

DISTANCE BETWEEN TWO POINTS

THEOREM 1 *The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula*

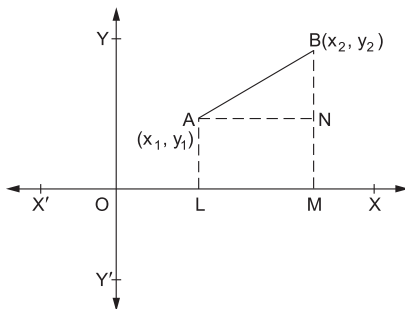
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

PROOF Let $X'OX$ and YOY' be the x -axis and y -axis respectively.

Then, O is the origin.

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the given points.

Draw $AL \perp OX$, $BM \perp OX$ and $AN \perp BM$.



Now, $OL = x_1$, $OM = x_2$, $AL = y_1$ and $BM = y_2$.

$$\therefore AN = LM = (OM - OL) = (x_2 - x_1),$$

$$BN = (BM - NM) = (BM - AL) = (y_2 - y_1) \quad [\because NM = AL].$$

From right $\triangle ANB$, by Pythagoras' theorem, we have

$$AB^2 = (AN^2 + BN^2)$$

$$\Rightarrow AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Hence, the distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

COROLLARY *The distance of the point $P(x, y)$ from the origin $O(0, 0)$ is given by*

$$OP = \sqrt{(x - 0)^2 + (y - 0)^2}, \text{ i.e., } OP = \sqrt{x^2 + y^2}.$$

SUMMARY

1. The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. The distance of the point $P(x, y)$ from the origin $O(0, 0)$ is given by

$$OP = \sqrt{x^2 + y^2}.$$

SOLVED EXAMPLES

EXAMPLE 1 Find the distance between the points $A(7, 13)$ and $B(10, 9)$.

SOLUTION The given points are $A(7, 13)$ and $B(10, 9)$.

Then, $x_1 = 7$, $y_1 = 13$ and $x_2 = 10$, $y_2 = 9$.

$$\begin{aligned} \therefore AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(10 - 7)^2 + (9 - 13)^2} = \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units.} \end{aligned}$$

EXAMPLE 2 Find the distance between the points $P(-4, 7)$ and $Q(2, -5)$.

SOLUTION The given points are $P(-4, 7)$ and $Q(2, -5)$.

Then, $x_1 = -4$, $y_1 = 7$ and $x_2 = 2$, $y_2 = -5$.

$$\begin{aligned} \therefore PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\{2 - (-4)\}^2 + \{-5 - 7\}^2} = \sqrt{6^2 + (-12)^2} \\ &= \sqrt{36 + 144} = \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5} \text{ units.} \end{aligned}$$

EXAMPLE 3 Find the distance of the point $P(6, -6)$ from the origin.

SOLUTION Let $P(6, -6)$ be the given point and $O(0, 0)$ be the origin.

$$\begin{aligned} \text{Then, } OP &= \sqrt{(6 - 0)^2 + (-6 - 0)^2} = \sqrt{6^2 + (-6)^2} \\ &= \sqrt{36 + 36} = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2} \text{ units.} \end{aligned}$$

EXAMPLE 4 Find the values of y for which the distance between the points $A(3, -1)$ and $B(11, y)$ is 10 units. [CBSE 2011]

SOLUTION We have

$$\begin{aligned} AB = 10 &\Rightarrow AB^2 = 100 \\ \Rightarrow (11 - 3)^2 + (y + 1)^2 &= 100 \\ \Rightarrow 8^2 + (y + 1)^2 &= 100 \\ \Rightarrow (y + 1)^2 &= 100 - 64 = 36 = 6^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow y+1 &= \pm 6 \\ \Rightarrow y+1 &= 6 \text{ or } y+1 = -6 \\ \Rightarrow y &= 5 \text{ or } y = -7. \end{aligned}$$

Hence, the required values of y are 5 and -7 .

EXAMPLE 5 *If the point $P(k-1, 2)$ is equidistant from the points $A(3, k)$ and $B(k, 5)$, find the values of k .* [CBSE 2014]

SOLUTION It is being given that $P(k-1, 2)$ is equidistant from the points $A(3, k)$ and $B(k, 5)$. So, we have

$$\begin{aligned} PA &= PB \Rightarrow PA^2 = PB^2 \\ \Rightarrow (k-1-3)^2 + (2-k)^2 &= (k-1-k)^2 + (2-5)^2 \\ \Rightarrow (k-4)^2 + (2-k)^2 &= (-1)^2 + (-3)^2 \\ \Rightarrow 2k^2 - 12k + 20 &= 1 + 9 \Rightarrow 2k^2 - 12k + 10 = 0 \\ \Rightarrow k^2 - 6k + 5 &= 0 \Rightarrow k^2 - 5k - k + 5 = 0 \\ \Rightarrow k(k-5) - (k-5) &= 0 \Rightarrow (k-5)(k-1) = 0 \\ \Rightarrow k &= 1 \text{ or } k = 5. \end{aligned}$$

Hence, the required values of k are 1 and 5.

EXAMPLE 6 *Find the relation between x and y such that the point $P(x, y)$ is equidistant from the points $A(1, 4)$ and $B(-1, 2)$.* [CBSE 2012]

SOLUTION We have

$$\begin{aligned} PA &= PB \Rightarrow PA^2 = PB^2 \\ \Rightarrow (x-1)^2 + (y-4)^2 &= (x+1)^2 + (y-2)^2 \\ \Rightarrow x^2 + y^2 - 2x - 8y + 17 &= x^2 + y^2 + 2x - 4y + 5 \\ \Rightarrow 4x + 4y - 12 &= 0 \Rightarrow x + y = 3 \Rightarrow y = 3 - x. \end{aligned}$$

Hence, $y = 3 - x$ is the desired relation between x and y .

TWO IMPORTANT NOTES

- (i) Any point on the x -axis is of the form $(x, 0)$.
- (ii) Any point on the y -axis is of the form $(0, y)$.

EXAMPLE 7 *Find those points on x -axis, each of which is at a distance of 5 units from the point $A(5, -3)$.* [CBSE 2011]

SOLUTION Let the required point on the x -axis be $P(x, 0)$. Then,

$$\begin{aligned} PA &= 5 \Rightarrow PA^2 = 25 \\ \Rightarrow (x-5)^2 + (0+3)^2 &= 25 \\ \Rightarrow x^2 - 10x + 25 + 9 &= 25 \Rightarrow x^2 - 10x + 9 = 0 \\ \Rightarrow x^2 - x - 9x + 9 &= 0 \Rightarrow x(x-1) - 9(x-1) = 0 \end{aligned}$$

$$\Rightarrow (x-1)(x-9) = 0 \Rightarrow x-1 = 0 \text{ or } x-9 = 0$$

$$\Rightarrow x = 1 \text{ or } x = 9.$$

Hence, the required points on the x -axis are $B(1, 0)$ and $C(9, 0)$.

EXAMPLE 8

Find those points on the y -axis, each of which is at a distance of 13 units from the point $A(-5, 7)$.

SOLUTION

Let the required point on the y -axis be $P(0, y)$. Then,

$$PA = 13 \Rightarrow PA^2 = 169$$

$$\Rightarrow (0+5)^2 + (y-7)^2 = 169$$

$$\Rightarrow y^2 - 14y + 74 = 169 \Rightarrow y^2 - 14y - 95 = 0$$

$$\Rightarrow y^2 - 19y + 5y - 95 = 0 \Rightarrow y(y-19) + 5(y-19) = 0$$

$$\Rightarrow (y-19)(y+5) = 0 \Rightarrow y-19 = 0 \text{ or } y+5 = 0$$

$$\Rightarrow y = 19 \text{ or } y = -5.$$

Hence, the required points on the y -axis are $B(0, 19)$ and $C(0, -5)$.

EXAMPLE 9

Find the point on x -axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$. [CBSE 2009]

SOLUTION

Let the given points be $A(5, -2)$ and $B(-3, 2)$ and let the required point be $P(x, 0)$. Then,

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-5)^2 + (0+2)^2 = (x+3)^2 + (0-2)^2$$

$$\Rightarrow (x-5)^2 + 4 = (x+3)^2 + 4$$

$$\Rightarrow (x-5)^2 - (x+3)^2 = 0$$

$$\Rightarrow (x^2 - 10x + 25) - (x^2 + 6x + 9) = 0$$

$$\Rightarrow -16x + 16 = 0$$

$$\Rightarrow 16x = 16 \Rightarrow x = 1.$$

Hence, the required point is $P(1, 0)$.

EXAMPLE 10

Find the point on y -axis which is equidistant from the points $(-5, 2)$ and $(9, -2)$. [CBSE 2009C]

SOLUTION

Let the given points be $A(-5, 2)$ and $B(9, -2)$, and let the required point be $P(0, y)$. Then,

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (0+5)^2 + (y-2)^2 = (0-9)^2 + (y+2)^2$$

$$\Rightarrow 5^2 + (y-2)^2 = (-9)^2 + (y+2)^2$$

$$\Rightarrow 25 + (y-2)^2 = 81 + (y+2)^2$$

$$\Rightarrow (y+2)^2 - (y-2)^2 = 25 - 81$$

$$\Rightarrow 8y = -56 \Rightarrow y = -7.$$

Hence, the required point is $P(0, -7)$.

EXAMPLE 11 Find the coordinates of the point equidistant from three given points $A(5, 1)$, $B(-3, -7)$ and $C(7, -1)$. [CBSE 2006]

SOLUTION Let the required point be $P(x, y)$. Then,

$$PA = PB = PC \Rightarrow PA^2 = PB^2 = PC^2$$

$$\Rightarrow PA^2 = PB^2 \text{ and } PB^2 = PC^2$$

$$\Rightarrow \begin{cases} (x-5)^2 + (y-1)^2 = (x+3)^2 + (y+7)^2, \\ (x+3)^2 + (y+7)^2 = (x-7)^2 + (y+1)^2 \end{cases}$$

$$\Rightarrow \begin{cases} x^2 + y^2 - 10x - 2y + 26 = x^2 + y^2 + 6x + 14y + 58, \\ x^2 + y^2 + 6x + 14y + 58 = x^2 + y^2 - 14x + 2y + 50. \end{cases}$$

$$\text{Now, } x^2 + y^2 - 10x - 2y + 26 = x^2 + y^2 + 6x + 14y + 58$$

$$\Rightarrow 16x + 16y = -32 \Rightarrow x + y = -2. \quad \dots \text{ (i)}$$

$$\text{And, } x^2 + y^2 + 6x + 14y + 58 = x^2 + y^2 - 14x + 2y + 50$$

$$\Rightarrow 20x + 12y = -8 \Rightarrow 5x + 3y = -2. \quad \dots \text{ (ii)}$$

Multiplying (i) by 5 and subtracting (ii) from the result, we get

$$2y = -8 \Rightarrow y = -4.$$

Putting $y = -4$ in (i), we get $x = 2$.

$$\therefore x = 2 \text{ and } y = -4.$$

Hence, the required point is $P(2, -4)$.

EXAMPLE 12 Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$. Find the values of y . Hence, find the radius of the circle. [CBSE 2014]

SOLUTION Since the radii of a circle are equal, we have

$$OA = OB \Rightarrow OA^2 = OB^2$$

$$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$$

$$\Rightarrow 3^2 + (-4y)^2 = (-3)^2 + (3y+7)^2$$

$$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y \quad (-1, y)$$

$$\Rightarrow 7y^2 - 42y - 49 = 0$$

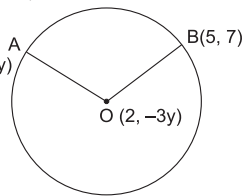
$$\Rightarrow y^2 - 6y - 7 = 0$$

$$\Rightarrow y^2 - 7y + y - 7 = 0$$

$$\Rightarrow y(y-7) + (y-7) = 0 \Rightarrow (y-7)(y+1) = 0$$

$$\Rightarrow y-7 = 0 \text{ or } y+1 = 0 \Rightarrow y=7 \text{ or } y=-1.$$

\therefore centre is either $(2, 3)$ or $(2, -21)$.



CASE I When centre is (2, 3)

$$\begin{aligned}\text{In this case, radius} &= \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \text{ units.}\end{aligned}$$

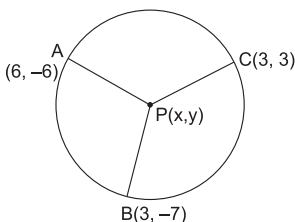
CASE II When centre is (2, -21)

$$\begin{aligned}\text{In this case, radius} &= \sqrt{(5-2)^2 + \{7 - (-21)\}^2} \\ &= \sqrt{3^2 + (28)^2} = \sqrt{9+784} \\ &= \sqrt{793} = 28.1 \text{ units.}\end{aligned}$$

EXAMPLE 13 Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).

SOLUTION Let $P(x, y)$ be the centre of a circle passing through the points $A(6, -6)$, $B(3, -7)$ and $C(3, 3)$. Then,

$$PA^2 = PB^2 \text{ and } PB^2 = PC^2$$



$$\begin{aligned}\Rightarrow & \begin{cases} (x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2, \\ (x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2 \end{cases} \\ \Rightarrow & \begin{cases} x^2 + y^2 - 12x + 12y + 72 = x^2 + y^2 - 6x + 14y + 58, \\ x^2 + y^2 - 6x + 14y + 58 = x^2 + y^2 - 6x - 6y + 18 \end{cases} \\ \Rightarrow & 6x + 2y = 14 \text{ and } 20y = -40 \\ \Rightarrow & 3x + y = 7 \text{ and } y = -2 \\ \Rightarrow & x = 3 \text{ and } y = -2.\end{aligned}$$

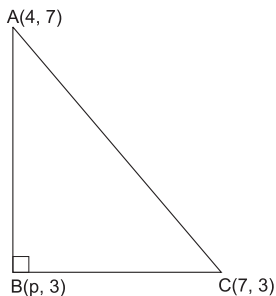
Hence, the centre of the circle is $P(3, -2)$.

EXAMPLE 14 The points $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$ are the vertices of a right triangle, right angled at B . Find the value of p . [CBSE 2015]

SOLUTION Since $\triangle ABC$ is a right angled at B , we have

$$\begin{aligned}AB^2 + BC^2 &= AC^2 \\ \Rightarrow & \{(p-4)^2 + (3-7)^2\} + \{(7-p)^2 + (3-3)^2\} = (7-4)^2 + (3-7)^2 \\ \Rightarrow & (p-4)^2 + (-4)^2 + (7-p)^2 + 0 = 3^2 + (-4)^2\end{aligned}$$

$$\begin{aligned} \Rightarrow 2p^2 - 22p + (16 + 49 + 16) &= 9 + 16 \\ \Rightarrow 2p^2 - 22p + 56 &= 0 \\ \Rightarrow p^2 - 11p + 28 &= 0 \\ \Rightarrow p^2 - 7p - 4p + 28 &= 0 \\ \Rightarrow p(p-7) - 4(p-7) &= 0 \\ \Rightarrow (p-7)(p-4) &= 0 \\ \Rightarrow p-7 = 0 \text{ or } p-4 = 0 \\ \Rightarrow p = 7 \text{ or } p = 4. \end{aligned}$$



When $p = 7$ then the points B and C coincide and so no triangle is formed in this case.

So, $p \neq 7$. Hence, $p = 4$.

EXAMPLE 15 Show that the points (a, a) , $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle. Find its area. [CBSE 2015]

SOLUTION Let the given points be $A(a, a)$, $B(-a, -a)$ and $C(-\sqrt{3}a, \sqrt{3}a)$. Then, $AB^2 = (-a - a)^2 + (-a - a)^2 = (-2a)^2 + (-2a)^2 = 8a^2$,

$$\begin{aligned} BC^2 &= (-\sqrt{3}a + a)^2 + (\sqrt{3}a + a)^2 \\ &= (a - \sqrt{3}a)^2 + (a + \sqrt{3}a)^2 = 2(a^2 + 3a^2) = 8a^2 \end{aligned}$$

$$\text{and } AC^2 = (-\sqrt{3}a - a)^2 + (\sqrt{3}a - a)^2 = (\sqrt{3}a + a)^2 + (\sqrt{3}a - a)^2 = 2(3a^2 + a^2) = 8a^2.$$

$$\therefore AB = BC = AC = \sqrt{8a^2} = 2\sqrt{2}a.$$

Hence, $\triangle ABC$ is equilateral and each of its sides is $2\sqrt{2}a$.

Area of equilateral $\triangle ABC$

$$\begin{aligned} &= \left\{ \frac{\sqrt{3}}{4} \times (\text{side})^2 \right\} \text{ sq units} = \left\{ \frac{\sqrt{3}}{4} \times (2\sqrt{2}a)^2 \right\} \text{ sq units} \\ &= \left\{ \frac{\sqrt{3}}{4} \times 8a^2 \right\} \text{ sq units} = 2\sqrt{3}a^2 \text{ sq units.} \end{aligned}$$

EXAMPLE 16 Prove that the points $A(-3, 0)$, $B(1, -3)$ and $C(4, 1)$ are the vertices of an isosceles right-angled triangle. Find the area of this triangle.

SOLUTION Let $A(-3, 0)$, $B(1, -3)$ and $C(4, 1)$ be the given points. Then,

$$AB = \sqrt{[1 - (-3)]^2 + (-3 - 0)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5 \text{ units,}$$

$$BC = \sqrt{(4 - 1)^2 + [1 - (-3)]^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ units}$$

$$\text{and } AC = \sqrt{[4 - (-3)]^2 + (1 - 0)^2} = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2} \text{ units.}$$

Thus, $AB = BC = 5$ units.

