7 Triangles

CONGRUENT FIGURES *Two geometric figures which have the same shape and size are known as congruent figures.*

Congruent figures are alike in every respect.

SIMILAR FIGURES *Geometric figures which have the same shape but different sizes are known as similar figures.*

Two congruent figures are always similar but two similar figures need not be congruent.

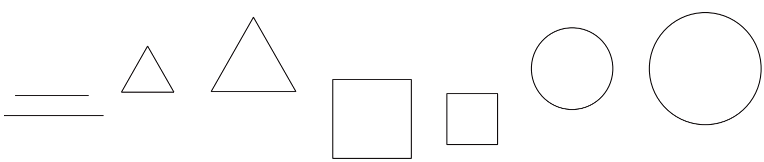
Examples

(i) Any two line segments are similar.

(ii) Any two equivalent triangles are similar.

(iii) Any two squares are similar.

(iv) Any two circles are similar.



SIMILAR POLYGONS

Two polygons having the same number of sides are said to be similar, if

(i) their corresponding angles are equal, and

(ii) the lengths of their corresponding sides are proportional.

If two polygons *ABCDE* and *PQRST* are similar, we write, *ABCDE* ~ *PQRST*, where the symbol '~' stands for '*is similar to*'.

The constant ratio between the corresponding sides of two similar figures is known as the *scale factor*, or the *representative fraction*. Since triangles are also polygons, so the same set of conditions apply for the similarity of triangles.

EQUIANGULAR TRIANGLES

Two triangles are said to be equiangular if their corresponding angles are equal.

SIMILAR TRIANGLES

Two triangles are said to be similar to each other if

(i) their corresponding angles are equal, and

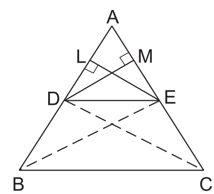
(ii) their corresponding sides are proportional.

RESULTS ON SIMILAR TRIANGLES

(BASIC-PROPORTIONALITY THEOREM) OR (THALES' THEOREM)

THEOREM 1 *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same ratio.* [CBSE 2002C, '04C, '05, '06C, '07, '09, '10]

GIVEN A $\triangle ABC$ in which $DE \parallel BC$ and DE intersects AB and AC at D and E respectively.



TO PROVE $\dfrac{AD}{DB} = \dfrac{AE}{EC}$.

CONSTRUCTION Join BE and CD.

Draw $EL \perp AB$ and $DM \perp AC$.

PROOF We have

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EL \qquad [\because \triangle = \frac{1}{2} \times \text{base} \times \text{height}]$$

and $\text{ar}(\triangle DBE) = \frac{1}{2} \times DB \times EL$.

$$\therefore \quad \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB}. \qquad \ldots \text{(i)}$$

Again, $\text{ar}(\triangle ADE) = \text{ar}(\triangle AED) = \frac{1}{2} \times AE \times DM$

and $\text{ar}(\triangle ECD) = \frac{1}{2} \times EC \times DM$.

$$\therefore \quad \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ECD)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}. \qquad \ldots \text{(ii)}$$

Now, $\triangle DBE$ and $\triangle ECD$ being on the same base DE and between the same parallels DE and BC, we have

$$\text{ar}(\triangle DBE) = \text{ar}(\triangle ECD) \qquad \ldots \text{(iii)}$$

From (i), (ii) and (iii), we have

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

COROLLARY *In a $\triangle ABC$, a line $DE \parallel BC$ intersects AB in D and AC in E, then prove that*

$$\text{(i) } \frac{AB}{DB} = \frac{AC}{EC} \qquad \text{(ii) } \frac{AD}{AB} = \frac{AE}{AC}.$$

PROOF (i) From Basic-Proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \Rightarrow \frac{AB}{DB} = \frac{AC}{EC}.$$

(ii) From Basic-Proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow \left(1 + \frac{DB}{AD}\right) = \left(1 + \frac{EC}{AE}\right)$$

$$\Rightarrow \frac{(AD + DB)}{AD} = \frac{(AE + EC)}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{AD}{AB} = \frac{AE}{AC}.$$

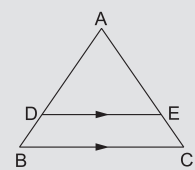
SUMMARY

In $\triangle ABC$, let $DE \parallel BC$. Then,

(i) $\frac{AD}{DB} = \frac{AE}{EC}$ (B.P.T.)

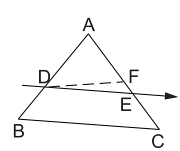
(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

(iii) $\frac{AD}{AB} = \frac{AE}{AC}$



THEOREM 2 **(Converse of Thales' theorem)** *If a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.*

GIVEN A $\triangle ABC$ and a line l intersecting AB at D and AC at E, such that $\frac{AD}{DB} = \frac{AE}{EC}$.



TO PROVE $DE \parallel BC$.

PROOF If possible, let DE not be parallel to BC. Then, there must be another line through D, which is parallel to BC. Let $DF \parallel BC$.

Then, by Thales' theorem, we have

$$\frac{AD}{DB} = \frac{AF}{FC}. \qquad \ldots \text{(i)}$$

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (given). … (ii)

From (i) and (ii), we get

$$\frac{AF}{FC}=\frac{AE}{EC} \Rightarrow \frac{AF}{FC}+1=\frac{AE}{EC}+1 \Rightarrow \frac{AF+FC}{FC}=\frac{AE+EC}{EC}$$

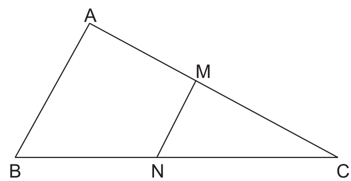
$$\Rightarrow \frac{AC}{FC}=\frac{AC}{EC} \Rightarrow \frac{1}{FC}=\frac{1}{EC} \Rightarrow FC=EC.$$

This is possible only when E and F coincide.

Hence, $DE \parallel BC$.

SOLVED EXAMPLES

EXAMPLE 1 *In the given figure,* $MN \parallel AB$, $BC = 7.5$ cm, $AM = 4$ cm *and* $MC = 2$ cm. *Find the length of* BN. [CBSE 2010]



SOLUTION In $\triangle ABC$, $MN \parallel AB$.

$\therefore \quad \frac{MC}{AC}=\frac{NC}{BC}$ [by Thales' theorem]

$\Rightarrow \quad \frac{MC}{AM+MC}=\frac{NC}{BC}$

$\Rightarrow \quad \frac{2}{4+2}=\frac{x}{7.5}$, where $NC = x$ cm

$\Rightarrow \quad x=\frac{2\times 7.5}{6}=\frac{15}{6}=2.5$

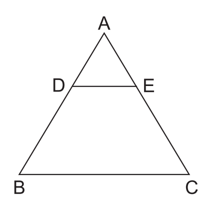
$\Rightarrow \quad NC = 2.5$ cm.

Hence, $BN = BC - NC = (7.5 - 2.5)$ cm $= 5$ cm.

EXAMPLE 2 *In the given figure,* $DE \parallel BC$ *and* $\frac{AD}{DB}=\frac{3}{5}$.

If $AC = 4.8$ cm, *find the length of* AE.

[CBSE 2008C]



SOLUTION Let $AE = x$ cm.

Then, $EC = (AC - AE) = (4.8 - x)$ cm.

Now, in $\triangle ABC$, $DE \parallel BC$.

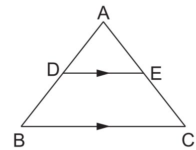
$\therefore \quad \frac{AD}{DB}=\frac{AE}{EC} \Rightarrow \frac{3}{5}=\frac{x}{(4.8-x)}$

$\Rightarrow \quad 3(4.8-x)=5x \Rightarrow 8x=14.4$

$\Rightarrow \quad x = 1.8.$

Hence, $AE = 1.8$ cm.

EXAMPLE 3 *In the given figure, in $\triangle ABC, DE \parallel BC$ so that $AD = (4x-3)$ cm, $AE = (8x-7)$ cm, $BD = (3x-1)$ cm and $CE = (5x-3)$ cm. Find the value of x.* [CBSE 2002C]



SOLUTION In $\triangle ABC, DE \parallel BC$.

$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE}$ [by Thales' theorem]

$\Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3} \Rightarrow (4x-3)(5x-3) = (3x-1)(8x-7)$

$\Rightarrow \quad 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\Rightarrow \quad 4x^2 - 2x - 2 = 0 \Rightarrow 2x^2 - x - 1 = 0$

$\Rightarrow \quad 2x^2 - 2x + x - 1 = 0 \Rightarrow 2x(x-1) + (x-1) = 0$

$\Rightarrow \quad (x-1)(2x+1) = 0$

$\Rightarrow \quad (x-1) = 0 \quad \text{or} \quad (2x+1) = 0$

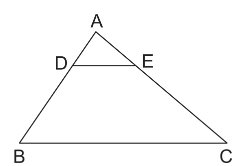
$\Rightarrow \quad x = 1 \text{ or } x = \frac{-1}{2}.$

But, $x = \frac{-1}{2} \Rightarrow AD = \left[4 \times \left(\frac{-1}{2}\right) = 3\right] = -5.$

And, distance can never be negative. So, $x \neq \frac{-1}{2}.$

Hence, $x = 1$.

EXAMPLE 4 *If D and E are points on the sides AB and AC respectively of $\triangle ABC$ such that $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm, show that $DE \parallel BC$.*



SOLUTION Given, $AB = 5.6$ cm, $AD = 1.4$ cm,

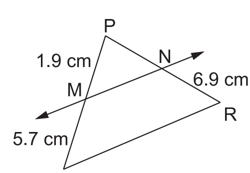
$AC = 7.2$ cm and $AE = 1.8$ cm.

$\therefore \quad \frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$ and $\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4}$

$\Rightarrow \quad \frac{AD}{AB} = \frac{AE}{AC}.$

Hence, by the converse of Thales' theorem, $DE \parallel BC$.

EXAMPLE 5 *In the adjoining figure, $MN \parallel QR$. Find (i) PN and (ii) PR.*



SOLUTION In $\triangle PQR$, $MN \parallel QR$.

$$\therefore \quad \frac{PM}{MQ} = \frac{PN}{NR} \quad \text{[by Thales' theorem]}$$

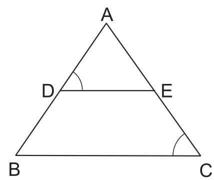
$$\Rightarrow \quad \frac{1.9}{5.7} = \frac{x}{6.9}, \text{ where } PN = x \text{ cm}$$

$$\Rightarrow \quad x = \frac{1.9 \times 6.9}{5.7} = 2.3.$$

Hence, (i) $PN = x$ cm $= 2.3$ cm

and (ii) $PR = PN + NR = (2.3 + 6.9)$ cm $= 9.2$ cm.

EXAMPLE 6 *In the given figure, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle ACB$. Prove that $\triangle ABC$ is an isosceles triangle.*



SOLUTION We have

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow DE \parallel BC \text{ [by the converse of Thales' theorem]}$$

$\therefore \quad \angle ADE = \angle ABC$ (corresponding $\angle$s).

But, $\angle ADE = \angle ACB$ (given).

$\therefore \quad \angle ABC = \angle ACB$.

So, $AB = AC$ [sides opposite to equal angles].

Hence, $\triangle ABC$ is an isosceles triangle.

EXAMPLE 7 *M and N are points on the sides AC and BC respectively of a $\triangle ABC$. In each of the following cases, state whether $MN \parallel AB$.*

(i) $CM = 4.2$ cm, $MA = 2.8$ cm, $NB = 3.6$ cm, $CN = 5.7$ cm

(ii) $CB = 6.92$ cm, $CN = 1.04$ cm, $CA = 1.73$ cm, $CM = 0.26$ cm

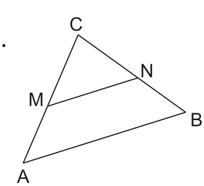
(iii) $CM = 5.1$ cm, $CA = 6.8$ cm, $CB = 5.6$ cm, $NB = 1.4$ cm

SOLUTION (i) We have

$$\frac{CM}{MA} = \frac{4.2}{2.8} = \frac{3}{2} \text{ and } \frac{CN}{NB} = \frac{5.7}{3.6} = \frac{19}{12}.$$

Since $\frac{CM}{MA} \neq \frac{CN}{NB}$.

So, MN is not parallel to AB.



(ii) We have

$$MA = CA - CM = (1.73 - 0.26) \text{ cm} = 1.47 \text{ cm}$$

and $NB = CB - CN = (6.92 - 1.04)$ cm $= 5.88$ cm.

$\therefore \quad \frac{CM}{MA} = \frac{0.26}{1.47} = \frac{26}{147}$ and $\frac{CN}{NB} = \frac{1.04}{5.88} = \frac{26}{147}$.

Clearly, $\frac{CM}{MA} = \frac{CN}{NB}$ and so $MN \parallel AB$

[by the converse of Thales' theorem].

(iii) We have

$$MA = CA - CM = (6.8 - 5.1) \text{ cm} = 1.7 \text{ cm}$$

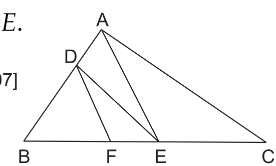
and $CN = CB - NB = (5.6 - 1.4)$ cm $= 4.2$ cm.

$\therefore \quad \frac{CM}{MA} = \frac{5.1}{1.7} = \frac{3}{1}$ and $\frac{CN}{NB} = \frac{4.2}{1.4} = \frac{3}{1}$.

Clearly, $\frac{CM}{MA} = \frac{CN}{NB}$ and so $MN \parallel AB$

[by the converse of Thales' theorem].

EXAMPLE 8 *In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.* [CBSE 2005, '07]



SOLUTION In $\triangle BAE$, $DF \parallel AE$.

