \sqrt{3})$, write the other zero. [CBSE 2010]

42. If one root of the quadratic equation $3x^2 - 10x + k = 0$ is reciprocal of the other, find the value of k. [CBSE 2014]

43. If the roots of the quadratic equation $px(x - 2) + 6 = 0$ are equal, find the value of p. [CBSE 2013]

44. Find the values of k so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots. [CBSE 2012]

45. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots. [CBSE 2014]

Short-Answer Questions

46. Solve: $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$. [CBSE 2015]

47. Solve: $2x^2 + ax - a^2 = 0$. [CBSE 2014]

48. Solve: $3x^2 + 5\sqrt{5}x - 10 = 0$. [CBSE 2014]

49. Solve: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$. [CBSE 2014]

50. Solve: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0.$ [CBSE 2014]

51. Solve: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0.$ [CBSE 2013]

52. Solve: $4x^2 + 4bx - (a^2 - b^2) = 0.$ [CBSE 2015]

53. Solve: $x^2 + 5x - (a^2 + a - 6) = 0.$ [CBSE 2015]

54. $x^2 + 6x - (a^2 + 2a - 8) = 0$ [CBSE 2015]

55. $x^2 - 4ax + 4a^2 - b^2 = 0$ [CBSE 2012]

ANSWERS (TEST YOURSELF)

1. (d) **2.** (b) **3.** (c) **4.** (b) **5.** (b) **6.** (c) **7.** (c) **8.** (d) **9.** (d)
10. (d) **11.** (d) **12.** (b) **13.** (a) **14.** (c) **15.** (c) **16.** (d) **17.** (c) **18.** (a)
19. (d) **20.** (a) **21.** (b) **22.** (d) **23.** (b) **24.** (d) **25.** (c) **26.** (c) **27.** (b)
28. (a) **29.** (c) **30.** (c) **31.** (b) **32.** 3 and 5

35. $k = \frac{-9}{4}$ **36.** $x = 2$ or $x = \frac{-3}{2}$ **37.** $x = -\sqrt{3}$ or $x = \frac{-1}{3\sqrt{3}}$ **38.** $k = 8$

39. $p = 3$ **40.** $ab = 3$ **41.** $(2 - \sqrt{3})$ **42.** $k = 3$ **43.** $p = 6$

44. $k = 0$ or $k = \frac{1}{4}$ **45.** $k = 0$ or $k = 1$ **46.** $x = \sqrt{3}$ or $x = 1$

47. $x = -a$ or $x = \frac{a}{2}$ **48.** $x = -2\sqrt{5}$ or $x = \frac{\sqrt{5}}{3}$ **49.** $x = -4\sqrt{3}$ or $x = \frac{2}{\sqrt{3}}$

50. $x = \sqrt{6}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}$ **51.** $x = \frac{-2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$

52. $x = \frac{-(b+a)}{2}$ or $x = \frac{(a-b)}{2}$ **53.** $x = -(a+3)$ or $x = (a-2)$

54. $x = -(a+4)$ or $x = (a-2)$ **55.** $x = (2a+b)$ or $x = (2a-b)$

HINTS TO SOME SELECTED QUESTIONS

7. Given equation is $x^2 - 3x + (k - 10) = 0.$

   Product of roots $= (k - 10)$. So, $k - 10 = -2 \Rightarrow k = 8.$

8. Required ratio $= \frac{12}{7} : \frac{18}{7} = 2 : 3.$

9. Let the other root be α. Then, $\frac{1}{3} \times \alpha = \frac{3}{3} = 1 \Rightarrow \alpha = 3.$

   So, the other root is 3.

10. Let the roots be α and $\frac{1}{\alpha}\cdot$ Then,

    product of roots $= \left(\alpha \times \frac{1}{\alpha}\right) = 1.$ So, $\frac{k}{5} = 1 \Rightarrow k = 5.$

11. Sum of roots $= \frac{-2}{k}$ and product of roots $= \frac{3k}{k} = 3.$

$\therefore \quad \frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}.$

12. Sum of the roots $= 5 + (-2) = 3$, product of roots $= 5 \times (-2) = -10.$

$\therefore \quad x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0.$

Hence, $x^2 - 3x - 10 = 0.$

13. Required equation is $x^2 - 6x + 6 = 0.$

14. We have $\alpha + \beta = \frac{-8}{3}$ and $\alpha\beta = \frac{2}{3}.$

$\therefore \quad \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{(\alpha + \beta)}{\alpha\beta} = \frac{-8}{3} \times \frac{3}{2} = -4.$