$\therefore \triangle ABC$ is isosceles.

Also, $(AB^2 + BC^2) = (5^2 + 5^2) = 50$ and

$$AC^2 = (5\sqrt{2})^2 = 50.$$

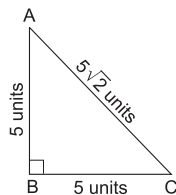
Thus, $AB^2 + BC^2 = AC^2$.

This shows that $\triangle ABC$ is right-angled at B .

In $\triangle ABC$, we have

base = $BC = 5$ units and height = $AB = 5$ units.

$$\begin{aligned} \therefore \text{area of } \triangle ABC &= \left(\frac{1}{2} \times \text{base} \times \text{height}\right) = \left(\frac{1}{2} \times 5 \times 5\right) \text{ sq units} \\ &= 12.5 \text{ sq units.} \end{aligned}$$



PROPERTIES OF VARIOUS TYPES OF QUADRILATERALS

A quadrilateral is a

- (i) *rectangle* if its opposite sides are equal and the diagonals are equal.
- (ii) *square* if all its sides are equal and the diagonals are equal.
- (iii) *parallelogram* if its opposite sides are equal.
- (iv) *parallelogram but not a rectangle* if its opposite sides are equal and the diagonals are not equal.
- (v) *rhombus but not a square* if all its sides are equal and the diagonals are not equal.

EXAMPLE 17 Show that the points $(1, 1)$, $(-1, 5)$, $(7, 9)$ and $(9, 5)$ taken in that order are the vertices of a rectangle. Also, find the area of the rectangle. [CBSE 2009C]

SOLUTION Let $A(1, 1)$, $B(-1, 5)$, $C(7, 9)$ and $D(9, 5)$ be the vertices of quad. $ABCD$. Then,

$$\begin{aligned} AB^2 &= (-1 - 1)^2 + (5 - 1)^2 \\ &= (-2)^2 + 4^2 = 4 + 16 = 20. \end{aligned}$$

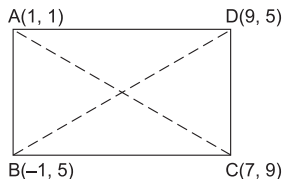
$$\begin{aligned} \therefore AB &= \sqrt{20} \text{ units} = \sqrt{4 \times 5} \text{ units} \\ &= 2\sqrt{5} \text{ units.} \end{aligned}$$

$$\begin{aligned} BC^2 &= (7 + 1)^2 + (9 - 5)^2 \\ &= 8^2 + 4^2 = 64 + 16 = 80. \end{aligned}$$

$$\therefore BC = \sqrt{80} \text{ units} = \sqrt{16 \times 5} \text{ units} = 4\sqrt{5} \text{ units.}$$

$$\begin{aligned} CD^2 &= 9 - 7^2 + 5 - 9^2 \\ &= 2^2 + (-4)^2 = 4 + 16 = 20. \end{aligned}$$

$$\therefore CD = \sqrt{20} \text{ units} = \sqrt{4 \times 5} \text{ units} = 2\sqrt{5} \text{ units.}$$



$$\begin{aligned}AD^2 &= (9-1)^2 + (5-1)^2 \\ &= 8^2 + 4^2 = 64 + 16 = 80.\end{aligned}$$

$$\therefore AD = \sqrt{80} \text{ units} = \sqrt{16 \times 5} \text{ units} = 4\sqrt{5} \text{ units.}$$

Thus, $AB = CD = 2\sqrt{5}$ units and $BC = AD = 4\sqrt{5}$ units.

$$\begin{aligned}\text{Also, } AC^2 &= (7-1)^2 + (9-1)^2 \\ &= 6^2 + 8^2 = 36 + 64 = 100.\end{aligned}$$

$$\therefore AC = \sqrt{100} \text{ units} = 10 \text{ units.}$$

$$\text{And, } BD^2 = (9+1)^2 + (5-5)^2 = 10^2 + 0^2 = 100.$$

$$\therefore BD = \sqrt{100} \text{ units} = 10 \text{ units.}$$

\therefore diagonal $AC =$ diagonal BD .

Thus, $ABCD$ is a quadrilateral whose opposite sides are equal and the diagonals are equal.

Hence, quad. $ABCD$ is a rectangle.

$$\begin{aligned}\text{Area of rectangle } ABCD &= \text{length} \times \text{breadth} \\ &= 4\sqrt{5} \times 2\sqrt{5} \text{ sq units} \\ &= 40 \text{ sq units.}\end{aligned}$$

EXAMPLE 18 Show that the points $A(3, 5)$, $B(6, 0)$, $C(1, -3)$ and $D(-2, 2)$ are the vertices of a square $ABCD$. [CBSE 2012]

SOLUTION Let $A(3, 5)$, $B(6, 0)$, $C(1, -3)$ and $D(-2, 2)$ be the angular points of a quadrilateral $ABCD$. Join AC and BD .

$$\text{Now, } AB = \sqrt{(6-3)^2 + (0-5)^2}$$

$$= \sqrt{3^2 + (-5)^2}$$

$$= \sqrt{9 + 25} = \sqrt{34} \text{ units,}$$

$$BC = \sqrt{(1-6)^2 + (-3-0)^2} = \sqrt{(-5)^2 + (-3)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34} \text{ units,}$$

$$CD = \sqrt{(-2-1)^2 + (2+3)^2} = \sqrt{(-3)^2 + 5^2}$$

$$= \sqrt{9 + 25} = \sqrt{34} \text{ units,}$$

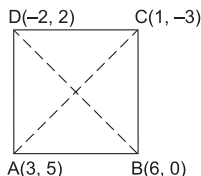
$$\text{and } DA = \sqrt{(3+2)^2 + (5-2)^2} = \sqrt{5^2 + 3^2}$$

$$= \sqrt{25 + 9} = \sqrt{34} \text{ units.}$$

Thus, $AB = BC = CD = DA$.

$$\text{Diag. } AC = \sqrt{(1-3)^2 + (-3-5)^2} = \sqrt{(-2)^2 + (-8)^2}$$

$$= \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17} \text{ units.}$$



$$\begin{aligned}\text{Diag. } BD &= \sqrt{(-2-6)^2 + (2-0)^2} \\ &= \sqrt{(-8)^2 + 2^2} = \sqrt{64+4} \\ &= \sqrt{68} = 2\sqrt{17} \text{ units.}\end{aligned}$$

\therefore diag. $AC =$ diag. BD .

Thus, $ABCD$ is a quadrilateral in which all sides are equal and the diagonals are equal.

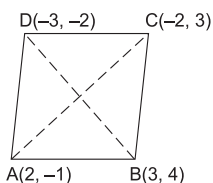
Hence, quad. $ABCD$ is a square.

EXAMPLE 19

If $A(2, -1)$, $B(3, 4)$, $C(-2, 3)$ and $D(-3, -2)$ be four points in a plane, show that $ABCD$ is a rhombus but not a square. Find the area of the rhombus. [CBSE 2013]

SOLUTION

Let $A(2, -1)$, $B(3, 4)$, $C(-2, 3)$ and $D(-3, -2)$ be the angular points of a quad. $ABCD$. Join AC and BD . Then,



$$\begin{aligned}AB &= \sqrt{(3-2)^2 + (4+1)^2} \\ &= \sqrt{1^2 + 5^2} = \sqrt{26} \text{ units,}\end{aligned}$$

$$BC = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{26} \text{ units,}$$

$$CD = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26} \text{ units,}$$

$$DA = \sqrt{(-3-2)^2 + (-2+1)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{26} \text{ units.}$$

$\therefore AB = BC = CD = DA = \sqrt{26}$ units.

$$\begin{aligned}\text{Now, } AC &= \sqrt{(-2-2)^2 + (3+1)^2} = \sqrt{(-4)^2 + 4^2} \\ &= \sqrt{32} = 4\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}\text{and } BD &= \sqrt{(-3-3)^2 + (-2-4)^2} = \sqrt{(-6)^2 + (-6)^2} \\ &= \sqrt{72} = 6\sqrt{2} \text{ units.}\end{aligned}$$

\therefore diagonal $AC \neq$ diagonal BD .

Thus, $ABCD$ is a quadrilateral in which all sides are equal but diagonals are not equal.

$\therefore ABCD$ is a rhombus but not a square.

$$\begin{aligned}\therefore \text{ area of the rhombus } ABCD &= \frac{1}{2} \times (\text{product of diagonals}) \\ &= \left(\frac{1}{2} \times AC \times BD \right) \\ &= \left(\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \right) \text{ sq units} \\ &= 24 \text{ sq units.}\end{aligned}$$

Hence, the area of rhombus $ABCD$ is 24 sq units.

COLLINEAR POINTS Three points A, B, C are said to be collinear if they lie on the same straight line.

TEST FOR COLLINEARITY OF THREE POINTS

In order to show that three given points A, B, C are collinear, we find the distances AB, BC and AC . If the sum of any two of these distances is equal to the third distance then the given points are collinear.

EXAMPLE 20 Prove that the points $A(1, 1), B(-2, 7)$ and $C(3, -3)$ are collinear.

SOLUTION Let $A(1, 1), B(-2, 7)$ and $C(3, -3)$ be the given points. Then,



$$AB = \sqrt{(-2-1)^2 + (7-1)^2} = \sqrt{(-3)^2 + 6^2} = \sqrt{45} = 3\sqrt{5} \text{ units,}$$

$$BC = \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{5^2 + (-10)^2} = \sqrt{125} = 5\sqrt{5} \text{ units,}$$

$$AC = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5} \text{ units.}$$

$$\therefore AB + AC = (3\sqrt{5} + 2\sqrt{5}) \text{ units} = 5\sqrt{5} \text{ units} = BC.$$

Thus, $AB + AC = BC$.

Hence, the given points A, B, C are collinear.

EXERCISE 6A

- Find the distance between the points:
 - $A(9, 3)$ and $B(15, 11)$
 - $A(7, -4)$ and $B(-5, 1)$
 - $A(-6, -4)$ and $B(9, -12)$
 - $A(1, -3)$ and $B(4, -6)$
 - $P(a+b, a-b)$ and $Q(a-b, a+b)$
 - $P(a \sin \alpha, a \cos \alpha)$ and $Q(a \cos \alpha, -a \sin \alpha)$
- Find the distance of each of the following points from the origin:
 - $A(5, -12)$
 - $B(-5, 5)$
 - $C(-4, -6)$.
- Find all possible values of x for which the distance between the points $A(x, -1)$ and $B(5, 3)$ is 5 units.
- Find all possible values of y for which the distance between the points $A(2, -3)$ and $B(10, y)$ is 10 units.
- Find the values of x for which the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units. [CBSE 2011]
- If the point $A(x, 2)$ is equidistant from the points $B(8, -2)$ and $C(2, -2)$, find the value of x . Also, find the length of AB .
- If the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$, find the value of p . Also, find the length of AB . [CBSE 2014]

8. Find the coordinates of the point on x -axis which is equidistant from the points $(-2, 5)$ and $(2, -3)$. [CBSE 2017]
9. Find points on the x -axis, each of which is at a distance of 10 units from the point $A(11, -8)$.
10. Find the point on the y -axis which is equidistant from the points $A(6, 5)$ and $B(-4, 3)$.
11. If the point $P(x, y)$ is equidistant from the points $A(5, 1)$ and $B(-1, 5)$, prove that $3x = 2y$. [CBSE 2005]
12. If $P(x, y)$ is a point equidistant from the points $A(6, -1)$ and $B(2, 3)$, show that $x - y = 3$. [CBSE 2008C]
13. Find the coordinates of the point equidistant from three given points $A(5, 3)$, $B(5, -5)$ and $C(1, -5)$. [CBSE 2006]
14. If the points $A(4, 3)$ and $B(x, 5)$ lie on a circle with the centre $O(2, 3)$, find the value of x . [CBSE 2009]
- HINT** $OA^2 = OB^2$.
15. If the point $C(-2, 3)$ is equidistant from the points $A(3, -1)$ and $B(x, 8)$, find the values of x . Also, find the distance BC . [CBSE 2013C]
16. If the point $P(2, 2)$ is equidistant from the points $A(-2, k)$ and $B(-2k, -3)$, find k . Also, find the length of AP . [CBSE 2014]
17. (i) If the point (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that $bx = ay$. [CBSE 2011]
- (ii) If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal then prove that $3x = 2y$. [CBSE 2017]
18. Using the distance formula, show that the given points are collinear:
- (i) $(1, -1)$, $(5, 2)$ and $(9, 5)$ (ii) $(6, 9)$, $(0, 1)$ and $(-6, -7)$
- (iii) $(-1, -1)$, $(2, 3)$ and $(8, 11)$ (iv) $(-2, 5)$, $(0, 1)$ and $(2, -3)$.
19. Show that the points $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$ are the vertices of an isosceles right triangle. [CBSE 2007, '13]
20. Show that the points $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ are the vertices of an isosceles right triangle.
21. If $A(5, 2)$, $B(2, -2)$ and $C(-2, t)$ are the vertices of a right triangle with $\angle B = 90^\circ$ then find the value of t . [CBSE 2015]
22. Prove that the points $A(2, 4)$, $B(2, 6)$ and $C(2 + \sqrt{3}, 5)$ are the vertices of an equilateral triangle. [CBSE 2013C]
23. Show that the points $(-3, -3)$, $(3, 3)$ and $(-3\sqrt{3}, 3\sqrt{3})$ are the vertices of an equilateral triangle. [CBSE 2012]
24. Show that the points $A(-5, 6)$, $B(3, 0)$ and $C(9, 8)$ are the vertices of an isosceles right-angled triangle. Calculate its area.

25. Show that the points $O(0, 0)$, $A(3, \sqrt{3})$ and $B(3, -\sqrt{3})$ are the vertices of an equilateral triangle. Find the area of this triangle.
26. Show that the following points are the vertices of a square:
- (i) $A(3, 2)$, $B(0, 5)$, $C(-3, 2)$ and $D(0, -1)$ [CBSE 2008C]
 (ii) $A(6, 2)$, $B(2, 1)$, $C(1, 5)$ and $D(5, 6)$ [CBSE 2006]
 (iii) $A(0, -2)$, $B(3, 1)$, $C(0, 4)$ and $D(-3, 1)$
27. Show that the points $A(-3, 2)$, $B(-5, -5)$, $C(2, -3)$ and $D(4, 4)$ are the vertices of a rhombus. Find the area of this rhombus.
- HINT** Area of a rhombus = $\frac{1}{2} \times$ (product of its diagonals).
28. Show that the points $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$ are the vertices of a rhombus. Find its area.
29. Show that the points $A(6, 1)$, $B(8, 2)$, $C(9, 4)$ and $D(7, 3)$ are the vertices of a rhombus. Find its area.
30. Show that the points $A(2, 1)$, $B(5, 2)$, $C(6, 4)$ and $D(3, 3)$ are the angular points of a parallelogram. Is this figure a rectangle?
31. Show that $A(1, 2)$, $B(4, 3)$, $C(6, 6)$ and $D(3, 5)$ are the vertices of a parallelogram. Show that $ABCD$ is not a rectangle.
32. Show that the following points are the vertices of a rectangle:
- (i) $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ and $D(2, 3)$
 (ii) $A(2, -2)$, $B(14, 10)$, $C(11, 13)$ and $D(-1, 1)$
 (iii) $A(0, -4)$, $B(6, 2)$, $C(3, 5)$ and $D(-3, -1)$
33. Show that $\triangle ABC$ with vertices $A(-2, 0)$, $B(0, 2)$ and $C(2, 0)$ is similar to $\triangle DEF$ with vertices $D(-4, 0)$, $F(4, 0)$ and $E(0, 4)$. [CBSE 2017]
34. Show that $\triangle ABC$, where $A(22, 0)$, $B(2, 0)$, $C(0, 2)$ and $\triangle PQR$ where $P(-4, 0)$, $Q(4, 0)$, $R(0, 4)$ are similar triangles. [CBSE 2017]

ANSWERS (EXERCISE 6A)

1. (i) 10 units (ii) 13 units (iii) 17 units
 (iv) $3\sqrt{2}$ units (v) $2\sqrt{2}b$ units (vi) $\sqrt{2}a$ units
2. (i) 13 units (ii) $5\sqrt{2}$ units (iii) $5\sqrt{2}$ units
3. $x = 8$ or $x = 2$ 4. $y = 3$ or $y = -9$ 5. $x = 1$ or $x = 17$
6. $x = 5$, $AB = 5$ units 7. $p = 1$, $AB = \sqrt{10}$ units 8. $(-2, 0)$
9. $(5, 0)$ and $(17, 0)$ 10. $(0, 9)$ 13. $P(3, -1)$
14. $x = 2$ 15. $x = 2$ or $x = -6$, $BC = \sqrt{41}$ units
16. $(k = -3 \Rightarrow AP = \sqrt{41})$, $(k = -1 \Rightarrow AP = 5)$ 21. $t = 1$

24. 50 sq units 25. $3\sqrt{3}$ sq units 27. 45 sq units
 28. 24 sq units 29. 3 sq units 30. No

HINTS TO SOME SELECTED QUESTIONS

15. $AC = BC \Rightarrow AC^2 = BC^2$

$$\Rightarrow (3 + 2)^2 + (-4)^2 = (x + 2)^2 + 5^2$$

$$\Rightarrow (x + 2)^2 = 16 \Rightarrow x + 2 = 4 \text{ or } x + 2 = -4$$

$$\Rightarrow x = 2 \text{ or } x = -6.$$

Distance between $B(2, 8)$ and $C(-2, 3) = \sqrt{4^2 + 5^2} = \sqrt{41}$ units.