$\therefore \quad \frac{BD}{DA} = \frac{BF}{FE}$. ... (i) [by Thales' theorem]

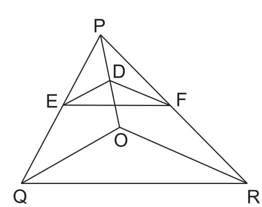
In $\triangle BAC$, $DE \parallel AC$.

$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC}$. ... (ii) [by Thales' theorem]

From (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BE}{EC} \quad \left[\text{each equal to } \frac{BD}{DA}\right].$$

EXAMPLE 9 *In the figure given along side, $DE \parallel OQ$* and $DF \parallel OR$. Show that $EF \parallel QR$.



SOLUTION In $\triangle POQ$, $DE \parallel OQ$.

$\therefore \quad \frac{PD}{DO} = \frac{PE}{EQ}$. ... (i) [by Thales' theorem]

In $\triangle POR$, $DF \parallel OR$.

$\therefore \quad \dfrac{PD}{DO} = \dfrac{PF}{FR}$. ... (ii)

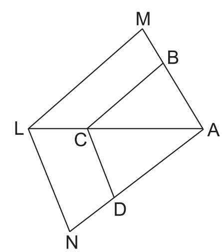
From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \left[\text{each equal to } \frac{PD}{DO}\right].$$

Thus, in $\triangle PQR$, E and F are points on PQ and PR respectively such that $\dfrac{PE}{EQ} = \dfrac{PF}{FR}$.

Hence, $EF \parallel QR$ [by the converse of Thales' theorem].

EXAMPLE 10 *In the given figure, $LM \parallel CB$ and $LN \parallel CD$. Prove that $\dfrac{AM}{AB} = \dfrac{AN}{AD}$.*



SOLUTION In $\triangle ALM$, $LM \parallel CB$.

$\therefore \quad \dfrac{AB}{AM} = \dfrac{AC}{AL} \Rightarrow \dfrac{AM}{AB} = \dfrac{AL}{AC}$. ... (i) [by Thales' theorem]

In $\triangle ALN$, $LN \parallel CD$.

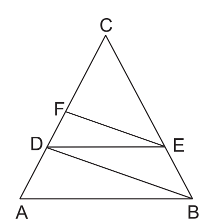
$\therefore \quad \dfrac{AC}{AL} = \dfrac{AD}{AN} \Rightarrow \dfrac{AL}{AC} = \dfrac{AN}{AD}$. ... (ii) [by Thales' theorem]

From (i) and (ii), we get

$$\frac{AM}{AB} = \frac{AN}{AD}.$$

EXAMPLE 11 *In the given figure, $AB \parallel DE$ and $BD \parallel EF$. Prove that $DC^2 = CF \times AC$.*

[CBSE 2004C, '10]



SOLUTION In $\triangle ABC$, $AB \parallel DE$.

$\therefore \quad \dfrac{CD}{DA} = \dfrac{CE}{EB}$. ... (i) [by Thales' theorem]

In $\triangle CDB$, $BD \parallel EF$.

$\therefore \quad \dfrac{CF}{FD} = \dfrac{CE}{EB}$. ... (ii) [by Thales' theorem]

From (i) and (ii), we get

$$\frac{CD}{DA} = \frac{CF}{FD}$$

$\Rightarrow \quad \frac{DA}{DC} = \frac{FD}{CF}$ [taking reciprocals]

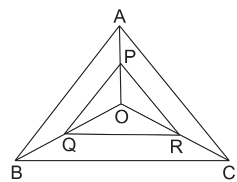
$\Rightarrow \quad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$

$\Rightarrow \quad \frac{DA + DC}{DC} = \frac{FD + CF}{CF}$

$\Rightarrow \quad \frac{AC}{DC} = \frac{DC}{CF}$

$\Rightarrow \quad DC^2 = CF \times AC.$

EXAMPLE 12 *In the given figure, PQ || AB and PR || AC. Prove that QR || BC.* [CBSE 2002, '05C]



SOLUTION In $\triangle OAB$, $PQ \parallel AB$.

$\therefore \quad \frac{OP}{PA} = \frac{OQ}{QB}.$ … (i) [by Thales' theorem]

In $\triangle AOC$, $PR \parallel AC$.

$\therefore \quad \frac{OP}{PA} = \frac{OR}{RC}.$ … (ii) [by Thales' theorem]

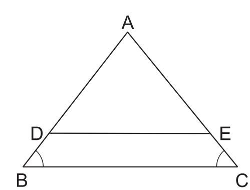
From (i) and (ii), we get

$\frac{OQ}{QB} = \frac{OR}{RC}$ in $\triangle OBC$.

Thus, in $\triangle OBC$, Q and R are points on OB and OC respectively such that $\frac{OQ}{QB} = \frac{OR}{RC}$.

Hence, by the converse of Thales' theorem, $QR \parallel BC$.

EXAMPLE 13 *In the given figure, in $\triangle ABC$, $\angle B = \angle C$ and $BD = CE$. Prove that $DE \parallel BC$.*



SOLUTION GIVEN A $\triangle ABC$ in which $\angle B = \angle C$ and $BD = CE$.

TO PROVE $DE \parallel BC$.

PROOF In $\triangle ABC$, $\angle B = \angle C \Rightarrow AB = AC$

[sides opposite equal ∠s are equal].

Now, $AB = AC \Rightarrow (AD + BD) = (AE + CE)$

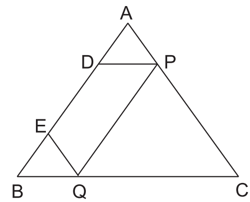
$\Rightarrow AD = AE$ [∵ $BD = CE$ (given)]

$\Rightarrow \frac{AD}{BD} = \frac{AE}{CE}$ [∵ $BD = CE$].

Thus, $\frac{AD}{BD} = \frac{AE}{CE}$.

Hence, $DE \parallel BC$ [by the converse of Thales' theorem].

EXAMPLE 14 *In $\triangle ABC$, D and E are two points on AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, prove that $PQ \parallel AB$.*



SOLUTION GIVEN A $\triangle ABC$ and D, E are two points on AB such that $AD = BE$. Also, $DP \parallel BC$, and $EQ \parallel AC$.

TO PROVE $PQ \parallel AB$.

PROOF In $\triangle ABC$, $DP \parallel BC$.

$\therefore \quad \frac{AD}{DB} = \frac{AP}{PC}$. … (i) [by Thales' theorem]

In $\triangle CBA$, $EQ \parallel AC$.

$\therefore \quad \frac{BE}{EA} = \frac{BQ}{QC}$ [by Thales' theorem]

$\Rightarrow \quad \frac{AD}{DB} = \frac{BQ}{QC}$. … (ii)

[$\because$ $BE = AD$, $EA = (ED + DA) = (DE + EB) = DB$]

From (i) and (ii), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}.$$

$\therefore \quad PQ \parallel AB$ [by the converse of Thales' theorem].

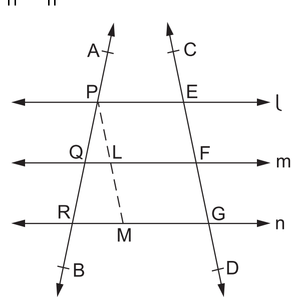
EXAMPLE 15 *If three or more parallel lines are intersected by two transversals, prove that the intercepts made by them on the transversals are proportional.*

SOLUTION GIVEN Three lines l, m, n such that $l \parallel m \parallel n$.

These lines are cut by the transversals AB and CD in P, Q, R and E, F, G respectively.

TO PROVE $\frac{PQ}{QR} = \frac{EF}{FG}$.

CONSTRUCTION Draw $PM \parallel CD$, meeting the lines m and n at L and M respectively.



PROOF $PL \parallel EF$ and $PE \parallel LF \Rightarrow PLFE$ is a ||gm

$\Rightarrow PL = EF.$ ... (i)

(opp. sides of a ||gm)

Also, $LM \parallel FG$ and $LF \parallel MG \Rightarrow LMGF$ is a ||gm

$\Rightarrow LM = FG.$ ... (ii)

(opp. sides of a ||gm)

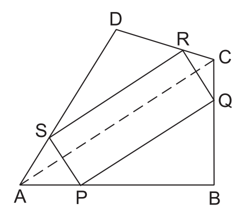
In $\triangle PRM$, $QL \parallel RM$ and therefore, by Thales' theorem, we have

$$\frac{PQ}{QR} = \frac{PL}{LM} = \frac{EF}{FG} \text{ [using (i) and (ii)].}$$

$$\therefore \quad \frac{PQ}{QR} = \frac{EF}{FG}.$$

EXAMPLE 16 *ABCD is a quadrilateral and P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively and are adjacent to A and C. Prove that PQRS is a parallelogram.*

SOLUTION GIVEN A quadrilateral $ABCD$ in which P, Q, R, S are the points of trisection of AB, BC, CD and DA respectively (as shown).



TO PROVE $PQRS$ is a parallelogram.

CONSTRUCTION Join AC.

PROOF In $\triangle BAC$, we have $\dfrac{BP}{PA} = \dfrac{BQ}{QC} = \dfrac{2}{1}.$

$\therefore \quad PQ \parallel AC.$ ... (i) [by the converse of Thales' theorem]

Also, in $\triangle DAC$, we have

$$\frac{DS}{SA} = \frac{DR}{RC} = \frac{2}{1}.$$

$\therefore \quad SR \parallel AC.$ ... (ii) [by the converse of Thales' theorem]

Thus, $PQ \parallel SR$ [from (i) and (ii)].

Similarly, by joining BD we can prove that $SP \parallel RQ$.

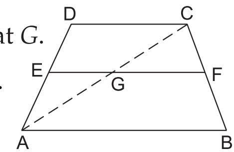
Hence, $PQRS$ is a parallelogram.

EXAMPLE 17 *ABCD is a trapezium with $AB \parallel DC$. E and F are points on non-parallel sides AD and BC respectively such that $EF \parallel AB$. Show that* $\dfrac{AE}{ED} = \dfrac{BF}{FC}.$

SOLUTION GIVEN A trap. $ABCD$ in which $AB \parallel DC$. E and F are points on AD and BC respectively such that $EF \parallel AB$.

TO PROVE $\frac{AE}{ED} = \frac{BF}{FC}$.

CONSTRUCTION Join AC, intersecting EF at G.



PROOF $EF \parallel AB$ and $AB \parallel DC \Rightarrow EF \parallel DC$.

Now, in $\triangle ADC$, $EG \parallel DC$.

$\therefore \quad \frac{AE}{ED} = \frac{AG}{GC}$. ... (i) [by Thales' theorem]

Similarly, in $\triangle CAB$, $GF \parallel AB$.

$\therefore \quad \frac{CG}{GA} = \frac{CF}{FB}$ [by Thales' theorem]

$\Rightarrow \quad \frac{AG}{GC} = \frac{BF}{FC}$. ... (ii)

From (i) and (ii), we get

$\frac{AE}{ED} = \frac{BF}{FC}$.

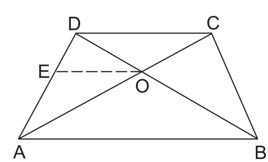
EXAMPLE 18 *ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O.*

Prove that $\frac{AO}{OC} = \frac{BO}{OD}$. [CBSE 2004]

SOLUTION GIVEN A trapezium $ABCD$ in which $AB \parallel DC$ and its diagonals AC and BD intersect at O.

TO PROVE $\frac{AO}{OC} = \frac{BO}{OD}$.

CONSTRUCTION Through O, draw $EO \parallel AB$, meeting AD at E.



PROOF In $\triangle ADC$, $EO \parallel DC$ [$\because$ $EO \parallel AB \parallel DC$].

$\therefore \quad \frac{AE}{ED} = \frac{AO}{OC}$ ... (i) [by Thales' theorem]

In $\triangle DAB$, $EO \parallel AB$.

$\therefore \quad \frac{DE}{EA} = \frac{DO}{OB}$ [by Thales' theorem]

$\Rightarrow \quad \frac{AE}{ED} = \frac{BO}{OD}$. ... (ii)

From (i) and (ii), we get

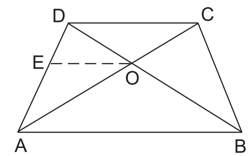
$\frac{AO}{OC} = \frac{BO}{OD}$.

EXAMPLE 19 *The diagonals of a quadrilateral ABCD intersect each other at the point O such that* $\frac{AO}{OC}=\frac{BO}{OD}$. *Show that ABCD is a trapezium.*

[CBSE 2005, '08]

SOLUTION GIVEN A quadrilateral $ABCD$ whose diagonals AC and BD intersect at a point O such that

$$\frac{AO}{OC}=\frac{BO}{OD}.$$



TO PROVE $ABCD$ is a trapezium, i.e., $AB \parallel DC$.

CONSTRUCTION Draw $EO \parallel DC$, meeting AD at E.

PROOF In $\triangle ACD$, $EO \parallel DC$.

$\therefore \quad \frac{AO}{OC}=\frac{AE}{ED}$ [by Thales' theorem].

But, $\frac{AO}{OC}=\frac{BO}{OD}$ (given)

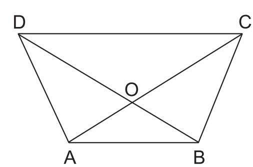
$\therefore \quad \frac{BO}{OD}=\frac{AE}{ED} \Rightarrow \frac{DO}{OB}=\frac{DE}{EA}$ in $\triangle DAB$.

So, $EO \parallel AB$ [by the converse of Thales' theorem].

But, $EO \parallel DC$.

Hence, $AB \parallel DC$, i.e., $ABCD$ is a trapezium.

