5 Arithmetic Progression

S RAMANUJAN (1887–1920)

Srinivasa Aaiyangar Ramanujan was born in Erode, a small village near Chennai in 1887. His father was a clerk at a cloth shop and his mother was a housewife. On the basis of his good school work, he was admitted to the University of Madras. But, in first year he passed in mathematics and failed in other subjects. What earned him fame, reputation and success was his research work on numbers.

HIS RESEARCH WORK AND CONTRIBUTION ON NUMBERS

(i) In 1900, he began to work on *Geometric* and *Arithmetic Series*. He gave new methods to solve *Cubic* and *Quadratic Equations*.

(ii) He investigated the series $\Sigma\frac{1}{n}$ and found *Bernoulli's Numbers*.

(iii) In 1908, he studied *Continued Fractions* and *Divergent Series*.

(iv) In 1911, he produced a research paper on *Bernoulli's Numbers*.

In 1918, he was given a fellowship by Trinity College, Cambridge for 7 years. He was a mathematical genius, despite his lack of university education.

He died at an early age of 33 years only.

SEQUENCE *Some numbers arranged in a definite order, according to a definite rule, are said to form a sequence.*

The number occurring at the nth place of a sequence is called its nth term, denoted by T_n or a_n.

Example Consider the rule, $T_n = (2n+1)$.

Putting $n = 1, 2, 3, 4, 5, \ldots$, we get

$T_1 = 3, T_2 = 5, T_3 = 7, T_4 = 9, T_5 = 11$, and so on.

Thus, the numbers 3, 5, 7, 9, 11, … form a sequence.

In this sequence, the first term is 3, the second term is 5, the third term is 7, and so on.

ARITHMETIC PROGRESSION (AP)

A sequence in which each term differs from its preceding term by a constant is called an arithmetic progression, written as AP.

This constant is called the *common difference* of the AP.

EXAMPLE 1 *Show that the progression* 8, 11, 14, 17, 20, ... *is an AP. Find its first term and the common difference.*

SOLUTION The given progression is 8, 11, 14, 17, 20, ...

Clearly, $(11-8) = (14-11) = (17-14) = (20-17) = 3$ (constant).

Thus, each term differs from its preceding term by 3.

So, the given progression is an AP.

Its first term= 8 and common difference = 3.

EXAMPLE 2 *Show that the progression* 11, 6, 1, –4, –9, ... *is an AP. Find its first term and the common difference.*

SOLUTION Clearly, $(6-11) = (1-6) = (-4-1) = (-9+4) = -5$ (constant).

Thus, each term differs from its preceding term by –5.

So, the given progression is an AP.

Its first term = 11 and common difference = –5.

EXAMPLE 3 *Show that each of the following progressions is an AP. Find the common difference and the next term of each.*

(i) $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$ [CBSE 2014] *(ii)* $\sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots$ [CBSE 2012]

SOLUTION (i) We may write the given terms as

$$\sqrt{7}, \sqrt{4\times 7}, \sqrt{9\times 7}, \ldots$$

i.e., $\sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, \ldots$

Clearly, $2\sqrt{7}-\sqrt{7} = 3\sqrt{7}-2\sqrt{7} = \sqrt{7}$.

So, the given progression is an AP with common difference $\sqrt{7}$.

Next term is $4\sqrt{7} = \sqrt{16\times 7} = \sqrt{112}$.

(ii) We may write the given terms as

$$\sqrt{9\times 2}, \sqrt{25\times 2}, \sqrt{49\times 2}, \ldots$$

i.e., $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \ldots$.

Clearly, $(5\sqrt{2}-3\sqrt{2}) = (7\sqrt{2}-5\sqrt{2}) = 2\sqrt{2}$.

So, the given progression is an AP with common difference $2\sqrt{2}$.

It is clear that the next term is $9\sqrt{2} = \sqrt{9\times 9\times 2} = \sqrt{162}$.

EXAMPLE 4 *Find a and b such that the numbers a, 9, b, 25 form an AP.* [CBSE 2014]

SOLUTION Since the numbers a, 9, b, 25 form an AP, we have

$$9-a = b-9 = 25-b.$$

Now, $b-9 = 25-b \Rightarrow 2b = 34 \Rightarrow b = 17$.

And, $9-a = b-9 \Rightarrow a+b = 18 \Rightarrow a+17 = 18 \Rightarrow a = 1$.

Hence, $a = 1$ and $b = 17$.

ARITHMETIC SERIES

By adding the terms of an AP, we get the corresponding arithmetic series.

Example On adding the terms of the AP 3, 7, 11, 15, 19, ..., we get the arithmetic series (3 + 7 + 11 + 15 + 19 + ...).

TO FIND THE GENERAL TERM OF AN AP

THEOREM 1 *If the first term of an AP is a and its common difference is d then show that its nth term is given by*

$$T_n = a+(n-1)d.$$

PROOF In the given AP, we have

first term = a and common difference = d.

So, the given AP may be written as

$$a, a+d, a+2d, a+3d, a+4d, \ldots .$$

In this AP, we have

first term, $T_1 = a = a+(1-1)d$;

second term, $T_2 = a+d = a+(2-1)d$;

third term, $T_3 = a+2d = a+(3-1)d$;

fourth term, $T_4 = a+3d = a+(4-1)d$;

… … … … … …

… … … … … …

$\therefore$ nth term, $T_n = a + (n-1)d$.

Hence, $T_n = a + (n-1)d$.

NOTE The nth term of an AP is called its *general term*.

General term of an AP

In an AP with first term a and common difference d, the nth term is given by

$$T_n = a + (n-1)d.$$

SOLVED EXAMPLES

EXAMPLE 1 *Find the (i) nth term and (ii)* 16*th term of the AP* 3, 5, 7, 9, 11, ...

SOLUTION The given AP is 3, 5, 7, 9, 11, ...

Its first term = 3 and common difference = (5 − 3) = 2.

$\therefore$ $a = 3$ and $d = (5-3) = 2$.

(i) Its nth term is given by

$$T_n = a + (n-1)d$$

$$= 3 + (n-1) \times 2 = (2n+1) \qquad [\because a = 3 \text{ and } d = 2].$$

$\therefore$ nth term = $(2n+1)$.

(ii) 16th term of the given AP is

$$T_{16} = a + (16-1)d = (a + 15d)$$

$$= (3 + 15 \times 2) = 33 \qquad [\because a = 3 \text{ and } d = 2].$$

$\therefore$ 16th term = 33.

EXAMPLE 2 *Find the (i) nth term and (ii)* 12*th term of the AP* 14, 9, 4, –1, –6, … .

SOLUTION The given AP is 14, 9, 4, –1, –6, … .

Its first term = 14 and common difference = (9 − 14) = −5.

$\therefore$ $a = 14$ and $d = -5$.

(i) The nth term of the AP is given by

$$T_n = a + (n-1)d$$

$$= 14 + (n-1) \times (-5) = (19 - 5n) \ [\because a = 14 \text{ and } d = -5].$$

$\therefore$ nth term = $(19 - 5n)$.

(ii) 12th term of the given AP is

$$T_{12} = a + (12-1)d = (a + 11d)$$

$= 14 + 11 \times (-5) = -41$ $\quad [\because\ a = 14 \text{ and } d = -5]$.

$\therefore$ 12th term $= -41$.

EXAMPLE 3 *Find the 105th term of the AP* $4, 4\frac{1}{2}, 5, 5\frac{1}{2}, 6, \ldots$.

SOLUTION The given AP is $4, \frac{9}{2}, 5, \frac{11}{2}, 6, \ldots$.

Its first term = 4 and common difference $= \left(\frac{9}{2} - 4\right) = \frac{1}{2}$.

$\therefore \quad a = 4$ and $d = \frac{1}{2}$.

The nth term of the AP is given by

$$T_n = a + (n-1)d$$

$$\Rightarrow \quad T_{105} = a + (105 - 1)d = (a + 104d)$$

$$4 + \left(104 \times \frac{1}{2}\right) = 56 \quad \left[\because\ a = 4 \text{ and } d = \frac{1}{2}\right].$$

Hence, 105th term = 56.

EXAMPLE 4 *Find the 25th term of the AP* $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$. [CBSE 2015]

SOLUTION The given AP is $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$.

Its first term $= -5$ and common difference $= \left(\frac{5}{2} - 0\right) = \frac{5}{2}$.

$\therefore \quad a = -5$ and $d = \frac{5}{2}$.

$$\Rightarrow \quad T_{25} = a + (25 - 1)d$$

$$= a + 24d = (-5) + \left(24 \times \frac{5}{2}\right) = -5 + 60 = 55.$$

Hence, 25th term = 55.

EXAMPLE 5 *If the nth term of an AP is* $(5n - 2)$, *find its (i) first term, (ii) common difference and (iii) 19th term.*

SOLUTION $T_n = (5n - 2)$ (given)

$\Rightarrow \quad T_1 = (5 \times 1 - 2) = 3$ and $T_2 = (5 \times 2 - 2) = 8$.

Thus, we have

(i) first term = 3.

(ii) common difference $= (T_2 - T_1) = (8 - 3) = 5$.

(iii) 19th term $= a + (19-1)d$, where $a = 3$ and $d = 5$

$= (3 + 18 \times 5) = 93.$

EXAMPLE 6 *If the seventh term of an AP is* $\frac{1}{9}$ *and its ninth term is* $\frac{1}{7}$, *find its 63rd term.* [CBSE 2014]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9} \quad \ldots \text{(i)}$$

$$T_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad \ldots \text{(ii)}$$

On subtracting (i) from (ii), we get

$$2d = \left(\frac{1}{7} - \frac{1}{9}\right) = \frac{2}{63} \Rightarrow d = \left(\frac{1}{2} \times \frac{2}{63}\right) = \frac{1}{63}.$$

Putting $d = \frac{1}{63}$ in (i), we get

$$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9} \Rightarrow a + \frac{2}{21} = \frac{1}{9} \Rightarrow a = \left(\frac{1}{9} - \frac{2}{21}\right) = \left(\frac{7-6}{63}\right) = \frac{1}{63}.$$

Thus, $a = \frac{1}{63}$ and $d = \frac{1}{63}$.

$$\therefore \quad T_{63} = a + (63-1)d = (a + 62d)$$

$$= \left(\frac{1}{63} + 62 \times \frac{1}{63}\right) = \left(\frac{1}{63} + \frac{62}{63}\right) = 1.$$

Hence, 63rd term of the given AP is 1.

EXAMPLE 7 *The sum of the 4th and 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44. Find the first three terms of the AP.*

[CBSE 2008, '09, '12]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_4 + T_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow 2a + 10d = 24$$

$$\Rightarrow a + 5d = 12 \quad \ldots \text{(i)}$$

and $T_6 + T_{10} = 44 \Rightarrow (a + 5d) + (a + 9d) = 44$

$$\Rightarrow 2a + 14d = 44$$

$$\Rightarrow a + 7d = 22. \quad \ldots \text{(ii)}$$

On solving (i) and (ii), we get $a = -13$ and $d = 5$.

$\therefore$ first three terms of the given AP are –13, –8 and –3.

EXAMPLE 8 *Which term of the AP* 5, 9, 13, 17, ... *is* 81? [CBSE 2005C]

SOLUTION In the given AP, we have

first term = 5 and common difference = (9 − 5) = 4.

$\therefore$ $a = 5$ and $d = 4$.

Let its nth term be 81. Then,

$$T_n = 81 \Rightarrow a + (n-1)d = 81$$
$$\Rightarrow 5 + (n-1) \times 4 = 81 \qquad [\because a = 5 \text{ and } d = 4]$$
$$\Rightarrow 4n = 80 \Rightarrow n = 20.$$

Hence, the 20th term of the given AP is 81.

EXAMPLE 9 *Which term of the AP* 3, 15, 27, 39, ... *will be* 120 *more than its* 21*st term?* [CBSE 2009]

SOLUTION Here $a = 3$ and $d = (15 - 3) = 12$.

$\therefore$ 21st term is given by

$$T_{21} = a + (21-1)d = a + 20d = (3 + 20 \times 12) = 243.$$

Required term = (243 + 120) = 363.

Let it be nth term. Then,

$$T_n = 363 \Rightarrow a + (n-1)d = 363$$
$$\Rightarrow 3 + (n-1) \times 12 = 363$$
$$\Rightarrow 12n = 372 \Rightarrow n = 31.$$

Hence, 31st term is the required term.

EXAMPLE 10 *Is* 51 *a term of the AP* 5, 8, 11, 14, …?

SOLUTION Here $a = 5$ and $d = (8 - 5) = 3$.

Let the nth term of the given AP be 51. Then,

$$T_n = 51 \Rightarrow a + (n-1)d = 51$$
$$\Rightarrow 5 + (n-1) \times 3 = 51 \qquad [\because a = 5 \text{ and } d = 3]$$
$$\Rightarrow 3n = 49 \Rightarrow n = 16\frac{1}{3}.$$

But, the number of terms cannot be a fraction.

$\therefore$ 51 is not a term of the given AP.

EXAMPLE 11 *How many terms are there in the AP* 7, 11, 15, …, 139?

SOLUTION In the given AP, we have $a = 7$ and $d = (11 - 7) = 4$.

Suppose there are n terms in the given *AP*. Then,

$$T_n = 139 \Rightarrow a + (n-1)d = 139$$
$$\Rightarrow 7 + (n-1) \times 4 = 139$$
$$\Rightarrow 4n = 136 \Rightarrow n = 34.$$

Hence, there are 34 terms in the given AP.

EXAMPLE 12 *Find the middle term of the AP* 213, 205, 197, …, 37. [CBSE 2015]

SOLUTION Here $a = 213$ and $d = (205 - 213) = -8$.

Let the given AP contain n terms. Then,

$$T_n = 37 \Rightarrow a + (n-1)d = 37$$
$$\Rightarrow 213 + (n-1) \times (-8) = 37$$
$$\Rightarrow 221 - 8n = 37$$
$$\Rightarrow 8n = 221 - 37 = 184 \Rightarrow n = 23.$$

Thus, the given AP contains 23 terms.

$\therefore$ its middle term $= \frac{1}{2}(23 + 1)$th term = 12th term.

So, middle term $= T_{12} = (a + 11d)$

$$= 213 + 11 \times (-8) = 213 - 88 = 125.$$

Hence, the middle term is 125.

EXAMPLE 13 *Which term of the AP* 24, 21, 18, 15, ... *is the first negative term?*

SOLUTION Here $a = 24$ and $d = (21 - 24) = -3$.

Let the nth term of the given AP be the first negative term.

Then, $T_n < 0 \Rightarrow \{a + (n-1)d\} < 0$

$$\Rightarrow \{24 + (n-1) \times (-3)\} < 0$$
$$\Rightarrow (27 - 3n) < 0 \Rightarrow 27 < 3n$$
$$\Rightarrow 3n > 27 \Rightarrow n > 9.$$

$\therefore$ $n = 10$.

Hence, the 10th term is the first negative term of the given AP.

EXAMPLE 14 *For what value of n are the nth terms of the following two APs the same* 13, 19, 25, … *and* 69, 68, 67, …? *Also, find this term.*

[CBSE 2006C, '11]

SOLUTION Let nth terms of the given progressions be t_n and T_n respectively.