Distance between $B(-6, 8)$ and $C(-2, 3) = \sqrt{4^2 + (-5)^2} = \sqrt{41}$ units.

16. $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 + (2 + 3)^2$$

$$\Rightarrow (2 - k)^2 - (2 + 2k)^2 = 25 - 16 = 9$$

$$\Rightarrow (4 + k^2 - 4k) - (4 + 4k^2 + 8k) = 9$$

$$\Rightarrow 3k^2 + 12k + 9 = 0$$

$$\Rightarrow k^2 + 4k + 3 = 0 \Rightarrow (k + 3)(k + 1) = 0$$

$$\Rightarrow k = -3 \text{ or } k = -1.$$

Case 1. When $k = -3$, we have

$$AP = \text{distance between } A(-2, -3) \text{ and } P(2, 2) = \sqrt{41} \text{ units.}$$

Case 2. When $k = -1$, we have

$$AP = \text{distance between } A(-2, -1) \text{ and } P(2, 2) = \sqrt{25} = 5 \text{ units.}$$

17. (i) It is given that the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$. Then,

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow \{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$$

$$\Rightarrow \{(x - a) + b\}^2 - \{(x - a) - b\}^2 = \{(y - b) + a\}^2 - \{(y - b) - a\}^2$$

$$\Rightarrow 4(x - a)b = 4(y - b)a \Rightarrow (x - a)b = (y - b)a \Rightarrow bx = ay.$$

21. $AB^2 + BC^2 = AC^2 \Rightarrow 25 + \{16 + (t + 2)^2\} = 49 + (t - 2)^2$

$$\Rightarrow (t + 2)^2 - (t - 2)^2 = 8 \Rightarrow 4 \times t \times 2 = 8 \Rightarrow t = 1.$$

23. Let the given points be $A(-3, -3)$, $B(3, 3)$ and $C(-3\sqrt{3}, 3\sqrt{3})$. Then,

$$AB^2 = (6^2 + 6^2) = 72,$$

$$BC^2 = (-3\sqrt{3} - 3)^2 + (3\sqrt{3} - 3)^2 = (-3)^2(\sqrt{3} + 1)^2 + 3^2(\sqrt{3} - 1)^2$$

$$= 9 \times [(\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2] = 9 \times 2 \times [(\sqrt{3})^2 + 1^2] = 72,$$

$$AC^2 = (-3\sqrt{3} + 3)^2 + (3\sqrt{3} + 3)^2 = (-3)^2 \times (\sqrt{3} - 1)^2 + 3^2 \times (\sqrt{3} + 1)^2$$

$$= 9 \times [(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2] = 9 \times 2 \times [(\sqrt{3})^2 + 1^2] = 72.$$

$$\therefore AB = BC = AC = \sqrt{72} = 6\sqrt{2} \text{ units.}$$

25. Here, $a = \sqrt{12}$. So, area = $\left\{ \frac{\sqrt{3}}{4} \times a^2 \right\} = \left(\frac{\sqrt{3}}{4} \times 12 \right) = 3\sqrt{3}$ sq units.

27. Diagonal $AC = \sqrt{(-5)^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$, and

diagonal $BD = \sqrt{9^2 + 9^2} = \sqrt{162} = 9\sqrt{2}$.

$$\therefore \text{area} = \left(\frac{1}{2} \times AC \times BD\right) = \left(\frac{1}{2} \times 5\sqrt{2} \times 9\sqrt{2}\right) \text{sq units} = 45 \text{ sq units.}$$

33. Find the length of sides AB, BC and CA of $\triangle ABC$ and of sides DE, EF and FD of $\triangle DEF$.

Then, show that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

SECTION FORMULA

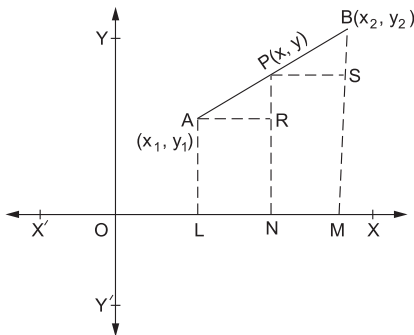
THEOREM 2 (Section formula) *The coordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by*

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

PROOF Let $X'OX$ and YOY' be the coordinate axes.

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the end points of the given line segment AB .

Let $P(x, y)$ be the point which divides AB in the ratio $m : n$.



Then, $\frac{AP}{PB} = \frac{m}{n}$.

Draw $AL \perp OX; BM \perp OX; PN \perp OX; AR \perp PN$; and $PS \perp BM$.

Now, $AR = LN = ON - OL = (x - x_1)$;

$$PS = NM = OM - ON = (x_2 - x);$$

$$PR = PN - RN = PN - AL = (y - y_1);$$

$$BS = BM - SM = BM - PN = (y_2 - y).$$

Clearly, $\triangle ARP$ and $\triangle PSB$ are similar and, therefore, their sides are proportional.

$$\begin{aligned}
 \therefore \frac{AP}{PB} &= \frac{AR}{PS} = \frac{PR}{BS} \\
 \Rightarrow \frac{m}{n} &= \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y} \\
 \Rightarrow \frac{m}{n} &= \frac{x - x_1}{x_2 - x} \quad \text{and} \quad \frac{m}{n} = \frac{y - y_1}{y_2 - y} \\
 \Rightarrow mx_2 - mx &= nx - nx_1 \quad \text{and} \quad my_2 - my = ny - ny_1 \\
 \Rightarrow (m+n)x &= (mx_2 + nx_1) \quad \text{and} \quad (m+n)y = (my_2 + ny_1) \\
 \Rightarrow x &= \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}.
 \end{aligned}$$

Hence, the coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

COROLLARY (Midpoint formula) *The coordinates of the midpoint M of a line segment AB with end points $A(x_1, y_1)$ and $B(x_2, y_2)$ are*

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

PROOF Let M be the midpoint of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Then, M divides AB in the ratio 1 : 1.

So, by the section formula, the coordinates of M are

$$\left(\frac{1 \cdot x_2 + 1 \cdot x_1}{1+1}, \frac{1 \cdot y_2 + 1 \cdot y_1}{1+1}\right), \text{ i.e., } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Hence, the coordinates of the midpoint of AB are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

SUMMARY

(i) If $P(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ then

$$x = \frac{mx_2 + nx_1}{m+n} \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}.$$

(ii) The midpoint of the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

SOLVED EXAMPLES

EXAMPLE 1 Find the coordinates of the point which divides the line segment joining the points $A(4, -3)$ and $B(9, 7)$ in the ratio 3 : 2.

SOLUTION The end points of AB are $A(4, -3)$ and $B(9, 7)$.

$$\therefore (x_1 = 4, y_1 = -3) \text{ and } (x_2 = 9, y_2 = 7).$$

Also, $m = 3$ and $n = 2$.



Let the required point be $P(x, y)$.

By the section formula, we have

$$x = \frac{(mx_2 + nx_1)}{(m+n)}, \quad y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$\Rightarrow x = \frac{(3 \times 9 + 2 \times 4)}{(3+2)}, \quad y = \frac{[3 \times 7 + 2 \times (-3)]}{(3+2)}$$

$$\Rightarrow x = 7, y = 3.$$

Hence, the required point is $P(7, 3)$.

EXAMPLE 2 Find the coordinates of the midpoint of the line segment joining the points $A(-5, 4)$ and $B(7, -8)$.

SOLUTION Let $M(x, y)$ be the midpoint of AB . Then,

$$x = \frac{[(-5) + 7]}{2} = 1 \text{ and } y = \frac{[4 + (-8)]}{2} = -2.$$

Hence, the required point is $M(1, -2)$.

EXAMPLE 3 Find the coordinates of the points of trisection of the line segment joining the points $A(-5, 6)$ and $B(4, -3)$. [CBSE 2009C]

SOLUTION Let P and Q be the points of trisection of AB .



Then, P divides AB in the ratio $1 : 2$.

So, the coordinates of P are

$$P\left(\frac{1 \times 4 + 2 \times (-5)}{1+2}, \frac{1 \times (-3) + 2 \times 6}{1+2}\right), \text{ i.e., } P(-2, 3).$$

Also, Q divides AB in the ratio $2 : 1$.

So, the coordinates of Q are

$$Q\left(\frac{2 \times 4 + 1 \times (-5)}{2+1}, \frac{2 \times (-3) + 1 \times 6}{2+1}\right), \text{ i.e., } Q(1, 0).$$

Hence, the points of trisection of AB are $P(-2, 3)$ and $Q(1, 0)$.

EXAMPLE 4 Find the coordinates of a point P on the line segment joining $A(1, 2)$ and $B(6, 7)$ such that $AP = \frac{2}{5}AB$. [CBSE 2015]

SOLUTION $AP = \frac{2}{5}AB \Rightarrow \frac{AP}{AB} = \frac{2}{5}$

$$\begin{aligned}\Rightarrow \frac{AP}{AP+PB} &= \frac{2}{5} \Rightarrow 5AP = 2AP + 2PB \\ \Rightarrow 3AP &= 2PB \Rightarrow \frac{AP}{PB} = \frac{2}{3} \Rightarrow AP:PB = 2:3.\end{aligned}$$

So, P divides AB in the ratio $2:3$.

\therefore coordinates of P are

$$\left(\frac{2 \times 6 + 3 \times 1}{2+3}, \frac{2 \times 7 + 3 \times 2}{2+3} \right) = \left(\frac{15}{5}, \frac{20}{5} \right) = (3, 4).$$

EXAMPLE 5

Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k . [CBSE 2010]

SOLUTION

$$\begin{aligned}\frac{AP}{AB} = \frac{1}{3} &\Rightarrow \frac{AP}{AP+PB} = \frac{1}{3} \\ \Rightarrow 3AP &= AP + PB \Rightarrow 2AP = PB \\ \Rightarrow \frac{AP}{PB} &= \frac{1}{2} \Rightarrow AP:PB = 1:2.\end{aligned}$$

So, P divides AB in the ratio $1:2$.

\therefore coordinates of P are

$$\left(\frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times (-8) + 2 \times 1}{1+2} \right) = \left(\frac{9}{3}, \frac{-6}{3} \right) = (3, -2).$$

Since $P(3, -2)$ lies on $2x - y + k = 0$, we have

$$2 \times 3 - (-2) + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8.$$

Hence, the required value of k is -8 .

EXAMPLE 6

In what ratio does the point $P(2, -5)$ divide the line segment joining $A(-3, 5)$ and $B(4, -9)$?

SOLUTION

Let the required ratio be $k:1$.

Then, by the section formula, the coordinates of P are

$$P \left(\frac{4k-3}{k+1}, \frac{-9k+5}{k+1} \right).$$

$$\therefore \frac{4k-3}{k+1} = 2 \text{ and } \frac{-9k+5}{k+1} = -5 \quad [\because P(2, -5) \text{ is given}]$$

$$\Rightarrow 4k - 3 = 2k + 2 \text{ and } -9k + 5 = -5k - 5$$

$$\Rightarrow 2k = 5 \text{ and } 4k = 10$$

$$\Rightarrow k = \frac{5}{2} \text{ in each case.}$$

So, the required ratio is $\frac{5}{2} : 1$, which is $5 : 2$.

Hence, P divides AB in the ratio $5 : 2$.

EXAMPLE 7

Find the ratio in which the point $P(x, 2)$ divides the line segment joining the points $A(12, 5)$ and $B(4, -3)$. Also, find the value of x .

[CBSE 2014]

SOLUTION

Let the required ratio be $k : 1$.

Then, by section formula, the coordinates of P are

$$P\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right).$$

But, this point is given as $P(x, 2)$.

$$\therefore \frac{-3k+5}{k+1} = 2 \Rightarrow -3k+5 = 2k+2 \Rightarrow 5k = 3 \Rightarrow k = \frac{3}{5}.$$

So, the required ratio is $3 : 5$.

Putting $k = \frac{3}{5}$ in P , we get

$$x = \frac{\left\{4 \times \frac{3}{5} + 12\right\}}{\left(\frac{3}{5} + 1\right)} = \frac{72}{8} = 9.$$

Hence, $x = 9$.

EXAMPLE 8

Find the ratio in which the point $P(11, y)$ divides the line segment joining the points $A(15, 5)$ and $B(9, 20)$. Also, find the value of y .

[CBSE 2013C]

SOLUTION

Let the required ratio be $k : 1$.

Then, by the section formula, the coordinates of P are

$$P\left(\frac{9k+15}{k+1}, \frac{20k+5}{k+1}\right).$$

But, this point is given as $P(11, y)$.

$$\therefore \frac{9k+15}{k+1} = 11 \Rightarrow 9k+15 = 11k+11 \Rightarrow 2k = 4 \Rightarrow k = 2.$$

So, the required ratio is $2 : 1$.

Putting $k = 2$ in P , we get

$$y = \frac{20 \times 2 + 5}{(2+1)} = \frac{45}{3} = 15.$$

Hence, $y = 15$.

EXAMPLE 9 If the point $P(-1, 2)$ divides the line segment joining the points $A(2, 5)$ and B in the ratio $3 : 4$, find the coordinates of B .

SOLUTION Let the coordinates of B be (x_1, y_1) .
 Then, P divides the join of $A(2, 5)$ and $B(x_1, y_1)$ in the ratio $3 : 4$.

So, by the section formula, the coordinates of P are

$$P\left(\frac{3x_1 + 8}{3 + 4}, \frac{3y_1 + 20}{3 + 4}\right), \text{ i.e., } P\left(\frac{3x_1 + 8}{7}, \frac{3y_1 + 20}{7}\right).$$

But, the coordinates of P are $(-1, 2)$.

$$\therefore \frac{3x_1 + 8}{7} = -1 \text{ and } \frac{3y_1 + 20}{7} = 2$$

$$\Rightarrow 3x_1 + 8 = -7 \text{ and } 3y_1 + 20 = 14$$

$$\Rightarrow 3x_1 = -15 \text{ and } 3y_1 = -6$$

$$\Rightarrow x_1 = -5 \text{ and } y_1 = -2.$$

Hence, the coordinates of B are $(-5, -2)$.

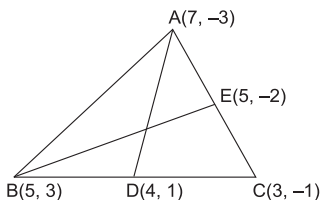
EXAMPLE 10 Find the lengths of the medians AD and BE of $\triangle ABC$ whose vertices are $A(7, -3)$, $B(5, 3)$ and $C(3, -1)$. [CBSE 2014]

SOLUTION Since D and E are the midpoints of BC and AC respectively, so their coordinates are

$$D\left(\frac{5+3}{2}, \frac{3+(-1)}{2}\right)$$

$$\text{and } E\left(\frac{7+3}{2}, \frac{-3-1}{2}\right),$$

i.e., $D(4, 1)$ and $E(5, -2)$.



$$\begin{aligned} \therefore AD &= \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \text{ units.} \end{aligned}$$

$$\therefore BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0 + (-5)^2} = \sqrt{25} = 5 \text{ units.}$$

EXAMPLE 11 The three vertices of a parallelogram $ABCD$ taken in order are $A(3, -4)$, $B(-1, -3)$ and $C(-6, 2)$. Find the coordinates of the fourth vertex D . [CBSE 2013]

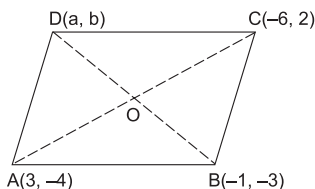
SOLUTION Let $A(3, -4)$, $B(-1, -3)$ and $C(-6, 2)$ be the three given vertices of a parallelogram $ABCD$ and let its fourth vertex be $D(a, b)$.

Join AC and BD .

Let AC and BD intersect at the point O .

We know that the diagonals of a $\parallel\text{gm}$ bisect each other.

So, O is the midpoint of each one of AC and BD .



Midpoint of AC is $\left(\frac{3+(-6)}{2}, \frac{-4+2}{2}\right)$, i.e., $O\left(\frac{-3}{2}, -1\right)$

and midpoint of BD is $O\left(\frac{-1+a}{2}, \frac{-3+b}{2}\right)$.

$$\therefore \frac{-1+a}{2} = \frac{-3}{2} \Rightarrow -1+a = -3 \Rightarrow a = -2$$

$$\text{and } \frac{-3+b}{2} = -1 \Rightarrow -3+b = -2 \Rightarrow b = 1.$$

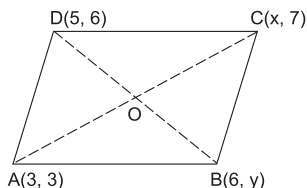
Hence, the fourth vertex of $\parallel\text{gm } ABCD$ is $D(-2, 1)$.

EXAMPLE 12

If $(3, 3)$, $(6, y)$, $(x, 7)$ and $(5, 6)$ are the vertices of a parallelogram taken in order, find the values of x and y . [CBSE 2011]

SOLUTION

Let $A(3, 3)$, $B(6, y)$, $C(x, 7)$ and $D(5, 6)$ be the vertices of a $\parallel\text{gm } ABCD$. Join AC and BD , intersecting each other at the point O .



We know that the diagonals of $\parallel\text{gm}$ bisect each other.