15. Product of roots $= \frac{c}{a}.$ Also, $\left(\alpha \times \frac{1}{\alpha}\right) = 1.$

$\therefore \quad \frac{c}{a} = 1 \Rightarrow c = a.$

16. Since the roots are equal, we have $D = 0.$

$\therefore \quad b^2 - 4ac = 0 \Rightarrow 4ac = b^2 \Rightarrow c = \frac{b^2}{4a}.$

17. Since the roots are equal, we have $D = 0.$

$\therefore \quad 36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or $-2.$

18. Since the roots are equal, we have $D = 0.$

$\therefore \quad 4(k+2)^2 - 36k = 0 \Rightarrow (k+2)^2 - 9k = 0$

$\therefore \quad k^2 - 5k + 4 = 0 \Rightarrow k^2 - 4k - k + 4 = 0 \Rightarrow k(k-4) - (k-4) = 0$

$\Rightarrow (k-4)(k-1) = 0 \Rightarrow k = 4$ or $k = 1.$

19. Since the roots are equal, we have $D = 0.$

$\therefore \quad 9k^2 - 16 = 0 \Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \frac{4}{3}$ or $k = \frac{-4}{3}.$

20. The roots of $ax^2 + bx + c = 0, a \neq 0$ are real and unequal only when $(b^2 - 4ac) > 0.$

21. When $D > 0$, the roots of the given quadratic equation are real and unequal.

22. $D = (-6)^2 - 4 \times 2 \times 7 = (36 - 56) = -20 < 0.$

So, the roots of the given equation are imaginary.

23. Given equation is $2x^2 - 6x + 3 = 0.$

$\therefore \quad D = (-6)^2 - 4 \times 2 \times 3 = (36 - 24) = 12,$

which is greater than 0 and not a perfect square.

$\therefore$ the roots are real, unequal and irrational.

24. The roots of $5x^2 - kx + 1 = 0$ are real and distinct.

$\therefore \quad (k^2 - 4 \times 5 \times 1) > 0 \Rightarrow k^2 > 20 \Rightarrow k > \sqrt{20}$ or $k < -\sqrt{20}$

$\Rightarrow k > 2\sqrt{5}$ or $k < -2\sqrt{5}.$

25. For no real roots, we must have $b^2 - 4ac < 0.$

$\therefore \quad (25k^2 - 4 \times 16) < 0 \Rightarrow 25k^2 < 64 \Rightarrow k^2 < \frac{64}{25} \Rightarrow \frac{-8}{5} < k < \frac{8}{5}.$

26. For no real roots, we must have: $b^2 - 4ac < 0$.

$$k^2 - 4 < 0 \Rightarrow k^2 < 4 \Rightarrow -2 < k < 2.$$

27. For real roots, we must have $b^2 - 4ac \geq 0$.

$\therefore \quad (-6)^2 - 4 \times k \times (-2) \geq 0 \Rightarrow 36 + 8k \geq 0$

$$\Rightarrow 8k \geq -36 \Rightarrow k \geq \frac{-9}{2}.$$

28. Let the required number be x. Then,

$$x + \frac{1}{x} = \frac{41}{20} \Rightarrow 20x^2 - 41x + 20 = 0$$
$$\Rightarrow 20x^2 - 25x - 16x + 20 = 0$$
$$\Rightarrow 5x(4x-5) - 4(4x-5) = 0$$
$$\Rightarrow (4x-5)(5x-4) = 0$$
$$\Rightarrow x = \frac{5}{4} \text{ or } x = \frac{4}{5}.$$

29. $2(l+b) = 82 \Rightarrow l + b = 41 \Rightarrow l = (41 - b)$.

And, $lb = 400 \Rightarrow (41-b)b = 400$

$$\Rightarrow b^2 - 41b + 400 = 0 \Rightarrow b^2 - 25b - 16b + 400 = 0$$
$$\Rightarrow b(b-25) - 16(b-25) = 0$$
$$\Rightarrow (b-25)(b-16) = 0$$

$\therefore \quad b = 25$ or $b = 16$.

But $b = 25 \Rightarrow l = (41 - 25) = 16 < b$.

$\therefore$ breadth = 16 m.

30. Let the breadth of the field be x m. Then, length = $(x + 8)$ m.

$\therefore \quad (x+8) \times x = 240 \Rightarrow x^2 + 8x - 240 = 0 \Rightarrow x^2 + 20x - 12x - 240 = 0$

$$\Rightarrow x(x+20) - 12(x+20) = 0$$
$$\Rightarrow (x+20)(x-12) = 0.$$

32. $\alpha + \beta = 8$ and $\alpha\beta = 15$.