The first AP is 13, 19, 25, … .

Let its first term be a and common difference be d. Then,

$a = 13$ and $d = (19 - 13) = 6$.

So, its nth term is given by

$t_n = a + (n-1)d$

$\Rightarrow \quad t_n = 13 + (n-1) \times 6$

$\Rightarrow \quad t_n = 6n + 7 \qquad \ldots \text{(i)}$

The second AP is 69, 68, 67, … .

Let its first term be A and common difference be D. Then,

$A = 69$ and $D = (68 - 69) = -1$.

So, it nth term is given by

$T_n = A + (n-1) \times D$

$\Rightarrow \quad T_n = 69 + (n-1) \times (-1)$

$\Rightarrow \quad T_n = 70 - n \qquad \ldots \text{(ii)}$

Now, $t_n = T_n \Rightarrow 6n + 7 = 70 - n$

$\Rightarrow 7n = 63 \Rightarrow n = 9$.

Hence, the 9th term of each AP is the same.

This term $= 70 - 9 = 61 \quad [\because T_n = (70 - n)]$.

EXAMPLE 15 *If seven times the 7th term of an AP is equal to eleven times the 11th term then what will be its 18th term?* [CBSE 2017]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$7T_7 = 11T_{11}$

$\Rightarrow \quad 7(a + 6d) = 11(a + 10d) \Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow \quad 4a + 68d = 0 \Rightarrow a + 17d = 0$

$\Rightarrow \quad a + (18-1)d = 0 \Rightarrow T_{18} = 0$.

Hence, the 18th term of the given AP is zero.

EXAMPLE 16 *If the nth term of a progression be a linear expression in n then prove that this progression is an AP.*

SOLUTION Let the nth term of a given progression be given by

$T_n = an + b$, where a and b are constants.

Then, $T_{n-1} = a(n-1) + b = [(an+b) - a]$.

$\therefore \quad T_n - T_{n-1} = (an+b) - [(an+b-a] = a,$ which is a constant.

Hence, the given progression is an AP.

EXAMPLE 17 *In a given AP if pth term is q and the qth term is p then show that the nth term is* $(p+q-n)$. [CBSE 2008]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_p = a + (p-1)d \text{ and } T_q = a + (q-1)d.$$

Now, $T_p = q$ and $T_q = p$ (given).

$\therefore \quad a + (p-1)d = q$ ... (i)

and $a + (q-1)d = p$ ... (ii)

On subtracting (i) from (ii), we get

$$(q-p)d = (p-q) \Rightarrow d = -1.$$

Putting $d = -1$ in (i), we get $a = (p+q-1)$.

Thus, $a = (p+q-1)$ and $d = -1$.

$\therefore \quad n\text{th term} = a + (n-1)d = (p+q-1) + (n-1) \times (-1)$

$= p + q - n.$

Hence, nth term $= (p+q-n)$.

EXAMPLE 18 *If m times the mth term of an AP is equal to n times the nth term and* $m \neq n$, *show that its* $(m+n)$*th term is zero.* [CBSE 2004C, '08]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_m = a + (m-1)d \text{ and } T_n = a + (n-1)d.$$

Now, $(m \cdot T_m) = (n \cdot T_n) \Rightarrow m \cdot \{a + (m-1)d\} = n \cdot \{a + (n-1)d\}$

$\Rightarrow a \cdot (m-n) + \{(m^2 - n^2) - (m-n)\} \cdot d = 0$

$\Rightarrow (m-n) \cdot \{a + (m+n-1)\}d.$

$\Rightarrow (m-n) \cdot T_{m+n} = 0$

$\Rightarrow T_{m+n} = 0 \quad [\because (m-n) \neq 0].$

Hence, the $(m+n)$th term is zero.

EXAMPLE 19 *If the pth, qth and rth terms of an AP be a, b, c respectively then show that*

$$a(q-r) + b(r-p) + c(p-q) = 0.$$

SOLUTION Let x be the first term and d be the common difference of the given AP. Then,

$$T_p = x + (p-1)d,\ T_q = x + (q-1)d \text{ and } T_r = x + (r-1)d.$$

$$\therefore \quad x + (p-1)d = a \qquad \text{... (i)}$$

$$x + (q-1)d = b \qquad \text{... (ii)}$$

$$x + (r-1)d = c. \qquad \text{... (iii)}$$

On multiplying (i) by $(q-r)$, (ii) by $(r-p)$ and (iii) by $(p-q)$, and adding, we get

$$a(q-r) + b(r-p) + c(p-q)$$
$$= x \cdot \{(q-r) + (r-p) + (p-q)\}$$
$$+ d \cdot \{(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)\}$$
$$= (x \times 0) + (d \times 0) = 0.$$

Hence, $a(q-r) + b(r-p) + c(p-q) = 0.$

EXAMPLE 20 *If the mth term of an AP be* $\frac{1}{n}$ *and its nth term be* $\frac{1}{m}$ *then show that its (mn)th term is* 1.

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_m = a + (m-1)d \text{ and } T_n = a + (n-1)d.$$

Now, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$ (given).

$$\therefore \quad a + (m-1)d = \frac{1}{n} \qquad \text{... (i)}$$

and $a + (n-1)d = \frac{1}{m}$ ... (ii)

On subtracting (ii) from (i), we get

$$(m-n)d = \left(\frac{1}{n} - \frac{1}{m}\right) = \left(\frac{m-n}{mn}\right) \Rightarrow d = \frac{1}{mn}.$$

Putting $d = \frac{1}{mn}$ in (i), we get

$$a + \frac{(m-1)}{mn} = \frac{1}{n} \Rightarrow a = \left\{\frac{1}{n} - \frac{(m-1)}{mn}\right\} = \frac{1}{mn}.$$

Thus, $a = \frac{1}{mn}$ and $d = \frac{1}{mn}$.

$$\therefore \quad (mn)\text{th term} = a + (mn-1)d$$
$$= \left\{\frac{1}{mn} + \frac{(mn-1)}{mn}\right\} \quad \left[\because a = \frac{1}{mn}\right]$$
$$= 1.$$

Hence, the (mn)th term of the given AP is 1.

TO FIND THE *n*TH TERM FROM THE END OF AN AP

THEOREM 2 *If a be the first term, d be the common difference and l be the last term of a given AP then show that its nth term from the end is* $\{l-(n-1)d\}$.

PROOF We may write the given AP as

$$a, (a+d), (a+2d), \ldots, (l-2d), (l-d), l.$$

Thus, we have

last term $= l = l-(1-1)d$;

2nd term from the end $= (l-d) = \{l-(2-1)d\}$;

3rd term from the end $= (l-2d) = \{l-(3-1)d\}$;

4th term from the end $= (l-3d) = \{l-(4-1)d\}$.

$\therefore$ nth term from the end $= \{l-(n-1)d\}$.

> ***n*th term from the end of an AP**
>
> Let a be the first term, d be the common difference and l be the last term of an AP. Then,
>
> *nth term from the end* $= \{l-(n-1)d\}$.

EXAMPLE 21 *Find the* 11*th term from the end of the AP* 10, 7, 4, …, −62.

SOLUTION We have

$$a = 10,\ d = (7-10) = -3,\ l = -62 \text{ and } n = 11.$$

$\therefore$ 11th term from the end $= \{l-(n-1)\times d\}$

$$= \{-62-(11-1)\times(-3)\}$$

$$= (-62+30) = -32.$$

Hence, the 11th term from the end of the given AP is –32.

WORD PROBLEMS

EXAMPLE 22 *How many three-digit numbers are divisible by* 7*?* [CBSE 2013]

SOLUTION All 3-digit numbers divisible by 7 are

105, 112, 119, …, 994.

Clearly, these numbers form an AP with

$$a = 105,\ d = (112-105) = 7 \text{ and } l = 994.$$

Let it contain n terms. Then,

$$T_n = 994 \Rightarrow a+(n-1)\times d = 994$$

$\Rightarrow 105 + (n-1) \times 7 = 994$

$\Rightarrow 98 + 7n = 994 \Rightarrow 7n = 896 \Rightarrow n = 128.$

Hence, there are 128 three-digit numbers divisible by 7.

EXAMPLE 23 *How many multiples of* 4 *lie between* 10 *and* 250? [CBSE 2011]

SOLUTION All multiples of 4 lying between 10 and 250 are

12, 16, 20, 24, …, 248.

Clearly, these numbers form an AP with

$a = 12$, $d = (16 - 12) = 4$ and $l = 248$.

Let it contain n terms. Then,

$T_n = 248 \Rightarrow a + (n-1)d = 248$

$\Rightarrow 12 + (n-1) \times 4 = 248 \Rightarrow 4n = 240 \Rightarrow n = 60.$

Hence, there are 60 multiples of 4 lying between 10 and 250.

EXAMPLE 24 *A sum of* ₹ 1000 *is invested at* 8% *per annum simple interest. Calculate the interest at the end of* 1, 2, 3, … *years. Is the sequence of interest an AP? Find the interest at the end of* 30 *years.*

SOLUTION Here $P =$ ₹ 1000, $R = 8\%$ per annum. Let I_n be the SI at the end of n years.

Then, $I_n = \frac{P \times R \times n}{100} =$ ₹ $\left(\frac{1000 \times 8 \times n}{100}\right) =$ ₹ $(80n)$. … (i)

Putting $n = 1, 2, 3, \ldots$ in (i), we get

$I_1 =$ ₹ $(80 \times 1) =$ ₹ 80, $I_2 =$ ₹ $(80 \times 2) =$ ₹ 160,

$I_3 =$ ₹ $(80 \times 3) =$ ₹ 240, …

Thus, $I_1 =$ ₹ 80, $I_2 =$ ₹ 160, $I_3 =$ ₹ 240, $I_4 =$ ₹ 320, … .

Thus, ₹ 80, ₹ 160, ₹ 240, ₹ 320, … form an AP with

$a = 80$ and $d = (160 - 80) = 80$.

$\therefore \quad T_{30} = a + (30 - 1) \times d$

$= 80 + 29 \times 80 = 80 + 2320 = 2400.$

Hence, the interest at the end of 30 years is ₹ 2400.

EXAMPLE 25 *Tanvy joined her job in a company in the year* 2015 *on a monthly salary of* ₹ 40000 *with an annual increment of* ₹ 2500. *In which year will she get* ₹ 65000 *as monthly salary?*

SOLUTION Monthly salary received by Tanvy in 2015, 2016, 2017, 2018, … is respectively ₹ 40000, ₹ 42500, ₹ 45000, ₹ 47500, … .

This is an AP with $a = 40000$, $d = 2500$ and $l = 65000$.

Let the number of terms of this AP be n. Then,

$$T_n = 65000 \Rightarrow a + (n-1)d = 65000$$
$$\Rightarrow 40000 + (n-1) \times 2500 = 65000$$
$$\Rightarrow (n-1) \times 2500 = 65000 - 40000 = 25000$$
$$\Rightarrow (n-1) = \frac{25000}{2500} = 10 \Rightarrow n = 11.$$

Thus, the 11th annual salary received by Tanvy will be ₹ 65000. Thus, after 10 years, i.e., in the year 2025, her annual salary will be ₹ 65000.

EXAMPLE 26 *In a new year, Reenu saved* ₹ 50 *in the first week and then increased her weekly savings by* ₹ 17.50. *If in the nth week, her weekly saving becomes* ₹ 207.50, *find the value of n.*

SOLUTION Weekly savings by Reenu in successive weeks are ₹ 50, ₹ 67.50, ₹ 85, ₹ 102.50, …, ₹ 207.50.

This is clearly an AP with $a = 50$, $d = 17.50$ and $l = 207.50$.

Let the number of terms of this AP be n. Then,

$$T_n = 207.50 \Rightarrow a + (n-1)d = 207.50$$
$$\Rightarrow 50 + (n-1) \times 17.50 = 207.50$$
$$\Rightarrow (n-1) \times 17.50 = 207.50 - 50 = 157.50$$
$$\Rightarrow (n-1) = \frac{157.50}{17.50} = \frac{15750}{1750} = 9 \Rightarrow n = 10.$$

Hence, Reenu's weekly savings will be ₹ 207.50 in 10th week.

EXERCISE 5A

1. Show that each of the progressions given below is an AP. Find the first term, common difference and next term of each.

(i) 9, 15, 21, 27, … .

(ii) 11, 6, 1, –4, … .

(iii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \ldots.$

(iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots.$

(v) $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \ldots.$

2. Find:
   (i) the 20th term of the AP 9, 13, 17, 21, … .
   (ii) the 35th term of the AP 20, 17, 14, 11, … .
   (iii) the 18th term of the AP $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots$.
   (iv) the 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots$.
   (v) the 15th term of the AP –40, –15, 10, 35, … .
3. (i) Find the 37th term of the AP $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots$.
   (ii) Find the 25th term of the AP $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \ldots$.
4. Find the value of p for which the numbers $2p-1, 3p+1, 11$ are in AP. Hence, find the numbers. [CBSE 2017]
5. Find the nth term of each of the following APs:
   (i) 5, 11, 17, 23, … . (ii) 16, 9, 2, –5, … .
6. If the nth term of a progression is $(4n-10)$ show that it is an AP. Find its (i) first term, (ii) common difference, and (iii) 16th term.
7. How many terms are there in the AP 6, 10, 14, 18, …, 174?
8. How many terms are there in the AP 41, 38, 35, …, 8?
9. How many terms are there in the AP $18, 15\frac{1}{2}, 13, \ldots, -47$?
10. Which term of the AP 3, 8, 13, 18, … is 88?
11. Which term of the AP 72, 68, 64, 60, … is 0?
12. Which term of the AP $\frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}, \ldots$ is 3?
13. Which term of the AP 21, 18, 15, … is –81?
14. Which term of the AP 8, 14, 20, 26, ... will be 72 more than its 41st term? [CBSE 2017]
15. Which term of the AP 5, 15, 25, … will be 130 more than its 31st term? [CBSE 2006C]
16. If the 10th term of an AP is 52 and 17th term is 20 more than its 13th term, find the AP. [CBSE 2009C, 2017]
17. Find the middle term of the AP 6, 13, 20, …, 216. [CBSE 2015]
18. Find the middle term of the AP 10, 7, 4, …, (–62). [CBSE 2009C]
19. Find the sum of two middle most terms of the AP $-\frac{4}{3}, -1, \frac{-2}{3}, \ldots, 4\frac{1}{3}$. [CBSE 2013C]
20. Find the 8th term from the end of the AP 7, 10, 13, ..., 184. [CBSE 2005]

21. Find the 6th term from the end of the AP 17, 14, 11, ..., (–40). [CBSE 2005]
22. Is 184 a term of the AP 3, 7, 11, 15, …?
23. Is –150 a term of the AP 11, 8, 5, 2, …?
24. Which term of the AP 121, 117, 113, … is its first negative term?
25. Which term of the AP $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is its first negative term? [CBSE 2017]
26. The 7th term of an AP is –4 and its 13th term is –16. Find the AP. [CBSE 2014]
27. The 4th term of an AP is zero. Prove that its 25th term is triple its 11th term. [CBSE 2005]
28. If the sixth term of an AP is zero then show that its 33rd term is three times its 15th term. [CBSE 2017]
29. The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference. [CBSE 2015]
30. The 9th term of an AP is –32 and the sum of its 11th and 13th terms is –94. Find the common difference of the AP. [CBSE 2015]
31. Determine the nth term of the AP whose 7th term is –1 and 16th term is 17. [CBSE 2014]
32. If 4 times the 4th term of an AP is equal to 18 times its 18th term then find its 22nd term. [CBSE 2012]
33. If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.
34. Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms. [CBSE 2012]
35. The sum of the 2nd and the 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find the AP. [CBSE 2014]
36. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, … and 3, 10, 17, … are equal? [CBSE 2008, ’17]
37. The 17th term of AP is 5 more than twice its 8th term. If the 11th term of the AP is 43, find its nth term. [CBSE 2012]
38. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term. [CBSE 2013]
39. The 19th term of an AP is equal to 3 times its 6th term. If its 9th term is 19, find the AP. [CBSE 2013]
40. If the pth term of an AP is q and its qth term is p then show that its $(p+q)$th term is zero.