EXAMPLE 20 *In the given figure, ABCD is a trapezium in which* $AB \parallel DC$ *and its diagonals intersect at O. If* $AO=(3x-1)$ cm, $OC=(5x-3)$ cm, $BO=(2x+1)$ cm *and* $OD=(6x-5)$ cm, *find the value of x.*



SOLUTION We know that $AB \parallel DC$ in trapezium $ABCD$ and its diagonals intersect at O. Then, we have

$$\frac{AO}{OC}=\frac{BO}{OD} \Rightarrow \frac{3x-1}{5x-3}=\frac{2x+1}{6x-5}$$
$$\Rightarrow (3x-1)(6x-5)=(2x+1)(5x-3)$$
$$\Rightarrow 18x^2-21x+5=10x^2-x-3$$
$$\Rightarrow 8x^2-20x+8=0 \Rightarrow 2x^2-5x+2=0$$
$$\Rightarrow (x-2)(2x-1)=0$$
$$\Rightarrow x=2 \text{ or } x=\frac{1}{2}.$$

But, $x=\frac{1}{2}$ will make $OC=(5x-3)$ cm $=\left(5\times\frac{1}{2}-3\right)$ cm $=-\frac{1}{2}$ cm.

And, the distance cannot be negative.

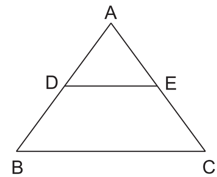
$\therefore \quad x \neq \frac{1}{2}.$

Hence, $x = 2$.

MIDPOINT THEOREM

EXAMPLE 21 *Prove that the line segment joining the midpoints of any two sides of a triangle is parallel to the third side.*

SOLUTION GIVEN A $\triangle ABC$ in which D and E are the midpoints of AB and AC respectively.



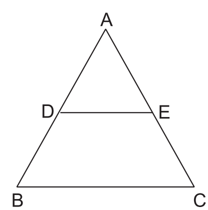
TO PROVE $DE \parallel BC$.

PROOF Since D and E are the midpoints of AB and AC respectively, we have $AD = DB$ and $AE = EC$.

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [each equal to 1].

Hence, by the converse of Thales' theorem, $DE \parallel BC$.

EXAMPLE 22 *Prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side.*



SOLUTION GIVEN A $\triangle ABC$ in which D is the midpoint of AB and $DE \parallel BC$, meeting AC at E.

TO PROVE $AE = EC$.

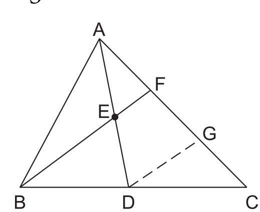
PROOF Since $DE \parallel BC$, by Thales' theorem, we have

$$\frac{AE}{EC} = \frac{AD}{DB} = 1 \quad [\because AD = DB \text{ (given)}]$$

$$\Rightarrow \quad \frac{AE}{EC} = 1 \Rightarrow AE = EC.$$

EXAMPLE 23 *In $\triangle ABC$, AD is a median and E is the midpoint of AD. If BE is produced, it meets AC in F. Show that $AF = \frac{1}{3}AC$.* [CBSE 2006C]

SOLUTION GIVEN A $\triangle ABC$ in which AD is a median and E is the midpoint of AD. Also, BE is produced to meet AC at F.



TO PROVE $AF = \frac{1}{3}AC$.

CONSTRUCTION From D, draw $DG \parallel EF$, meeting AC at G.

PROOF In $\triangle BCF$, D is the midpoint of BC and $DG \parallel BF$.

$\therefore$ G is the midpoint of CF.

So, $FG = GC$.

In $\triangle ADG$, E is the midpoint of AD and $EF \parallel DG$.

$\therefore$ F is the midpoint of AG.

So, $AF = FG$.

Thus, $AF = FG = GC$.

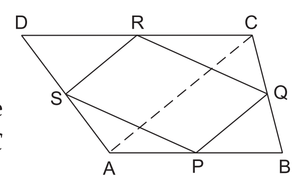
$\therefore$ $AC = (AF + FG + GC) = 3AF$.

Hence, $AF = \frac{1}{3}AC$.

EXAMPLE 24 *Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.*

SOLUTION GIVEN A quadrilateral $ABCD$ in which P, Q, R, S are the midpoints of AB, BC, CD and DA respectively.

TO PROVE $PQRS$ is a parallelogram.



CONSTRUCTION Join AC.

PROOF In $\triangle ABC$, P and Q are the midpoints of AB and BC respectively.

$\therefore$ $PQ \parallel AC$. … (i) [by midpoint theorem]

In $\triangle DAC$, S and R are the midpoints of AD and CD respectively.

$\therefore$ $SR \parallel AC$. … (ii) [by midpoint theorem]

From (i) and (ii), we get $PQ \parallel SR$.

Similarly, by joining BD, we can prove that $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram.

ANGLE-BISECTOR THEOREM

EXAMPLE 25 *Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.*

SOLUTION GIVEN A $\triangle ABC$ in which AD, the bisector of $\angle A$, meets BC in D.

TO PROVE $\frac{BD}{DC} = \frac{AB}{AC}$.

CONSTRUCTION Draw $CE \parallel DA$, meeting BA produced at E.

PROOF Since $DA \parallel CE$, we have

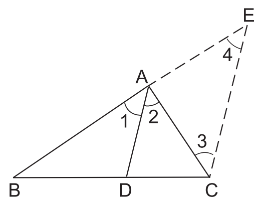
$\angle 2 = \angle 3$ [alternate int. $\angle$s]

and $\angle 1 = \angle 4$ [corresponding $\angle$s].

But, $\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle A$]

$\therefore$ $\angle 3 = \angle 4$.

So, $AE = AC$.



Now, in $\triangle BCE$, $DA \parallel CE$.

$\therefore$ $\dfrac{BD}{DC} = \dfrac{AB}{AE}$

$\Rightarrow$ $\dfrac{BD}{DC} = \dfrac{AB}{AC}$ [$\because$ $AE = AC$].

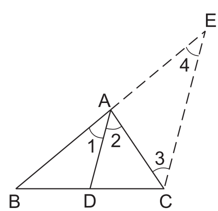
Hence, $\dfrac{BD}{DC} = \dfrac{AB}{AC}$.

EXAMPLE 26 *In a $\triangle ABC$, let D be a point on BC such that $\dfrac{BD}{DC} = \dfrac{AB}{AC}$. Prove that AD is the bisector of $\angle A$.*

SOLUTION GIVEN A $\triangle ABC$ in which D is a point on BC such that $\dfrac{BD}{DC} = \dfrac{AB}{AC}$.

TO PROVE AD is the bisector of $\angle A$.

CONSTRUCTION Produce BA to E such that $AE = AC$. Join EC.



PROOF $\dfrac{BD}{DC} = \dfrac{AB}{AC}$ (given)

$\Rightarrow$ $\dfrac{BD}{DC} = \dfrac{AB}{AE}$ [$\because$ $AC = AE$]

$\Rightarrow$ $DA \parallel CE$ [by the converse of Thales' theorem].

$\therefore$ $\angle 2 = \angle 3$ ... (i) [alternate int. $\angle$s]

and $\angle 1 = \angle 4$ ... (ii) [corresponding $\angle$s]

Also, $AE = AC \Rightarrow \angle 3 = \angle 4$. ... (iii)

$\therefore$ $\angle 1 = \angle 2$ [from (i), (ii) and (iii)].

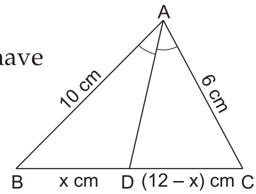
Hence, AD is the bisector of $\angle A$.

EXAMPLE 27 *In the given figure, AD is the bisector of $\angle BAC$. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, find BD and DC.* [CBSE 2001]

SOLUTION Let $BD = x$ cm. Then,

$DC = (BC - BD) = (12 - x)$ cm.

In $\triangle ABC$, AD is the bisector of $\angle BAC$.



So, by the angle-bisector theorem, we have

$$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{12-x} = \frac{10}{6}$$

$$\Rightarrow 6x = 10(12-x)$$

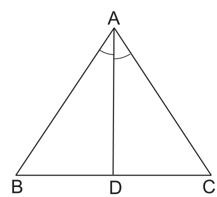
$$\Rightarrow 16x = 120 \Rightarrow x = 7.5.$$

Hence, $BD = 7.5$ cm, and $DC = (12-7.5)$ cm $= 4.5$ cm.

EXAMPLE 28 *If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.* [CBSE 2000, '01, '02]

SOLUTION GIVEN A $\triangle ABC$ in which AD is the bisector of $\angle BAC$ such that $BD = DC$.

TO PROVE $AB = AC$.



PROOF Since AD is the bisector of $\angle A$, by the angle-bisector theorem, we have

$$\frac{AB}{AC} = \frac{BD}{DC} = 1 \qquad [\because \; BD = DC]$$

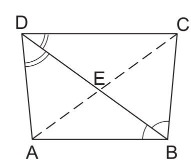
$$\Rightarrow \frac{AB}{AC} = 1 \Rightarrow AB = AC.$$

Hence, $\triangle ABC$ is an isosceles triangle.

EXAMPLE 29 *If the diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$, prove that* $\frac{AB}{BC} = \frac{AD}{CD}$.

SOLUTION GIVEN A quad. $ABCD$ in which diagonal BD bisects both $\angle B$ and $\angle D$.

TO PROVE $\frac{AB}{BC} = \frac{AD}{CD}$.



CONSTRUCTION Join AC, intersecting BD at E.

PROOF In $\triangle CBA$, BE is the bisector of $\angle ABC$.

$$\therefore \frac{AE}{EC} = \frac{AB}{BC}. \qquad \ldots \text{(i)} \quad \text{[by the angle-bisector theorem]}$$

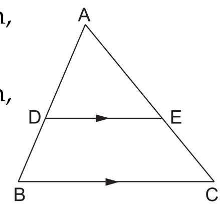
In $\triangle ADC$, DE is the bisector of $\angle ADC$.

$$\therefore \frac{AE}{EC} = \frac{AD}{CD}. \qquad \ldots \text{(ii)} \quad \text{[by the angle-bisector theorem]}$$

From (i) and (ii), we get $\frac{AB}{BC} = \frac{AD}{CD}$.

EXERCISE 7A

1. D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$.



   (i) If $AD = 3.6$ cm, $AB = 10$ cm and $AE = 4.5$ cm, find EC and AC.

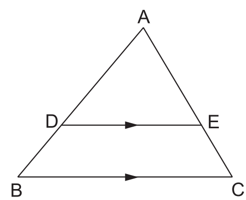
   (ii) If $AB = 13.3$ cm, $AC = 11.9$ cm and $EC = 5.1$ cm, find AD.

   (iii) If $\frac{AD}{DB} = \frac{4}{7}$ and $AC = 6.6$ cm, find AE.

   (iv) If $\frac{AD}{AB} = \frac{8}{15}$ and $EC = 3.5$ cm, find AE.

2. D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \parallel BC$.

   Find the value of x, when

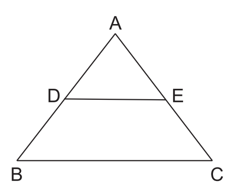


   (i) $AD = x$ cm, $DB = (x - 2)$ cm, $AE = (x + 2)$ cm and $EC = (x - 1)$ cm.

   (ii) $AD = 4$ cm, $DB = (x - 4)$ cm, $AE = 8$ cm and $EC = (3x - 19)$ cm.

   (iii) $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x + 4)$ cm and $EC = 3x$ cm.

3. D and E are points on the sides AB and AC respectively of a $\triangle ABC$. In each of the following cases, determine whether $DE \parallel BC$ or not.



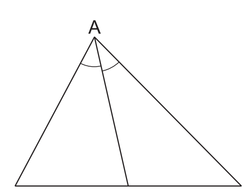
   (i) $AD = 5.7$ cm, $DB = 9.5$ cm, $AE = 4.8$ cm and $EC = 8$ cm.

   (ii) $AB = 11.7$ cm, $AC = 11.2$ cm, $BD = 6.5$ cm and $AE = 4.2$ cm.

   (iii) $AB = 10.8$ cm, $AD = 6.3$ cm, $AC = 9.6$ cm and $EC = 4$ cm.

   (iv) $AD = 7.2$ cm, $AE = 6.4$ cm, $AB = 12$ cm and $AC = 10$ cm.

4. In a $\triangle ABC$, AD is the bisector of $\angle A$.



   (i) If $AB = 6.4$ cm, $AC = 8$ cm and $BD = 5.6$ cm, find DC.

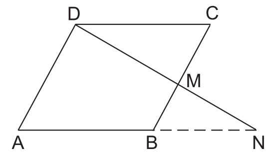
   (ii) If $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, find BD and DC.

   (iii) If $AB = 5.6$ cm, $BD = 3.2$ cm and $BC = 6$ cm, find AC. [CBSE 2001C]

   (iv) If $AB = 5.6$ cm, $AC = 4$ cm and $DC = 3$ cm, find BC. [CBSE 1999C]

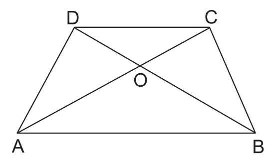
5. M is a point on the side BC of a parallelogram $ABCD$. DM when produced meets AB produced at N. Prove that

   (i) $\frac{DM}{MN} = \frac{DC}{BN}$, (ii) $\frac{DN}{DM} = \frac{AN}{DC}$.



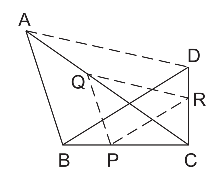
6. Show that the line segment which joins the midpoints of the oblique sides of a trapezium is parallel to the parallel sides.

7. In the adjoining figure, $ABCD$ is a trapezium in which $CD \parallel AB$ and its diagonals intersect at O. If $AO = (2x+1)$ cm, $OC = (5x-7)$ cm, $DO = (7x-5)$ cm and $OB = (7x+1)$ cm, find the value of x.