The quadratic equation whose roots are α and β is given by

$x^2 - 8x + 15 = 0$

$\Rightarrow \quad x^2 - 5x - 3x + 15 = 0$

$\Rightarrow \quad x(x-5) - 3(x-5) = 0$

$\Rightarrow \quad (x-3)(x-5) = 0$

$\therefore \quad x = 3$ or $x = 5$.

33. Putting $x = -3$ in the given equation, we get

LHS $= (-3)^2 + 6 \times (-3) + 9 = (9 - 18 + 9) = 0 =$ RHS.

Hence, $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.

34. Putting $x = -2$ in the given equation, we get

LHS $= 3 \times (-2)^2 + 13 \times (-2) + 14 = (12 - 26 + 14) = 0 =$ RHS.

Hence, $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.

35. Since $x = \frac{-1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$, we have

$$3 \times \left(\frac{-1}{2}\right)^2 + 2k \times \left(\frac{-1}{2}\right) - 3 = 0$$

$$\Rightarrow \quad \frac{3}{4} - k - 3 = 0 \Rightarrow k = \left(\frac{3}{4} - 3\right) = \frac{-9}{4}.$$

36. $2x^2 - x - 6 = 0 \Rightarrow 2x^2 - 4x + 3x - 6 = 0 \Rightarrow 2x(x-2) + 3(x-2) = 0$

$$\Rightarrow (x-2)(2x+3) = 0 \Rightarrow x = 2 \text{ or } x = \frac{-3}{2}.$$

37. $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0 \Rightarrow 3\sqrt{3}x^2 + 9x + x + \sqrt{3} = 0$

$$\Rightarrow 3\sqrt{3}x(x+\sqrt{3}) + (x+\sqrt{3}) = 0 \Rightarrow (x+\sqrt{3})(3\sqrt{3}x+1) = 0$$

$$\Rightarrow x = -\sqrt{3} \text{ or } x = \frac{-1}{3\sqrt{3}}.$$

38. $D = (8)^2 - 4 \times 2 \times k = (64 - 8k)$.

$\therefore \quad D = 0 \Rightarrow 64 - 8k = 0 \Rightarrow 8k = 64 \Rightarrow k = 8.$

39. The given equation is of the form $ax^2 + bx + c = 0$, where

$a = p$, $b = -2\sqrt{5}p$ and $c = 15$.

$\therefore \quad D = (b^2 - 4ac) = (20p^2 - 60p)$.

So, $D = 0 \Rightarrow 20p^2 - 60p = 0 \Rightarrow 20p(p-3) = 0 \Rightarrow p = 3. \quad [\because \ p \neq 0]$

40. Since 1 is a root of each of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, we have

$a \times (1)^2 + a \times 1 + 3 = 0$ and $1^2 + 1 + b = 0$

$\Rightarrow \quad a + a + 3 = 0$ and $2 + b = 0 \Rightarrow a = \frac{-3}{2}$ and $b = -2$.

$\therefore \quad ab = \left(\frac{-3}{2}\right) \times (-2) = 3.$

41. Let the other zero be α.

Sum of the zeros of $x^2 - 4x + 1$ is 4.

$\therefore \quad \alpha + (2+\sqrt{3}) = 4 \Rightarrow \alpha = 4 - (2+\sqrt{3}) = (2-\sqrt{3})$.

42. Let the roots of $3x^2 - 10x + k = 0$ be α and $\frac{1}{\alpha}$.

Then, product of roots $= \frac{k}{3}$.

$\therefore \quad \left(\alpha \times \frac{1}{\alpha}\right) = \frac{k}{3} \Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3.$

43. Given equation is $px^2 - 2px + 6 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = p$, $b = -2p$ and $c = 6$.

$\therefore \quad D = (b^2 - 4ac) = 4p^2 - 24p$.

For equal roots, we have $D = 0$.

$\therefore \quad 4p^2 - 24p = 0 \Rightarrow 4p(p-6) = 0 \Rightarrow p = 6. \quad [\because \ p \neq 0]$

44. $D = (-4k)^2 - 4k = 16k^2 - 4k = 4k(4k-1)$.

For equal roots, we must have $D = 0$.