41. The first and last terms of an AP are a and l respectively. Show that the sum of the nth term from the beginning and the nth term from the end is $(a+l)$.
42. How many two-digit numbers are divisible by 6? [CBSE 2011]
43. How many two-digit numbers are divisible by 3? [CBSE 2012]
44. How many three-digit numbers are divisible by 9? [CBSE 2013]
45. How many numbers are there between 101 and 999, which are divisible by both 2 and 5? [CBSE 2014]
46. In a flower bed, there are 43 rose plants in the first row, 41 in the second, 39 in the third, and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed?
47. A sum of ₹ 2800 is to be used to award four prizes. If each prize after the first is ₹ 200 less than the preceding prize, find the value of each of the prizes.
48. Find how many integers between 200 and 500 are divisible by 8. [CBSE 2017]

ANSWERS (EXERCISE 5A)

1. (i) $a=9, d=6, T_5=33$ (ii) $a=11, d=-5, T_5=-9$

(iii) $a=-1, d=\frac{1}{6}, T_5=\frac{-1}{3}$ (iv) $a=\sqrt{2}, d=\sqrt{2}, T_5=\sqrt{50}$

(v) $a=\sqrt{20}, d=\sqrt{5}, T_5=\sqrt{180}$

2. (i) 85 (ii) –82 (iii) $35\sqrt{2}$ (iv) $4\frac{3}{4}$ (v) 310

3. (i) 69 (ii) –7 **4.** $p=2;$ 3, 7, 11 **5.** (i) $(6n-1)$ (ii) $(23-7n)$

6. (i) –6 (ii) 4 (iii) 54 **7.** 43 **8.** 12 **9.** 27 **10.** 18th

11. 19th **12.** 14th **13.** 35th **14.** 53rd **15.** 44th **16.** 7, 12, 17, 22, …

17. 111 **18.** –26 **19.** 3 **20.** 163 **21.** –25

22. No **23.** No **24.** 32nd **25.** 28th **26.** 8, 6, 4, 2, 0, …

29. 3 **30.** –5 **31.** $T_n=(2n-15)$ **32.** 0 **34.** 2 **35.** 1, 5, 9, 13, …

36. $n=13$ **37.** $T_n=(4n-1)$ **39.** 3, 5, 7, 9, … **42.** 15

43. 30 **44.** 100 **45.** 89 **46.** 17 **47.** ₹ 1000, ₹ 800, ₹ 600, ₹ 400 **48.** 37

HINTS TO SOME SELECTED QUESTIONS

1. (iv) Given progression is $\sqrt{2}, \sqrt{4\times 2}, \sqrt{9\times 2}, \sqrt{16\times 2}, \ldots$

i.e., $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$

$T_5=5\sqrt{2}=\sqrt{5\times 5\times 2}=\sqrt{50}$.

(v) Given progression is $\sqrt{4\times5}, \sqrt{9\times5}, \sqrt{16\times5}, \sqrt{25\times5}, \ldots$

i.e., $2\sqrt{5}, 3\sqrt{5}, 4\sqrt{5}, 5\sqrt{5}, \ldots$

$T_5 = 6\sqrt{5} = \sqrt{6\times6\times5} = \sqrt{180}$.

3. (i) $a = 6, d = \left(\frac{31}{4} - 6\right) = \frac{7}{4}$.

(ii) $a = 5, d = \left(\frac{9}{2} - 5\right) = \frac{-1}{2}$.

19. $a = \frac{-4}{3}, d = \left(-1 + \frac{4}{3}\right) = \frac{1}{3}$ and $T_n = \frac{13}{3}$.

$\therefore \quad T_n = \frac{13}{3} \Rightarrow a + (n-1)d = \frac{13}{3} \Rightarrow \frac{-4}{3} + (n-1)\times\frac{1}{3} = \frac{13}{3} \Rightarrow n = 18.$

$\therefore$ sum of two middle most terms

$$= (T_9 + T_{10}) = (a+8d) + (a+9d) = (2a+17d) = \left(\frac{-8}{3} + \frac{17}{3}\right) = \frac{9}{3} = 3.$$

22. Let $T_n = 184$. Then, $a + (n-1)d = 184$.

$\therefore \quad 3 + (n-1)\times4 = 184 \Rightarrow n - 1 = \frac{181}{4} \Rightarrow n = \left(\frac{181}{4} + 1\right) = \frac{185}{4} = 46\frac{1}{4}$.

Thus, n is not a natural number.

So, 184 is not a term of the given AP.

24. Let $T_n < 0$. Then, $121 + (n-1)\times(-4) < 0$.

$\therefore \quad -4n + 125 < 0 \Rightarrow 125 < 4n \Rightarrow 4n > 125 \Rightarrow n = 31\frac{1}{4}$.

So, 32nd term is the first negative term.

27. $a + 3d = 0 \Rightarrow a = -3d$.

25th term $= a + 24d = -3d + 24d = 21d$.

3(11th term) $= 3(a+10d) = 3(-3d+10d) = 21d$.

Hence, 25th term = 3(11th term).

28. $a + 5d = 0 \Rightarrow a = -5d$.

33th term $= a + 32d = -5d + 32d = 27d$.

3(15th term) $= 3(a+14d) = 3(-5d+14d) = 3\times9d = 27d$.

29. $a + 3d = 11, T_5 + T_7 = 34 \Rightarrow 2a + 10d = 34 \Rightarrow a + 5d = 17$.

Solve $a + 3d = 11$ and $a + 5d = 17$.

32. $4(a+3d) = 18(a+17d) \Rightarrow 14a + 294d = 0 \Rightarrow a + 21d = 0 \Rightarrow T_{22} = 0$.

34. $\{5 + (5+d) + (5+2d) + (5+3d)\} = \frac{1}{2}\{(5+4d) + (5+5d) + (5+6d) + (5+7d)\}$

$\Rightarrow \quad 2(20+6d) = (20+22d) \Rightarrow 10d = 20 \Rightarrow d = 2.$

35. $T_2 + T_7 = 30 \Rightarrow (a+d) + (a+6d) = 30 \Rightarrow 2a + 7d = 30.$ … (i)

$T_{15} = 2T_8 - 1 \Rightarrow (a+14d) = 2(a+7d) - 1 \Rightarrow a = 1.$

Putting $a = 1$ in (i), we get $d = 4$.

36. $T_n = t_n \Rightarrow 63 + (n-1)\times2 = 3 + (n-1)\times7 \Rightarrow 5n = 65 \Rightarrow n = 13.$

37. $T_{17} = 2T_8 + 5 \Rightarrow a + 16d = 2(a+7d) + 5 \Rightarrow a - 2d = -5.$ … (i)

Also, $a + 10d = 43$. ... (ii)

Solve (i) and (ii), we get $a = 3$ and $d = 4$.

38. $T_{24} = 2T_{10} \Rightarrow a + 23d = 2(a + 9d) \Rightarrow a - 5d = 0 \Rightarrow a = 5d$. ... (i)

$T_{72} = a + 71d = 5d + 71d = 76d$.

$T_{15} = a + 14d = 5d + 14d = 19d$.

Hence, $T_{72} = 4 \times T_{15}$.

40. $T_p = q \Rightarrow a + (p-1)d = q$. ... (i)

$T_q = p \Rightarrow a + (q-1)d = p$. ... (ii)

On subtracting (ii) from (i), we get $(p-q)d = (q-p) \Rightarrow d = -1$.

Putting $d = -1$ in (i), we get $a = p + q - 1$.

$\therefore \quad T_{p+q} = a + (p+q-1)d = (p+q-1) - (p+q-1) = 0$.

41. Required sum $= \{a + (n-1)d\} + \{l - (n-1)d\} = (a + l)$.

42. The given numbers are 12, 18, 24, 30, ..., 96.

Let their number be n. Then, $12 + (n-1) \times 6 = 96$. Find n.

44. The given numbers are 108, 117, 126, ..., 999.

45. Clearly, each given number is divisible by 10.

So, these numbers are 110, 120, 130, ..., 990.

46. Let there be n rows. Then, the number of plants in various rows are 43, 41, 39, ..., 11 respectively.

Here, $a = 43$, $d = -2$ and $T_n = 11$. Find n.

47. Let the values of these prizes be ₹ x, ₹ $(x - 200)$, ₹ $(x - 400)$ and ₹ $(x - 600)$. Their sum is ₹ 2800.

ARITHMETIC MEAN

ARITHMETIC MEAN (AM) BETWEEN TWO NUMBERS

If a, A, b are in AP we say that A is the arithmetic mean between a and b and it is abbreviated as AM.

TO FIND THE ARITHMETIC MEAN BETWEEN TWO NUMBERS

Let the given numbers be a and b.

Let A be the AM between a and b. Then,

$$a, A, b \text{ are in } AP \Rightarrow (A - a) = (b - A)$$

$$\Rightarrow A = \frac{1}{2}(a + b).$$

$\therefore$ AM between a and $b = \frac{1}{2}(a+b)$.

Arithmetic mean between a and b is $\frac{1}{2}(a+b)$.

SOLVED EXAMPLES

EXAMPLE 1 *Find the AM between*

(i) 13 *and* 19 *(ii)* $(a-b)$ *and* $(a+b)$

SOLUTION (i) AM between 13 and 19 $= \frac{1}{2}(13+19) = 16$.

(ii) AM between $(a-b)$ and $(a+b) = \frac{1}{2}[(a-b)+(a+b)] = a$.

EXAMPLE 2 *If the numbers* $(2n-1)$, $(3n+2)$ *and* $(6n-1)$ *are in AP, find* n *and hence find these numbers.* [CBSE 2013C]

SOLUTION Since $(2n-1)$, $(3n+2)$ and $(6n-1)$ are in AP, we have

$$(3n+2)-(2n-1) = (6n-1)-(3n+2)$$

$\Rightarrow$ $n+3 = 3n-3 \Rightarrow 2n = 6 \Rightarrow n = 3$.

Hence, $n = 3$ and these numbers are 5, 11 and 17.

AN IMPORTANT RESULT

It is always convenient to make a choice of

(i) 3 numbers in AP as $(a-d), a, (a+d)$;

(ii) 4 numbers in AP as $(a-3d), (a-d), (a+d), (a+3d)$;

(iii) 5 numbers in AP as $(a-2d), (a-d), a, (a+d), (a+2d)$.

EXAMPLE 3 *The sum of three numbers in AP is* 21 *and their product is* 231. *Find the numbers.*

SOLUTION Let the required numbers be $(a-d)$, a and $(a+d)$.

Then, $(a-d)+a+(a+d) = 21 \Rightarrow 3a = 21 \Rightarrow a = 7$.

And, $(a-d)\cdot a\cdot(a+d) = 231 \Rightarrow a(a^2-d^2) = 231$

$\Rightarrow 7(49-d^2) = 231$ [$\because$ $a = 7$]

$\Rightarrow 7d^2 = 343-231 = 112$

$\Rightarrow d^2 = 16 \Rightarrow d = \pm 4$.

Thus, $a = 7$ and $d = \pm 4$.

Hence, the required numbers are (3, 7, 11) or (11, 7, 3).

EXAMPLE 4 *Find four numbers in AP whose sum is 20 and the sum of whose squares is 120.*

SOLUTION Let the required numbers be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

Then, $(a-3d)+(a-d)+(a+d)+(a+3d) = 20$

$\Rightarrow \quad 4a = 20 \Rightarrow a = 5.$

And, $(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2 = 120$

$\Rightarrow \quad 4(a^2+5d^2) = 120$

$\Rightarrow \quad (a^2+5d^2) = 30 \Rightarrow 25+5d^2 = 30$ $[\because a = 5]$

$\Rightarrow \quad 5d^2 = 5 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1.$

Thus, $a = 5$ and $d = \pm 1$.

Hence, the required numbers are (2, 4, 6, 8) or (8, 6, 4, 2).

EXERCISE 5B

1. Determine k so that $(3k-2)$, $(4k-6)$ and $(k+2)$ are three consecutive terms of an AP. [CBSE 2009C]
2. Find the value of x for which the numbers $(5x+2)$, $(4x-1)$ and $(x+2)$ are in AP.
3. If $(3y-1)$, $(3y+5)$ and $(5y+1)$ are three consecutive terms of an AP then find the value of y. [CBSE 2014]
4. Find the value of x for which $(x+2)$, $2x$, $(2x+3)$ are three consecutive terms of an AP. [CBSE 2009C]
5. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.
6. Find three numbers in AP whose sum is 15 and product is 80.

   HINT Let the numbers be $(a-d)$, a, $(a+d)$.
7. The sum of three numbers in AP is 3 and their product is –35. Find the numbers.
8. Divide 24 in three parts such that they are in AP and their product is 440.
9. The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms.

10. The angles of a quadrilateral are in AP whose common difference is 10°. Find the angles.

HINT Let these angles be x°, $(x+10)^\circ$, $(x+20)^\circ$ and $(x+30)^\circ$.
Their sum is 360°.

11. Find four numbers in AP whose sum is 28 and the sum of whose squares is 216.

12. Divide 32 into four parts which are the four terms of an AP such that the product of the first and the fourth terms is to the product of the second and the third terms as 7 : 15. [CBSE 2014]

HINT Let these parts be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

13. The sum of first three terms of an AP is 48. If the product of first and second terms exceeds 4 times the third term by 12. Find the AP. [CBSE 2013C]

HINT Let these terms be $(a-d)$, a, $(a+d)$.

ANSWERS (EXERCISE 5B)

1. $k=3$ **2.** $x=3$ **3.** $y=5$ **4.** $x=5$
6. (2, 5, 8) or (8, 5, 2) **7.** (–5, 1, 7) or (7, 1, –5) **8.** (5, 8, 11) or (11, 8, 5)
9. (4, 7, 10) or (10, 7, 4) **10.** 75°, 85°, 95°, 105°
11. (4, 6, 8, 10) or (10, 8, 6, 4) **12.** (2, 6, 10, 14) or (14, 10, 6, 2) **13.** 7, 16, 25

SUM OF *n* TERMS OF AN AP

TO FIND THE SUM OF *n* TERMS OF AN AP

THEOREM 3 *Prove that the sum of n terms of an AP in which first term = a, common difference = d and last term = l, is given by*

$$S_n = \frac{n}{2}(a+l) \text{ and } S_n = \frac{n}{2}\cdot\{2a+(n-1)d\}.$$

PROOF Consider an AP having n terms in which

first term = a, common difference = d and last term = l.