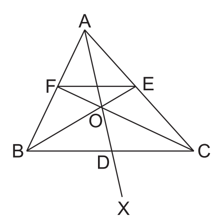


8. In a $\triangle ABC$, M and N are points on the sides AB and AC respectively such that $BM = CN$. If $\angle B = \angle C$ then show that $MN \parallel BC$.

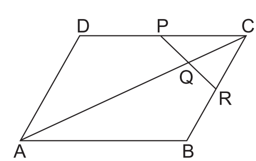
9. $\triangle ABC$ and $\triangle DBC$ lie on the same side of BC, as shown in the figure. From a point P on BC, $PQ \parallel AB$ and $PR \parallel BD$ are drawn, meeting AC at Q and CD at R respectively. Prove that $QR \parallel AD$.



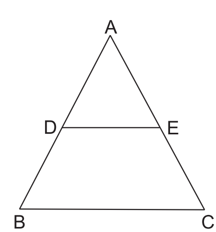
10. In the given figure, side BC of $\triangle ABC$ is bisected at D and O is any point on AD. BO and CO produced meet AC and AB at E and F respectively, and AD is produced to X so that D is the midpoint of OX. Prove that $AO : AX = AF : AB$ and show that $EF \parallel BC$.



11. $ABCD$ is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meets BC at R, prove that R is the midpoint of BC.

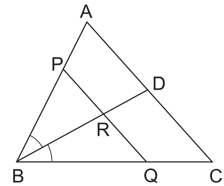


12. In the adjoining figure, ABC is a triangle in which $AB = AC$. If D and E are points on AB and AC respectively such that $AD = AE$, show that the points B, C, E and D are concyclic.



13. In $\triangle ABC$, the bisector of $\angle B$ meets AC at D. A line $PQ \parallel AC$ meets AB, BC and BD at P, Q and R respectively.

Show that $PR \times BQ = QR \times BP$.



ANSWERS (EXERCISE 7A)

1. (i) $EC = 8$ cm, $AC = 12.5$ cm (ii) $AD = 7.6$ cm (iii) $AE = 2.4$ cm (iv) $AE = 4$ cm

2. (i) $x = 4$ (ii) $x = 11$ (iii) $x = 4$

3. (i) Yes (ii) No (iii) Yes (iv) No

4. (i) $DC = 7$ cm (ii) $BD = 2.5$ cm, $DC = 3.5$ cm (iii) $AC = 4.9$ cm (iv) $BC = 7.2$ cm

7. $x = 2$

HINTS TO SOME SELECTED QUESTIONS

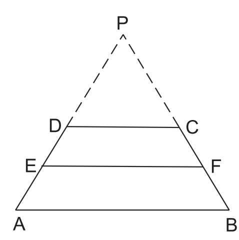
5. (i) In $\triangle NDA$, $MB \parallel DA$

$$\Rightarrow \frac{NM}{MD} = \frac{NB}{BA} \Rightarrow \frac{DM}{MN} = \frac{AB}{BN} \Rightarrow \frac{DM}{MN} = \frac{DC}{BN} \quad [\because \; AB = DC].$$

(ii) $$\frac{NM}{MD} = \frac{NB}{BA} \Rightarrow \frac{NM}{MD} + 1 = \frac{NB}{BA} + 1 \Rightarrow \frac{NM + MD}{MD} = \frac{NB + BA}{BA}$$

$$\Rightarrow \frac{DN}{DM} = \frac{AN}{AB} \Rightarrow \frac{DN}{DM} = \frac{AN}{DC} \quad [\because \; AB = DC].$$

6. Let E and F be the midpoints of the sides AD and BC of a trapezium $ABCD$ having $AB \parallel CD$. Produce AD and BC to meet at P.



In $\triangle PAB$, $DC \parallel AB$.

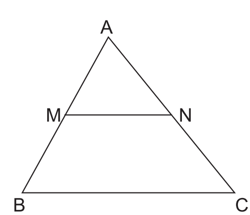
$$\therefore \quad \frac{PD}{DA} = \frac{PC}{CB} \Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF} \Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$$

$$\Rightarrow DC \parallel EF.$$

8. $\angle B = \angle C \Rightarrow AB = AC \Rightarrow AM + BM = AN + CN \Rightarrow AM = AN \quad [\because \; BM = CN]$.

Now, $AM = AN$, $BM = CN \Rightarrow \dfrac{AM}{BM} = \dfrac{AN}{CN}$

$$\Rightarrow \frac{AM}{MB} = \frac{AN}{NC} \Rightarrow MN \parallel BC.$$

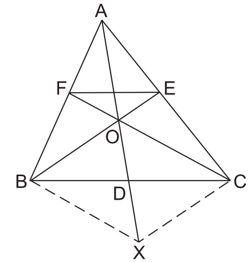


9. In $\triangle CAB$, $QP \parallel AB \Rightarrow \dfrac{CP}{PB} = \dfrac{CQ}{QA}$.

In $\triangle CDB$, $RP \parallel DB \Rightarrow \frac{CP}{PB} = \frac{CR}{RD}$.

$\therefore \quad \frac{CQ}{QA} = \frac{CR}{RD} \Rightarrow QR \parallel AD \quad$ (in $\triangle CDA$).

10. Join BX and CX.



Clearly, $BD = DC$ and $OD = DX$ (given).

Thus, the diagonals of quad. $OBXC$ bisect each other.

$\therefore \quad OBXC$ is a parallelogram.

$\therefore \quad BX \parallel CF$ and so, $OF \parallel BX$.

Similarly, $OE \parallel XC$.

In $\triangle ABX$, $OF \parallel BX$.

$\therefore \quad \frac{AO}{AX} = \frac{AF}{AB}$. … (i)

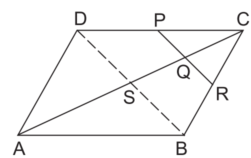
In $\triangle ACX$, $OE \parallel XC$.

$\therefore \quad \frac{AO}{AX} = \frac{AE}{AC}$. … (ii)

From (i) and (ii), we get $\frac{AF}{AB} = \frac{AE}{AC}$.

Hence, $FE \parallel BC$.

11. Join BD. Suppose it meets AC at S.



Since the diagonals of ||gm bisect each other, $CS = \frac{1}{2}AC$.

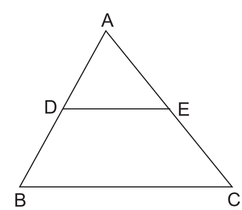
Now, $CS = \frac{1}{2}AC$ and $CQ = \frac{1}{4}AC \Rightarrow CQ = \frac{1}{2}CS$.

$\therefore \quad Q$ is the midpoint of CS.

So, $PQ \parallel DS$ and therefore, $QR \parallel SB$.

In $\triangle CSB$, Q is the midpoint of CS and $QR \parallel SB$, so R is the midpoint of BC.

12. $AB = AC$ and $AD = AE$



$\Rightarrow \quad AB - AD = AC - AE \Rightarrow DB = EC$

$\Rightarrow \quad \frac{AD}{AE} = \frac{DB}{EC} \quad$ [each equal to 1]

$\Rightarrow \quad DE \parallel BC \quad$ [by the converse of Thales' theorem]

$\Rightarrow \quad \angle DEC + \angle ECB = 180^\circ$

$\Rightarrow \quad \angle DEC + \angle CBD = 180^\circ \quad [\because AB = AC \Rightarrow \angle C = \angle B]$

$\Rightarrow \quad$ quad. $BCED$ is cyclic.

Hence, the points B, C, E, D are concyclic.

13. In $\triangle BQP$, BR is the bisector of $\angle B$.

$\therefore \quad \frac{QR}{PR} = \frac{BQ}{BP} \quad$ [by angle-bisector theorem].

CRITERIA FOR SIMILARITY OF TWO TRIANGLES

Two triangles are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are proportional.

Thus, $\triangle ABC \sim \triangle DEF$, *if*

(i) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

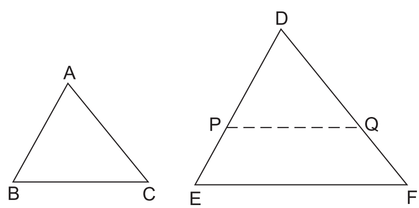
and (ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

THEOREM 1 **(AAA-similarity)** *If in two triangles, the corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar.*

GIVEN $\triangle ABC$ and $\triangle DEF$ such that $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

TO PROVE $\triangle ABC \sim \triangle DEF$.

CONSTRUCTION Cut $DP = AB$ and $DQ = AC$. Join PQ.



PROOF In $\triangle ABC$ and $\triangle DPQ$, we have

$AB = DP$ [by construction]

$AC = DQ$ [by construction]

$\angle A = \angle D$ [given]

$\therefore$ $\triangle ABC \cong \triangle DPQ$ [by SAS-congruence]

$\Rightarrow$ $\angle B = \angle P$

$\Rightarrow$ $\angle E = \angle P$ [$\because$ $\angle B = \angle E$ (given)]

$\Rightarrow$ $PQ \parallel EF$ [$\because$ corresponding $\angle$s are equal]

$\Rightarrow$ $\frac{DP}{DE} = \frac{DQ}{DF}$

$\Rightarrow$ $\frac{AB}{DE} = \frac{CA}{FD}$ [$\because$ $DP = AB$ and $DQ = AC$]

Similarly, $\frac{AB}{DE} = \frac{BC}{EF}$.

$\therefore$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

Thus, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

Hence, $\triangle ABC \sim \triangle DEF$.

IMPORTANT REMARK Two $\triangle$s are similar $\Leftrightarrow$ they are equiangular.

COROLLARY **(AA-similarity)** *If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar.*

PROOF In $\triangle ABC$ and $\triangle DEF$, let $\angle A = \angle D$ and $\angle B = \angle E$.

Then, 3rd $\angle C$ = 3rd $\angle F$.

Thus, the two triangles are equiangular and hence similar.

REMARK AA-similarity is the same as AAA-similarity.

THEOREM 2 **(SSS-similarity)** *If the corresponding sides of two triangles are proportional then their corresponding angles are equal, and hence the two triangles are similar.*

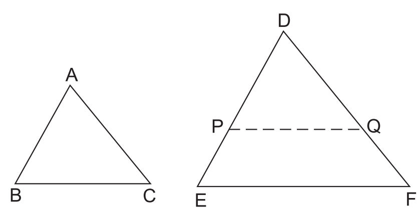
GIVEN $\triangle ABC$ and $\triangle DEF$ in which $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$.

TO PROVE $\triangle ABC \sim \triangle DEF$.

CONSTRUCTION Let us take $\triangle ABC$ and $\triangle DEF$ such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} (< 1).$$

Cut $DP = AB$ and $DQ = AC$. Join PQ.



PROOF $\dfrac{AB}{DE} = \dfrac{AC}{DF} \Rightarrow \dfrac{DP}{DE} = \dfrac{DQ}{DF}$ [$\because$ $AB = DP$ and $AC = DQ$].

So, by the converse of Thales' theorem, $PQ \parallel EF$.

$\therefore$ $\angle P = \angle E$ [corresponding $\angle$s]

$\angle Q = \angle F$ [corresponding $\angle$s]

$\therefore$ $\triangle DPQ \sim \triangle DEF$ [by AA-similarity]

$\Rightarrow$ $\dfrac{DP}{DE} = \dfrac{PQ}{EF}$

$\Rightarrow$ $\dfrac{AB}{DE} = \dfrac{PQ}{EF}$. … (i) [$\because$ $DP = AB$]

But, $\dfrac{AB}{DE} = \dfrac{BC}{EF}$. … (ii) [given]

$\therefore$ $\dfrac{PQ}{EF} = \dfrac{BC}{EF}$ [from (i) and (ii)]

$\Rightarrow$ $BC = PQ$.

Thus, $AB = DP$, $AC = DQ$ and $BC = PQ$.

$\therefore$ $\triangle ABC \cong \triangle DPQ$ [by SSS-congruence]

$\therefore$ $\angle A = \angle D, \angle B = \angle P = \angle E$ and $\angle C = \angle Q = \angle F$

$\Rightarrow$ $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$.

Thus, the given triangles are equiangular and hence similar.

THEOREM 3 **(SAS-similarity)** *If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.*

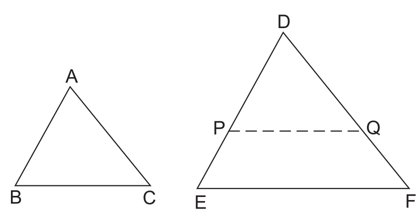
GIVEN $\triangle ABC$ and $\triangle DEF$ in which $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$.

TO PROVE $\triangle ABC \sim \triangle DEF$.

CONSTRUCTION Let us take $\triangle ABC$ and $\triangle DEF$ such that

$$\frac{AB}{DE} = \frac{AC}{DF} \ (< 1) \text{ and } \angle A = \angle D.$$

Cut $DP = AB$ and $DQ = AC$. Join PQ.



PROOF In $\triangle ABC$ and $\triangle DPQ$, we have

$AB = DP$ [by construction]

$\angle A = \angle D$ (given)

$AC = DQ$ [by construction]

$\therefore$ $\triangle ABC \cong \triangle DPQ$ [by SAS-congruence]

$\therefore$ $\angle A = \angle D, \angle B = \angle P$ and $\angle C = \angle Q$.

Now, $\frac{AB}{DE} = \frac{AC}{DF}$ (given)

$\Rightarrow$ $\frac{DP}{DE} = \frac{DQ}{DF}$ [$\because$ $AB = DP$ and $AC = DQ$]

$\Rightarrow$ $PQ \parallel EF$ [by the converse of Thales' theorem]

$\Rightarrow$ $\angle P = \angle E$ and $\angle Q = \angle F$ [corresponding $\angle$s]

$\therefore$ $\angle A = \angle D, \angle B = \angle P = \angle E$ and $\angle C = \angle Q = \angle F$.

Thus, $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$.

So, the given triangles are equiangular and hence similar.