Therefore, O is the midpoint of AC as well as that of BD .

$$\therefore \text{midpoint of } AC \text{ is } \left(\frac{x+3}{2}, \frac{7+3}{2}\right), \text{ i.e., } \left(\frac{x+3}{2}, 5\right)$$

$$\text{and midpoint of } BD \text{ is } \left(\frac{5+6}{2}, \frac{6+y}{2}\right), \text{ i.e., } \left(\frac{11}{2}, \frac{6+y}{2}\right).$$

But, these points coincide at the point O .

$$\therefore \frac{x+3}{2} = \frac{11}{2} \text{ and } \frac{6+y}{2} = 5$$

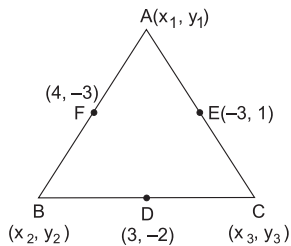
$$\Rightarrow x+3 = 11 \text{ and } 6+y = 10$$

$$\Rightarrow x = 8 \text{ and } y = 4.$$

Hence, $x = 8$ and $y = 4$.

EXAMPLE 13 Let $D(3, -2)$, $E(-3, 1)$ and $F(4, -3)$ be the midpoints of the sides BC , CA and AB respectively of $\triangle ABC$. Then, find the coordinates of the vertices A , B and C .

SOLUTION Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the given $\triangle ABC$, and let $D(3, -2)$, $E(-3, 1)$ and $F(4, -3)$ be the midpoints of BC , CA and AB respectively.



D is the midpoint of BC

$$\Rightarrow \frac{x_2 + x_3}{2} = 3, \frac{y_2 + y_3}{2} = -2$$

$$\Rightarrow \begin{cases} x_2 + x_3 = 6 & \dots \text{(i)} \\ y_2 + y_3 = -4. & \dots \text{(ii)} \end{cases}$$

E is the midpoint of CA

$$\Rightarrow \frac{x_1 + x_3}{2} = -3, \frac{y_1 + y_3}{2} = 1$$

$$\Rightarrow \begin{cases} x_1 + x_3 = -6 & \dots \text{(iii)} \\ y_1 + y_3 = 2. & \dots \text{(iv)} \end{cases}$$

F is the midpoint of AB

$$\Rightarrow \frac{x_1 + x_2}{2} = 4, \frac{y_1 + y_2}{2} = -3$$

$$\Rightarrow \begin{cases} x_1 + x_2 = -8 & \dots \text{(v)} \\ y_1 + y_2 = -6. & \dots \text{(vi)} \end{cases}$$

Adding (i), (iii) and (v), we get

$$2(x_1 + x_2 + x_3) = 8 \Rightarrow (x_1 + x_2 + x_3) = 4. \dots \text{(vii)}$$

Using (i), (iii) and (v) with (vii), we get

$$x_1 = -2, x_2 = 10 \text{ and } x_3 = -4.$$

Adding (ii), (iv) and (vi), we get

$$2(y_1 + y_2 + y_3) = -8 \Rightarrow y_1 + y_2 + y_3 = -4. \dots \text{(viii)}$$

Using (ii), (iv) and (vi) with (viii), we get

$$y_1 = 0, y_2 = -6 \text{ and } y_3 = 2.$$

Hence, the vertices of $\triangle ABC$ are $A(-2, 0)$, $B(10, -6)$ and $C(-4, 2)$.

EXAMPLE 14 The coordinates of one end point of a diameter AB of a circle are $A(4, -1)$ and the coordinates of the centre of the circle are $C(1, -3)$. Find the coordinates of B .

SOLUTION Let $C(1, -3)$ be the centre of the given circle and let $A(4, -1)$ and $B(a, b)$ be the two end points of the given diameter AB .

Then, clearly C is the midpoint of AB .

$$\therefore \text{ the coordinates of } C \text{ are } C\left(\frac{4+a}{2}, \frac{-1+b}{2}\right).$$

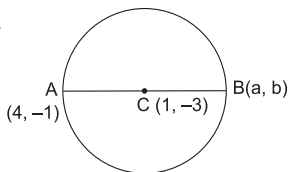
But, the coordinates of C are $(1, -3)$.

$$\therefore \frac{4+a}{2} = 1 \text{ and } \frac{-1+b}{2} = -3$$

$$\Rightarrow 4+a = 2 \text{ and } -1+b = -6$$

$$\Rightarrow a = -2 \text{ and } b = -5.$$

Hence, the coordinates of B are $B(-2, -5)$.



CENTROID OF A TRIANGLE *The point of intersection of the medians of a triangle is called its centroid.*

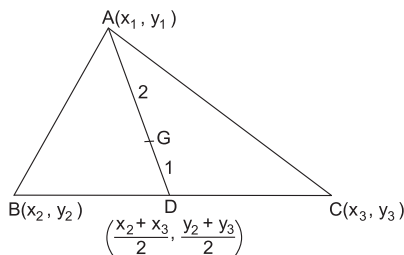
TO FIND THE COORDINATES OF THE CENTROID OF A TRIANGLE

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a $\triangle ABC$.

Let D be the midpoint of BC .

$$\text{Then, the coordinates of } D \text{ are } D\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right).$$

Let $G(x, y)$ be the centroid of $\triangle ABC$.



Then, G divides AD in the ratio $2 : 1$.

$$\therefore x = \frac{\left\{ \frac{2(x_2+x_3)}{2} + x_1 \right\}}{(2+1)} = \frac{(x_1+x_2+x_3)}{3},$$

$$y = \frac{\left\{ \frac{2(y_2+y_3)}{2} + y_1 \right\}}{(2+1)} = \frac{(y_1+y_2+y_3)}{3}.$$

\therefore the coordinates of G are $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

EXAMPLE 15 Find the centroid of $\triangle ABC$ whose vertices are $A(-3, 0)$, $B(5, -2)$ and $C(-8, 5)$.

SOLUTION Here, $(x_1 = -3, y_1 = 0)$, $(x_2 = 5, y_2 = -2)$ and $(x_3 = -8, y_3 = 5)$.

Let $G(x, y)$ be the centroid of $\triangle ABC$. Then,

$$x = \frac{1}{3}(x_1 + x_2 + x_3) = \frac{1}{3}(-3 + 5 - 8) = -2,$$

$$y = \frac{1}{3}(y_1 + y_2 + y_3) = \frac{1}{3}(0 - 2 + 5) = 1.$$

Hence, the centroid of $\triangle ABC$ is $G(-2, 1)$.

EXAMPLE 16 Two vertices of a $\triangle ABC$ are given by $A(6, 4)$ and $B(-2, 2)$, and its centroid is $G(3, 4)$. Find the coordinates of the third vertex C of $\triangle ABC$. [CBSE 2005C]

SOLUTION It is being given that the two vertices of $\triangle ABC$ are $A(6, 4)$ and $B(-2, 2)$. Let its third vertex be $C(x, y)$.

Then, the coordinates of the centroid of $\triangle ABC$ are

$$G\left(\frac{6 + (-2) + x}{3}, \frac{4 + 2 + y}{3}\right), \text{ i.e., } G\left(\frac{4 + x}{3}, \frac{6 + y}{3}\right).$$

But, it is given that the centroid of $\triangle ABC$ is $G(3, 4)$.

$$\therefore \frac{4 + x}{3} = 3 \text{ and } \frac{6 + y}{3} = 4$$

$$\Rightarrow 4 + x = 9 \text{ and } 6 + y = 12.$$

$$\Rightarrow x = 5 \text{ and } y = 6.$$

Hence, the coordinates of the third vertex is $C(5, 6)$.

EXERCISE 6B

- (i) Find the coordinates of the point which divides the join of $A(-1, 7)$ and $B(4, -3)$ in the ratio $2 : 3$.
(ii) Find the coordinates of the point which divides the join of $A(-5, 11)$ and $B(4, -7)$ in the ratio $7 : 2$.
- Find the coordinates of the points of trisection of the line segment joining the points $A(7, -2)$ and $B(1, -5)$. [CBSE 2017]
- If the coordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the coordinates of the point P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB . [CBSE 2015]

4. Point A lies on the line segment PQ joining $P(6, -6)$ and $Q(-4, -1)$ in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If the point A also lies on the line $3x + k(y + 1) = 0$, find the value of k . [CBSE 2015]
5. Points P, Q, R and S divide the line segment joining the points $A(1, 2)$ and $B(6, 7)$ in five equal parts. Find the coordinates of the points P, Q and R . [CBSE 2014]
6. Points P, Q and R in that order are dividing a line segment joining $A(1, 6)$ and $B(5, -2)$ in four equal parts. Find the coordinates of P, Q and R . [CBSE 2009C]
7. The line segment joining the points $A(3, -4)$ and $B(1, 2)$ is trisected at the points $P(p, -2)$ and $Q\left(\frac{5}{3}, q\right)$. Find the values of p and q . [CBSE 2005]
8. Find the coordinates of the midpoint of the line segment joining
(i) $A(3, 0)$ and $B(-5, 4)$ (ii) $P(-11, -8)$ and $Q(8, -2)$.
9. If $(2, p)$ is the midpoint of the line segment joining the points $A(6, -5)$ and $B(-2, 11)$, find the value of p . [CBSE 2010]
10. The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $C(1, 2a + 1)$. Find the values of a and b .
11. The line segment joining $A(-2, 9)$ and $B(6, 3)$ is a diameter of a circle with centre C . Find the coordinates of C .
12. Find the coordinates of a point A , where AB is a diameter of a circle with centre $C(2, -3)$ and the other end of the diameter is $B(1, 4)$.
13. In what ratio does the point $P(2, 5)$ divide the join of $A(8, 2)$ and $B(-6, 9)$?
14. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$. [CBSE 2015]
15. Find the ratio in which the point $P(m, 6)$ divides the join of $A(-4, 3)$ and $B(2, 8)$. Also, find the value of m . [CBSE 2004]
16. Find the ratio in which the point $(-3, k)$ divides the join of $A(-5, -4)$ and $B(-2, 3)$. Also, find the value of k . [CBSE 2007]
17. In what ratio is the line segment joining $A(2, -3)$ and $B(5, 6)$ divided by the x -axis? Also, find the coordinates of the point of division.

18. In what ratio is the line segment joining the points $A(-2, -3)$ and $B(3, 7)$ divided by the y -axis? Also, find the coordinates of the point of division. [CBSE 2006C]
19. In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points $A(3, -1)$ and $B(8, 9)$? [CBSE 2007]
20. Find the lengths of the medians of a $\triangle ABC$ whose vertices are $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$.
21. Find the centroid of $\triangle ABC$ whose vertices are $A(-1, 0)$, $B(5, -2)$ and $C(8, 2)$. [CBSE 2003]
22. If $G(-2, 1)$ is the centroid of a $\triangle ABC$ and two of its vertices are $A(1, -6)$ and $B(-5, 2)$, find the third vertex of the triangle.
23. Find the third vertex of a $\triangle ABC$ if two of its vertices are $B(-3, 1)$ and $C(0, -2)$, and its centroid is at the origin. [CBSE 2004C]
24. Show that the points $A(3, 1)$, $B(0, -2)$, $C(1, 1)$ and $D(4, 4)$ are the vertices of a parallelogram $ABCD$.
25. If the points $P(a, -11)$, $Q(5, b)$, $R(2, 15)$ and $S(1, 1)$ are the vertices of a parallelogram $PQRS$, find the values of a and b .
26. If three consecutive vertices of a parallelogram $ABCD$ are $A(1, -2)$, $B(3, 6)$ and $C(5, 10)$, find its fourth vertex D .
27. In what ratio does y -axis divide the line segment joining the points $(-4, 7)$ and $(3, -7)$? [CBSE 2012]
28. If the point $P\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points $A(3, -5)$ and $B(-7, 9)$ then find the ratio in which P divides AB . Also, find the value of y . [CBSE 2010]
29. Find the ratio in which the line segment joining the points $A(3, -3)$ and $B(-2, 7)$ is divided by x -axis. Also, find the point of division. [CBSE 2014]
30. The base QR of an equilateral triangle PQR lies on x -axis. The coordinates of the point Q are $(-4, 0)$ and origin is the midpoint of the base. Find the coordinates of the points P and R . [CBSE 2015]
31. The base BC of an equilateral triangle ABC lies on y -axis. The coordinates of point C are $(0, -3)$. The origin is the midpoint of the base. Find the coordinates of the points A and B . Also, find the coordinates of another point D such that $ABCD$ is a rhombus. [CBSE 2015]
32. Find the ratio in which the point $P(-1, y)$ lying on the line segment joining points $A(-3, 10)$ and $B(6, -8)$ divides it. Also, find the value of y . [CBSE 2013]

33. $ABCD$ is a rectangle formed by the points $A(-1, -1)$, $B(-1, 4)$, $C(5, 4)$ and $D(5, -1)$. If P , Q , R and S be the midpoints of AB , BC , CD and DA respectively, show that $PQRS$ is a rhombus.
34. The midpoint P of the line segment joining the points $A(-10, 4)$ and $B(-2, 0)$ lies on the line segment joining the points $C(-9, -4)$ and $D(-4, y)$. Find the ratio in which P divides CD . Also find the value of y .
[CBSE 2014]
35. A line intersects the y -axis and x -axis at the points P and Q respectively. If $(2, -5)$ is the midpoint of PQ then find the coordinates of P and Q .
[CBSE 2017]
36. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? Also, find the value of y .
[CBSE 2017]
37. The midpoints of the sides BC , CA and AB of a $\triangle ABC$ are $D(3, 4)$, $E(8, 9)$ and $F(6, 7)$ respectively. Find the coordinates of the vertices of the triangle.
[CBSE 2017]
38. If two adjacent vertices of a parallelogram are $(3, 2)$ and $(-1, 0)$ and the diagonals intersect at $(2, -5)$ then find the coordinates of the other two vertices.
[CBSE 2017]

ANSWERS (EXERCISE 6B)

1. (i) $(1, 3)$ (ii) $(2, -3)$ 2. $(5, -3), (3, -4)$ 3. $P\left(\frac{-2}{7}, \frac{-20}{7}\right)$ 4. $k = 2$
5. $P(2, 3), Q(3, 4), R(4, 5)$ 6. $P(2, 4), Q(3, 2), R(4, 0)$
7. $p = \frac{7}{3}, q = 0$ 8. (i) $(-1, 2)$ (ii) $\left(\frac{-3}{2}, -5\right)$ 9. $p = 3$ 10. $a = 2, b = 2$
11. $C(2, 6)$ 12. $A(3, -10)$ 13. $3 : 4$ 14. $1 : 5$ 15. $3 : 2, m = \frac{-2}{5}$
16. $2 : 1, k = \frac{2}{3}$ 17. $(1 : 2), (3, 0)$ 18. $(2 : 3), (0, 1)$ 19. $2 : 3$
20. $AD = \sqrt{10}$ units, $BE = 2$ units, $CF = \sqrt{10}$ units 21. $G(4, 0)$
22. $C(-2, 7)$ 23. $A(3, 1)$ 25. $a = 4, b = 3$ 26. $D(3, 2)$
27. $4 : 3$ 28. $1 : 3, y = \frac{-3}{2}$ 29. $(3 : 7), \left(\frac{3}{2}, 0\right)$
30. $P(0, 4\sqrt{3})$ or $P(0, -4\sqrt{3})$ and $R(4, 0)$
31. $A(3\sqrt{3}, 0)$ or $A(-3\sqrt{3}, 0)$ and $B(0, 3)$;
[$a(3\sqrt{3}, 0), b(0, 3), d(3\sqrt{3}, -6)$] or [$a(-3\sqrt{3}, 0), b(0, 3), d(-3\sqrt{3}, -6)$]
32. $2 : 7, y = 6$ 34. $3 : 2, y = 6$ 35. $P(0, -10), Q(4, 0)$ 36. $2 : 9; y = -\frac{4}{11}$
37. $A(11, 12), B(1, 2), C(5, 8)$ 38. $(1, -12); (5, -10)$

HINTS TO SOME SELECTED QUESTIONS

$$\begin{aligned}
 3. \quad \frac{AP}{AB} = \frac{3}{7} &\Rightarrow \frac{AP}{AP+PB} = \frac{3}{7} \Rightarrow \frac{AP}{AP+PB} = \frac{7}{3} \\
 &\Rightarrow \frac{AP}{AP} + \frac{PB}{AP} = \frac{7}{3} \Rightarrow 1 + \frac{PB}{AP} = \frac{7}{3} \\
 &\Rightarrow \frac{PB}{AP} = \left(\frac{7}{3} - 1\right) = \frac{4}{3} \Rightarrow \frac{AP}{PB} = \frac{3}{4} \Rightarrow AP : PB = 3 : 4.
 \end{aligned}$$

\therefore coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$, where $m = 3, n = 4$,

$$\text{i.e., } P\left(\frac{3 \times 2 + 4 \times (-2)}{3+4}, \frac{3 \times (-4) + 4 \times (-2)}{3+4}\right) = P\left(\frac{-2}{7}, \frac{-20}{7}\right).$$

$$\begin{aligned}
 4. \quad \frac{PA}{PQ} = \frac{2}{5} &\Rightarrow \frac{PQ}{PA} = \frac{5}{2} \Rightarrow \frac{PA+AQ}{PA} = \frac{5}{2} \\
 &\Rightarrow 1 + \frac{AQ}{PA} = \frac{5}{2} \Rightarrow \frac{AQ}{PA} = \left(\frac{5}{2} - 1\right) = \frac{3}{2} \\
 &\Rightarrow \frac{PA}{AQ} = \frac{2}{3} \Rightarrow PA : AQ = 2 : 3.
 \end{aligned}$$

\therefore coordinates of A are $\left(\frac{2 \times (-4) + 3 \times 6}{2+3}, \frac{2 \times (-1) + 3 \times (-6)}{2+3}\right) = A(2, -4)$.