Then, $l = a+(n-1)d$ ... (i) [$\because$ l = nth term]

Also, we may write the given AP as

$a, (a+d), (a+2d), \ldots, (l-2d), (l-d), l.$

Let S_n be the sum of the first n terms of this AP. Then,

$$S_n = a+(a+d)+(a+2d)+\ldots+(l-2d)+(l-d)+l \qquad \ldots \text{(ii)}$$

Writing the above series in the reverse order, we get

$$S_n = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a \qquad \ldots \text{(iii)}$$

Adding the corresponding terms of (ii) and (iii), we get

$$2S_n = (a + l) + (a + l) + (a + l) + \ldots n \text{ times} = n(a + l)$$

$$\Rightarrow \quad S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad S_n = \frac{n}{2}[a + a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d] \quad \text{[using (i)]}.$$

Hence, $S_n = \frac{n}{2}(a + l)$ and $S_n = \frac{n}{2}[2a + (n - 1)d]$.

> **SUMMARY**
>
> Sum of n terms of an AP $a, a + d, a + 2d, \ldots, l$ is given by
>
> $$S_n = \frac{n}{2}(a + l) \text{ or } S_n = \frac{n}{2} \times \{2a + (n - 1)d\}.$$

SOLVED EXAMPLES

EXAMPLE 1 *Find the sum of first* 24 *terms of the AP* 5, 8, 11, 14, … .

SOLUTION Here $a = 5$, $d = (8 - 5) = 3$ and $n = 24$.

Using the formula, $S_n = \frac{n}{2} \cdot \{2a + (n - 1)d\}$, we get

$$S_{24} = \frac{24}{2} \cdot \{2 \times 5 + (24 - 1) \times 3\} \qquad [\because\ a = 5, d = 3 \text{ and } n = 24]$$

$$= 12 \times (10 + 69) = 948.$$

Hence, the sum of first 24 terms of the given AP is 948.

EXAMPLE 2 *Find the sum* 25 + 28 + 31 + … + 100. [CBSE 2006C]

SOLUTION Here $a = 25$, $d = (28 - 25) = 3$ and $l = 100$.

Let the total number of terms be n. Then,

$$T_n = 100 \Rightarrow a + (n - 1)d = 100$$

$$\Rightarrow 25 + (n - 1) \times 3 = 100$$

$$\Rightarrow 3n + 22 = 100$$

$$\Rightarrow 3n = 78 \Rightarrow n = 26.$$

Required sum $= \frac{n}{2} \cdot (a + l)$

$$= \frac{26}{2} \cdot (25 + 100) = 13 \times 125 = 1625.$$

Hence, the required sum is 1625.

EXAMPLE 3 *Find the sum* $18+15\frac{1}{2}+13+\ldots+\left(-49\frac{1}{2}\right)\cdot$ [CBSE 2013]

SOLUTION We have, $\frac{31}{2}-18=13-\frac{31}{2}=\frac{-5}{2}\cdot$

So, the given series is an arithmetic series.

Here, $a=18$, $d=\frac{-5}{2}$ and $l=\frac{-99}{2}\cdot$

Let the given series contain n terms. Then,

$$T_n=\frac{-99}{2}\Rightarrow a+(n-1)d=\frac{-99}{2}$$

$$\Rightarrow 18+(n-1)\times\left(\frac{-5}{2}\right)=\frac{-99}{2}$$

$$\Rightarrow 36-5n+5=-99$$

$$\Rightarrow -5n=-140\Rightarrow n=28.$$

$\therefore$ required sum $=\frac{n}{2}(a+l)$

$$=\frac{28}{2}\cdot\left\{18-\frac{99}{2}\right\}=7(36-99)=-441.$$

Hence, the sum of the given series is –441.

EXAMPLE 4 *Find the sum of first n terms of an AP whose nth term is* $(5n-1)$. *Hence, find the sum of first* 20 *terms.* [CBSE 2011]

SOLUTION The nth term of the given AP is given by, $T_n=(5n-1)$.

Let a be the first term and d be the common difference of this AP. Then,

$$a=T_1=(5\times1-1)=4,\ T_2=(5\times2-1)=9.$$

$\therefore$ $d=(T_2-T_1)=(9-4)=5.$

$\therefore$ sum of first n terms $=\frac{n}{2}\cdot\{2a+(n-1)d\}$

$$=\frac{n}{2}\times\{2\times4+(n-1)\times5\}=\frac{1}{2}n(5n+3).$$

$\therefore$ sum of first 20 terms $=\frac{1}{2}\times20\times(5\times20+3)=1030.$

Hence, $S_n=\frac{1}{2}n(5n+3)$ and $S_{20}=1030.$

EXAMPLE 5 *If the sum of first n terms of an AP is* $\frac{1}{2}(3n^2+7n)$ *then find its nth term and hence, write its* 20*th term.* [CBSE 2015]

SOLUTION We have, $S_n=\frac{1}{2}(3n^2+7n)$.

$\therefore \quad S_{n-1} = \frac{1}{2} \cdot \{3(n-1)^2 + 7(n-1)\} = \frac{1}{2}(3n^2 + n - 4).$

$\therefore \quad T_n = (S_n - S_{n-1}) = \frac{1}{2}(3n^2 + 7n) - \frac{1}{2}(3n^2 + n - 4)$

$= \frac{1}{2}(6n + 4) = (3n + 2).$

$\therefore \quad n$th term $= (3n + 2)$.

Hence, 20th term $= (3 \times 20 + 2) = 62$.

EXAMPLE 6 *How many terms of the AP* 3, 5, 7, 9, … *must be added to get the sum* 120?

SOLUTION Here, $a = 3$ and $d = (5 - 3) = 2$.

Let the required number of terms be n. Then,

$S_n = 120 \Rightarrow \frac{n}{2} \cdot \{2a + (n-1)d\} = 120$

$\Rightarrow \frac{n}{2} \cdot \{2 \times 3 + (n-1) \times 2\} = 120$

$\Rightarrow \frac{n}{2} \cdot (2n + 4) = 120 \Rightarrow n^2 + 2n - 120 = 0$

$\Rightarrow n^2 + 12n - 10n - 120 = 0$

$\Rightarrow n(n+12) - 10(n+12) = 0 \Rightarrow (n+12)(n-10) = 0$

$\Rightarrow n + 12 = 0$ or $n - 10 = 0 \Rightarrow n = -12$ or $n = 10$

$\Rightarrow n = 10$ [$\because$ number of terms cannot be negative]

Hence, the required number of terms is 10.

EXAMPLE 7 *How many terms of the AP* 17, 15, 13, 11, ... *must be added to get the sum* 72? *Explain the double answer.*

SOLUTION Here, $a = 17$ and $d = (15 - 17) = -2$.

Let the sum of n terms be 72. Then,

$S_n = 72 \Rightarrow \frac{n}{2} \cdot \{2a + (n-1)d\} = 72$

$\Rightarrow n \cdot \{2 \times 17 + (n-1) \times (-2)\} = 144$

$\Rightarrow n(36 - 2n) = 144 \Rightarrow 2n^2 - 36n + 144 = 0$

$\Rightarrow n^2 - 18n + 72 = 0 \Rightarrow n^2 - 12n - 6n + 72 = 0$

$\Rightarrow n(n - 12) - 6(n - 12) = 0 \Rightarrow (n-12)(n-6) = 0$

$\Rightarrow n = 6$ or $n = 12$.

$\therefore$ sum of first 6 terms = sum of first 12 terms = 72.

This means that the sum of all terms from 7th to 12th is zero.

EXAMPLE 8 *The first and the last terms of an AP are 7 and 49 respectively. If sum of all its terms is 420, find its common difference.* [CBSE 2014]

SOLUTION Let the given AP contain n terms.

Here $a = 7, l = 49$ and $S_n = 420$.

$$\therefore \quad S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad \frac{n}{2}(7 + 49) = 420 \Rightarrow \frac{n}{2} \times 56 = 420$$

$$\Rightarrow \quad 28n = 420 \Rightarrow n = \frac{420}{28} = 15.$$

Thus, the given AP contains 15 terms and $T_{15} = 49$.

Let d be the common difference of the given AP. Then,

$$T_{15} = 49 \Rightarrow a + 14d = 49$$

$$\Rightarrow 7 + 14d = 49$$

$$\Rightarrow 14d = 42 \Rightarrow d = 3.$$

Hence, the common difference of the given AP is 3.

EXAMPLE 9 *The sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161. Find the 28th term of this AP.* [CBSE 2014]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then, using $S_n = \frac{n}{2} \cdot [2a + (n - 1)d]$, we get

$$S_7 = \frac{7}{2}(2a + 6d) \Rightarrow 7(a + 3d) = 63 \qquad [\because S_7 = 63]$$

$$\Rightarrow a + 3d = 9. \qquad \ldots \text{(i)}$$

Clearly, the sum of first 14 terms $= 63 + 161 = 224$.

$$\therefore \quad S_{14} = 224 \Rightarrow \frac{14}{2}(2a + 13d) = 224$$

$$\Rightarrow 7(2a + 13d) = 224$$

$$\Rightarrow 2a + 13d = 32. \qquad \ldots \text{(ii)}$$

Multiplying (i) by 2 and subtracting the result from (ii), we get

$$7d = 14 \Rightarrow d = 2.$$

Putting $d = 2$ in (i), we get $a = 9 - 6 = 3$.

Thus, $a = 3$ and $d = 2$.

$\therefore$ 28th term of this AP is given by

$$T_{28} = (a + 27d) = (3 + 27 \times 2) = 57.$$

Hence, the 28th term of the given AP is 57.

EXAMPLE 10 *The 14th term of an AP is twice its 8th term. If its 6th term is –8 then find the sum of its first 20 terms.* [CBSE 2015]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_{14} = 2 \times T_8 \Rightarrow a + 13d = 2(a + 7d)$$
$$\Rightarrow a + d = 0. \quad \dots \text{(i)}$$

Also, $T_6 = -8 \Rightarrow a + 5d = -8.$ ... (ii)

On solving (i) and (ii), we get $a = 2$ and $d = -2$.

The sum of first 20 terms is given by

$$S_{20} = \frac{n}{2} \cdot [2a + (n-1)d], \text{ where } n = 20$$
$$= \left(\frac{20}{2}\right) \times \{(2 \times 2 + 19 \times (-2)\}$$
$$= 10 \times (4 - 38) = 10 \times (-34) = -340.$$

Hence, the required sum is –340.

EXAMPLE 11 *The sum of 4th and 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44. Find the sum of first ten terms of the AP.* [CBSE 2012]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_4 + T_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24$$
$$\Rightarrow 2a + 10d = 24$$
$$\Rightarrow a + 5d = 12. \quad \dots \text{(i)}$$

And, $T_6 + T_{10} = 44 \Rightarrow (a + 5d) + (a + 9d) = 44$

$$\Rightarrow 2a + 14d = 44$$
$$\Rightarrow a + 7d = 22. \quad \dots \text{(ii)}$$

On solving (i) and (ii), we get $a = -13$ and $d = 5$.

$\therefore$ the sum of first 10 terms of the given AP is given by

$$S_{10} = \left(\frac{10}{2}\right) \cdot (2a + 9d) \qquad \left[\text{using } S_n = \frac{n}{2}\{(2a + (n-1)d\}\right]$$
$$= 5 \times \{2 \times (-13) + 9 \times 5\} = 5(-26 + 45) = 5 \times 19 = 95.$$

Hence, the sum of first 10 terms of the given AP is 95.

EXAMPLE 12 *Sum of first 14 terms of an AP is 1505 and its first term is 10. Find its 25th term.* [CBSE 2012]

SOLUTION Here $a = 10$ and let d be the common difference. Then,

$$S_{14} = 1505 \Rightarrow \frac{n}{2}[2a + (n-1)d] = 1505, \text{ where } n = 14 \text{ and } a = 10$$

$$\Rightarrow \frac{14}{2} \cdot (20 + 13d) = 1505 \Rightarrow (20 + 13d) = \frac{1505}{7} = 215$$

$$\Rightarrow 13d = 195 \Rightarrow d = 15.$$

Thus, $a = 10$ and $d = 15$.

$\therefore \quad T_{25} = (a + 24d) = (10 + 24 \times 15) = 370.$

Hence, the 25th term is 370.

EXAMPLE 13 *In an AP of* 50 *terms, the sum of first* 10 *terms is* 210 *and the sum of its last* 15 *terms is* 2565. *Find the AP.* [CBSE 2014]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then, the sum of first n terms is given by

$$S_n = \frac{n}{2} \cdot \{2a + (n-1)d\}.$$

$$\therefore \quad S_{10} = \frac{10}{2} \cdot (2a + 9d) \Rightarrow 5(2a + 9d) = 210$$

$$\Rightarrow 2a + 9d = 42. \quad \ldots \text{(i)}$$

Sum of last 15 terms $= (S_{50} - S_{35})$.

$\therefore \quad (S_{50} - S_{35}) = 2565$

$\Rightarrow \quad \frac{50}{2}(2a + 49d) - \frac{35}{2}(2a + 34d) = 2565$

$\Rightarrow \quad 25(2a + 49d) - 35(a + 17d) = 2565$

$\Rightarrow \quad (50a - 35a) + (1225d - 595d) = 2565$

$\Rightarrow \quad 15a + 630d = 2565 \Rightarrow a + 42d = 171. \quad \ldots$ (ii)

On solving (i) and (ii), we get $a = 3$ and $d = 4$.

Hence, the required AP is 3, 7, 11, 15, 19, … .

EXAMPLE 14 *The sum of first* 6 *terms of an AP is* 42. *The ratio of its* 10*th term to* 30*th term is* 1 : 3. *Find the first and the* 13*th term of the AP.*

[CBSE 2009]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$T_{10} = a + 9d \text{ and } T_{30} = a + 29d$$

$$\therefore \quad \frac{T_{10}}{T_{30}} = \frac{1}{3} \Rightarrow \frac{a + 9d}{a + 29d} = \frac{1}{3}$$

$$\Rightarrow 3a + 27d = a + 29d$$

$$\Rightarrow 2a = 2d \Rightarrow a = d.$$

Also, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \quad S_6 = \frac{6}{2}(2a + 5d)$$

$$= (6a + 15d) = (6a + 15a) \qquad [\because d = a]$$

$$= 21a.$$

But, $S_6 = 42$ (given)

$\therefore \quad 21a = 42 \Rightarrow a = 2.$

Thus, $a = 2$ and $d = 2$.

$\therefore \quad$ 13th term, $T_{13} = (a + 12d) = (2 + 12 \times 2) = 26.$

Hence, the first term is 2 and the 13th term is 26.