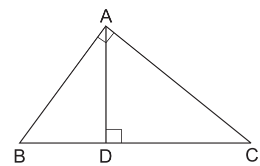
<u>THEOREM 4</u> *If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.*

GIVEN A $\triangle ABC$ in which $\angle BAC = 90^\circ$ and $AD \perp BC$.

TO PROVE (i) $\triangle DBA \sim \triangle ABC$

(ii) $\triangle DAC \sim \triangle ABC$

(iii) $\triangle DBA \sim \triangle DAC$.



PROOF (i) In $\triangle DBA$ and $\triangle ABC$, we have

$\angle BDA = \angle BAC = 90^\circ$

$\angle DBA = \angle ABC$ (common)

$\therefore$ $\triangle DBA \sim \triangle ABC$ [by AA-similarity].

(ii) In $\triangle DAC$ and $\triangle ABC$, we have

$\angle CDA = \angle CAB = 90^\circ$

$\angle DCA = \angle ACB$ (common)

$\therefore$ $\triangle DAC \sim \triangle ABC$ [by AA-similarity].

(iii) In $\triangle DBA$ and $\triangle DAC$, we have

$\angle ADB = \angle CDA = 90^\circ$.

$$\therefore \left.\begin{aligned} &\angle B + \angle BAD = 90^\circ \\ &\angle C + \angle CAD = 90^\circ \\ &\angle BAD + \angle CAD = 90^\circ \end{aligned}\right\} \Rightarrow \angle B = \angle CAD \text{ and } \angle C = \angle BAD.$$

Thus, in $\triangle DBA$ and $\triangle DAC$, we have:

$\angle ADB = \angle CDA$ [each equal to 90°]

$\angle B = \angle CAD$

$\angle BAD = \angle C$

$\therefore$ $\triangle DBA \sim \triangle DAC$ [by AAA-similarity].

SUMMARY

(i) If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, then $\triangle ABC \sim \triangle DEF$. (AAA-similarity)

(ii) If $\angle A = \angle D$, $\angle B = \angle E$, then $\triangle ABC \sim \triangle DEF$. (AA-similarity)

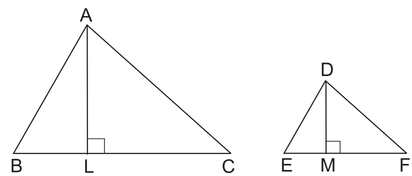
(iii) If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$. (SSS-similarity)

(iv) If $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$. (SAS-similarity)

SOME MORE RESULTS

THEOREM 1 *If two triangles are equiangular, prove that the ratio of their corresponding sides is the same as the ratio of the corresponding altitudes.*

GIVEN $\triangle ABC$ and $\triangle DEF$ in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and $AL \perp BC$ and $DM \perp EF$.



TO PROVE $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.

PROOF Since $\triangle ABC$ and $\triangle DEF$ are equiangular, $\triangle ABC \sim \triangle DEF$.

$\therefore \quad \dfrac{AB}{DE} = \dfrac{BC}{EF}$. … (i)

In $\triangle ALB$ and $\triangle DME$, we have

$\angle ALB = \angle DME = 90^\circ$ and $\angle B = \angle E$ (given).

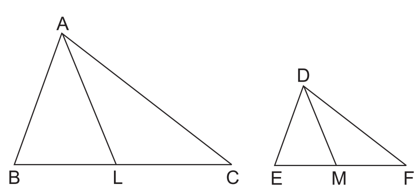
$\therefore \quad \triangle ALB \sim \triangle DME$ [by AA-similarity]

$\therefore \quad \dfrac{AB}{DE} = \dfrac{AL}{DM}$. … (ii)

From (i) and (ii), we get $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.

THEOREM 2 *If two triangles are equiangular, prove that the ratio of their corresponding sides is the same as the ratio of the corresponding medians.*

GIVEN $\triangle ABC$ and $\triangle DEF$ in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and AL and DM are the medians.



TO PROVE $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.

PROOF Since $\triangle ABC$ and $\triangle DEF$ are equiangular, we have $\triangle ABC \sim \triangle DEF$.

$\therefore \quad \dfrac{AB}{DE} = \dfrac{BC}{EF}$. … (i)

But, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{2BL}{2EM} = \dfrac{BL}{EM}$.

Now, in $\triangle ABL$ and $\triangle DEM$, we have

$\dfrac{AB}{DE} = \dfrac{BL}{EM}$ and $\angle B = \angle E$ (given).

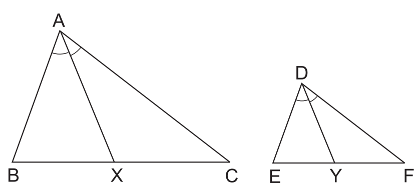
$\therefore \quad \triangle ABL \sim \triangle DEM$ [by SAS-similarity]

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AL}{DM}$. … (ii)

From (i) and (ii), we get $\dfrac{BC}{EF} = \dfrac{AL}{DM}$.

THEOREM 3 *If two triangles are equiangular, show that the ratio of the corresponding sides is the same as the ratio of the corresponding angle-bisector segments.*

GIVEN $\triangle ABC$ and $\triangle DEF$ in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, and AX and DY are the bisectors of $\angle A$ and $\angle D$ respectively.



TO PROVE $\dfrac{BC}{EF} = \dfrac{AX}{DY}$.

PROOF Since $\triangle ABC$ and $\triangle DEF$ are equiangular, we have $\triangle ABC \sim \triangle DEF$.

$\therefore \quad \dfrac{AB}{DE} = \dfrac{BC}{EF}$. … (i)

Now, $\angle A = \angle D \Rightarrow \dfrac{1}{2}\angle A = \dfrac{1}{2}\angle D \Rightarrow \angle BAX = \angle EDY$.

Thus, in $\triangle ABX$ and $\triangle DEY$, we have

$\angle BAX = \angle EDY$ (proved)

$\angle B = \angle E$ (given)

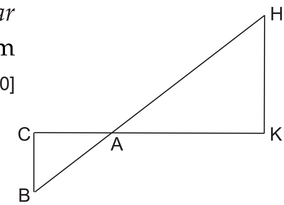
$\therefore \quad \triangle ABX \sim \triangle DEY$ [by AA-similarity]

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AX}{DY}$. … (ii)

From (i) and (ii), we get $\dfrac{BC}{EF} = \dfrac{AX}{DY}$.

SOLVED EXAMPLES

EXAMPLE 1 *In the adjoining figure, $\triangle AHK$ is similar to $\triangle ABC$. If $AK = 10$ cm, $BC = 3.5$ cm and $HK = 7$ cm, find AC.* [CBSE 2010]



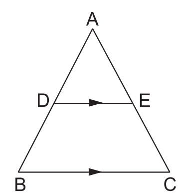
SOLUTION $\triangle AHK \sim \triangle ABC$

$\Rightarrow \dfrac{AK}{AC} = \dfrac{HK}{BC} \Rightarrow \dfrac{10}{x} = \dfrac{7}{3.5}$, where $AC = x$ cm

$\Rightarrow x = \dfrac{10 \times 3.5}{7} = 5.$

$\therefore AC = 5$ cm.

EXAMPLE 2 *In the given figure, $DE \parallel BC$, $AD = 2$ cm, $BD = 2.5$ cm, $AE = 3.2$ cm and $DE = 4$ cm. Find AC and BC.* [CBSE 2001C]



SOLUTION Since $DE \parallel BC$, we have

$\angle ADE = \angle ABC$ (corresponding $\angle$s)

and $\angle AED = \angle ACB$ (corresponding $\angle$s).

$\therefore \triangle ADE \sim \triangle ABC$ [by AA-similarity].

So, the corresponding sides of $\triangle ADE$ and $\triangle ABC$ are proportional.

$\therefore \dfrac{AD}{AB} = \dfrac{DE}{BC} = \dfrac{AE}{AC}$. ... (i)

Now, $\dfrac{AD}{AB} = \dfrac{DE}{BC} \Rightarrow \dfrac{2}{4.5} = \dfrac{4}{BC}$ [$\because AB = AD + BD = 4.5$ cm]

$\Rightarrow BC = \left(\dfrac{4 \times 4.5}{2}\right)$ cm $= 9$ cm.

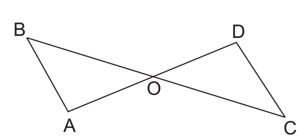
Again, $\dfrac{DE}{BC} = \dfrac{AE}{AC}$ [from (i)]

$\Rightarrow \dfrac{4}{9} = \dfrac{3.2}{AC}$ [$\because BC = 9$ cm]

$\Rightarrow AC = \left(\dfrac{9 \times 3.2}{4}\right)$ cm $= 7.2$ cm.

Hence, $AC = 7.2$ cm and $BC = 9$ cm.

EXAMPLE 3 *In the given figure, $AB \parallel CD$. Prove that $\triangle AOB \sim \triangle DOC$.*



SOLUTION $AB \parallel CD$ (given).

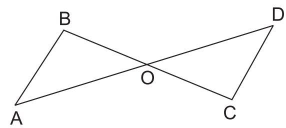
$\therefore \quad \angle OAB = \angle ODC$ (alternate angles)

$\angle OBA = \angle OCD$ (alternate angles)

$\angle AOB = \angle DOC$ (vertical opposite $\angle$s)

$\therefore \quad \triangle AOB \sim \triangle DOC$ [by AAA-similarity].

EXAMPLE 4 *In the given figure, $\triangle AOB \sim \triangle DOC$. Prove that $AB \parallel CD$.*



SOLUTION $\triangle AOB \sim \triangle DOC$.

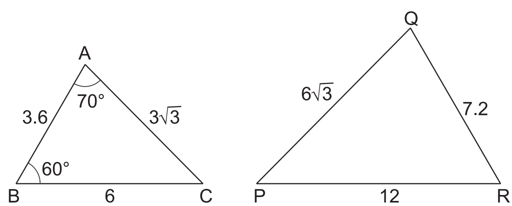
So, the given triangles are equiangular.

$\therefore \quad \angle OAB = \angle ODC$.

But, these are alternate angles.

$\therefore \quad AB \parallel CD$.

EXAMPLE 5 *Find $\angle P$ in the adjoining figure.*



SOLUTION In $\triangle ABC$ and $\triangle QRP$, we have

$$\frac{AB}{QR} = \frac{3.6}{7.2} = \frac{1}{2}, \frac{BC}{RP} = \frac{6}{12} = \frac{1}{2} \text{ and } \frac{CA}{PQ} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

Thus, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$ and so

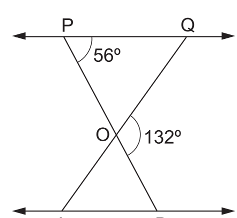
$\triangle ABC \sim \triangle QRP$ [by SSS-similarity].

$\therefore \quad \angle C = \angle P$ [corresponding angles of similar triangles].

But, $\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$.

$\therefore \quad \angle P = 50^\circ$.

EXAMPLE 6 *In the given figure, $\triangle OQP \sim \triangle OAB$, $\angle OPQ = 56^\circ$ and $\angle BOQ = 132^\circ$. Find $\angle OAB$.*



SOLUTION In $\triangle OPQ$, we have

$\angle BOQ = \angle OPQ + \angle OQP$

[$\because$ the exterior angle is equal to the sum of the two interior opposite angles]

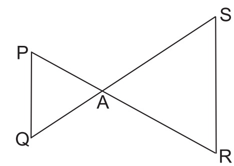
$\Rightarrow \quad \angle OQP = \angle BOQ - \angle OPQ = 132^\circ - 56^\circ = 76^\circ.$

Now, $\triangle OQP \sim \triangle OAB$

$\Rightarrow \quad \angle OQP = \angle OAB$ [corresponding angles of similar triangles].

$\therefore \quad \angle OAB = \angle OQP = 76^\circ.$

EXAMPLE 7 *In the given figure,* $AP \cdot AR = AS \cdot AQ$. *Prove that* $\angle P = \angle S$ and $\angle Q = \angle R$.



SOLUTION We have

$\angle PAQ = \angle SAR.$ ... (i) [vertically opposite angles]

Also, $AP \cdot AR = AS \cdot AQ$ (given)

$\Rightarrow \quad \dfrac{AP}{AS} = \dfrac{AQ}{AR}.$ ... (ii)

From (i) and (ii), we have

$\triangle PAQ \sim \triangle SAR$ [by SAS-similarity]

$\therefore \quad \angle P = \angle S$ and $\angle Q = \angle R$

[corresponding angles of similar triangles].

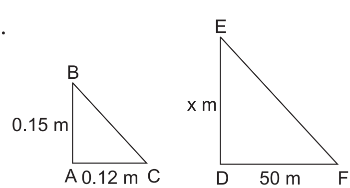
EXAMPLE 8 *A vertical stick which is* 15 cm *long casts a* 12-cm-*long shadow on the ground. At the same time, a vertical tower casts a* 50-m-*long shadow on the ground. Find the height of the tower.*

SOLUTION Let AB be the vertical stick and let AC be its shadow.

Then, $AB = 0.15$ m and $AC = 0.12$ m.

Let DE be the vertical tower and let DF be its shadow.

Then, $DF = 50$ m. Let $DE = x$ m.



Now, in $\triangle BAC$ and $\triangle EDF$, we have

$\angle BAC = \angle EDF = 90^\circ$

$\angle ACB = \angle DFE$

[angular elevation of the sun at the same time]

$\therefore \quad \triangle BAC \sim \triangle EDF$

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AC}{DF} \Rightarrow \dfrac{0.15}{x} = \dfrac{0.12}{50}$

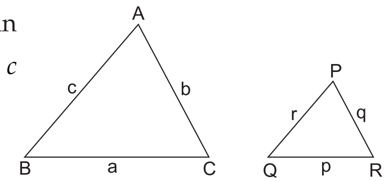
$\Rightarrow \quad x = \dfrac{(0.15 \times 50)}{0.12} = 62.5.$

Hence, the height of the tower is 62.5 m.