Since the point $A(2, -4)$ lies on the line $3x + k(y + 1) = 0$, we have

$$(3 \times 2) + k(-4 + 1) = 0 \Rightarrow 3k = 6 \Rightarrow k = 2.$$

5. P divides AB in the ratio $1 : 4$.

Q divides AB in the ratio $2 : 3$.

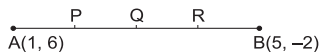
R divides AB in the ratio $3 : 2$.



6. P divides AB in the ratio $1 : 3$.

Q divides AB in the ratio $2 : 2 = 1 : 1$.

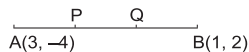
R divides AB in the ratio $3 : 1$.



7. P divides AB in the ratio $1 : 2$.

This point is $P\left(\frac{1 \times 1 + 2 \times 3}{1+2}, \frac{1 \times 2 + 2 \times (-4)}{1+2}\right)$.

$$\therefore \left(\frac{7}{3}, -2\right) \equiv (p, -2) \Rightarrow p = \frac{7}{3}.$$



Q divides AB in the ratio $2 : 1$. So, we get

$$Q\left(\frac{2 \times 1 + 1 \times 3}{2+1}, \frac{2 \times 2 + 1 \times (-4)}{2+1}\right), \text{ i.e., } \left(\frac{5}{3}, 0\right)$$

$$\therefore \left(\frac{5}{3}, 0\right) \equiv \left(\frac{5}{3}, q\right) \Rightarrow q = 0.$$

14. Let the required ratio be $k : 1$.

Then, the coordinates of P are $\left(\frac{2k + \frac{1}{2}}{k+1}, \frac{-5k + \frac{3}{2}}{k+1}\right)$.

$$\begin{aligned}\therefore \frac{2k + \frac{1}{2}}{k + 1} = \frac{3}{4} &\Rightarrow \frac{4k + 1}{2(k + 1)} = \frac{3}{4} \Rightarrow 16k + 4 = 6k + 6 \\ &\Rightarrow 10k = 2 \Rightarrow k = \frac{1}{5}.\end{aligned}$$

\therefore required ratio is $\frac{1}{5} : 1$, i.e., $1 : 5$.

15. Let the required ratio be $k : 1$.

Then, the coordinates of P are $\left(\frac{2k - 4}{k + 1}, \frac{8k + 3}{k + 1}\right)$.

$$\therefore \frac{8k + 3}{k + 1} = 6 \Rightarrow 8k + 3 = 6k + 6 \Rightarrow 2k = 3 \Rightarrow k = \frac{3}{2}.$$

So, the required ratio is $\frac{3}{2} : 1$, i.e., $3 : 2$.

$$\therefore m = \frac{2k - 4}{k + 1} = \frac{\left(2 \times \frac{3}{2} - 4\right)}{\left(\frac{3}{2} + 1\right)} = \frac{-2}{5}.$$

17. Let the x -axis be cut by the join of $A(2, -3)$ and $B(5, 6)$ in the ratio $k : 1$ at the point $P(x, 0)$.

$$\text{Then, } \frac{6k - 3}{k + 1} = 0 \Rightarrow 6k - 3 = 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{3}{6} = \frac{1}{2}.$$

\therefore required ratio is $\frac{1}{2} : 1$, i.e., $1 : 2$.

Point of division is $P\left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, 0\right)$, i.e., $(3, 0)$.

19. Let the required ratio be $k : 1$.

Then, the point $P\left(\frac{8k + 3}{k + 1}, \frac{9k - 1}{k + 1}\right)$ lies on the line $x - y - 2 = 0$.

$$\begin{aligned}\therefore \frac{8k + 3}{k + 1} - \frac{9k - 1}{k + 1} - 2 &= 0 \Rightarrow (8k + 3) - (9k - 1) - 2(k + 1) = 0 \\ &\Rightarrow (8k - 9k - 2k) + (3 + 1 - 2) = 0 \\ &\Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}.\end{aligned}$$

Hence, the required ratio is $\left(\frac{2}{3} : 1\right)$, i.e., $2 : 3$.

27. Let the required ratio be $k : 1$.

Then, x -coordinate of the point of division is 0.

$$\therefore \frac{3k - 4}{k + 1} = 0 \Rightarrow 3k - 4 = 0 \Rightarrow k = \frac{4}{3}.$$

Hence, the required ratio is $\left(\frac{4}{3} : 1\right)$, i.e., 4 : 3.

28. Let the required ratio be $k : 1$.

Then, we get $P\left(\frac{-7k+3}{k+1}, \frac{9k-5}{k+1}\right)$.

$$\begin{aligned} \therefore \frac{-7k+3}{k+1} &= \frac{1}{2} \Rightarrow -14k+6 = k+1 \\ &\Rightarrow 15k=5 \Rightarrow k = \frac{1}{3}. \end{aligned}$$

So, the required ratio is $\left(\frac{1}{3} : 1\right)$, i.e., 1 : 3.

Putting $k = \frac{1}{3}$, we get

$$y = \frac{9k-5}{k+1} = \frac{\left(9 \times \frac{1}{3}\right) - 5}{\left(\frac{1}{3} + 1\right)} = \frac{-6}{4} = \frac{-3}{2}.$$

29. Let the required ratio be $k : 1$.

Then, the point of division is $P\left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$.

Since this point lies on the x -axis, its y -coordinates must be 0.

$$\therefore \frac{7k-3}{k+1} = 0 \Rightarrow 7k-3=0 \Rightarrow k = \frac{3}{7}.$$

So, the required ratio is $\left(\frac{3}{7} : 1\right)$, i.e., 3 : 7.

Now, put $k = \frac{3}{7}$, to get

$$P\left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right) \text{ as } P\left(\frac{3}{2}, 0\right).$$

30. Since R lies on the x -axis, its coordinates are of the form $R(a, 0)$ and clearly O , the origin is the midpoint of QR . So,

$$\frac{-4+a}{2} = 0 \Rightarrow -4+a=0 \Rightarrow a=4.$$

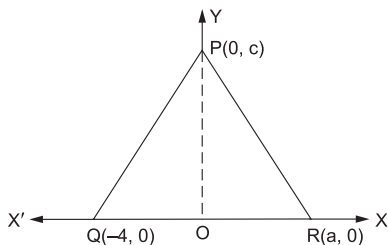
So, the coordinates of R are $R(4, 0)$.

Clearly, P lies on the y -axis. Let the coordinates of P be $P(0, c)$.

Since $\triangle PQR$ is equilateral, we have

$$PQ^2 = PR^2 = QR^2.$$

$$\therefore 16 + c^2 = 64 \Rightarrow c^2 = 48 \Rightarrow c = \pm\sqrt{48} = \pm 4\sqrt{3}.$$



31. Since B lies on the y -axis, its coordinates are of the form $B(0, b)$. Since $O(0, 0)$ is the midpoint of BC , we have

$$\frac{-3 + b}{2} = 0 \Rightarrow -3 + b = 0 \Rightarrow b = 3.$$

So, the coordinates of B are $B(0, 3)$.

Clearly, the vertex A lies on the x -axis.

Let this point be $A(a, 0)$.

Since $\triangle ABC$ is equilateral, we have

$$AC^2 = AB^2 = BC^2.$$

$$\therefore a^2 + 9 = 36 \Rightarrow a^2 = 27$$

$$\Rightarrow a = \pm 3\sqrt{3}.$$

So, the coordinates of A are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$.

32. Let the required ratio be $k : 1$.

Then, we get $P\left(\frac{6k - 3}{k + 1}, \frac{-8k + 10}{k + 1}\right)$.

$$\text{But, } \frac{6k - 3}{k + 1} = -1 \Rightarrow 6k - 3 = -k - 1 \Rightarrow 7k = 2 \Rightarrow k = \frac{2}{7}.$$

So, the required ratio is $\left(\frac{2}{7} : 1\right) = (2 : 7)$.

34. The point P is $P\left(\frac{-10 - 2}{2}, \frac{4 + 0}{2}\right)$, i.e., $(-6, 2)$.

Let $P(-6, 2)$ divide CD in the ratio $k : 1$.

$$\text{Then, } \frac{-4k - 9}{k + 1} = -6 \text{ and } \frac{ky - 4}{k + 1} = 2$$

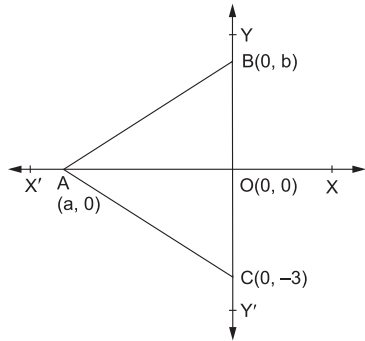
$$\Rightarrow -4k - 9 = -6k - 6 \text{ and } ky - 4 = 2k + 2$$

$$\Rightarrow 2k = 3 \text{ and } k(y - 2) = 6$$

$$\Rightarrow k = \frac{3}{2} \text{ and } (y - 2) = \frac{6}{k} = \left(6 \times \frac{2}{3}\right) = 4$$

$$\Rightarrow k = \frac{3}{2} \text{ and } y = 4 + 2 = 6.$$

Hence, the ratio is $3 : 2$ and $y = 6$.

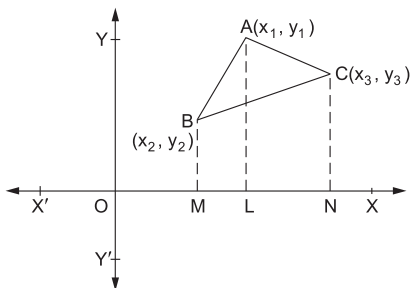


AREA OF A TRIANGLE

THEOREM 1 *The area of a $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by*

$$\text{area}(\triangle ABC) = \left| \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} \right|.$$

PROOF Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the given $\triangle ABC$.



Draw AL , BM and CN perpendiculars to the x -axis.

Then, $ML = (x_1 - x_2)$, $LN = (x_3 - x_1)$ and $MN = (x_3 - x_2)$.

\therefore area of $\triangle ABC$

$$\begin{aligned}
 &= \text{ar}(\text{trap. } BMLA) + \text{ar}(\text{trap. } ALNC) - \text{ar}(\text{trap. } BMNC) \\
 &= \left\{ \frac{1}{2} (AL + BM) \times ML \right\} + \left\{ \frac{1}{2} (AL + CN) \times LN \right\} \\
 &\quad - \left\{ \frac{1}{2} (BM + CN) \times MN \right\} \\
 &= \frac{1}{2} (y_1 + y_2)(x_1 - x_2) + \frac{1}{2} (y_1 + y_3)(x_3 - x_1) - \frac{1}{2} (y_2 + y_3)(x_3 - x_2) \\
 &= \frac{1}{2} [x_1(y_1 + y_2 - y_1 - y_3) + x_2(y_2 + y_3 - y_1 - y_2) \\
 &\quad + x_3(y_1 + y_3 - y_2 - y_3)] \\
 &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)].
 \end{aligned}$$

Since the area is never negative, we have

$$\text{area}(\triangle ABC) = \frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|.$$

Condition for Collinearity of Three Points

Let the given points be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then, A , B and C are collinear

$$\Rightarrow \text{area of } \triangle ABC = 0$$

$$\Rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0.$$

SUMMARY OF THE RESULTS

1. Area of $\triangle ABC$ with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is given by

$$\text{ar}(\triangle ABC) = \frac{1}{2} | [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] |.$$

2. Three points $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are collinear only when $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.

SOLVED EXAMPLES

EXAMPLE 1 Find the area of $\triangle ABC$ whose vertices are $A(2, 7)$, $B(3, -1)$ and $C(-5, 6)$.

SOLUTION The coordinates of the vertices of the given triangle are $A(x_1 = 2, y_1 = 7)$, $B(x_2 = 3, y_2 = -1)$ and $C(x_3 = -5, y_3 = 6)$.

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) | \\ &= \frac{1}{2} | 2 \cdot (-1 - 6) + 3 \cdot (6 - 7) - 5 \cdot (7 + 1) | \\ &= \frac{1}{2} | 2 \times (-7) + 3 \times (-1) - 5 \times 8 | \\ &= \frac{1}{2} | -14 - 3 - 40 | = \frac{1}{2} | -57 | = \frac{57}{2} = 28.5 \text{ sq units.} \end{aligned}$$

Hence, the area of the given triangle is 28.5 sq units.

EXAMPLE 2 Find the values of k so that the area of the triangle with vertices $(1, -1)$, $(-4, 2k)$ and $(-k, -5)$ is 24 square units. [CBSE 2015]

SOLUTION Let $A(1, -1)$, $B(-4, 2k)$ and $C(-k, -5)$ be the vertices of the given $\triangle ABC$.

Then, $A(x_1 = 1, y_1 = -1)$, $B(x_2 = -4, y_2 = 2k)$, and $C(x_3 = -k, y_3 = -5)$.

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) | \\ &= \frac{1}{2} | 1 \cdot (2k + 5) - 4 \cdot (-5 + 1) - k(-1) - 2k | \\ &= \frac{1}{2} | 2k + 5 + 16 + k + 2k^2 | \\ &= \frac{1}{2} | 2k^2 + 3k + 21 | \text{ sq units.} \end{aligned}$$

But, it is given that $\text{ar}(\triangle ABC) = 24$ sq units.

$$\therefore \frac{1}{2} \left| 2k^2 + 3k + 21 \right| = 24 \Rightarrow \left| k^2 + \frac{3}{2}k + \frac{21}{2} \right| = 24. \quad \dots \text{(i)}$$

$$\begin{aligned} \text{But } \left\{ k^2 + \frac{3}{2}k + \frac{21}{2} \right\} &= \left(k^2 + \frac{3}{2}k + \frac{9}{16} \right) + \left(\frac{21}{2} - \frac{9}{16} \right) \\ &= \left\{ \left(k + \frac{3}{4} \right)^2 + \frac{159}{16} \right\} > 0. \end{aligned}$$

So, we may write (i) as

$$\begin{aligned} k^2 + \frac{3}{2}k + \frac{21}{2} = 24 &\Rightarrow 2k^2 + 3k + 21 = 48 \\ \Rightarrow 2k^2 + 3k - 27 = 0 &\Rightarrow 2k^2 + 9k - 6k - 27 = 0 \\ \Rightarrow k(2k + 9) - 3(2k + 9) = 0 &\Rightarrow (2k + 9)(k - 3) = 0 \\ \Rightarrow k - 3 = 0 \text{ or } 2k + 9 = 9 &\Rightarrow k = 3 \text{ or } k = -\frac{9}{2}. \end{aligned}$$

Hence, $k = 3$ or $k = -\frac{9}{2}$.

EXAMPLE 3

If $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$ are the vertices of a $\triangle ABC$ and AD is its median. Prove that the median AD divides $\triangle ABC$ into two triangles of equal areas. [CBSE 2014]

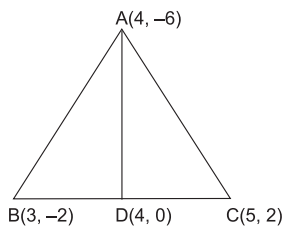
SOLUTION

It is being given that $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$ are the vertices of a $\triangle ABC$ and AD is its median.

Clearly, D is the midpoint of BC .

\therefore the coordinates of D are

$$\left(\frac{3+5}{2}, \frac{-2+2}{2} \right) = (4, 0).$$



For $\triangle ABD$, we have

$$A(x_1 = 4, y_1 = -6), B(x_2 = 3, y_2 = -2) \text{ and } D(x_3 = 4, y_3 = 0).$$

$$\begin{aligned} \text{ar}(\triangle ABD) &= \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| \\ &= \frac{1}{2} \left| 4 \cdot (-2 - 0) + 3 \cdot (0 + 6) + 4 \cdot (-6 + 2) \right| \\ &= \frac{1}{2} \left| 4 \times (-2) + 3 \times 6 + 4 \times (-4) \right| \\ &= \frac{1}{2} \left| -8 + 18 - 16 \right| = \frac{1}{2} \left| -6 \right| \\ &= \left(\frac{1}{2} \times 6 \right) = 3 \text{ sq units.} \end{aligned}$$

For $\triangle ADC$, we have

$$(x_1 = 4, y_1 = -6), (x_2 = 4, y_2 = 0) \text{ and } (x_3 = 5, y_3 = 2).$$

$$\begin{aligned} \therefore \text{ar}(\triangle ADC) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |4 \cdot (0 - 2) + 4 \cdot (2 + 6) + 5 \cdot (-6 - 0)| \\ &= \frac{1}{2} |4 \times (-2) + 4 \times 8 + 5 \times (-6)| \\ &= \frac{1}{2} |-8 + 32 - 30| = \frac{1}{2} |-6| \\ &= \left(\frac{1}{2} \times 6\right) = 3 \text{ sq units.} \end{aligned}$$

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ADC).$$

Hence, the median AD divides $\triangle ABC$ into two triangles of equal areas.

EXAMPLE 4

Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are $A(2, 2)$, $B(4, 4)$ and $C(2, 6)$.