EXAMPLE 15 *If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$.* [CBSE 2015]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$S_n = \frac{n}{2} \cdot [2a + (n-1)d].$$

$$\therefore \quad 3(S_8 - S_4) = 3\left[\frac{8}{2}(2a + 7d) - \frac{4}{2}(2a + 3d)\right]$$

$$= 3[4(2a + 7d) - 2(2a + 3d)] = 6(2a + 11d)$$

$$= \frac{12}{2} \cdot (2a + 11d) = S_{12}.$$

Hence, $S_{12} = 3(S_8 - S_4)$.

EXAMPLE 16 *If the sum of first n, $2n$ and $3n$ terms of an AP be S_1, S_2 and S_3 respectively then prove that $S_3 = 3(S_2 - S_1)$.*

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

S_1 = sum of first n terms of the given AP,

S_2 = sum of first $2n$ terms of the given AP,

S_3 = sum of first $3n$ terms of the given AP.

$\therefore \quad S_1 = \frac{n}{2} \cdot \{2a + (n-1)d\}$, $S_2 = \frac{2n}{2} \cdot \{2a + (2n-1)d\}$, and

$$S_3 = \frac{3n}{2} \cdot \{2a + (3n-1)d\}$$

$$\Rightarrow \quad 3(S_2 - S_1) = 3 \cdot \left[\{2na + n(2n-1)d\} - \left\{ na + \frac{1}{2}n(n-1)d \right\} \right]$$

$$= 3 \cdot \left[na + \frac{3}{2}n^2 d - \frac{1}{2}nd \right] = \frac{3n}{2} \cdot [2a + 3nd - d]$$

$$= \frac{3n}{2} \cdot \{2a + (3n-1)d\} = S_3.$$

Hence, $S_3 = 3(S_2 - S_1)$.

EXAMPLE 17 *If the sum of the first p terms of an AP is the same as the sum of its first q terms (where $p \neq q$) then show that the sum of its first $(p+q)$ terms is zero.*

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$S_p = S_q \Rightarrow \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$

$$\Rightarrow (p-q)(2a) = (q-p)(q+p-1)d$$

$$\Rightarrow 2a = (1-p-q)d \qquad \ldots \text{(i)}$$

Sum of the first $(p+q)$ terms of the given AP

$$= \frac{(p+q)}{2} \cdot \{2a + (p+q-1)d\}$$

$$= \frac{(p+q)}{2} \cdot \{(1-p-q)d + (p+q-1)d\} \quad \text{[using (i)]}$$

$$= 0.$$

EXAMPLE 18 *If the sum of the first m terms of an AP be n and the sum of its first n terms be m then show that the sum of its first $(m+n)$ terms is $-(m+n)$.*

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

$$S_m = n \Rightarrow \frac{m}{2}[2a + (m-1)d] = n$$

$$\Rightarrow 2am + m(m-1)d = 2n. \qquad \ldots \text{(i)}$$

And, $S_n = m \Rightarrow \frac{n}{2}[2a + (n-1)d] = m$

$$\Rightarrow 2an + n(n-1)d = 2m. \qquad \ldots \text{(ii)}$$

On subtracting (ii) from (i), we get

$$2a(m-n) + [(m^2 - n^2) - (m-n)]d = 2(n-m)$$

$$\Rightarrow \quad (m-n)[2a + (m+n-1)d] = 2(n-m)$$

$$\Rightarrow \quad 2a + (m+n-1)d = -2 \qquad \ldots \text{(iii)}$$

Sum of the first $(m+n)$ terms of the given AP

$$= \frac{(m+n)}{2} \cdot \{2a + (m+n-1)d\}$$

$$= \frac{(m+n)}{2} \cdot (-2) = -(m+n) \text{ [using (iii)].}$$

Hence, the sum of first $(m+n)$ terms of the given AP is $-(m+n)$.

EXAMPLE 19 *The ratio of the sums of first m and first n terms of an AP is* $m^2 : n^2$. *Show that the ratio of its mth and nth terms is* $(2m-1):(2n-1)$.

[CBSE 2017]

SOLUTION Let a be the first term and d be the common difference of the given AP. Then,

S_m = sum of first m terms of the given AP;

S_n = sum of first n terms of the given AP.

$$\frac{S_m}{S_n} = \frac{m^2}{n^2} \Rightarrow \frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n}$$

$$\Rightarrow 2an + mnd - nd = 2am + mnd - md$$

$$\Rightarrow 2an - 2am = nd - md$$

$$\Rightarrow 2a(n-m) = d(n-m) \Rightarrow 2a = d. \qquad \text{... (i)}$$

$$\therefore \quad \frac{T_m}{T_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1) \cdot 2a}{a + (n-1) \cdot 2a} \qquad \text{[from (i)]}$$

$$= \frac{a + 2am - 2a}{a + 2an - 2a} = \frac{2am - a}{2an - a} = \frac{a(2m-1)}{a(2n-1)} = \frac{2m-1}{2n-1}.$$

EXAMPLE 20 *The ratio of the* 11*th term to the* 18*th term of an AP is* 2 : 3. *Find the ratio of the* 5*th term to the* 21*st term, and also the ratio of the sum of the first* 5 *terms to the sum of first* 21 *terms.* [CBSE 2017]

SOLUTION Let a be the first term and d be the common difference of the given AP.

Then, $\frac{T_{11}}{T_{18}} = \frac{2}{3} \Rightarrow \frac{a + (11-1)d}{a + (18-1)d} = \frac{2}{3}$

$$\Rightarrow \frac{a + 10d}{a + 17d} = \frac{2}{3} \Rightarrow 3a + 30d = 2a + 34d$$

$$\Rightarrow a = 4d. \qquad \text{... (i)}$$

Ratio of 5th term to 21st term $= \dfrac{T_5}{T_{21}} = \dfrac{a+(5-1)d}{a+(21-1)d}$

$$= \frac{a+4d}{a+20d} = \frac{4d+4d}{4d+20d} \quad \text{[from (i)]}$$

$$= \frac{8d}{24d} = \frac{1}{3} = 1:3.$$

Ratio of sum of first 5 terms to sum of first 21 terms

$$= \frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+(5-1)d]}{\frac{21}{2}[2a+(21-1)d]} = \frac{5(2a+4d)}{21(2a+20d)}$$

$$= \frac{10(a+2d)}{42(a+10d)} = \frac{10(4d+2d)}{42(4d+10d)} \quad \text{[from (i)]}$$

$$= \frac{60d}{588d} = \frac{60}{588} = \frac{5}{49} = 5:49.$$

EXAMPLE 21 *If the ratio of the sum of the first n terms of two APs is* $(7n+1):(4n+27)$ *then find the ratio of their 9th terms.* [CBSE 2017]

SOLUTION Let a_1 and a_2 be the first terms and d_1 and d_2 be the common difference of the two APs respectively.

Let S_n and S'_n be the sums of the first n terms of the two APs and T_n and T'_n be their nth terms respectively.

Then, $\dfrac{S_n}{S'_n} = \dfrac{7n+1}{4n+27} \Rightarrow \dfrac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \dfrac{7n+1}{4n+27}$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}. \quad \dots \text{(i)}$$

To find the ratio of mth terms, we replace n by $(2m-1)$ in the above expression.

Replacing n by $(2\times 9-1)$, i.e., 17 on both sides in (i), we get

$$\frac{2a_1+(17-1)d_1}{2a_2+(17-1)d_2} = \frac{7\times 17+1}{4\times 17+27} \Rightarrow \frac{2a_1+16d_1}{2a_2+16d_2} = \frac{120}{95}$$

$$\Rightarrow \frac{a_1+8d_1}{a_2+8d_2} = \frac{24}{19}$$

$$\Rightarrow \frac{a_1+(9-1)d_1}{a_2+(9-1)d_2} = \frac{24}{19}$$

$$\Rightarrow \frac{T_n}{T'_n} = \frac{24}{19}.$$

$\therefore$ required ratio = 24 : 19.

WORD PROBLEMS

EXAMPLE 22 *Find the sum of all multiples of 7 lying between 500 and 900.* [CBSE 2012]

SOLUTION All multiples of 7 lying between 500 and 900 are

504, 511, 518, ..., 896

This is an AP in which $a = 504$, $d = 7$ and $l = 896$.

Let the given AP contain n terms. Then,

$$T_n = 896 \Rightarrow a + (n-1)d = 896 \Rightarrow 504 + (n-1) \times 7 = 896$$

$$\Rightarrow 497 + 7n = 896 \Rightarrow 7n = 399 \Rightarrow n = 57.$$

$\therefore$ required sum $= \frac{n}{2}(a + l)$

$$= \frac{57}{2} \cdot (504 + 896) = \left(\frac{57}{2} \times 1400\right) = 39900.$$

Hence, the required sum is 39900.

EXAMPLE 23 *Find the sum of all 3-digit natural numbers which are multiples of 11.* [CBSE 2009, '12]

SOLUTION All 3-digit natural numbers, which are multiples of 11 are given as 110, 121, 132, ..., 990.

This is an AP in which $a = 110$, $d = 11$ and $l = 990$.

Let the given AP contain n terms. Then,

$$T_n = 990 \Rightarrow a + (n-1)d = 990 \Rightarrow 110 + (n-1) \times 11 = 990$$

$$\Rightarrow 99 + 11n = 990 \Rightarrow 11n = 891 \Rightarrow n = 81.$$

$\therefore$ required sum $= \frac{n}{2}(a + l) = \frac{81}{2} \times (110 + 990)$

$$= \left(\frac{81}{2} \times 1100\right) = 44550.$$

Hence, the required sum is 44550.

EXAMPLE 24 *Ramkali required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly saving by ₹ 20 every week. Find whether she will be able to send her daughter to school after 12 weeks. What value is generated in the above situation?* [CBSE 2015]

SOLUTION The amounts saved by Ramkali in successive weeks are ₹ 100, ₹ 120, ₹ 140, ₹ 160, ... up to 12 terms.

These amounts form an AP in which $a = 100$, $d = 20$ and $n = 12$.

Using $S_n = \frac{n}{2} \times [2a + (n-1)d]$, we get

$$S_{12} = \frac{12}{2} \times [2 \times 100 + 11 \times 20] = (6 \times 420) = 2520 > 2500.$$

Thus, Ramkali will be able to deposit her daughter's fees and so she can send her to school.

The above situation shows that saving is a good habit as it helps preserve and collect money for a good cause.

EXAMPLE 25 *200 logs are stacked in such a way that there are 20 logs in the bottom row, 19 in the next row, 18 in the next row, and so on. In how many rows 200 logs are placed and how many logs are there in the top row?*

SOLUTION Let the required number of rows be n. Then,

$$20 + 19 + 18 + \ldots \text{ to } n \text{ terms} = 200.$$

This is an arithmetic series in which

$$a = 20,\ d = (19 - 20) = -1 \text{ and } S_n = 200.$$

We know that $S_n = \frac{n}{2} \cdot \{2a + (n-1)d\}$.

$\therefore \quad \frac{n}{2} \cdot \{2 \times 20 + (n-1) \times (-1)\} = 200$

$\Rightarrow \quad n(41 - n) = 400 \Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow \quad n^2 - 25n - 16n + 400 = 0 \Rightarrow n(n-25) - 16(n-25) = 0$

$\Rightarrow \quad (n-25)(n-16) = 0 \Rightarrow n - 25 = 0 \text{ or } n - 16 = 0$

$\Rightarrow \quad n = 25 \text{ or } n = 16.$

Now, $T_{25} = (a + 24d) = 20 + 24 \times (-1) = -4$.

This is meaningless as the number of logs cannot be negative.

So, we reject the value $n = 25$.

$\therefore \quad n = 16$. Thus, there are 16 rows in the whole stack.

Now, $T_{16} = (a + 15d) = 20 + 15 \times (-1) = 20 - 15 = 5$.

Hence, there are 5 logs in the top row.

EXAMPLE 26 *The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year. Find the production during (i) first year (ii) 8th year (iii) first 6 years.*

SOLUTION Let the production during first year be a and let d be the increase in production every year. Then,

$$T_6 = 16000 \Rightarrow a + 5d = 16000 \quad \ldots \text{(i)}$$

and $T_9 = 22600 \Rightarrow a + 8d = 22600.$ ... (ii)

On subtracting (i) from (ii), we get

$$3d = 6600 \Rightarrow d = 2200.$$

Putting $d = 2200$ in (i), we get

$$a + 5 \times 2200 = 16000 \Rightarrow a + 11000 = 16000$$

$$\Rightarrow \quad a = 16000 - 11000 = 5000.$$

Thus, $a = 5000$ and $d = 2200$.

(i) Production during first year, $a = 5000$.

(ii) Production during 8th year is given by

$$T_8 = (a + 7d) = (5000 + 7 \times 2200) = (5000 + 15400) = 20400.$$

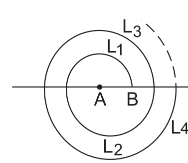
(iii) Production during first 6 years is given by

$$S_6 = \frac{6}{2}\{2a + 5d\} = 3(2 \times 5000 + 5 \times 2200)$$
$$= 3(10000 + 11000) = 63000.$$

EXAMPLE 27 *A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii* 0.5 cm, 1 cm, 1.5 cm, 2 cm, ... *as shown in the given figure. What is the total length of such a spiral made up of* 13 *consecutive semicircles? (Take* $\pi = \frac{22}{7}$*)*

SOLUTION Let $L_1, L_2, L_3, L_4, \ldots, L_{13}$ be the lengths of semicircles of radii 0.5 cm, 1 cm, 1.5 cm, 2 cm, ... and $\frac{13}{2}$ cm respectively.

Then, we have



$$L_1 = (\pi \times 0.5) \text{ cm} = \frac{\pi}{2} \text{ cm},$$

$$L_2 = (\pi \times 1) \text{ cm} = 2\left(\frac{\pi}{2}\right) \text{ cm},$$

$$L_3 = (\pi \times 1.5) \text{ cm} = 3\left(\frac{\pi}{2}\right) \text{ cm},$$

$$L_4 = (\pi \times 2) \text{ cm} = 4\left(\frac{\pi}{2}\right) \text{ cm},$$

... ...

... ...

$$L_{13} = \left(\pi \times \frac{13}{2}\right) \text{ cm} = 13\left(\frac{\pi}{2}\right) \text{ cm}.$$

$\therefore$ total length of the spiral

$$= L_1 + L_2 + L_3 + L_4 + \ldots + L_{13}$$

$$= \left\{\frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) + 3\left(\frac{\pi}{2}\right) + 4\left(\frac{\pi}{2}\right) + \ldots + 13\left(\frac{\pi}{2}\right)\right\} \text{ cm}$$

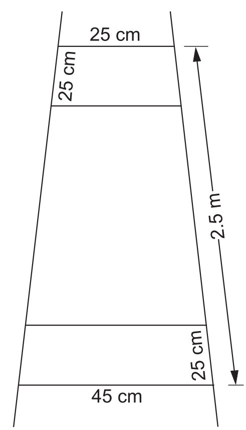
$$= \frac{\pi}{2}(1 + 2 + 3 + 4 + \ldots + 13) \text{ cm}$$

$$= \frac{\pi}{2} \times \frac{13}{2} \times (1 + 13) \text{ cm} \qquad \left[S_n = \frac{n}{2}(a + l)\right]$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times \frac{13}{2} \times 14\right) \text{ cm} = 143 \text{ cm}.$$

Hence, the required length of the spiral is 143 cm.