Now, $D = 0 \Rightarrow 4k(4k-1) = 0 \Rightarrow k = 0$ or $4k - 1 = 0 \Rightarrow k = 0$ or $k = \frac{1}{4}\cdot$

45. $D = (-3k)^2 - 9k = 9k^2 - 9k = 9k(k-1)$.

For equal roots, we must have $D = 0$.

Now, $D = 0 \Rightarrow 9k(k-1) = 0 \Rightarrow k = 0$ or $k - 1 = 0 \Rightarrow k = 0$ or $k = 1$.

46. $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$\Rightarrow \quad x^2 - \sqrt{3}x - x + \sqrt{3} = 0 \Rightarrow x(x-\sqrt{3}) - (x-\sqrt{3}) = 0$

$\Rightarrow \quad (x-\sqrt{3})(x-1) = 0 \Rightarrow x = \sqrt{3}$ or $x = 1$.

47. $2x^2 + ax - a^2 = 0 \Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x+a) - a(x+a) = 0$

$\Rightarrow (x+a)(2x-a) = 0 \Rightarrow x = -a$ or $x = \frac{a}{2}\cdot$

48. $3x^2 + 5\sqrt{5}x - 10 = 0$

$\Rightarrow \quad 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$

$\Rightarrow \quad 3x(x+2\sqrt{5}) - \sqrt{5}(x+2\sqrt{5}) = 0$

$\Rightarrow \quad (x+2\sqrt{5})(3x-\sqrt{5}) = 0 \Rightarrow x = -2\sqrt{5}$ or $x = \frac{\sqrt{5}}{3}\cdot$

49. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x^2 + 12x - 2x - 8\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x(x+4\sqrt{3}) - 2(x+4\sqrt{3}) = 0$

$\Rightarrow \quad (x+4\sqrt{3})(\sqrt{3}x-2) = 0 \Rightarrow x = -4\sqrt{3}$ or $x = \frac{2}{\sqrt{3}}\cdot$

50. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x^2 - 3\sqrt{2}x - \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x(x-\sqrt{6}) - \sqrt{2}(x-\sqrt{6}) = 0$

$\Rightarrow \quad (x-\sqrt{6})(\sqrt{3}x-\sqrt{2}) = 0 \Rightarrow x = \sqrt{6}$ or $x = \frac{\sqrt{2}}{\sqrt{3}}\cdot$

51. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4x(\sqrt{3}x+2) - \sqrt{3}(\sqrt{3}x-2) = 0$

$\Rightarrow \quad (\sqrt{3}x+2)(4x-\sqrt{3}) = 0 \Rightarrow x = \frac{-2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}\cdot$

52. $4x^2 + 4bx + (b^2 - a^2) = 0$

$\Rightarrow \quad 4x^2 + 2(b+a)x + 2(b-a)x + (b^2 - a^2) = 0$

$\Rightarrow \quad 2x\{2x + (b+a)\} + (b-a)\{2x + (b+a)\} = 0$

$\Rightarrow \quad \{2x + (b+a)\}\{2x + (b-a)\} = 0$

$\Rightarrow \quad x = \frac{-(b+a)}{2}$ or $x = \frac{a-b}{0}\cdot$

53. $x^2 + 5x - (a+3)(a-2) = 0$

$\Rightarrow \quad x^2 + (a+3)x - (a-2)x - (a+3)(a-2) = 0$

$\Rightarrow \quad x\{x + (a+3)\} - (a-2)\{x + (a+3)\} = 0$

$\Rightarrow \quad \{x + (a+3)\}\{x - (a-2)\} = 0$

$\Rightarrow \quad x = -(a+3)$ or $x = (a-2)$.

54. $x^2 + 6x - (a+4)(a-2) = 0$

$\Rightarrow \quad x^2 + (a+4)x - (a-2)x - (a+4)(a-2) = 0$

$\Rightarrow \quad x\{x(a+4)\} - (a-2)\{x + (a+4)\} = 0$

$\Rightarrow \quad \{x + (a+4)\}\{x - (a-2)\} = 0$

$\Rightarrow \quad x = -(a+4)$ or $x = (a-2)$.

55. $x^2 - 4ax + (2a+b)(2a-b) = 0$

$\Rightarrow \quad x^2 - (2a+b)x - (2a-b)x + (2a+b)(2a-b) = 0$

$\Rightarrow \quad x\{x - (2a+b)\} - (2a-b)\{x - (2a+b)\} = 0$

$\Rightarrow \quad \{x - (2a+b)\}\{x - (2a-b)\} = 0$

$\Rightarrow \quad x = (2a+b)$ or $x = (2a-b)$.

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