[CBSE 2009C]

SOLUTION

Let $A(2, 2)$, $B(4, 4)$ and $C(2, 6)$ be the vertices of the given $\triangle ABC$. Let D , E and F be the midpoints of AB , BC and CA respectively.

Then, the coordinates of D , E and F are

$$D\left(\frac{2+4}{2}, \frac{2+4}{2}\right), E\left(\frac{4+2}{2}, \frac{4+6}{2}\right) \text{ and } F\left(\frac{2+2}{2}, \frac{2+6}{2}\right),$$

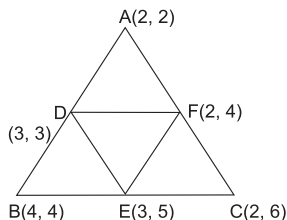
i.e., $D(3, 3)$, $E(3, 5)$ and $F(2, 4)$.

For $\triangle DEF$, we have

$$(x_1 = 3, y_1 = 3), (x_2 = 3, y_2 = 5) \text{ and } (x_3 = 2, y_3 = 4).$$

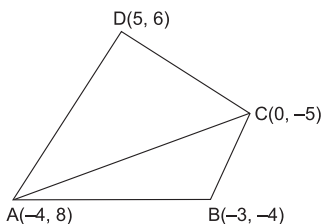
$$\begin{aligned} \therefore \text{ar}(\triangle DEF) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |3 \cdot (5 - 4) + 3 \cdot (4 - 3) + 2 \cdot (3 - 5)| \\ &= \frac{1}{2} |(3 \times 1) + (3 \times 1) + 2 \times (-2)| \\ &= \frac{1}{2} |3 + 3 - 4| = \left(\frac{1}{2} \times 2\right) = 1 \text{ sq unit.} \end{aligned}$$

Hence, the area of $\triangle DEF$ is 1 sq unit.



EXAMPLE 5 Find the area of a quadrilateral $ABCD$ whose vertices are $A(-4, 8)$, $B(-3, -4)$, $C(0, -5)$ and $D(5, 6)$. [CBSE 2015]

SOLUTION Area of quad. $ABCD = \text{ar}(\triangle ABC) + \text{ar}(\triangle ACD)$.



For $\triangle ABC$, we have

$$(x_1 = -4, y_1 = 8), (x_2 = -3, y_2 = -4) \text{ and } (x_3 = 0, y_3 = -5).$$

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |(-4) \cdot (-4 + 5) + (-3) \cdot (-5 - 8) + 0 \cdot (8 + 4)| \\ &= \frac{1}{2} |(-4) \times 1 + (-3) \times (-13) + 0| \\ &= \frac{1}{2} |-4 + 39 + 0| = \frac{35}{2} \text{ sq units.} \end{aligned}$$

For $\triangle ACD$, we have

$$(x_1 = -4, y_1 = 8), (x_2 = 0, y_2 = -5) \text{ and } (x_3 = 5, y_3 = 6).$$

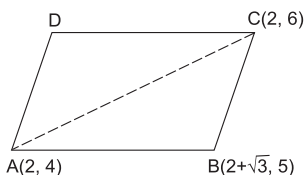
$$\begin{aligned} \therefore \text{ar}(\triangle ACD) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |(-4) \cdot (-5 - 6) + 0 \cdot (6 - 8) + 5 \cdot (8 + 5)| \\ &= \frac{1}{2} |(-4) \times (-11) + 0 \times (-2) + 5 \times 13| \\ &= \frac{1}{2} |44 + 0 + 65| = \frac{109}{2} \text{ sq unit.} \end{aligned}$$

$$\begin{aligned} \therefore \text{ar}(\text{quad. } ABCD) &= \text{ar}(\triangle ABC) + \text{ar}(\triangle ACD) \\ &= \left(\frac{35}{2} + \frac{109}{2} \right) \text{ sq units} = \frac{144}{2} \text{ sq units} \\ &= 72 \text{ sq units.} \end{aligned}$$

Hence, the area of the given quadrilateral $ABCD$ is 72 sq units.

EXAMPLE 6 Find the area of a parallelogram $ABCD$ if three of its vertices are $A(2, 4)$, $B(2 + \sqrt{3}, 5)$ and $C(2, 6)$. [CBSE 2013]

SOLUTION Let $ABCD$ be the given parallelogram, three of whose vertices are $A(2, 4)$, $B(2 + \sqrt{3}, 5)$ and $C(2, 6)$.



In $\triangle ABC$, we have

$$(x_1 = 2, y_1 = 4), (x_2 = 2 + \sqrt{3}, y_2 = 5), (x_3 = 2, y_3 = 6).$$

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |2(5 - 6) + (2 + \sqrt{3})(6 - 4) + 2(4 - 5)| \\ &= \frac{1}{2} |2 \times (-1) + (4 + 2\sqrt{3}) - 2| \\ &= \frac{1}{2} (2\sqrt{3}) = \sqrt{3} \text{ sq unit} \end{aligned}$$

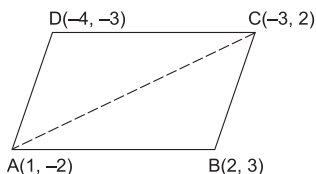
$$\begin{aligned} \Rightarrow \text{ar}(\text{||gm } ABCD) &= 2 \times \text{ar}(\triangle ABC) = (2 \times \sqrt{3}) \text{ sq units} \\ &= (2\sqrt{3}) \text{ sq units.} \end{aligned}$$

Hence, the area of the given parallelogram is $2\sqrt{3}$ sq units.

EXAMPLE 7 If the points $A(1, -2)$, $B(2, 3)$, $C(-3, 2)$ and $D(-4, -3)$ are the vertices of a parallelogram $ABCD$ then taking AB as the base, find the height of the parallelogram. [CBSE 2013]

SOLUTION Let $A(1, -2)$, $B(2, 3)$, $C(-3, 2)$ and $D(-4, -3)$ be the vertices of the given ||gm $ABCD$. Join AC . Then,

$$\text{ar}(\text{||gm } ABCD) = 2 \times \text{ar}(\triangle ABC).$$



In $\triangle ABC$, we have

$$(x_1 = 1, y_1 = -2), (x_2 = 2, y_2 = 3) \text{ and } (x_3 = -3, y_3 = 2).$$

$$\begin{aligned} \therefore \text{ar}(\triangle ABC) &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |1 \cdot (3 - 2) + 2 \cdot (2 + 2) - 3 \cdot (-2 - 3)| \end{aligned}$$

$$= \frac{1}{2} |1 + 8 + 15| = 12 \text{ sq units.}$$

$$\therefore \text{ar}(\text{llgm } ABCD) = 2 \times \text{ar}(\triangle ABC) = 2 \times 12 = 24 \text{ sq units.}$$

$$\text{Base } AB = \sqrt{(2-1)^2 + (3+2)^2} = \sqrt{1^2 + 5^2} = \sqrt{26} \text{ units.}$$

Let h be the height of the llgm $ABCD$. Then,

$$\text{ar}(\text{llgm } ABCD) = (\text{base} \times \text{height}) = (\sqrt{26} \times h) \text{ sq units.}$$

$$\therefore \sqrt{26} \times h = 24 \Rightarrow h = \left\{ \frac{24}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}} \right\} = \left(\frac{24}{26} \times \sqrt{26} \right) \text{ units}$$

$$\Rightarrow h = \left(\frac{12}{13} \times 5.09 \right) = \frac{61.08}{13} = 4.69 \text{ units.}$$

Hence, the height of the parallelogram is 4.69 units.

EXAMPLE 8 Show that the points $A(-1, 1)$, $B(5, 7)$ and $C(8, 10)$ are collinear.

SOLUTION Let $A(-1, 1)$, $B(5, 7)$ and $C(8, 10)$ be the given points. Then,

$$(x_1 = -1, y_1 = 1), (x_2 = 5, y_2 = 7) \text{ and } (x_3 = 8, y_3 = 10).$$

$$\begin{aligned} \therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \\ &= (-1)(7 - 10) + 5(10 - 1) + 8(1 - 7) \\ &= (3 + 45 - 48) = 0. \end{aligned}$$

Hence, the given points are collinear.

EXAMPLE 9 Show that the points $A(a, b + c)$, $B(b, c + a)$ and $C(c, a + b)$ are collinear. [CBSE 2010]

SOLUTION Let $A(a, b + c)$, $B(b, c + a)$ and $C(c, a + b)$ be the given points.

Then, $(x_1 = a, y_1 = b + c)$, $(x_2 = b, y_2 = c + a)$ and $(x_3 = c, y_3 = a + b)$.

$$\begin{aligned} \therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \\ &= a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a) \\ &= a(c - b) + b(a - c) + c(b - a) = 0. \end{aligned}$$

Hence, the given points are collinear.

EXAMPLE 10 If the area of $\triangle ABC$ with vertices $A(x, y)$, $B(1, 2)$ and $C(2, 1)$ is 6 sq units then prove that $x + y = 15$ or $x + y + 9 = 0$. [CBSE 2013]

SOLUTION The vertices of $\triangle ABC$ are $A(x, y)$, $B(1, 2)$ and $C(2, 1)$.

Here, $(x_1 = x, y_1 = y)$, $(x_2 = 1, y_2 = 2)$ and $(x_3 = 2, y_3 = 1)$.

$$\therefore \text{ar}(\triangle ABC) = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\begin{aligned}
 &= \frac{1}{2} |x \cdot (2-1) + 1 \cdot (1-y) + 2 \cdot (y-2)| \\
 &= \frac{1}{2} |x+1-y+2y-4| = \frac{1}{2} |x+y-3|.
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{1}{2} |x+y-3| = 6 &\Rightarrow |x+y-3| = 12 \\
 &\Rightarrow x+y-3 = 12 \text{ or } x+y-3 = -12 \\
 &\Rightarrow x+y = 15 \text{ or } x+y+9 = 0.
 \end{aligned}$$

Hence, $x+y = 15$ or $x+y+9 = 0$.

EXAMPLE 11 Find the value of k for which the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear. [CBSE 2015, '17]

SOLUTION The given points are $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$.

Here, $(x_1 = k+1, y_1 = 2k)$, $(x_2 = 3k, y_2 = 2k+3)$ and $(x_3 = 5k-1, y_3 = 5k)$.

Let the given points be collinear. Then,

$$\begin{aligned}
 &x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \\
 \Rightarrow &(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)[2k-(2k+3)] = 0 \\
 \Rightarrow &(k+1)(3-3k) + 3k \times 3k + (5k-1) \times (-3) = 0 \\
 \Rightarrow &3-3k^2+9k^2-15k+3 = 0 \Rightarrow 6k^2-15k+6 = 0 \\
 \Rightarrow &2k^2-5k+2 = 0 \Rightarrow 2k^2-4k-k+2 = 0 \\
 \Rightarrow &2k(k-2)-(k-2) = 0 \Rightarrow (k-2)(2k-1) = 0 \\
 \Rightarrow &k-2 = 0 \text{ or } 2k-1 = 0 \Rightarrow k = 2 \text{ or } k = \frac{1}{2}.
 \end{aligned}$$

Hence, $k = 2$ or $k = \frac{1}{2}$.

EXAMPLE 12 If the points $A(-1, -4)$, $B(b, c)$ and $C(5, -1)$ are collinear and $2b+c = 4$, find the values of b and c . [CBSE 2014]

SOLUTION Let $A(x_1 = -1, y_1 = -4)$, $B(x_2 = b, y_2 = c)$ and $C(x_3 = 5, y_3 = -1)$ be the given points.

Since these points A, B and C are collinear, we have

$$\begin{aligned}
 &x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \\
 \Rightarrow &(-1) \cdot (c+1) + b \cdot (-1+4) + 5 \cdot (-4-c) = 0 \\
 \Rightarrow &-c-1+3b-20-5c = 0 \\
 \Rightarrow &3b-6c = 21 \Rightarrow b-2c = 7. \qquad \dots \text{(i)}
 \end{aligned}$$

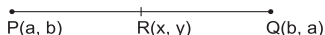
Also, it is given that $2b+c = 4$ (ii)

On solving (i) and (ii), we get $b = 3$ and $c = -2$.

Hence, $b = 3$ and $c = -2$.

EXAMPLE 13 If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$ then prove that $x + y = a + b$. [CBSE 2010]

SOLUTION It is being given that the points $P(a, b)$, $R(x, y)$ and $Q(b, a)$ lie on the same line segment and, therefore, these points are collinear.



Let $P(x_1 = a, y_1 = b)$, $R(x_2 = x, y_2 = y)$ and $Q(x_3 = b, y_3 = a)$ be the given collinear points. Then,

$$\begin{aligned} & x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \\ \Rightarrow & a(y - a) + x(a - b) + b(b - y) = 0 \\ \Rightarrow & ay - a^2 + ax - bx + b^2 - by = 0 \\ \Rightarrow & (a - b)y - (a^2 - b^2) + (a - b)x = 0 \\ \Rightarrow & y - (a + b) + x = 0 \Rightarrow x + y = a + b. \end{aligned}$$

Hence, $x + y = a + b$.

EXERCISE 6C

- Find the area of $\triangle ABC$ whose vertices are:
 - $A(1, 2)$, $B(-2, 3)$ and $C(-3, -4)$
 - $A(-5, 7)$, $B(-4, -5)$ and $C(4, 5)$ [CBSE 2008]
 - $A(3, 8)$, $B(-4, 2)$ and $C(5, -1)$ [CBSE 2008C]
 - $A(10, -6)$, $B(2, 5)$ and $C(-1, 3)$
- Find the area of quadrilateral $ABCD$ whose vertices are $A(3, -1)$, $B(9, -5)$, $C(14, 0)$ and $D(9, 19)$. [CBSE 2012]
- Find the area of quadrilateral $PQRS$ whose vertices are $P(-5, -3)$, $Q(-4, -6)$, $R(2, -3)$ and $S(1, 2)$. [CBSE 2015]
- Find the area of quadrilateral $ABCD$ whose vertices are $A(-3, -1)$, $B(-2, -4)$, $C(4, -1)$ and $D(3, 4)$. [CBSE 2013C]
- If $A(-7, 5)$, $B(-6, -7)$, $C(-3, -8)$ and $D(2, 3)$ are the vertices of a quadrilateral $ABCD$ then find the area of the quadrilateral. [CBSE 2017]
- Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are $A(2, 1)$, $B(4, 3)$ and $C(2, 5)$. [CBSE 2011]

7. $A(7, -3)$, $B(5, 3)$ and $C(3, -1)$ are the vertices of a $\triangle ABC$ and AD is its median. Prove that the median AD divides $\triangle ABC$ into two triangles of equal areas.
8. Find the area of $\triangle ABC$ with $A(1, -4)$ and midpoints of sides through A being $(2, -1)$ and $(0, -1)$. [CBSE 2015]
9. $A(6, 1)$, $B(8, 2)$ and $C(9, 4)$ are the vertices of a parallelogram $ABCD$. If E is the midpoint of DC , find the area of $\triangle ADE$. [CBSE 2013C]
10. (i) If the vertices of $\triangle ABC$ be $A(1, -3)$, $B(4, p)$ and $C(-9, 7)$ and its area is 15 square units, find the values of p . [CBSE 2012]
(ii) The area of a triangle is 5 sq units. Two of its vertices are $(2, 1)$ and $(3, -2)$. If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of y . [CBSE 2017]
11. Find the value of k so that the area of the triangle with vertices $A(k+1, 1)$, $B(4, -3)$ and $C(7, -k)$ is 6 square units. [CBSE 2015]
12. For what value of $k(k > 0)$ is the area of the triangle with vertices $(-2, 5)$, $(k, -4)$ and $(2k+1, 10)$ equal to 53 square units? [CBSE 2012]
13. Show that the following points are collinear:
 - (i) $A(2, -2)$, $B(-3, 8)$ and $C(-1, 4)$
 - (ii) $A(-5, 1)$, $B(5, 5)$ and $C(10, 7)$
 - (iii) $A(5, 1)$, $B(1, -1)$ and $C(11, 4)$
 - (iv) $A(8, 1)$, $B(3, -4)$ and $C(2, -5)$
14. Find the value of x for which the points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear. [CBSE 2015]
15. For what value of x are the points $A(-3, 12)$, $B(7, 6)$ and $C(x, 9)$ collinear?
16. For what value of y are the points $P(1, 4)$, $Q(3, y)$ and $R(-3, 16)$ are collinear?
17. Find the value of y for which the points $A(-3, 9)$, $B(2, y)$ and $C(4, -5)$ are collinear.
18. For what values of k are the points $A(8, 1)$, $B(3, -2k)$ and $C(k, -5)$ collinear. [CBSE 2015]
19. Find a relation between x and y , if the points $A(2, 1)$, $B(x, y)$ and $C(7, 5)$ are collinear. [CBSE 2009C]
20. Find a relation between x and y , if the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear. [CBSE 2015]
21. Prove that the points $A(a, 0)$, $B(0, b)$ and $C(1, 1)$ are collinear, if $\frac{1}{a} + \frac{1}{b} = 1$.