EXAMPLE 28 *A ladder has rungs* 25 cm *apart. The rungs decrease uniformly in length from* 45 cm *at the bottom to* 25 cm *at the top. If the top and bottom rungs are* 2.5 m *apart, what is the length of the wood required for the rungs?*



SOLUTION It is given that the top and bottom rungs are 250 cm apart and the gap between two consecutive rungs is 25 cm.

$\therefore$ number of rungs $= \left(\frac{250}{25} + 1\right) = 11.$

The largest rung is 45 cm long and the smallest one is 25 cm long.

It is given that the rungs are decreasing uniformly in length from 45 cm at the bottom to 25 cm at the top.

So, the lengths of the rungs form an AP with $a = 45$ cm and l = length of 11th rung = 25 cm.

$\therefore$ length of the wood required to form 11 rungs

$$= \frac{n}{2}(a + l) \text{ cm} = \frac{11}{2}(45 + 25) \text{ cm} = \left(\frac{11}{2} \times 70\right) \text{ cm} = 385 \text{ cm}.$$

Hence, the required length of the wood to form these rungs is 3.85 m.

EXAMPLE 29 *The houses of a row in a colony are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x.*

SOLUTION We are given an AP, namely

$$1, 2, 3, \ldots, (x-1), x, (x+1), \ldots, 49$$

such that $1+2+3+\ldots+(x-1) = (x+1)+(x+2)+\ldots+49$.

Thus, we have $S_{x-1} = S_{49} - S_x$. … (i)

Using the formula, $S_n = \frac{n}{2}(a+l)$ in (i), we have

$$\frac{(x-1)}{2}\cdot\{1+(x-1)\} = \frac{49}{2}\cdot(1+49) - \frac{x}{2}\cdot(1+x)$$

$$\Rightarrow \quad \frac{x(x-1)}{2} + \frac{x(x+1)}{2} = 1225$$

$$\Rightarrow \quad 2x^2 = 2450 \Rightarrow x^2 = 1225 \Rightarrow x = \sqrt{1225} = 35.$$

Hence, $x = 35$.

EXAMPLE 30 *Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3 when divided by 4. Also, find the sum of all numbers on both sides of the middle term.* [CBSE 2015]

SOLUTION The sequence formed by the given numbers is

$$103, 107, 111, 115, \ldots, 999.$$

This is an AP in which $a = 103$ and $d = (107 - 103) = 4$.

Let the total number of these terms be n. Then,

$$T_n = 999 \Rightarrow a + (n-1)d = 999$$

$$\Rightarrow 103 + (n-1)\times 4 = 999$$

$$\Rightarrow (n-1)\times 4 = 896 \Rightarrow (n-1) = 224 \Rightarrow n = 225.$$

$$\therefore \quad \text{middle term} = \left(\frac{n+1}{2}\right)\text{th term} = \left(\frac{225+1}{2}\right)\text{th term} = 113\text{th term}.$$

$$T_{113} = (a + 112d) = (103 + 112\times 4) = 551.$$

$$\Rightarrow \quad T_{112} = (551 - 4) = 547.$$

So, we have to find S_{112} and $(S_{225} - S_{113})$.

Using the formula $S_m = \frac{m}{2}(a+l)$ for each sum, we get

$$S_{112} = \frac{112}{2}(103 + 547) = (112 \times 325) = 36400.$$

$$(S_{225} - S_{113}) = \frac{225}{2}(103 + 999) - \frac{113}{2}(103 + 551)$$
$$= (225 \times 551) - (113 \times 327)$$
$$= 123975 - 36951 = 87024.$$

Sum of all numbers on LHS of the middle term is 36400.

Sum of all numbers on RHS of the middle term is 87024.

EXERCISE 5C

1. Find the sum of each of the following APs:
   (i) 2, 7, 12, 17, … to 19 terms.
   (ii) 9, 7, 5, 3, … to 14 terms.
   (iii) –37, –33, –29, … to 12 terms.
   (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.
   (v) 0.6, 1.7, 2.8, … to 100 terms.
2. Find the sum of each of the following arithmetic series:
   (i) $7 + 10\frac{1}{2} + 14 + \ldots + 84.$
   (ii) $34 + 32 + 30 + \ldots + 10.$
   (iii) $(-5) + (-8) + (-11) + \ldots + (-230)$
   (iv) $5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + \ldots + (-5) + 81 + (-3)$
3. Find the sum of first n terms of an AP whose nth term is $(5 - 6n)$. Hence, find the sum of its first 20 terms.
4. The sum of the first n terms of an AP is $(3n^2 + 6n)$. Find the nth term and the 15th term of this AP. [CBSE 2014]
5. The sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$. Find its (i) nth term, (ii) first term and (iii) common difference. [CBSE 2005C]
6. (i) The sum of the first n terms of an AP is $\left(\frac{5n^2}{2} + \frac{3n}{2}\right)$. Find the nth term and the 20th term of this AP. [CBSE 2006C]
   (ii) The sum of the first n terms of an AP is $\left(\frac{3n^2}{2} + \frac{5n}{2}\right)$. Find its nth term and the 25th term. [CBSE 2006C]
7. If mth term of an AP is $\frac{1}{n}$ and nth term is $\frac{1}{m}$ then find the sum of its first mn terms. [CBSE 2017]
8. How many terms of the AP 21, 18, 15, … must be added to get the sum 0?

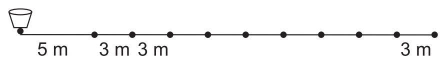
9. How many terms of the AP 9, 17, 25, … must be taken so that their sum is 636? [CBSE 2017]
10. How many terms of the AP 63, 60, 57, 54, … must be taken so that their sum is 693? Explain the double answer.
11. How many terms of the AP $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$ must be taken so that their sum is 300? Explain the double answer.
12. Find the sum of all odd numbers between 0 and 50. [CBSE 20013C}
13. Find the sum of all natural numbers between 200 and 400 which are divisible by 7. [CBSE 2012]
14. Find the sum of first forty positive integers divisible by 6. [CBSE 2012]
15. Find the sum of the first 15 multiples of 8.
16. Find the sum of all multiples of 9 lying between 300 and 700.
17. Find the sum of all three-digit natural numbers which are divisible by 13. [CBSE 2011]
18. Find the sum of first 100 even natural numbers which are divisible by 5. [CBSE 2007C]
19. Find the sum of n terms of the following series:

    $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots.$ [CBSE 2017]
20. In an AP, it is given that $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms. [CBSE 2015]
21. In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference. [CBSE 2010]
22. In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference. [CBSE 2011]
23. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
24. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms. [CBSE 2012, '14, '17]
25. In an AP, the first term is 22, nth term is –11 and sum of first n terms is 66. Find n and hence find the common difference. [CBSE 2008]
26. The 12th term of an AP is –13 and the sum of its first four terms is 24. Find the sum of its first 10 terms. [CBSE 2015]
27. The sum of the first 7 terms of an AP is 182. If its 4th and 17th terms are in the ratio 1 : 5, find the AP. [CBSE 2014]

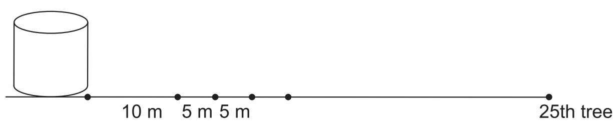
28. The sum of the first 9 terms of an AP is 81 and that of its first 20 terms is 400. Find the first term and the common difference of the AP. [CBSE 2009]
29. The sum of the first 7 terms of an AP is 49 and the sum of its first 17 terms is 289. Find the sum of its first n terms. [CBSE 2008C]
30. Two APs have the same common difference. If the first terms of these APs be 3 and 8 respectively, find the difference between the sums of their first 50 terms. [CBSE 2011]
31. The sum of first 10 terms of an AP is –150 and the sum of its next 10 terms is –550. Find the AP. [CBSE 2010]
32. The 13th term of an AP is 4 times its 3rd term. If its 5th term is 16, find the sum of its first 10 terms. [CBSE 2015]
33. The 16th term of an AP is 5 times its 3rd term. If its 10th term is 41, find the sum of its first 15 terms. [CBSE 2015]
34. (i) An AP 5, 12, 19, … has 50 terms. Find its last term. Hence, find the sum of its last 15 terms. [CBSE 2015]

    (ii) An AP 8, 10, 12, … has 60 terms. Find its last term. Hence, find the sum of its last 10 terms. [CBSE 2015]
35. The sum of first n terms of two APs are in the ratio $(3n+8):(7n+15)$. Find the ratio of their 12th terms.
36. The sum of the 4th and the 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44. Find the sum of its first 10 terms. [CBSE 2013C]
37. The sum of first m terms of an AP is $(4m^2 - m)$. If its nth term is 107, find the value of n. Also, find the 21st term of this AP. [CBSE 2013]
38. The sum of first q terms of an AP is $(63q - 3q^2)$. If its pth term is –60, find the value of p. Also, find the 11th term of its AP. [CBSE 2013]
39. Find the number of terms of the AP –12, –9, –6, …, 21. If 1 is added to each term of this AP then find the sum of all terms of the AP thus obtained. [CBSE 2013]
40. Sum of the first 14 terms of an AP is 1505 and its first term is 10. Find its 25th term. [CBSE 2012]
41. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
42. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by students. Which value is shown in the question? [CBSE 2014]

43. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are 10 potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and he continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



44. There are 25 trees at equal distances of 5 m in a line with a water tank, the distance of the water tank from the nearest tree being 10 m. A gardener waters all the trees separately, starting from the water tank and returning back to the water tank after watering each tree to get water for the next. Find the total distance covered by the gardener in order to water all the trees.



45. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each prize.
46. A man saved ₹ 33000 in 10 months. In each month after the first, he saved ₹ 100 more than he did in the preceding month. How much did he save in the first month?
47. A man arranges to pay off a debt of ₹ 36000 by 40 monthly instalments which form an arithmetic series. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid. Find the value of the first instalment.
48. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
49. A child puts one five-rupee coin of her savings in the piggy bank on the first day. She increases her saving by one five-rupee coin daily. If the piggy bank can hold 190 coins of five rupees in all, find the number of days she can contribute to put the five-rupee coins into it and find the total money she saved. [CBSE 2017]

ANSWERS (EXERCISE 5C)

1. (i) 893 (ii) –56 (iii) –180 (iv) $\frac{33}{20}$ (v) 5505

2. (i) $1046\frac{1}{2}$ (ii) 286 (iii) –8930 (iv) 420 **3.** $S_n = n(2-3n)$, $S_{20} = -1160$

4. $T_n = (6n+3)$, $T_{15} = 93$ **5.** (i) $T_n = (6n-4)$ (ii) $a = 2$ (iii) $d = 6$

6. (i) $T_n = (5n-1)$, $T_{20} = 99$ (ii) $T_n = (3n+1)$, $T_{25} = 76$ **7.** $\frac{1+mn}{2}$

8. 15 **9.** 12 **10.** $n = 21$ or $n = 22$, 22nd term is 0

11. $n = 25$ or $n = 36$, sum of last 11th terms is 0 **12.** 625 **13.** 8729

14. 4920 **15.** 960 **16.** 21978 **17.** 37674 **18.** 50500

19. $\frac{1}{2}(7n-1)$ **20.** 1, 6, 11, 16, … **21.** $d = 3$ **22.** $d = 3$

23. $n = 38$ and $S_n = 6973$ **24.** $d = \frac{8}{3}$ and $n = 16$ **25.** $n = 12$, $d = -3$

26. $S_{10} = 0$ **27.** 2, 10, 18, 26, … **28.** $a = 1$, $d = 2$ **29.** $S_n = n^2$

30. 250 **31.** 3, –1, –5, –9, … **32.** $S_{10} = 175$ **33.** $S_{15} = 495$

34. (i) $l = 348$, sum = 4485 (ii) $T_{60} = 126$, sum = 1170 **35.** 7 : 16

36. $S_{10} = 95$ **37.** $n = 14$, $T_{21} = 163$ **38.** $p = 21$, $T_{11} = 0$

39. $n = 12$, sum = 66 **40.** $T_{25} = 130$ **41.** 5610 **42.** 312

43. 370 m **44.** 3500 m **45.** ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60, ₹ 40

46. ₹ 2850 **47.** ₹ 510 **48.** ₹ 27750 **49.** 19, ₹ 950

HINTS TO SOME SELECTED QUESTIONS

2. (i) $a = 7, d = \left(\frac{21}{2} - 7\right) = \frac{7}{2}$ and $l = 84$.

$T_n = 84 \Rightarrow a + (n-1)d = 84 \Rightarrow 7 + (n-1) \times \frac{7}{2} = 84 \Rightarrow n = 23.$

$S_{23} = \frac{n}{2}(a+l) = \frac{23}{2}(7+84) = 1046\frac{1}{2}.$

3. $T_n = (5-6n) \Rightarrow T_1 = (5-6) = -1$ and $T_2 = (5 - 6 \times 2) = -7$

$\therefore \quad a = -1$ and $d = (-7+1) = -6.$

$S_n = \frac{n}{2} \times \{2 \times (-1) + (n-1) \times (-6)\} = n(2-3n).$

$\therefore \quad S_{20} = 20 \times (2 - 3 \times 20) = -1160.$

9. $4n^2 + 5n - 636 = 0 \Rightarrow 4n^2 + 53n - 48n - 636 = 0.$

10. $n^2 - 43n + 462 = 0 \Rightarrow n^2 - 21n - 22n + 462 = 0.$

11. $n^2 - 61n + 900 = 0 \Rightarrow n^2 - 25n - 36n + 900 = 0.$

12. Required sum $= (1 + 3 + 5 + 7 + \ldots + 49)$.

$T_n = 49 \Rightarrow 1 + (n-1) \times 2 = 49 \Rightarrow n = 25.$

Use $S_n = \frac{n}{2}(a+l)$.

13. Required sum $= (203 + 210 + 217 + \ldots + 399)$.

$T_n = 399 \Rightarrow 203 + (n-1) \times 7 = 399 \Rightarrow n = 29.$

Use $S_n = \frac{n}{2}(a+l)$.

14. Required sum $= (6 + 12 + 18 + \ldots + 240)$.

Here $n = 40$.

Use $S_n = \frac{n}{2}(a+l)$.

15. Required sum $= (8 + 16 + 24 + \ldots + 120)$.

Here $n = 15$. Use $S_n = \frac{n}{2}(a+l)$.