EXAMPLE 9 *Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.*

SOLUTION GIVEN $\triangle ABC$ and $\triangle PQR$ in which $BC = a$, $CA = b$, $AB = c$ and $QR = p$, $RP = q$, $PQ = r$.

Also, $\triangle ABC \sim \triangle PQR$.



TO PROVE $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$.

PROOF Since $\triangle ABC$ and $\triangle PQR$ are similar, therefore their corresponding sides are proportional.

$$\therefore \quad \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \text{ (say)} \qquad \ldots \text{(i)}$$

$$\Rightarrow \quad a = kp, b = kq \text{ and } c = kr.$$

$$\therefore \quad \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r}$$

$$= \frac{k(p+q+r)}{(p+q+r)} = k. \qquad \ldots \text{(ii)}$$

From (i) and (ii), we get

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle PQR} \quad \text{[each equal to } k\text{]}.$$

EXAMPLE 10 *The perimeters of two similar triangles are* 25 cm *and* 15 cm *respectively. If one side of the first triangle is* 9 cm, *find the corresponding side of the second triangle.* [CBSE 2002C]

SOLUTION Let $\triangle ABC \sim \triangle DEF$ given in such a way that perimeter of $\triangle ABC = 25$ cm, perimeter of $\triangle DEF = 15$ cm and $AB = 9$ cm. Then, we have to find DE. Let $DE = x$ cm.

We know that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

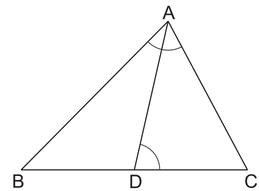
$$\therefore \quad \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{AB}{DE}$$

$$\Rightarrow \quad \frac{25}{15} = \frac{9}{x} \Rightarrow x = \left(\frac{9 \times 15}{25}\right) = 5.4.$$

$$\therefore \quad DE = 5.4 \text{ cm}.$$

Hence, the corresponding side of the second triangle is 5.4 cm.

<u>EXAMPLE 11</u> *In the given figure, D is a point on the side BC of* $\triangle ABC$ *such that* $\angle ADC = \angle BAC$. *Prove that* $CA^2 = CB \times CD$. [CBSE 2004]



SOLUTION GIVEN A $\triangle ABC$ in which D is a point on BC such that $\angle ADC = \angle BAC$.

TO PROVE $CA^2 = CB \times CD$.

PROOF In $\triangle ABC$ and $\triangle DAC$, we have

$\angle BAC = \angle ADC$ (given)

$\angle ACB = \angle DCA$ (common)

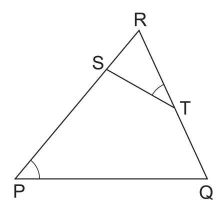
$\therefore$ $\triangle ABC \sim \triangle DAC$ [by AA-similarity].

So, the sides of $\triangle ABC$ and $\triangle DAC$ are proportional.

$\therefore$ $\dfrac{CA}{CB} = \dfrac{CD}{CA}$.

Hence, $CA^2 = CB \times CD$.

<u>EXAMPLE 12</u> *In the given figure, S and T are points on sides PR and QR of* $\triangle PQR$ *such that* $\angle P = \angle RTS$. *Show that* $\triangle RPQ \sim \triangle RTS$.



SOLUTION GIVEN $\triangle RPQ$ and $\triangle RTS$ in which $\angle P = \angle RTS$.

TO PROVE $\triangle RPQ \sim \triangle RTS$.

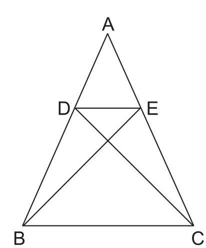
PROOF In $\triangle RPQ$ and $\triangle RTS$, we have

$\angle P = \angle RTS$ (given)

$\angle R = \angle R$ (common)

$\therefore$ $\triangle RPQ \sim \triangle RTS$ [by AA-similarity].

<u>EXAMPLE 13</u> *In the given figure, if* $\triangle ABE \cong \triangle ACD$, *show that* $\triangle ADE \sim \triangle ABC$.



SOLUTION $\triangle ABE \cong \triangle ACD$ (given)

$\therefore$ $AE = AD$ … (i) [cpct]

and $AB = AC$. … (ii) [cpct]

In $\triangle ADE$ and $\triangle ABC$, we have

$\angle DAE = \angle BAC$ (common)

and $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ [using (i) and (ii)].

$\therefore$ $\triangle ADE \sim \triangle ABC$ [by SAS-similarity].

EXAMPLE 14 *In the given figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P.*

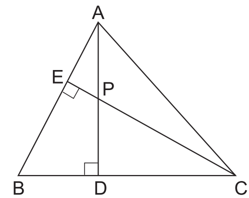
Show that:

(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$



SOLUTION (i) In $\triangle AEP$ and $\triangle CDP$, we have

$\angle AEP = \angle CDP$ [each equal to 90°]

$\angle APE = \angle CPD$ [vertically opposite $\angle$s]

$\therefore$ $\triangle AEP \sim \triangle CDP$ [by AA-similarity].

(ii) In $\triangle ABD$ and $\triangle CBE$, we have

$\angle ADB = \angle CEB = 90^\circ$

$\angle B = \angle B$ [common]

$\therefore$ $\triangle ABD \sim \triangle CBE$ [by AA-similarity].

(iii) In $\triangle AEP$ and $\triangle ADB$, we have

$\angle AEP = \angle ADB = 90^\circ$

$\angle EAP = \angle DAB$ (common)

Hence, $\triangle AEP \sim \triangle ADB$ [by AA-similarity].

(iv) In $\triangle PDC$ and $\triangle BEC$, we have

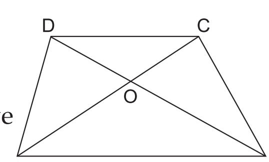
$\angle PDC = \angle BEC = 90^\circ$

$\angle PCD = \angle BCE$ (common)

$\therefore$ $\triangle PDC \sim \triangle BEC$ [by AA-similarity].

EXAMPLE 15 *Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at point O. Using a similarity criterion for two triangles, show that* $\frac{OA}{OC} = \frac{OB}{OD}$.

SOLUTION GIVEN A trapezium $ABCD$ in which $AB \parallel DC$. The diagonals AC and BD intersect at O.



TO PROVE $\frac{OA}{OC} = \frac{OB}{OD}$.

PROOF In $\triangle OAB$ and $\triangle OCD$, we have

$\angle OAB = \angle OCD$

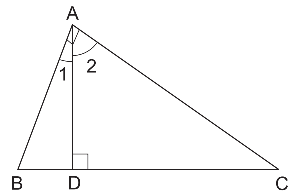
[alternate angles, since $AB \parallel DC$]

and $\angle OBA = \angle ODC$ [alternate angles, since $AB \parallel DC$].

$\therefore$ $\Delta OAB \sim \Delta OCD$ [by AA-similarity].

And so, $\frac{OA}{OC} = \frac{OB}{OD}$.

<ins>EXAMPLE 16</ins> *In ΔABC, $AD \perp BC$ and $AD^2 = BD \cdot CD$. Prove that $\angle BAC = 90°$.*



SOLUTION GIVEN A ΔABC in which $AD \perp BC$ and $AD^2 = BD \cdot CD$.

TO PROVE $\angle BAC = 90°$.

PROOF $AD^2 = BD \cdot CD \Rightarrow \frac{BD}{AD} = \frac{AD}{CD}$.

Now, in ΔDBA and ΔDAC, we have

$$\angle BDA = \angle ADC = 90° \text{ and } \frac{BD}{AD} = \frac{AD}{CD}.$$

$\therefore$ $\Delta DBA \sim \Delta DAC$ [by SAS-similarity]

$\therefore$ $\angle B = \angle 2$ and $\angle 1 = \angle C$

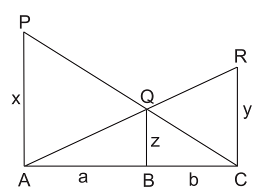
$\therefore$ $\angle 1 + \angle 2 = \angle B + \angle C \Rightarrow \angle A = \angle B + \angle C$

$\Rightarrow 2\angle A = \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A = \angle BAC = 90°$.

<ins>EXAMPLE 17</ins> *In the given figure PA, QB and RC each is perpendicular to AC such that $PA = x$, $RC = y$, $QB = z$, $AB = a$ and $BC = b$.*

Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$. [CBSE 2000C]



SOLUTION $PA \perp AC$ and $QB \perp AC \Rightarrow QB \parallel PA$.

Thus, in ΔPAC, $QB \parallel PA$. So, $\Delta QBC \sim \Delta PAC$.

$\therefore$ $\frac{QB}{PA} = \frac{BC}{AC} \Rightarrow \frac{z}{x} = \frac{b}{a+b}$. ... (i) [by the property of similar Δs]

In ΔRAC, $QB \parallel RC$. So, $\Delta QBA \sim \Delta RCA$.

$\therefore$ $\frac{QB}{RC} = \frac{AB}{AC} \Rightarrow \frac{z}{y} = \frac{a}{a+b}$. ... (ii) [by the property of similar Δs]

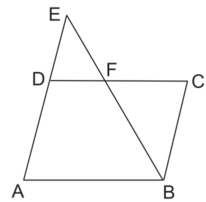
From (i) and (ii), we get

$$\frac{z}{x} + \frac{z}{y} = \left(\frac{b}{a+b} + \frac{a}{a+b}\right) = 1$$

$\Rightarrow \quad \frac{z}{x} + \frac{z}{y} = 1 \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}.$

Hence, $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}.$

EXAMPLE 18 *The side AD of a parallelogram ABCD is produced to a point E. BE intersects CD at F. Show that* $\triangle ABE \sim \triangle CFB$.



SOLUTION We have

$AD \parallel BC \Rightarrow AE \parallel BC$ [*ABCD* is a parallelogram]

$\therefore \quad \angle AEB = \angle CBF$ [alternate angles].

In $\triangle ABE$ and $\triangle CFB$, we have

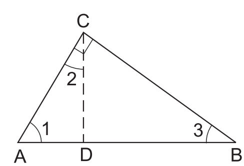
$\angle AEB = \angle CBF$ [proved above]

$\angle EAB = \angle BCF$ [$\because \angle DAB = \angle BCD$, being opposite angles of a parallelogram]

$\therefore \quad \triangle ABE \sim \triangle CFB$ [by AA-similarity].

EXAMPLE 19 *In the given figure,* $\angle ACB = 90^\circ$ *and* $CD \perp AB$. *Prove that*

$CD^2 = BD \cdot AD.$ [CBSE 2006]



SOLUTION GIVEN A $\triangle ABC$ in which

$\angle ACB = 90^\circ$ and $CD \perp AB$.

TO PROVE $CD^2 = BD \cdot AD$.

PROOF In right $\triangle ADC$, we have $\angle 1 + \angle 2 = 90^\circ$.

In right $\triangle ACB$, we have

$\angle 1 + \angle 3 = 90^\circ$.

$\therefore \quad \angle 1 + \angle 2 = \angle 1 + \angle 3 \Rightarrow \angle 2 = \angle 3.$

In $\triangle ADC$ and $\triangle CDB$, we have

$\angle 2 = \angle 3$ (proved)

and $\angle ADC = \angle CDB = 90^\circ$.

$\therefore \quad \triangle ADC \sim \triangle CDB$ [by AA-similarity]

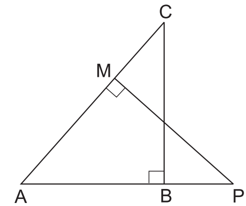
$\therefore \quad \frac{AD}{CD} = \frac{CD}{BD}.$

Hence, $CD^2 = BD \cdot AD$.

EXAMPLE 20 *In the given figure, $\triangle ABC$ and $\triangle AMP$ are right-angled at B and M respectively.*

Prove that: (i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



SOLUTION GIVEN $\triangle ABC$ and $\triangle AMP$ such that $\angle B = 90°$ and $\angle M = 90°$.

TO PROVE (i) $\triangle ABC \sim \triangle AMP$ (ii) $\frac{CA}{PA} = \frac{BC}{MP}$.

PROOF (i) In $\triangle ABC$ and $\triangle AMP$, we have

$\angle ABC = \angle AMP = 90°$

$\angle A = \angle A$ (common)

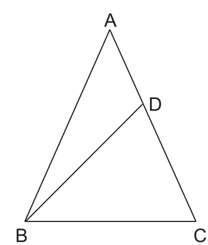
$\therefore$ $\triangle ABC \sim \triangle AMP$ [by AA-similarity].

(ii) Since $\triangle ABC \sim \triangle AMP$, their corresponding sides are proportional.

$\therefore$ $\frac{CA}{PA} = \frac{BC}{MP}$.

EXAMPLE 21 *In a $\triangle ABC$, $AB = AC$ and D is a point on AC such that $BC^2 = AC \times DC$. Prove that $BD = BC$.*

SOLUTION GIVEN A $\triangle ABC$ in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times DC$.



TO PROVE $BD = BC$.

PROOF $BC^2 = AC \times DC$ (given)

$\Rightarrow$ $\frac{BC}{DC} = \frac{AC}{BC}$.

Thus, in $\triangle ABC$ and $\triangle BDC$, we have

$\frac{BC}{DC} = \frac{AC}{BC}$ and $\angle C = \angle C$ (common).

$\therefore$ $\triangle ABC \sim \triangle BDC$ [by SAS-similarity]

$\Rightarrow$ $\frac{AC}{BC} = \frac{AB}{BD}$

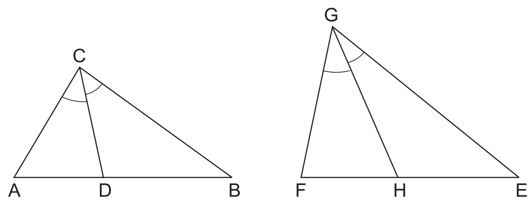
$\Rightarrow$ $\frac{AC}{BC} = \frac{AC}{BD}$ [$\because$ $AB = AC$ (given)]

$\Rightarrow$ $BD = BC$.