22. If the points $P(-3, 9)$, $Q(a, b)$ and $R(4, -5)$ are collinear and $a + b = 1$, find the values of a and b . [CBSE 2014]
23. Find the area of $\triangle ABC$ with vertices $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$. Also, find the area of the triangle formed by joining the midpoints of its sides. Show that the ratio of the areas of two triangles is $4 : 1$. [CBSE 2014]
24. If $a \neq b \neq c$, prove that (a, a^2) , (b, b^2) , $(0, 0)$ will not be collinear. [CBSE 2017]

ANSWERS (EXERCISE 6C)

1. (i) 11 sq units (ii) 53 sq units (iii) 37.5 sq units (iv) 24.5 sq units
2. 132 sq units 3. 28 sq units 4. 28 sq units 5. 77 sq units 6. 1 sq unit
8. 12 sq units 9. $\frac{3}{4}$ sq unit 10. (i) $p = -3$ or $p = -9$ (ii) $y = \frac{13}{2}$
11. $k = 3$ 12. $k = 3$ 14. $x = -63$ 15. $x = 2$
16. $y = -2$ 17. $y = -1$ 18. $k = 2$ or $k = \frac{11}{2}$ 19. $4x - 5y - 3 = 0$
20. $2x + y + 3 = 0$ 22. $a = 2, b = -1$ 23. 4 sq units, 1 sq unit

HINTS TO SOME SELECTED QUESTIONS

8. Let the vertices of $\triangle ABC$ be $A(1, -4)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Let $D(2, -1)$ and $E(0, -1)$ be the midpoints of AB and AC respectively. Then,

$$\frac{1+x_2}{2} = 2, \frac{-4+y_2}{2} = -1$$

$$\Rightarrow 1+x_2 = 4, -4+y_2 = -2$$

$$\Rightarrow x_2 = 3, y_2 = 2.$$

And, $\frac{1+x_3}{2} = 0, \frac{-4+y_3}{2} = -1$

$$\Rightarrow 1+x_3 = 0, -4+y_3 = -2$$

$$\Rightarrow x_3 = -1, y_3 = 2.$$

In $\triangle ABC$, we have

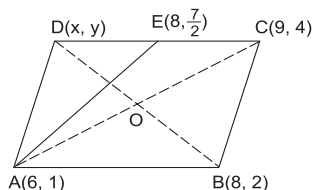
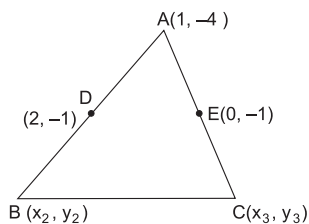
$$A(x_1 = 1, y_1 = -4), B(x_2 = 3, y_2 = 2) \text{ and } C(x_3 = -1, y_3 = 2).$$

Now, find $\text{ar}(\triangle ABC)$.

9. Let the fourth vertex be $D(x, y)$.

Midpoint of AC is $\left(\frac{6+9}{2}, \frac{4+1}{2}\right)$, i.e., $\left(\frac{15}{2}, \frac{5}{2}\right)$.

Midpoint of BD is $\left(\frac{8+x}{2}, \frac{2+y}{2}\right)$.



$$\therefore \frac{8+x}{2} = \frac{15}{2} \text{ and } \frac{2+y}{2} = \frac{5}{2} \Rightarrow x=7 \text{ and } y=3.$$

So, we get the point $D(7, 3)$.

$$\text{Midpoint of } DC \text{ is } E\left(\frac{7+9}{2}, \frac{3+4}{2}\right), \text{ i.e., } E\left(8, \frac{7}{2}\right).$$

Now, $A(6, 1)$, $D(7, 3)$ and $E\left(8, \frac{7}{2}\right)$ are vertices of $\triangle ADE$. Now find its area.

10. (i) Taking $A(x_1 = 1, y_1 = -3)$, $B(x_2 = 4, y_2 = p)$ and $C(x_3 = -9, y_3 = 7)$, we get
 $\text{ar}(\triangle ABC) = 15 \text{ sq units}$

$$\Rightarrow \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 15$$

$$\Rightarrow |1 \cdot (p - 7) + 4 \cdot (7 + 3) - 9 \cdot (-3 - p)| = 30$$

$$\Rightarrow |(p - 7) + 40 + 27 + 9p| = 30 \Rightarrow |10p + 60| = 30$$

$$\Rightarrow 10p + 60 = 30 \text{ or } 10p + 60 = -30 \Rightarrow p = -3 \text{ or } p = -9.$$

11. Let $A(x_1 = k + 1, y_1 = 1)$, $B(x_2 = 4, y_2 = -3)$ and $C(x_3 = 7, y_3 = -k)$ be the vertices of $\triangle ABC$. Then,

$$\begin{aligned} \text{ar}(\triangle ABC) &= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2}|(k + 1)(-3 + k) + 4(-k - 1) + 7(1 + 3)| \\ &= \frac{1}{2}|(k^2 - 6k + 21)| = \frac{1}{2}|(k^2 - 6k + 9) + 12| \\ &= \frac{1}{2}|(k - 3)^2 + 12| = \frac{1}{2}[(k - 3)^2 + 12]. \end{aligned}$$

$$\therefore \frac{1}{2}[(k - 3)^2 + 12] = 6 \Rightarrow (k - 3)^2 + 12 = 12 \Rightarrow (k - 3)^2 = 0.$$

Hence, $k = 3$.

12. Let $A(x_1 = -2, y_1 = 5)$, $B(x_2 = k, y_2 = -4)$ and $C(x_3 = 2k + 1, y_3 = 10)$. Then,
 $\text{ar}(\triangle ABC) = 53 \text{ sq units}$

$$\Rightarrow \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 53$$

$$\Rightarrow \frac{1}{2}|(-2)(-4 - 10) + k(10 - 5) + (2k + 1)(5 + 4)| = 53$$

$$\Rightarrow |28 + 5k + 18k + 9| = 106$$

$$\Rightarrow |23k + 37| = 106$$

$$\Rightarrow 23k + 37 = 106 \text{ or } 23k + 37 = -106$$

$$\Rightarrow k = 3 \text{ or } k = -\frac{143}{23}.$$

14. Here, $A(x_1 = x, y_1 = 2)$, $B(x_2 = -3, y_2 = -4)$ and $C(x_3 = 7, y_3 = -5)$.

$$\therefore \Delta = 0 \Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$$

$$\Rightarrow x + 21 + 42 = 0 \Rightarrow x = -63.$$

20. Here, $A(x_1 = x, y_1 = y)$, $B(x_2 = -5, y_2 = 7)$ and $C(x_3 = -4, y_3 = 5)$.

$$\therefore \Delta = 0 \Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow x(7 - 5) - 5(5 - y) - 4(y - 7) = 0$$

$$\Rightarrow 2x - 25 + 5y - 4y + 28 = 0$$

$$\Rightarrow 2x + y + 3 = 0.$$

21. Here, $A(x_1 = a, y_1 = 0)$, $B(x_2 = 0, y_2 = b)$ and $C(x_3 = 1, y_3 = 1)$.

$$\therefore \Delta = 0 \Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow a(b - 1) + 0 \cdot (1 - 0) + 1 \cdot (0 - b) = 0$$

$$\Rightarrow ab - a - b = 0 \Rightarrow a + b = ab$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = 1.$$

.....

EXERCISE 6D

Very-Short-Answer Questions

- Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$. Find the values of y . [CBSE 2014]
- If the point $A(0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$, find p . [CBSE 2014]
- $ABCD$ is a rectangle whose three vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$. Find the length of one of its diagonal. [CBSE 2014]
- If the point $P(k - 1, 2)$ is equidistant from the points $A(3, k)$ and $B(k, 5)$, find the values of k . [CBSE 2014]
- Find the ratio in which the point $P(x, 2)$ divides the join of $A(12, 5)$ and $B(4, -3)$. [CBSE 2014]
- Prove that the diagonals of a rectangle $ABCD$ with vertices $A(2, -1)$, $B(5, -1)$, $C(5, 6)$ and $D(2, 6)$ are equal and bisect each other. [CBSE 2014]
- Find the lengths of the medians AD and BE of $\triangle ABC$ whose vertices are $A(7, -3)$, $B(5, 3)$ and $C(3, -1)$. [CBSE 2014]
- If the point $C(k, 4)$ divides the join of $A(2, 6)$ and $B(5, 1)$ in the ratio $2 : 3$ then find the value of k . [CBSE 2013C]
- Find the point on x -axis which is equidistant from points $A(-1, 0)$ and $B(5, 0)$. [CBSE 2013C]
- Find the distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$. [CBSE 2009C]

11. Find the value of a , so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$. [CBSE 2009]
12. If the points $A(4, 3)$ and $B(x, 5)$ lie on the circle with centre $O(2, 3)$, find the value of x . [CBSE 2009]
13. If $P(x, y)$ is equidistant from the points $A(7, 1)$ and $B(3, 5)$, find the relation between x and y .
14. If the centroid of $\triangle ABC$ having vertices $A(a, b)$, $B(b, c)$ and $C(c, a)$ is the origin, then find the value of $(a + b + c)$.
15. Find the centroid of $\triangle ABC$ whose vertices are $A(2, 2)$, $B(-4, -4)$ and $C(5, -8)$.
16. In what ratio does the point $C(4, 5)$ divide the join of $A(2, 3)$ and $B(7, 8)$?
17. If the points $A(2, 3)$, $B(4, k)$ and $C(6, -3)$ are collinear, find the value of k .

ANSWERS (EXERCISE 6D)

1. $y = 7$ or $y = -1$ 2. $p = 1$ 3. 5 units 4. $k = 1$ or $k = 5$ 5. 3 : 5
7. $AD = 5$ units, $BE = 5$ units 8. $k = \frac{16}{5}$ 9. $P(2, 0)$ 10. 2 units
11. $a = \frac{1}{3}$ 12. $x = 2$ 13. $x - y = 2$ 14. $a + b + c = 0$ 15. $\left(1, \frac{-10}{3}\right)$
16. 2 : 3 17. $k = 0$

HINTS TO SOME SELECTED QUESTIONS

1. We know that the radii of a circle are equal.

$$\therefore OA = OB \Rightarrow OA^2 = OB^2$$

$$\Rightarrow (2 + 1)^2 + (-3y - y)^2 = (2 - 5)^2 + (-3y - 7)^2$$

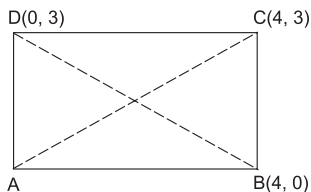
$$\Rightarrow 3^2 + 16y^2 = (-3)^2 + (9y^2 + 49 + 42y)$$

$$\Rightarrow 7y^2 - 42y - 49 = 0 \Rightarrow y^2 - 6y - 7 = 0$$

$$\Rightarrow (y - 7)(y + 1) = 0 \Rightarrow y = 7 \text{ or } y = -1$$

3. Diagonal $AC =$ diagonal BD

$$= \sqrt{(4 - 0)^2 + (0 - 3)^2} = \sqrt{25} = 5 \text{ units.}$$



$$4. PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$$

$$\Rightarrow (k-4)^2 + (2-k)^2 = 1+9=10$$

$$\Rightarrow 2k^2 - 12k + 10 = 0 \Rightarrow k^2 - 6k + 5 = 0$$

$$\Rightarrow (k-1)(k-5) = 0$$

$$\Rightarrow k = 1 \text{ or } k = 5.$$

5. Let the required ratio be $k : 1$. Then,

$$\frac{-3k+5}{k+1} = 2 \Rightarrow -3k+5 = 2k+2 \Rightarrow 5k = 3 \Rightarrow k = \frac{3}{5}.$$

$$\text{Required ratio} = \left(\frac{3}{5} : 1\right) = 3 : 5.$$

$$6. \text{ Diagonal } AC = \sqrt{(5-2)^2 + (6+1)^2}$$

$$= \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

$$\text{Diagonal } BD = \sqrt{(5-2)^2 + (-1-6)^2}$$

$$= \sqrt{3^2 + (-7)^2} = \sqrt{9+49} = \sqrt{58}.$$

\therefore Diag. $AC =$ Diag. BD .

$$\text{Midpoint of } AC = \left(\frac{5+2}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right).$$

$$\text{Midpoint of } BD = \left(\frac{5+2}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right).$$

7. Midpoint of BC is $D\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$, i.e., $D(4, 1)$.

Midpoint of AC is $E\left(\frac{7+3}{2}, \frac{-3-1}{2}\right)$, i.e., $E(5, -2)$.

$$AD = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5 \text{ units.}$$

$$BE = \sqrt{(5-5)^2 + (3+2)^2} = \sqrt{0+25} = \sqrt{25} = 5 \text{ units.}$$

$$8. k = \frac{(2 \times 5 + 3 \times 2)}{(2 + 3)} = \frac{16}{5}.$$

9. Let the required point be $P(x, 0)$. Then,

$$PA^2 = PB^2 \Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$$

$$\Rightarrow x^2 + 1 + 2x = x^2 + 25 - 10x$$

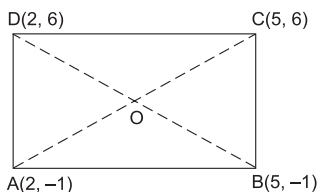
$$\Rightarrow 12x = 24 \Rightarrow x = 2.$$

\therefore the required point is $P(2, 0)$.

$$10. \text{ Given distance} = \sqrt{\left(\frac{-8}{5} - \frac{2}{5}\right)^2 - (2-2)^2} = \sqrt{(-2)^2 - 0^2} = \sqrt{4} = 2 \text{ units.}$$

11. Since $(3, a)$ lies on the line $2x - 3y = 5$, we have

$$2 \times 3 - 3a = 5 \Rightarrow 3a = 1 \Rightarrow a = \frac{1}{3}.$$



12. $(x-2)^2 + (5-3)^2 = (4-2)^2 + (3-3)^2$
 $\Rightarrow (x-2)^2 + 2^2 = 2^2 + 0^2$
 $\Rightarrow (x-2)^2 = 0 \Rightarrow x-2=0 \Rightarrow x=2.$
13. $(7-x)^2 + (1-y)^2 = (3-x)^2 + (5-y)^2$
 $\Rightarrow (49 + x^2 - 14x) + (1 + y^2 - 2y) = (9 + x^2 - 6x) + (25 + y^2 - 10y)$
 $\Rightarrow x^2 + y^2 - 14x - 2y + 50 = x^2 + y^2 - 6x - 10y + 34$
 $\Rightarrow 8x - 8y = 16 \Rightarrow x - y = 2.$
14. Centroid is $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}\right).$
 So, $\frac{a+b+c}{3} = 0 \Rightarrow a+b+c = 0.$
16. Let the required ratio be $k : 1$. Then, C is $\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right).$
 $\therefore \frac{7k+2}{k+1} = 4 \Rightarrow 7k+2 = 4k+4 \Rightarrow 3k=2 \Rightarrow k = \frac{2}{3}.$
 Required ratio is $\frac{2}{3} : 1$, i.e., $2 : 3.$
17. Let $(x_1 = 2, y_1 = 3), (x_2 = 4, y_2 = k)$ and $(x_3 = 6, y_3 = -3).$
 $\therefore \Delta = 0 \Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
 $\Rightarrow 2(k+3) + 4(-3-3) + 6(3-k) = 0$
 $\Rightarrow -4k = 0 \Rightarrow k = 0.$
-

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

- The distance of the point $P(-6, 8)$ from the origin is [CBSE 2013C]
 (a) 8 (b) $2\sqrt{7}$ (c) 6 (d) 10
- The distance of the point $(-3, 4)$ from x -axis is [CBSE 2012]
 (a) 3 (b) -3 (c) 4 (d) 5
- The point on x -axis which is equidistant from points $A(-1, 0)$ and $B(5, 0)$ is [CBSE 2013]
 (a) $(0, 2)$ (b) $(2, 0)$ (c) $(3, 0)$ (d) $(0, 3)$
- If $R(5, 6)$ is the midpoint of the line segment AB joining the points $A(6, 5)$ and $B(4, 4)$ then y equals [CASE 2014]
 (a) 5 (b) 7 (c) 12 (d) 6
- If the point $C(k, 4)$ divides the join of the points $A(2, 6)$ and $B(5, 1)$ in the ratio $2 : 3$ then the value of k is [CBSE 2013C]
 (a) 16 (b) $\frac{28}{5}$ (c) $\frac{16}{5}$ (d) $\frac{8}{5}$

6. The perimeter of the triangle with vertices $(0, 4)$, $(0, 0)$ and $(3, 0)$ is [CBSE 2014]
 (a) $(7 + \sqrt{5})$ (b) 5 (c) 10 (d) 12
7. If $A(1, 3)$, $B(-1, 2)$, $C(2, 5)$ and $D(x, 4)$ are the vertices of a ||gm $ABCD$ then the value of x is [CBSE 2012]
 (a) 3 (b) 4 (c) 0 (d) $\frac{3}{2}$
8. If the points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear then the value of x is [CBSE 2014]
 (a) -63 (b) 63 (c) 60 (d) -60
9. The area of a triangle with vertices $A(5, 0)$, $B(8, 0)$ and $C(8, 4)$ in square units is [CBSE 2012]
 (a) 20 (b) 12 (c) 6 (d) 16
10. The area of $\triangle ABC$ with vertices $A(a, 0)$, $O(0, 0)$ and $B(0, b)$ in square units is [CBSE 2011]
 (a) ab (b) $\frac{1}{2}ab$ (c) $\frac{1}{2}a^2b^2$ (d) $\frac{1}{2}b^2$
11. If $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6, 5)$ and $B(-2, 3)$ then the value of a is [CBSE 2011]
 (a) -8 (b) 3 (c) -4 (d) 4
12. $ABCD$ is a rectangle whose three vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$. The length of one of its diagonals is [CBSE 2014]
 (a) 5 (b) 4 (c) 3 (d) 25
13. The coordinates of the point P dividing the line segment joining the points $A(1, 3)$ and $B(4, 6)$ in the ratio $2 : 1$ is [CBSE 2012]
 (a) $(2, 4)$ (b) $(3, 5)$ (c) $(4, 2)$ (d) $(5, 3)$
14. If the coordinates of one end of a diameter of a circle are $(2, 3)$ and the coordinates of its centre are $(-2, 5)$, then the coordinates of the other end of the diameter are [CBSE 2012]
 (a) $(-6, 7)$ (b) $(6, -7)$ (c) $(4, 2)$ (d) $(5, 3)$
15. In the given figure $P(5, -3)$ and $Q(3, y)$ are the points of trisection of the line segment joining $A(7, -2)$ and $B(1, -5)$. Then, y equals [CBSE 2012]
 (a) 2 (b) 4
 (c) -4 (d) $-\frac{5}{2}$