16. Required sum $= (306 + 315 + 324 + \ldots + 693)$.

17. Required sum $= (104 + 117 + 130 + \ldots + 988)$.

18. Required sum $= (10 + 20 + 30 + \ldots + 1000)$.

19. Required sum $= (4 + 4 + \ldots \text{ to } n \text{ terms}) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + \frac{n}{n}\right)$

$$= 4n - \frac{n}{2}\left(\frac{1}{n} + \frac{n}{n}\right) \quad \left[\text{sum} = \frac{n}{2}(a+l)\right]$$

$$= 4n - \frac{(1+n)}{2} = \frac{1}{2}(7n-1).$$

20. $S_5 + S_7 = 167 \Rightarrow \frac{5}{2}(2a+4d) + \frac{7}{2}(2a+6d) = 167$

$\Rightarrow 12a + 31d = 167.$ ... (i)

$S_{10} = 235 \Rightarrow \frac{10}{2}(2a+9d) = 235 \Rightarrow 2a + 9d = 47.$ ... (ii)

Solve (i) and (ii), we get $a = 1$ and $d = 5$.

21. $S_n = \frac{n}{2}(a+l) \Rightarrow \frac{n}{2}(2+29) = 155 \Rightarrow n = 10.$

$T_{10} = 29 \Rightarrow 2 + 9d = 29 \Rightarrow d = 3.$

23. Let the number of terms be n. Then,

$T_n = 350 \Rightarrow a + (n-1)d = 350 \Rightarrow 17 + (n-1) \times 9 = 350 \Rightarrow n = 38.$

$S_n = \frac{n}{2}(a+l)$.

24. $S_n = \frac{n}{2}(a+l) \Rightarrow \frac{n}{2}(5+45) = 400 \Rightarrow n = 16.$

$T_{16} = 45 \Rightarrow a + 15d = 45 \Rightarrow 5 + 15d = 45 \Rightarrow d = \frac{8}{3}.$

25. $S_n = \frac{n}{2}(a+l) \Rightarrow \frac{n}{2}[22 + (-11)] = 66 \Rightarrow n = 12.$

$T_{12} = -11 \Rightarrow a + 11d = -11 \Rightarrow 22 + 11d = -11 \Rightarrow d = -3.$

26. $a + 11d = -13.$ ... (i) and $\frac{4}{2}(2a+3d) = 24 \Rightarrow 2a + 3d = 12.$ ... (ii)

Solve (i) and (ii), we get $a = 9$ and $d = -2$.

Now, find S_{10}.

27. $\frac{7}{2}(2a+6d) = 182 \Rightarrow a+3d = 26.$ ... (i)

$\frac{a+3d}{a+16d} = \frac{1}{5} \Rightarrow 5a+15d = a+16d \Rightarrow 4a-d = 0.$ ... (ii)

Solve (i) and (ii), we get $a = 2$ and $d = 8$.

28. $\frac{9}{2}(2a+8d) = 81 \Rightarrow a+4d = 9$ ... (i)

$\frac{20}{2}(2a+19d) = 400 \Rightarrow 2a+19d = 40.$ ... (ii)

30. Required difference $= \frac{50}{2}\cdot[2\times 8+49d] - \frac{50}{2}\cdot[2\times 3+49d] = 400-150 = 250.$

31. $S_{10} = -150$ and $S_{20} = (-150)+(-550) = -700.$

32. $T_{13} = 4\times T_3 \Rightarrow a+12d = 4(a+2d) \Rightarrow 3a-4d = 0.$ ... (i)

$T_5 = 16 \Rightarrow a+4d = 16.$ ... (ii)

Solve (i) and (ii), we get $a = 4$ and $d = 3$.

Now, find S_{10}.

34. (i) $l = T_{50} = (5+49\times 7) = 348.$

Required sum $= (S_{50}-S_{35})$. Now, $T_{35} = (5+34\times 7) = 243.$

$\therefore \quad (S_{50}-S_{35}) = \frac{50}{2}(5+348) - \frac{35}{2}(5+243) = 8825-4340 = 4485.$

(ii) $l = T_{60} = (8+59\times 2) = 126.$

Required sum $= (S_{60}-S_{50})$. Now, $T_{50} = (8+49\times 2) = 106.$

$\therefore \quad (S_{60}-S_{50}) = \frac{60}{2}(8+126) - \frac{50}{2}(8+106) = 4020-2850 = 1170.$

36. $T_4+T_8 = 24 \Rightarrow (a+3d)+(a+7d) = 24 \Rightarrow a+5d = 12.$ ... (i)

$T_6+T_{10} = 44 \Rightarrow (a+5d)+(a+9d) = 44 \Rightarrow a+7d = 22.$ ... (ii)

Solve (i) and (ii), we get $a = -13$ and $d = 5$.

37. $S_m = (4m^2-m) \Rightarrow S_{m-1} = 4(m-1)^2-(m-1) = (4m^2-9m+5).$

$\therefore \quad T_m = (S_m - S_{m-1}) = (4m^2-m)-(4m^2-9m+5) = (8m-5)$

$\Rightarrow \quad T_n = (8n-5) \Rightarrow 8n-5 = 107 \Rightarrow n = 14.$

$\therefore \quad T_{21} = (8\times 21-5) = 163.$

38. $S_q = (63q-3q^2) \Rightarrow S_{q-1} = 63(q-1)-3(q-1)^2 = (69q-3q^2-66).$

$\therefore \quad T_q = (S_q - S_{q-1}) = -6q+66 \Rightarrow T_p = -6p+66.$

Now, $-6p+66 = -60 \Rightarrow 6p = 126 \Rightarrow p = 21.$

Also, $T_{11} = (-6\times 11)+66 = 0.$

39. Here $a = -12$, $d = 3$ and $l = 21$.

$T_n = 21 \Rightarrow a+(n-1)d = 21 \Rightarrow -12+(n-1)\times 3 = 21 \Rightarrow n = 12.$

Required sum $= \frac{12}{2}(-12+21)+12 = 54+12 = 66.$

40. $\frac{n}{2}(a+l)=1505 \Rightarrow \frac{14}{2}(10+l)=1505 \Rightarrow l=205.$

$T_{14}=205 \Rightarrow a+13d=205 \Rightarrow 10+13d=205 \Rightarrow d=15.$

$T_{25}=(a+24d)=(10+24\times 5)=130.$

41. Let a be the first term of the given AP.

Then, $14-a=18-14 \Rightarrow 14-a=4 \Rightarrow a=10.$

$\therefore$ $a=10, d=4$ and $n=51$.

Now, find S_n.

42. Required sum $=2\times[2+4+6+8+\ldots+24]=2\times\frac{12}{2}(2+24)=312.$

Planting trees helps in reducing air pollution.

43. Total distance run to pick up the 1st, 2nd, 3rd, ..., 10th potato

$=[(2\times 5)+2\times(5+3)+2\times(5+3\times 2)+\ldots+2\times(5+3\times 9)]$ m

$=2\times[5+8+11+\ldots+32]$ m $=2\times\frac{10}{2}(5+32)$ m $=370$ m.

44. Total distance covered

$=[(2\times 10)+2\times(10+5\times 1)+2\times(10+5\times 2)+\ldots+2\times(10+5\times 24)]$ m

$=2\times[10+15+20+\ldots+130]$ m $=2\times\frac{25}{2}(10+130)$ m $=3500$ m.

45. Let the values of these prizes be ₹ x, ₹ $(x-20)$, ₹ $(x-40)$, ... and ₹ $(x-20\times 6)$ respectively.

This is an AP with $a=x, l=(x-120)$ and $n=7$.

$\therefore$ sum $=\frac{n}{2}(a+l)=\frac{7}{2}(x+x-120)=7(x-60)$.

$\Rightarrow$ $7x-420=700 \Rightarrow 7x=1120 \Rightarrow x=160.$

46. Suppose he saved ₹ x, ₹ $(x+100)$, ₹ $(x+200)$, ..., ₹ $(x+900)$ in 1st, 2nd, 3rd, ... and 10th month respectively. Then,

$x+(x+100)+(x+200)+\ldots+(x+900)=33000$

$\Rightarrow$ $10x+(100+200+300+\ldots+900)=33000$

$\Rightarrow$ $10x+\frac{9}{2}(100+900)=33000$ $\left[\because S_n=\frac{n}{2}(a+l)\right]$

$\Rightarrow$ $10x=33000-4500=28500 \Rightarrow x=2850.$

47. Let the first instalment be ₹ x and let it be increased by ₹ d each month. Then,

$x+(x+d)+(x+2d)+\ldots+(x+29d)=\frac{2}{3}\times 36000$

$\Rightarrow$ $30x+(1+2+3+\ldots+29)d=24000$

$\Rightarrow$ $30x+\frac{29}{2}(1+29)d=24000 \Rightarrow 2x+29d=1600.$ ... (i)

Also, $x+(x+d)+(x+2d)+\ldots+(x+39d)=36000$ gives

$40x+\frac{39}{2}(1+39)d=36000 \Rightarrow 2x+39d=1800.$ ... (ii)

On solving (i) and (ii), we get $d=20$ and $x=510$.

48. Total penalty = ₹ (200 + 250 + 300 + ... up to 30 terms).

$T_{30}=(200+29\times 50)=1650.$

$\therefore$ required sum = ₹ $\left\{\frac{30}{2}(200+1650)\right\}$ = ₹ 27750.

EXERCISE 5D

Very-Short and Short-Answer Questions

1. The first three terms of an AP are respectively $(3y-1)$, $(3y+5)$ and $(5y+1)$, find the value of y. [CBSE 2014]
2. If k, $(2k-1)$ and $(2k+1)$ are the three successive terms of an AP, find the value of k. [CBSE 2014]
3. If 18, a, $(b-3)$ are in AP, then find the value of $(2a-b)$. [CBSE 2014]
4. If the numbers a, 9, b, 25 form an AP, find a and b. [CBSE 2014]
5. If the numbers $(2n-1)$, $(3n+2)$ and $(6n-1)$ are in AP, find the value of n and the numbers. [CBSE 2013C]
6. How many three-digit natural numbers are divisible by 7? [CBSE 2013]
7. How many three-digit natural numbers are divisible by 9? [CBSE 2013]
8. If the sum of first m terms of an AP is $(2m^2+3m)$ then what is its second term? [CBSE 2011]
9. What is the sum of first n terms of the AP $a, 3a, 5a, \ldots$. [CBSE 2012]
10. What is the 5th term from the end of the AP 2, 7, 12, …, 47? [CBSE 2011]
11. If a_n denotes the nth term of the AP 2, 7, 12, 17, …, find the value of $(a_{30}-a_{20})$. [CBSE 2011]
12. The nth term of an AP is $(3n+5)$. Find its common difference. [CBSE 2009C]
13. The nth term of an AP is $(7-4n)$. Find its common difference. [CBSE 2008]
14. Write the next term of the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$. [CBSE 2008]
15. Write the next term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$. [CBSE 2008C]
16. Which term of the AP 21, 18, 15, … is zero? [CBSE 2008C]
17. Find the sum of first n natural numbers.
18. Find the sum of first n even natural numbers.
19. The first term of an AP is p and its common difference is q. Find its 10th term. [CBSE 2008]
20. If $\frac{4}{5}$, a, 2 are in AP, find the value of a. [CBSE 2009]
21. If $(2p+1)$, 13, $(5p-3)$ are in AP, find the value of p. [CBSE 2009]

22. If $(2p-1), 7, 3p$ are in AP, find the value of p. [CBSE 2009]

23. If the sum of first p terms of an AP is (ap^2+bp), find its common difference. [CBSE 2010]

24. If the sum of first n terms is $(3n^2+5n)$, find its common difference.

25. Find an AP whose 4th term is 9 and the sum of its 6th and 13th terms is 40. [CBSE 2011]

26. What is the common difference of an AP in which $a_{27}-a_7=84$? [CBSE 2017]

27. If $1+4+7+10+\ldots+x=287$, find the value of x. [CBSE 2017]

ANSWERS (EXERCISE 5D)

1. $y=5$ **2.** $k=3$ **3.** 15 **4.** $a=1, b=17$
5. $n=3, (5, 11, 17)$ **6.** 128 **7.** 100 **8.** 9 **9.** n^2a **10.** 27
11. 50 **12.** 3 **13.** –4 **14.** $\sqrt{50}$ **15.** $\sqrt{32}$ **16.** 8th
17. $\frac{1}{2}n(n+1)$ **18.** $n(n+1)$ **19.** $(p+9q)$ **20.** $a=\frac{7}{5}$ **21.** $p=4$
22. $p=3$ **23.** $d=2a$ **24.** $d=6$ **25.** 3, 5, 7, 9, 11, …
26. $d=4.2$ **27.** $x=40$

HINTS TO SOME SELECTED QUESTIONS

1. We have, $(3y+5)-(3y-1)=(5y+1)-(3y+5)$
   $\Rightarrow \quad 2y-4=6 \Rightarrow 2y=10 \Rightarrow y=5.$
2. We have, $(2k-1)-k=(2k+1)-(2k-1)$
   $\Rightarrow \quad k-1=2 \Rightarrow k=3.$
3. We have, $a-18=(b-3-a) \Rightarrow 2a-b=(18-3)=15.$
4. We have, $9-a=b-9=25-b$
   $\therefore \quad 2b=34 \Rightarrow b=17.$
   Also, $9-a=17-9=8 \Rightarrow a=9-8=1.$
5. We have, $(3n+2)-(2n-1)=(6n-1)-(3n+2)$
   $\therefore \quad n+3=3n-3 \Rightarrow 2n=6 \Rightarrow n=3.$
   The numbers are $(2\times 3-1), (3\times 3+2)$ and $(6\times 3-1)$, i.e., 5, 11, 17.
6. These numbers are 105, 112, 119, 126, …, 994.
   Let $T_n=994$. Then, $105+(n-1)\times 7=994 \Rightarrow n=128.$
7. These numbers are 108, 117, 126, …, 999.
   Let $T_n=999$. Then, $108+(n-1)\times 9=999 \Rightarrow n=100.$

8. $S_m = (2m^2 + 3m) \Rightarrow S_1 = 2 + 3 = 5$ and $S_2 = 2 \times 4 + 3 \times 2 = 14$.

$\therefore \quad T_2 = S_2 - S_1 = 14 - 5 = 9$.

9. Required sum $= \frac{n}{2}[2a + (n-1) \times 2a] = n(na) = n^2 a$.

10. nth term from the end $= l - (n-1)d$.

$\therefore \quad$ 5th term from the end $= 47 - 4 \times 5 = 47 - 20 = 27$.

11. $(a_{30} - a_{20}) = (2 + 29 \times 5) - (2 + 19 \times 5) = 147 - 97 = 50$.

12. $T_n = (3n + 5) \Rightarrow T_1 = (3 \times 1 + 5) = 8, T_2 = (3 \times 2 + 5) = 11$.

$\therefore \quad d = (T_2 - T_1) = 3$.

13. $T_n = (7 - 4n) \Rightarrow T_1 = (7 - 4 \times 1) = 3, T_2 = (7 - 4 \times 2) = -1$.

$\therefore \quad d = (T_2 - T_1) = (-1 - 3) = -4$.

14. Given terms are $\sqrt{4 \times 2}, \sqrt{9 \times 2}, \sqrt{16 \times 2}, \ldots$, i.e., $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$.

So, the next term is $5\sqrt{2} = \sqrt{5 \times 5 \times 2} = \sqrt{50}$.

15. Given terms are $\sqrt{2}, \sqrt{4 \times 2}, \sqrt{9 \times 2}, \ldots$, i.e., $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots$.

The next term is $4\sqrt{2} = \sqrt{4 \times 4 \times 2} = \sqrt{32}$.