Hence, $BD = BC$.

EXAMPLE 22 *In the given figure, CD and GH are respectively the bisectors of $\angle ACB$ and $\angle FGE$ of $\triangle ABC$ and $\triangle FEG$ respectively. If $\triangle ABC \sim \triangle FEG$, prove that:*

(a) $\triangle ADC \sim \triangle FHG$ *(b) $\triangle BCD \sim \triangle EGH$* *(c) $\frac{CD}{GH} = \frac{AC}{FG}$*



SOLUTION $\triangle ABC \sim \triangle FEG$ (given)

$\therefore \quad \angle ACB = \angle FGE$

[corresponding angles of similar triangles are equal]

$\Rightarrow \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\therefore \quad \angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$.

(a) In $\triangle ADC$ and $\triangle FHG$, we have

$\angle DAC = \angle HFG$ [$\because$ $\angle A = \angle F$ since $\triangle ABC \sim \triangle FEG$]

and $\angle ACD = \angle FGH$ [proved above]

$\therefore \quad \triangle ADC \sim \triangle FHG$ [by AA-similarity].

(b) In $\triangle BCD$ and $\triangle EGH$, we have

$\angle DBC = \angle HEG$ [$\because$ $\angle B = \angle E$ since $\triangle ABC \sim \triangle FEG$]

and $\angle DCB = \angle HGE$ [proved above]

$\therefore \quad \triangle BCD \sim \triangle EGH$.

(c) We have

$\triangle ADC \sim \triangle FHG$ [proved above].

And so, $\frac{CD}{GH} = \frac{AC}{FG}$

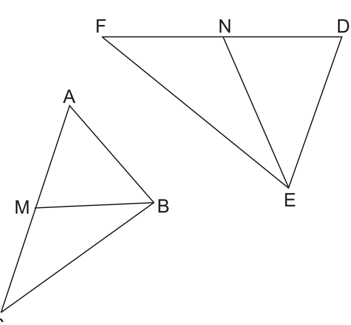
[corresponding sides of similar triangles are proportional].

EXAMPLE 23 *In the given figure, BM and EN are respectively the medians of $\triangle ABC$ and $\triangle DEF$. If $\triangle ABC \sim \triangle DEF$, prove that:*

(a) $\triangle AMB \sim \triangle DNE$

(b) $\triangle CMB \sim \triangle FNE$

(c) $\frac{BM}{EN} = \frac{AC}{DF}$



SOLUTION $\triangle ABC \sim \triangle DEF$ (given)

$\therefore$ $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ … (i)

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$. … (ii)

Since BM and EN are medians, we have

$CA = 2AM = 2CM$ and $FD = 2DN = 2FN$.

$\therefore$ from (ii), we have

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2AM}{2DN} = \frac{2CM}{2FN}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AM}{DN} = \frac{CM}{FN}. \quad \text{… (iii)}$$

(a) In $\triangle AMB$ and $\triangle DNE$, we have

$\angle BAM = \angle EDN$ [$\because$ $\angle A = \angle D$ from (i)]

and $\frac{AB}{DE} = \frac{AM}{DN}$ [from (iii)].

$\therefore$ $\triangle AMB \sim \triangle DNE$ [by SAS-similarity].

(b) In $\triangle CMB$ and $\triangle FNE$, we have

$\angle BCM = \angle EFN$ [$\because$ $\angle C = \angle F$ from (i)]

and $\frac{BC}{EF} = \frac{CM}{FN}$ [from (iii)].

$\therefore$ $\triangle CMB \sim \triangle FNE$ [by SAS-similarity].

(c) As proved above, $\triangle AMB \sim \triangle DNE$ and so

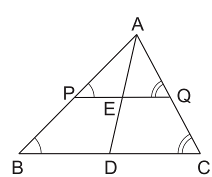
$\frac{AB}{DE} = \frac{BM}{EN}$. … (iv)

From (ii) and (iv), we get

$$\frac{BM}{EN} = \frac{AC}{FD}.$$

EXAMPLE 24 *In a $\triangle ABC$, P and Q are points on AB and AC respectively such that $PQ \parallel BC$. Prove that the median AD, drawn from A to BC, bisects PQ.*

SOLUTION GIVEN A $\triangle ABC$ in which P and Q are points on AB and AC respectively such that $PQ \parallel BC$ and AD is the median, cutting PQ at E.



TO PROVE $PE = EQ$.

PROOF In $\triangle APE$ and $\triangle ABD$, we have

$\angle PAE = \angle BAD$ [common]

$\angle APE = \angle ABD$ [corresponding $\angle$s]

$\therefore$ $\triangle APE \sim \triangle ABD$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$\therefore$ $\frac{AE}{AD} = \frac{PE}{BD}$. … (i)

In $\triangle AEQ$ and $\triangle ADC$, we have

$\angle QAE = \angle CAD$ [common]

$\angle AQE = \angle ACD$ [corresponding angles]

$\therefore$ $\triangle AEQ \sim \triangle ADC$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$\therefore$ $\frac{AE}{AD} = \frac{EQ}{DC}$. … (ii)

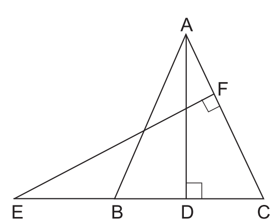
From (i) and (ii), we get $\frac{PE}{BD} = \frac{EQ}{DC}$ $\left[\text{each equal to } \frac{AE}{AD}\right]$.

But, $BD = DC$ [$\because$ AD is the median]

$\therefore$ $PE = EQ$.

EXAMPLE 25 *In the given figure, E is a point on side CB produced of an isosceles $\triangle ABC$ with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.*

SOLUTION GIVEN A $\triangle ABC$ in which $AB = AC$ and $AD \perp BC$. Side CB is produced to E and $EF \perp AC$.



TO PROVE $\triangle ABD \sim \triangle ECF$.

PROOF We know that the angles opposite to equal sides of a triangle are equal.

$\therefore$ $\angle B = \angle C$ [$\because$ $AB = AC$].

Now, in $\triangle ABD$ and $\triangle ECF$, we have

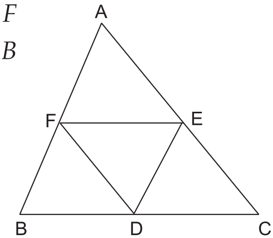
$\angle B = \angle C$ [proved above]

$\angle ADB = \angle EFC = 90°$.

$\therefore$ $\triangle ABD \sim \triangle ECF$ [by AA-similarity].

EXAMPLE 26 *Prove that the line segments joining the midpoints of the sides of a triangle form four triangles, each of which is similar to the original triangle.*

SOLUTION GIVEN A $\triangle ABC$ in which D, E, F are the midpoints of BC, CA and AB respectively.



TO PROVE $\triangle AFE \sim \triangle ABC$,
$\triangle FBD \sim \triangle ABC$,
$\triangle EDC \sim \triangle ABC$.
and $\triangle DEF \sim \triangle ABC$.

PROOF We shall first show that $\triangle AFE \sim \triangle ABC$.

Since F and E are the midpoints of AB and AC respectively, so by the midpoint theorem, we have $FE \parallel BC$.

$\therefore$ $\angle AFE = \angle B$ [corresponding ∠s]

Now, in $\triangle AFE$ and $\triangle ABC$, we have

$\angle AFE = \angle B$ [corresponding ∠s]

and $\angle A = \angle A$ (common).

$\therefore$ $\triangle AFE \sim \triangle ABC$ [by AA-similarity].

Similarly, $\triangle FBD \sim \triangle ABC$ and $\triangle EDC \sim \triangle ABC$.

Now, we shall show that $\triangle DEF \sim \triangle ABC$.

In the same manner as above, we can prove that

$ED \parallel AF$ and $DF \parallel EA$.

$\therefore$ $AFDE$ is a ||gm.

$\therefore$ $\angle EDF = \angle A$ [opposite angles of a ||gm].

Similarly, $BDEF$ is a ||gm.

$\therefore$ $\angle DEF = \angle B$ [opposite angles of a ||gm].

Thus, in $\triangle DEF$ and $\triangle ABC$, we have

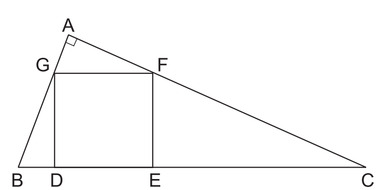
$\angle EDF = \angle A$ and $\angle DEF = \angle B$.

$\therefore$ $\triangle DEF \sim \triangle ABC$ [by AA-similarity].

Hence, the result follows.

EXAMPLE 27 *In the given figure, DEFG is a square and* $\angle BAC = 90°$. *Prove that*

(i) $\triangle AGF \sim \triangle DBG$
(ii) $\triangle AGF \sim \triangle EFC$
(iii) $\triangle DBG \sim \triangle EFC$
(iv) $DE^2 = BD \times EC$



[CBSE 2009]

SOLUTION GIVEN A $\triangle ABC$ in which $\angle BAC = 90^\circ$ and $DEFG$ is a square.

TO PROVE (i) $\triangle AGF \sim \triangle DBG$ (ii) $\triangle AGF \sim \triangle EFC$

(iii) $\triangle DBG \sim \triangle EFC$ (iv) $DE^2 = BD \times EC$

PROOF (i) In $\triangle AGF$ and $\triangle DBG$, we have

$\angle GAF = \angle BDG = 90^\circ$

$\angle AGF = \angle DBG$ [corresponding ∠s]

[∵ $GF \parallel BC$ and AB is the transversal]

$\therefore \quad \triangle AGF \sim \triangle DBG$.

(ii) In $\triangle AGF$ and $\triangle EFC$, we have

$\angle FAG = \angle CEF = 90^\circ$

$\angle GFA = \angle FCE$ [corresponding ∠s]

[∵ $GF \parallel BC$ and AC is the transversal]

$\therefore \quad \triangle AGF \sim \triangle EFC$.

(iii) $\triangle DBG \sim \triangle AGF$ and $\triangle AGF \sim \triangle EFC$

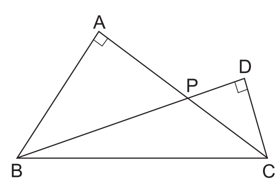
$\Rightarrow \quad \triangle DBG \sim \triangle EFC$.

(iv) $\triangle DBG \sim \triangle EFC \Rightarrow \dfrac{BD}{FE} = \dfrac{DG}{EC}$

$\Rightarrow \dfrac{BD}{DE} = \dfrac{DE}{EC}$ [∵ $DG = DE$ and $FE = DE$].

Hence, $DE^2 = BD \times EC$.

EXAMPLE 28 *Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that* $AP \times PC = BP \times PD$. [CBSE 2000C]



SOLUTION GIVEN Right triangles $\triangle ABC$ and $\triangle DBC$ are drawn on the same hypotenuse BC and on the same side of BC. Also, AC and BD intersect at P.

TO PROVE $AP \times PC = BP \times PD$.

PROOF In $\triangle BAP$ and $\triangle CDP$, we have

$\angle BAP = \angle CDP = 90^\circ$

$\angle BPA = \angle CPD$ (ver. opp. ∠s)

$\therefore \quad \triangle BAP \sim \triangle CDP$ [by AA-similarity]

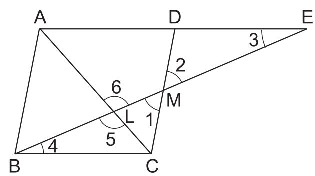
$\therefore \quad \dfrac{AP}{DP} = \dfrac{BP}{CP}$

$\Rightarrow \quad AP \times CP = BP \times DP \;\Rightarrow\; AP \times PC = BP \times PD.$

Hence, $AP \times PC = BP \times PD$.

EXAMPLE 29 *Through the midpoint M of the side CD of a parallelogram ABCD, the line BM is drawn, intersecting AC in L and AD produced in E. Prove that EL = 2BL.* [CBSE 2006C, '08, '09]

SOLUTION A ||gm $ABCD$ and M is the midpoint of CD. Line BM is drawn, intersecting AC in L and AD produced in E.



TO PROVE $EL = 2BL$.

PROOF In $\triangle BMC$ and $\triangle EMD$, we have

$\angle 1 = \angle 2$ (vert. opp. $\angle$s)

$MC = MD$ [M is the midpoint of CD]

$\angle BCM = \angle EDM$ [alternate interior $\angle$s]

$\therefore \quad \triangle BMC \cong \triangle EMD$

$\therefore \quad BC = DE$.

But, $BC = AD$ [opposite sides of a ||gm]

$\therefore \quad BC = AD = DE \Rightarrow AE = (AD + DE) = 2BC.$ … (i)

Now, in $\triangle AEL$ and $\triangle CBL$, we have

$\angle 6 = \angle 5$ (vert. opp. $\angle$s)

$\angle 3 = \angle 4$ [alternate interior $\angle$s]

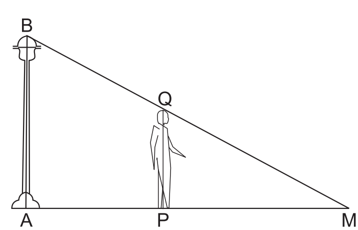
$\therefore \quad \triangle AEL \sim \triangle CBL$ [AA-similarity]

$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{BC} = \frac{2BC}{BC} = 2$ [using (i)]

$\Rightarrow \quad EL = 2BL$.

Hence, $EL = 2BL$.