16. The midpoint of segment AB is $P(0, 4)$. If the coordinates of B are $(-2, 3)$, then the coordinates of A are [CBSE 2011]
(a) $(2, 5)$ (b) $(-2, -5)$ (c) $(2, 9)$ (d) $(-2, 11)$
17. The point P which divides the line segment joining the points $A(2, -5)$ and $B(5, 2)$ in the ratio $2 : 3$ lies in the quadrant [CBSE 2011]
(a) I (b) II (c) III (d) IV
18. If $A(-6, 7)$ and $B(-1, -5)$ are two given points then the distance $2AB$ is [CBSE 2011]
(a) 13 (b) 26 (c) 169 (d) 238
19. Which point on x -axis is equidistant from the points $A(7, 6)$ and $B(-3, 4)$?
(a) $(0, 4)$ (b) $(-4, 0)$ (c) $(3, 0)$ (d) $(0, 3)$
20. The distance of $P(3, 4)$ from the x -axis is
(a) 3 units (b) 4 units (c) 5 units (d) 1 unit
21. In what ratio does the x -axis divide the join of $A(2, -3)$ and $B(5, 6)$?
(a) $2 : 3$ (b) $3 : 5$ (c) $1 : 2$ (d) $2 : 1$
22. In what ratio does the y -axis divide the join of $P(-4, 2)$ and $Q(8, 3)$?
(a) $3 : 1$ (b) $1 : 3$ (c) $2 : 1$ (d) $1 : 2$
23. If $P(-1, 1)$ is the midpoint of the line segment joining $A(-3, b)$ and $B(1, b + 4)$ then $b = ?$
(a) 1 (b) -1 (c) 2 (d) 0
24. The line $2x + y - 4 = 0$ divides the line segment joining $A(2, -2)$ and $B(3, 7)$ in the ratio
(a) $2 : 5$ (b) $2 : 9$ (c) $2 : 7$ (d) $2 : 3$
25. If $A(4, 2)$, $B(6, 5)$ and $C(1, 4)$ be the vertices of $\triangle ABC$ and AD is a median, then the coordinates of D are
(a) $\left(\frac{5}{2}, 3\right)$ (b) $\left(5, \frac{7}{2}\right)$ (c) $\left(\frac{7}{2}, \frac{9}{2}\right)$ (d) none of these
26. If $A(-1, 0)$, $B(5, -2)$ and $C(8, 2)$ are the vertices of a $\triangle ABC$ then its centroid is
(a) $(12, 0)$ (b) $(6, 0)$ (c) $(0, 6)$ (d) $(4, 0)$
27. Two vertices of $\triangle ABC$ are $A(-1, 4)$ and $B(5, 2)$ and its centroid is $G(0, -3)$. Then, the coordinates of C are
(a) $(4, 3)$ (b) $(4, 15)$ (c) $(-4, -15)$ (d) $(-15, -4)$
28. The points $A(-4, 0)$, $B(4, 0)$ and $C(0, 3)$ are the vertices of a triangle, which is
(a) isosceles (b) equilateral (c) scalene (d) right angled

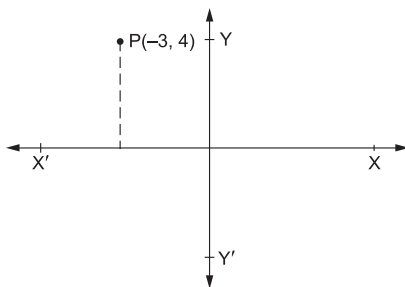
29. The points $P(0, 6)$, $Q(-5, 3)$ and $R(3, 1)$ are the vertices of a triangle, which is
 (a) equilateral (b) isosceles (c) scalene (d) right angled
30. If the points $A(2, 3)$, $B(5, k)$ and $C(6, 7)$ are collinear then
 (a) $k = 4$ (b) $k = 6$ (c) $k = \frac{-3}{2}$ (d) $k = \frac{11}{4}$
31. If the points $A(1, 2)$, $O(0, 0)$ and $C(a, b)$ are collinear then
 (a) $a = b$ (b) $a = 2b$ (c) $2a = b$ (d) $a + b = 0$
32. The area of $\triangle ABC$ with vertices $A(3, 0)$, $B(7, 0)$ and $C(8, 4)$ is
 (a) 14 sq units (b) 28 sq units
 (c) 8 sq units (d) 6 sq units
33. $AOBC$ is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$. The length of each of its diagonals is
 (a) 5 units (b) 3 units (c) 4 units (d) $\sqrt{34}$ units
34. If the distance between the points $A(4, p)$ and $B(1, 0)$ is 5 then
 (a) $p = 4$ only (b) $p = -4$ only (c) $p = \pm 4$ (d) $p = 0$

ANSWERS (MCQ)

1. (d) 2. (c) 3. (b) 4. (b) 5. (c) 6. (d) 7. (b) 8. (a)
 9. (c) 10. (b) 11. (a) 12. (a) 13. (b) 14. (a) 15. (c) 16. (a)
 17. (d) 18. (b) 19. (c) 20. (b) 21. (c) 22. (d) 23. (b) 24. (b)
 25. (c) 26. (d) 27. (c) 28. (a) 29. (d) 30. (b) 31. (c) 32. (c)
 33. (d) 34. (c)

HINTS TO SOME SELECTED QUESTIONS

- $OP^2 = (-6)^2 + 8^2 = 36 + 64 = 100 \Rightarrow OP = \sqrt{100} = 10$ units.
- Clearly, the distance of the point $P(-3, 4)$ from x -axis is 4 units.



3. Let the required point be $P(x, 0)$. Then,

$$\begin{aligned} PA^2 &= PB^2 \Rightarrow (x+1)^2 = (x-5)^2 \\ &\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25 \\ &\Rightarrow 12x = 24 \Rightarrow x = 2. \end{aligned}$$

So, the required point is $P(2, 0)$.

4. $\frac{5+y}{2} = 6 \Rightarrow 5+y = 12 \Rightarrow y = 7.$

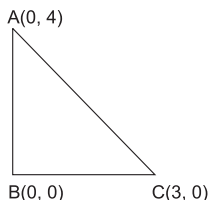
5. $k = \frac{(2 \times 5) + (3 \times 2)}{(2+3)} = \frac{16}{5}.$

6. $AB = 4$ units, $BC = 3$ units.

$$\begin{aligned} AC^2 &= (3-0)^2 + (0-4)^2 \\ &= (9+16) = 25. \end{aligned}$$

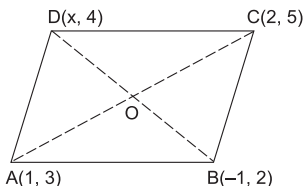
$\Rightarrow AC = 5$ units.

\therefore perimeter = $(4+3+5)$ units
= 12 units.



7. Midpoint of $BD =$ midpoint of AC

$$\Rightarrow \frac{-1+x}{2} = \frac{1+2}{2} \Rightarrow -1+x = 3 \Rightarrow x = 4.$$

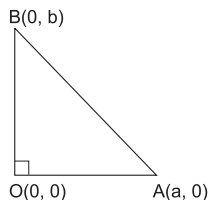


8. Here, $(x_1 = x, y_1 = 2)$, $(x_2 = -3, y_2 = -4)$ and $(x_3 = 7, y_3 = -5)$.

$$\begin{aligned} \therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) &= 0 \\ \Rightarrow x(-4+5) - 3(-5-2) + 7(2+4) &= 0 \\ \Rightarrow x + 21 + 42 = 0 \Rightarrow x &= -63. \end{aligned}$$

9. Here, $(x_1 = 5, y_1 = 0)$, $(x_2 = 8, y_2 = 0)$ and $(x_3 = 8, y_3 = 4)$.

$$\begin{aligned} \therefore \Delta &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} |5(0-4) + 8(4-0) + 8(0-0)| \\ &= \frac{1}{2} |-20 + 32| = \left(\frac{1}{2} \times 12\right) = 6 \text{ sq units.} \end{aligned}$$

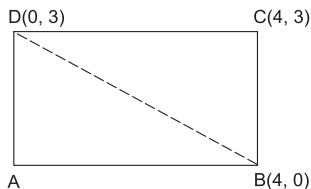


10. Here, base = a units and height = b units.

$$\begin{aligned} \therefore \text{area} &= \left(\frac{1}{2} \times \text{base} \times \text{height}\right) \\ &= \frac{1}{2} ab \text{ sq units.} \end{aligned}$$

$$11. \frac{a}{2} = \frac{(-6-2)}{2} = -4 \Rightarrow a = -8.$$

$$12. \text{Diagonal } BD = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{16+9} \\ = \sqrt{25} = 5 \text{ units.}$$



$$13. \text{Coordinates of } P \text{ are } \left(\frac{(2 \times 4 + 1 \times 1)}{2+1}, \frac{(2 \times 6 + 1 \times 3)}{2+1} \right) = (3, 5).$$

14. Let $C(-2, 5)$ be the centre of the given circle and $A(2, 3)$ and $B(x, y)$ be the end points of a diameter ACB .

Then, C is the midpoint of AB .

$$\therefore \frac{2+x}{2} = -2 \text{ and } \frac{3+y}{2} = 5$$

$$\Rightarrow 2+x = -4 \text{ and } 3+y = 10$$

$$\Rightarrow x = -6 \text{ and } y = 7.$$

So, the coordinates of B are $(-6, 7)$.

15. $Q(3, y)$ divides AB in the ratio $2 : 1$.

$$\text{So, } Q \text{ is } \left(\frac{2 \times 1 + 1 \times 7}{2+1}, \frac{2 \times (-5) + 1 \times (-2)}{2+1} \right), \text{ i.e., } (3, -4).$$

Hence, $y = -4$.

16. Let the point A be (a, b) . Then,

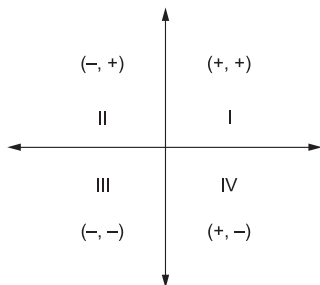
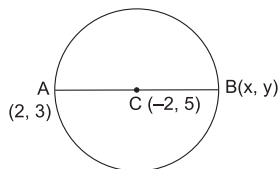
$$\frac{a+(-2)}{2} = 0 \text{ and } \frac{b+3}{2} = 4$$

$$\Rightarrow a-2=0 \text{ and } b+3=8 \Rightarrow a=2, b=5.$$

\therefore the point A is $(2, 5)$.

$$17. \text{The point } P \text{ is given by } P \left(\frac{2 \times 5 + 3 \times 2}{2+3}, \frac{2 \times 2 - 3 \times 5}{2+3} \right) = P \left(3, \frac{-11}{5} \right).$$

So, P lies in IV quadrant.



$$18. 2AB = 2 \times \sqrt{(-1+6)^2 + (-5-7)^2} = 2 \times \sqrt{(5)^2 + (-12)^2} \\ = 2 \times \sqrt{169} = 2 \times 13 = 26.$$

19. Let the required point be $P(x, 0)$. Then,

$$AP^2 = BP^2 \Rightarrow (x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2 \\ \Rightarrow x^2 - 14x + 85 = x^2 + 6x + 25 \\ \Rightarrow 20x = 60 \Rightarrow x = 3.$$

\therefore the required point is $P(3, 0)$.

21. Let the x -axis cut AB at $P(x, 0)$ in the ratio $k : 1$.

$$\text{Then, } \frac{6k-3}{k+1} = 0 \Rightarrow 6k-3-0 \Rightarrow 6k=3 \Rightarrow k = \frac{1}{2}.$$

$$\therefore \text{ required ratio} = \left(\frac{1}{2} : 1\right) = 1 : 2.$$

22. Let the y -axis cut AB at $P(0, y)$ in the ratio $k : 1$. Then,

$$P\left(\frac{8k-4}{k+1}, \frac{3k+2}{k+1}\right) = P(0, y) \Rightarrow \frac{8k-4}{k+1} = 0 \\ \Rightarrow 8k-4=0 \Rightarrow k = \frac{1}{2}.$$

$$\therefore \text{ required ratio} = \left(\frac{1}{2} : 1\right) = 1 : 2.$$

$$23. \text{ We have } \frac{b+(b+4)}{2} = 1 \Rightarrow 2b+4=2 \Rightarrow 2b=-2 \Rightarrow b=-1.$$

24. Let the required ratio be $k : 1$.

$$\text{Then, the point of division is } P\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right).$$

This point lies on the line $2x + y - 4 = 0$.

$$\therefore \frac{2(3k+2)}{k+1} + \frac{(7k-2)}{k+1} - 4 = 0 \Rightarrow 6k+4+7k-2-4k-4=0 \\ \Rightarrow 9k=2 \Rightarrow k = \frac{2}{9}.$$

So, the required ratio is $\left(\frac{2}{9} : 1\right)$, i.e., $(2 : 9)$.

$$25. \text{ Midpoint of } BC \text{ is } D\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = D\left(\frac{7}{2}, \frac{9}{2}\right).$$

$$26. \text{ Centroid is } G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = G\left(\frac{-1+5+8}{3}, \frac{0-2+2}{3}\right) = (4, 0).$$

27. Let the vertex C be $C(x, y)$. Then,

$$\frac{-1+5+x}{3} = 0 \text{ and } \frac{4+2+y}{3} = -3 \Rightarrow x+4=0 \text{ and } 6+y=-9$$

$$\therefore x = -4 \text{ and } y = -15.$$

So, the coordinates of C are $(-4, -15)$.

$$28. AB^2 = (4 + 4)^2 + (0 - 0)^2 = 8^2 + 0^2 = 64 + 0 = 64 \Rightarrow AB = \sqrt{64} = 8 \text{ units.}$$

$$BC^2 = (0 - 4)^2 + (3 - 0)^2 = (-4)^2 + 3^2 = 16 + 9 = 25 \Rightarrow BC = \sqrt{25} = 5 \text{ units.}$$

$$AC^2 = (0 + 4)^2 + (3 - 0)^2 = 4^2 + 3^2 = 16 + 9 = 25 \Rightarrow AC = \sqrt{25} = 5 \text{ units.}$$

$\therefore \triangle ABC$ is isosceles.

$$29. PQ^2 = (-5 - 0)^2 + (3 - 6)^2 = (-5)^2 + (-3)^2 = 25 + 9 = 34,$$

$$QR^2 = (3 + 5)^2 + (1 - 3)^2 = 8^2 + (-2)^2 = 64 + 4 = 68,$$

$$PR^2 = (3 - 0)^2 + (1 - 6)^2 = 3^2 + (-5)^2 = (9 + 25) = 34.$$

$$\therefore PQ^2 + PR^2 = QR^2.$$

Hence, $\triangle PQR$ is right-angled.

$$30. \text{ Here, } (x_1 = 2, y_1 = 3), (x_2 = 5, y_2 = k) \text{ and } (x_3 = 6, y_3 = 7).$$

Since the given points are collinear, we must have:

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 2(k - 7) + 5(7 - 3) + 6(3 - k) = 0 \Rightarrow 2k - 14 + 20 + 18 - 6k = 0$$

$$\Rightarrow 4k = 24 \Rightarrow k = 6.$$

$$31. \text{ Here, } (x_1 = 1, y_1 = 2), (x_2 = 0, y_2 = 0) \text{ and } (x_3 = a, y_3 = b).$$

Since the given points are collinear, we have

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$\Rightarrow 1 \cdot (0 - b) + 0 \cdot (b - 2) + a \cdot (2 - 0) = 0 \Rightarrow -b + 0 + 2a = 0 \Rightarrow 2a = b.$$

$$32. \text{ Here, } (x_1 = 3, y_1 = 0), (x_2 = 7, y_2 = 0) \text{ and } (x_3 = 8, y_3 = 4).$$

$$\therefore \text{ area of } \triangle ABC = \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2} \{3(0 - 4) + 7(4 - 0) + 8(0 - 0)\} = \frac{1}{2} \{-12 + 28 + 0\}$$

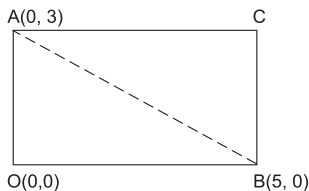
$$= \left(\frac{1}{2} \times 16\right) = 8 \text{ sq units.}$$

$$33. \text{ Diagonal } OC$$

$$= \text{diagonal } AB$$

$$= \sqrt{(5 - 0)^2 + (0 - 3)^2}$$

$$= \sqrt{25 + 9} = \sqrt{34} \text{ units.}$$



$$34. (4 - 1)^2 + (p - 0)^2 = 5^2 \Rightarrow p^2 = (25 - 9) = 16 \Rightarrow p = \pm 4.$$