16. Let $T_n = 0$. Then, $21 + (n-1) \times (-3) = 0 \Rightarrow n = 8$. So, 8th term is zero.

17. $S_n = 1 + 2 + 3 + \ldots + n = \frac{n}{2}(1 + n)$. $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

18. $S_n = 2 + 4 + 6 + \ldots + 2n = \frac{n}{2}(2 + 2n) = n(n+1)$. $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

19. Here $a = p$ and $d = q$.

$\therefore \quad T_{10} = a + 9d = (p + 9q)$.

20. We have, $a - \frac{4}{5} = 2 - a \Rightarrow 2a = \left(2 + \frac{4}{5}\right) = \frac{14}{5} \Rightarrow a = \frac{7}{5}$.

21. We have, $13 - (2p + 1) = (5p - 3) - 13$.

$\therefore \quad (5p - 3) + (2p + 1) = 26 \Rightarrow 7p = 28 \Rightarrow p = 4$.

22. We have, $7 - (2p - 1) = 3p - 7$.

$\therefore \quad (2p - 1) + 3p = 14 \Rightarrow 5p = 15 \Rightarrow p = 3$.

23. $T_1 = S_1 = a + b \quad$ [putting $p = 1$].

Now, $S_2 = (a \times 4 + b \times 2) = (4a + 2b)$.

$\therefore \quad T_2 = S_2 - S_1 = (4a + 2b) - (a + b) = (3a + b)$.

$\therefore \quad d = T_2 - T_1 = (3a + b) - (a + b) = 2a$.

24. $S_n = (3n^2 + 5n)$

$\Rightarrow \quad T_1 = S_1 = (3 \times 1 + 5 \times 1) = 8$.

Now, $S_2 = 3 \times 4 + 5 \times 2 = 22$.

$\therefore \quad T_2 = S_2 - S_1 = 22 - 8 = 14$.

$\Rightarrow \quad d = T_2 - T_1 = 14 - 8 = 6$.

25. $T_4 = 9 \Rightarrow a + 3d = 9.$ … (i)

$T_6 + T_{13} = 40 \Rightarrow (a + 5d) + (a + 12d) = 40 \Rightarrow 2a + 17d = 40.$ … (ii)

From (i) and (ii), we get $a = 3$ and $d = 2$.

$\therefore$ AP is 3, 5, 7, 9, 11, … .

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is [CBSE 2013]

   (a) p (b) $-p$ (c) -1 (d) 1

2. The common difference of the AP $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$ is [CBSE 2013]

   (a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) b (d) $-b$

3. The next term of the AP $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$ is [CBSE 2014]

   (a) $\sqrt{70}$ (b) $\sqrt{84}$ (c) $\sqrt{98}$ (d) $\sqrt{112}$

4. If 4, x_1, x_2, x_3, 28 are in AP then $x_3 = ?$

   (a) 19 (b) 23
   (c) 22 (d) Cannot be determined

5. If the nth term of an AP is $(2n + 1)$ then the sum of its first three terms is [CBSE 2012]

   (a) $6n + 3$ (b) 15 (c) 12 (d) 21

6. The sum of first n terms of an AP is $(3n^2 + 6n)$. The common difference of the AP is [CBSE 2014]

   (a) 6 (b) 9 (c) 15 (d) -3

7. The sum of first n terms of an AP is $(5n - n^2)$. The nth term of the AP is

   (a) $(5 - 2n)$ (b) $(6 - 2n)$ (c) $(2n - 5)$ (d) $(2n - 6)$

8. The sum of first n terms of an AP is $(4n^2 + 2n)$. The nth term of this AP is [CBSE 2014]

   (a) $(6n - 2)$ (b) $(7n - 3)$ (c) $(8n - 2)$ (d) $(8n + 2)$

9. The 7th term of an AP is -1 and its 16th term is 17. The nth term of the AP is [CBSE 2014]

   (a) $(3n + 8)$ (b) $(4n - 7)$ (c) $(15 - 2n)$ (d) $(2n - 15)$

10. The 5th term of an AP is –3 and its common difference is –4. The sum of its first 10 terms is [CBSE 2011]

(a) 50 (b) –50 (c) 30 (d) –30

11. The 5th term of an AP is 20 and the sum of its 7th and 11th terms is 64. The common difference of the AP is [CBSE 2015]

(a) 4 (b) 5 (c) 3 (d) 2

12. The 13th term of an AP is 4 times its 3rd term. If its 5th term is 16 then the sum of its first ten terms is [CBSE 2015]

(a) 150 (b) 175 (c) 160 (d) 135

13. An AP 5, 12, 19, … has 50 terms. Its last term is [CBSE 2015]

(a) 343 (b) 353 (c) 348 (d) 362

14. The sum of first 20 odd natural numbers is [CBSE 2012]

(a) 100 (b) 210 (c) 400 (d) 420

15. The sum of first 40 positive integers divisible by 6 is [CBSE 2014]

(a) 2460 (b) 3640 (c) 4920 (d) 4860

16. How many two-digit numbers are divisible by 3? [CBSE 2012]

(a) 25 (b) 30 (c) 32 (d) 36

17. How many three-digit numbers are divisible by 9? [CBSE 2013]

(a) 86 (b) 90 (c) 96 (d) 100

18. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) –8 (c) 4 (d) –4

19. If a_n denotes the nth term of the AP 3, 8, 13, 18, … then what is the value of $(a_{30} - a_{20})$?

(a) 40 (b) 36 (c) 50 (d) 56

20. Which term of the AP 72, 63, 54, … is 0?

(a) 8th (b) 9th (c) 10th (d) 11th

21. Which term of the AP 25, 20, 15, … is the first negative term?

(a) 10th (b) 9th (c) 8th (d) 7th

22. Which term of the AP 21, 42, 63, 84, … is 210?

(a) 9th (b) 10th (c) 11th (d) 12th

23. What is 20th term from the end of the AP 3, 8, 13, …, 253?

(a) 163 (b) 158 (c) 153 (d) 148

24. $(5 + 13 + 21 + \ldots + 181) = ?$

(a) 2476 (b) 2337 (c) 2219 (d) 2139

25. The sum of first 16 terms of the AP 10, 6, 2, … is
(a) 320 (b) –320 (c) –352 (d) –400

26. How many terms of the AP 3, 7, 11, 15, … will make the sum 406?
(a) 10 (b) 12 (c) 14 (d) 20

27. The 2nd term of an AP is 13 and its 5th term is 25. What is its 17th term?
(a) 69 (b) 73 (c) 77 (d) 81

28. The 17th term of an AP exceeds its 10th term by 21. The common difference of the AP is
(a) 3 (b) 2 (c) –3 (d) –2

29. The 8th term of an AP is 17 and its 14th term is 29. The common difference of the AP is
(a) 3 (b) 2 (c) 5 (d) –2

30. The 7th term of an AP is 4 and its common difference is –4. What is its first term?
(a) 16 (b) 20 (c) 24 (d) 28

ANSWERS (MCQ)

1. (c) **2.** (d) **3.** (d) **4.** (c) **5.** (b) **6.** (a) **7.** (b) **8.** (c) **9.** (d)
10. (b) **11.** (c) **12.** (b) **13.** (c) **14.** (c) **15.** (c) **16.** (b) **17.** (d) **18.** (a)
19. (c) **20.** (b) **21.** (d) **22.** (b) **23.** (b) **24.** (d) **25.** (b) **26.** (c) **27.** (b)
28. (a) **29.** (b) **30.** (d)

HINTS TO SOME SELECTED QUESTIONS

1. $d = \left\{\frac{1-p}{p} - \frac{1}{p}\right\} = \left(\frac{1-p-1}{p}\right) = \frac{-p}{p} = -1.$

2. $d = \left\{\frac{1-3b}{3} - \frac{1}{3}\right\} = \left(\frac{1-3b-1}{3}\right) = \frac{-3b}{3} = -b.$

3. Given terms are $\sqrt{7}, \sqrt{4\times 7}, \sqrt{9\times 7}, \ldots$, i.e., $\sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, \ldots$.
So, the next term is $4\sqrt{7} = \sqrt{4\times 4\times 7} = \sqrt{112}$.

4. Given AP is 4, x_1, x_2, x_3, 28. Clearly, $a = 4$ and $T_5 = 28$.
Now, $T_5 = 28 \Rightarrow a + 4d = 28 \Rightarrow 4 + 4d = 28 \Rightarrow 4d = 24 \Rightarrow d = 6$.
$\therefore \quad x_3 = T_4 = (a + 3d) = (4 + 3\times 6) = 22.$

5. $(T_1 + T_2 + T_3) = (2\times 1 + 1) + (2\times 2 + 1) + (2\times 3 + 1) = 3 + 5 + 7 = 15.$

6. Given $S_n = (3n^2 + 6n)$.

$\therefore \quad T_1 = S_1 = (3 \times 1^2 + 6 \times 1) = 9, S_2 = (3 \times 2^2 + 6 \times 2) = 24.$

$\therefore \quad T_2 = (S_2 - S_1) = (24 - 9) = 15.$

Hence, $d = (T_2 - T_1) = 15 - 9 = 6.$

7. $T_n = (S_n - S_{n-1}) = (5n - n^2) - \{5(n-1) - (n-1)^2\} = (5n - n^2) - (7n - n^2 - 6) = (6 - 2n)$.

8. $T_n = (S_n - S_{n-1}) = (4n^2 + 2n) - \{4(n-1)^2 + 2(n-1)\}$

$\quad = (4n^2 + 2n) - (4n^2 - 6n + 2) = (8n - 2)$.

9. $T_7 = -1 \Rightarrow a + 6d = -1.$ ... (i)

$T_{16} = 17 \Rightarrow a + 15d = 17.$ ... (ii)

On solving (i) and (ii), we get $a = -13$ and $d = 2$.

$\therefore \quad T_n = a + (n-1)d = -13 + (n-1) \times 2 = (2n - 15)$.

10. $a + 4d = -3$ and $d = -4$. So, $a = 13$.

$\therefore \quad S_{10} = \frac{10}{2}[2a + 9d] = 5[2 \times 13 + 9 \times (-4)] = -50.$

11. $T_5 = 20 \Rightarrow a + 4d = 20.$ ... (i)

$(T_7 + T_{11}) = 64 \Rightarrow (a + 6d) + (a + 10d) = 64 \Rightarrow a + 8d = 32.$ ... (ii)

On solving (i) and (ii), we get $d = 3$.

12. $T_{13} = 4 \times T_3 \Rightarrow a + 12d = 4(a + 2d) \Rightarrow 3a - 4d = 0.$ ... (i)

$T_5 = 16 \Rightarrow a + 4d = 16.$ ... (ii)

On solving (i) and (ii), we get $a = 4$ and $d = 3$.

$S_{10} = \frac{10}{2}(2a + 9d) = 5(2 \times 4 + 9 \times 3) = 175.$

13. 50th term $= a + (n-1)d = (5 + 49 \times 7) = 348.$

14. $S_{20} = 1 + 3 + 5 + 7 + \ldots$ up to 20 terms.

Here $a = 1$, $d = 2$. So, $T_{20} = (a + 19d) = (1 + 19 \times 2) = 38 = l$.

$\therefore \quad S_{20} = \frac{n}{2}(a + l) = \frac{20}{2}(1 + 39) = 400.$

15. Required sum $= 6 + 12 + 18 + \ldots + 240 = \frac{40}{2}(6 + 240) = 4920.$

16. Two-digit numbers divisible by 3 are 12, 15, 18, …, 99.

Let $T_n = 99$. Then, $12 + (n-1) \times 3 = 99 \Rightarrow (n-1) \times 3 = 87 \Rightarrow n = 30.$

17. Three-digit numbers divisible by 9 are 108, 117, 126, …, 999.

Let $T_n = 999$. Then, $108 + (n-1) \times 9 = 999$.

$\therefore \quad (n-1) \times 9 = 891 \Rightarrow (n-1) = 99 \Rightarrow n = 100.$

18. $a_{18} - a_{14} = 32 \Rightarrow (a + 17d) - (a + 13d) = 32 \Rightarrow 4d = 32 \Rightarrow d = 8.$

19. Here $a = 3$ and $d = 5$.

$\therefore \quad (a_{30} - a_{20}) = (3 + 29 \times 5) - (3 + 19 \times 5) = 148 - 98 = 50.$

20. Let the nth term of 72, 63, 54, … be 0. Then,

$T_n = 0 \Rightarrow 72 + (n-1) \times (-9) = 0 \Rightarrow 9n = 81 \Rightarrow n = 9.$

21. Let $T_n < 0$. Then, $25 + (n-1) \times (-5) < 0$.

$\therefore \quad 30 < 5n \Rightarrow 5n > 30 \Rightarrow n > 6$.

So, the required term is 7th.

22. Let $T_n = 210$. Then, $21 + (n-1) \times 21 = 210$

$\therefore \quad (n-1) \times 21 = 189 \Rightarrow n - 1 = 9 \Rightarrow n = 10$.

23. 20th term from the end $= (253 - 19 \times 5) = (253 - 95) = 158$. $\quad [l - (n-1)d]$

24. Let the number of terms be n. Then, $T_n = 181$.

$\therefore \quad 5 + (n-1) \times 8 = 181 \Rightarrow (n-1) \times 8 = 176 \Rightarrow (n-1) = 22 \Rightarrow n = 23$.

$\therefore \quad \text{sum} = \frac{n}{2}(a + l) = \frac{23}{2}(5 + 181) = 23 \times 93 = 2139$.

25. $l = T_{16} = 10 + 15 \times (-4) = -50$.

$\therefore \quad \text{sum} = \frac{n}{2}(a + l) = \frac{16}{2}(10 - 50) = 8 \times (-40) = -320$.

26. Let $S_n = 406$. Then, $\frac{n}{2}[2 \times 3 + (n-1) \times 4] = 406$

$\Rightarrow \quad \frac{n}{2}(6 + 4n - 4) = 406 \Rightarrow \frac{n}{2}(4n + 2) = 406$

$\Rightarrow \quad n(2n + 1) = 406 \Rightarrow 2n^2 + n - 406 = 0$.

$\therefore \quad n = \frac{-1 \pm \sqrt{1 + 3248}}{4} = \frac{-1 + \sqrt{3249}}{4}$ [neglecting negative value]

$\Rightarrow \quad n = \frac{-1 + 57}{4} = \frac{56}{4} = 14$.

27. $a + d = 13$ … (i) and $a + 4d = 25$ … (ii).

From (i) and (ii), we get $a = 9$ and $d = 4$.

$\therefore \quad T_{17} = (a + 16d) = (9 + 16 \times 4) = 73$.

28. $(T_{17} - T_{10}) = 21 \Rightarrow (a + 16d) - (a + 9d) = 21 \Rightarrow 7d = 21 \Rightarrow d = 3$.

29. $(T_{14} - T_8) = (29 - 17) = 12 \Rightarrow (a + 13d) - (a + 7d) = 12 \Rightarrow 6d = 12 \Rightarrow d = 2$.

30. $a + 6d = 4 \Rightarrow a + 6 \times (-4) = 4 \Rightarrow a = 28$.

✿