EXAMPLE 30 *A lamp is* 3.3 m *above the ground. A boy* 110 cm *tall walks away from the base of this lamp post at a speed of* 0.8 m/s. *Find the length of the shadow of the boy after* 4 *seconds.*



SOLUTION Let AB be the lamp post and PQ be the boy, where P is the position of the boy after 4 seconds.

AP = distance moved in 4 s at 0.8 m/s = (4×0.8) m = 3.2 m.

PM is the length of shadow of the boy.

Let $PM = x$ m.

In $\triangle AMB$ and $\triangle PMQ$, we have

$\angle MAB = \angle MPQ = 90^\circ$

[$\because$ both the lamp post and the boy stand vertically erect]

$\angle AMB = \angle PMQ$ (common)

$\therefore \quad \triangle AMB \sim \triangle PMQ$ [by AA-similarity].

And so, $\frac{AM}{PM} = \frac{AB}{PQ}$

[corres. sides of similar triangles are proportional]

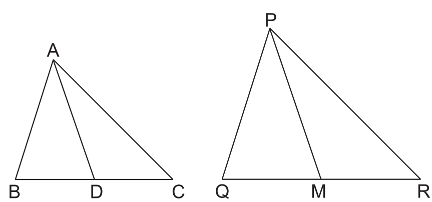
$\Rightarrow \quad \frac{AP + PM}{PM} = \frac{AB}{PQ} \Rightarrow \frac{3.2 + x}{x} = \frac{3.3}{1.1}$

[$\because$ $AB = 3.3$ m, $PQ = 110$ cm $= 1.1$ m]

$\Rightarrow \quad 3.2 + x = 3x \Rightarrow 2x = 3.2 \Rightarrow x = 1.6.$

$\therefore$ the length of the shadow of the boy after 4 seconds is 1.6 m.

EXAMPLE 31 *Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of another triangle PQR, as shown in the figure. Prove that* $\triangle ABC \sim \triangle PQR$.



SOLUTION We have

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}. \quad \ldots \text{(i)}$$

In $\triangle ABD$ and $\triangle PQM$, we have

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM} \quad \text{[from (i)]}$$

$\therefore \quad \triangle ABD \sim \triangle PQM$ [by SSS-similarity].

And so, $\angle B = \angle Q$ [corres. angles of similar triangles are equal].

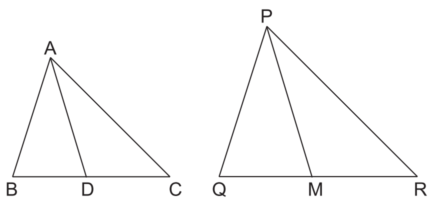
Now, in $\triangle ABC$ and $\triangle PQR$, we have

$\angle B = \angle Q$ [proved above]

and $\dfrac{AB}{PQ} = \dfrac{BD}{QM}$ [from (i)].

$\therefore$ $\triangle ABC \sim \triangle PQR$ [by SAS-similarity].

EXAMPLE 32 *Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Prove that* $\triangle ABC \sim \triangle PQR$.

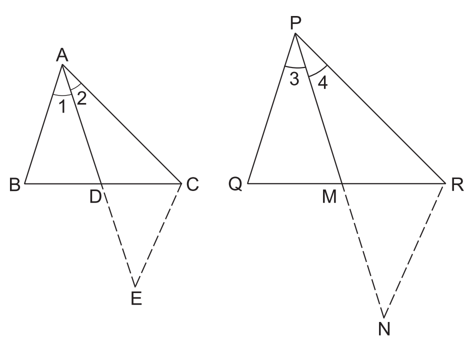


SOLUTION GIVEN AD and PM are medians of $\triangle ABC$ and $\triangle PQR$ respectively such that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

TO PROVE $\triangle ABC \sim \triangle PQR$.

CONSTRUCTION Produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$.
Join EC and NR.



PROOF In $\triangle ABD$ and $\triangle ECD$, we have

$BD = CD$ [$\because$ D is the midpoint of BC]

$AD = ED$ [by construction]

$\angle BDA = \angle CDE$ [vertically opposite angles]

$\therefore$ $\triangle ABD \cong \triangle ECD$ [by SAS-congruency].

And so, $AB = EC$. … (i) [cpct]

Similarly, $\triangle PQM \cong \triangle NRM$

and so, $PQ = NR$. … (ii) [cpct]

Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ (given)

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AD}{PM}$ [using (i) and (ii)]

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{2AD}{2PM} = \frac{AE}{PN}$ [$\because$ $2AD = AE$, $2PM = PN$]

$\Rightarrow \triangle ACE \sim \triangle PNR$ [SSS-similarity].

$\therefore \angle 2 = \angle 4$

[corresponding angles of similar triangles are equal].

Similarly, $\angle 1 = \angle 3$ [it can be proved by joining BE and QN and showing $\triangle ABE \sim \triangle PQN$].

$\therefore \angle 1 + \angle 2 = \angle 3 + \angle 4$, i.e., $\angle BAC = \angle QPR$. … (iii)

Now, in $\triangle ABC$ and $\triangle PQR$, we have

$\frac{AB}{PQ} = \frac{AC}{PR}$ [given]

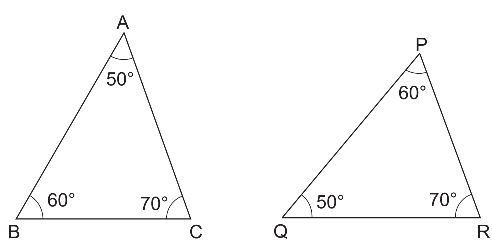
$\angle BAC = \angle QPR$ [from (iii)]

$\therefore \triangle ABC \sim \triangle PQR$ [by SAS-similarity].

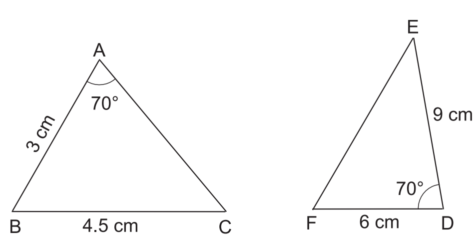
EXERCISE 7B

1. In each of the given pairs of triangles, find which pair of triangles are similar. State the similarity criterion and write the similarity relation in symbolic form.

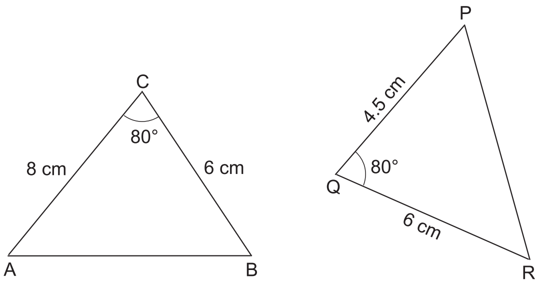
(i)



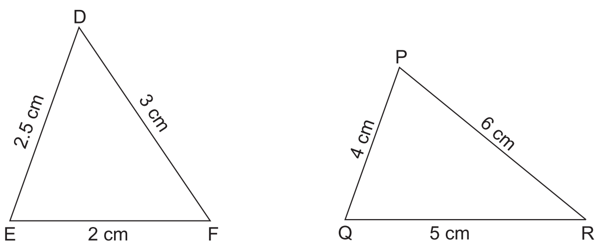
(ii)



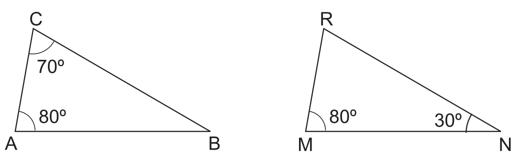
(iii)



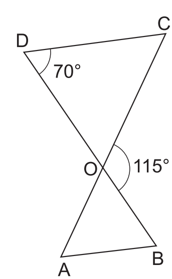
(iv)



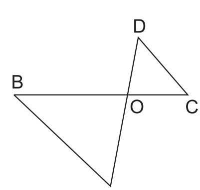
(v)



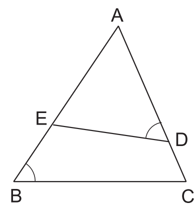
2. In the given figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 115^\circ$ and $\angle CDO = 70^\circ$.
   Find (i) $\angle DOC$ (ii) $\angle DCO$ (iii) $\angle OAB$ (iv) $\angle OBA$.



3. In the given figure, $\triangle OAB \sim \triangle OCD$. If $AB = 8$ cm, $BO = 6.4$ cm, $OC = 3.5$ cm and $CD = 5$ cm, find (i) OA (ii) DO.



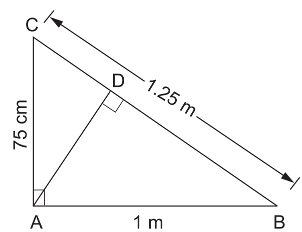
4. In the given figure, if $\angle ADE = \angle B$, show that $\triangle ADE \sim \triangle ABC$. If $AD = 3.8$ cm, $AE = 3.6$ cm, $BE = 2.1$ cm and $BC = 4.2$ cm, find DE.



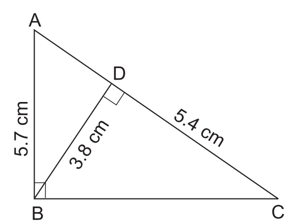
5. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm respectively. If $PQ = 12$ cm, find AB. [CBSE 2001]

6. The corresponding sides of two similar triangles ABC and DEF are $BC = 9.1$ cm and $EF = 6.5$ cm. If the perimeter of $\triangle DEF$ is 25 cm, find the perimeter of $\triangle ABC$.

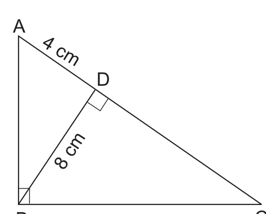
7. In the given figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. Show that $\triangle BDA \sim \triangle BAC$. If $AC = 75$ cm, $AB = 1$ m and $BC = 1.25$ m, find AD.



8. In the given figure, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AB = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, find BC.

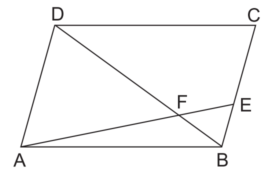


9. In the given figure, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm, $AD = 4$ cm, find CD.



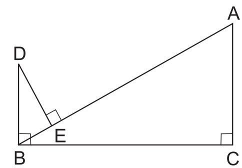
10. P and Q are points on the sides AB and AC respectively of a $\triangle ABC$. If $AP = 2$ cm, $PB = 4$ cm, $AQ = 3$ cm and $QC = 6$ cm, show that $BC = 3PQ$.

11. $ABCD$ is a parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that $AF \times FB = EF \times FD$.



12. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

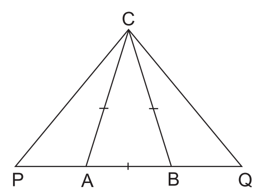
Prove that $\dfrac{BE}{DE} = \dfrac{AC}{BC}$. [CBSE 2008]



13. A vertical pole of length 7.5 m casts a shadow 5 m long on the ground and at the same time a tower casts a shadow 24 m long. Find the height of the tower.

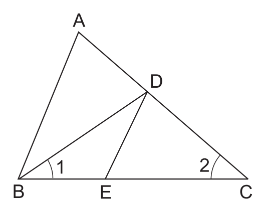
14. In an isosceles $\triangle ABC$, the base AB is produced both ways in P and Q such that $AP \times BQ = AC^2$.

Prove that $\triangle ACP \sim \triangle BCQ$.

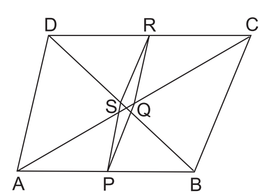


15. In the given figure, $\angle 1 = \angle 2$ and $\dfrac{AC}{BD} = \dfrac{CB}{CE}$.

Prove that $\triangle ACB \sim \triangle DCE$.

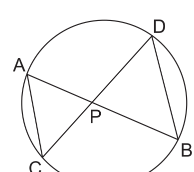


16. $ABCD$ is a quadrilateral in which $AD = BC$. If P, Q, R, S be the midpoints of AB, AC, CD and BD respectively, show that $PQRS$ is a rhombus.



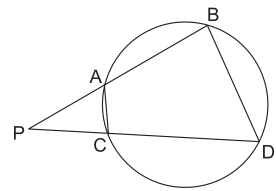
17. In a circle, two chords AB and CD intersect at a point P inside the circle. Prove that

(a) $\triangle PAC \sim \triangle PDB$ (b) $PA \cdot PB = PC \cdot PD$.



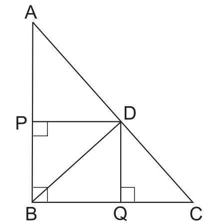
18. Two chords AB and CD of a circle intersect at a point P outside the circle. Prove that

(a) $\triangle PAC \sim \triangle PDB$ (b) $PA \cdot PB = PC \cdot PD$.



19. In a right triangle ABC, right-angled at B, D is a point on hypotenuse such that $BD \perp AC$. If $DP \perp AB$ and $DQ \perp BC$ then prove that

(a) $DQ^2 = DP \cdot QC$ (b) $DP^2 = DQ \cdot AP$.



ANSWERS (EXERCISE 7B)

1. (i) $\triangle ABC \sim \triangle QPR$ (AAA-similarity)

(ii) not similar

(iii) $\triangle CAB \sim \triangle QRP$ (SAS-similarity)

(iv) $\triangle FED \sim \triangle PQR$ (SSS-similarity)

(v) $\triangle ABC \sim \triangle MNR$ (AA-similarity)

2. (i) $\angle DOC = 65^\circ$ (ii) $\angle DCO = 45^\circ$ (iii) $\angle OAB = 45^\circ$ (iv) $\angle OBA = 70^\circ$

3. (i) $OA = 5.6$ cm (ii) $DO = 4$ cm **4.** 2.8 cm **5.** 16 cm

6. 35 cm **7.** $AD = 60$ cm **8.** $BC = 8.1$ cm **9.** $CD = 16$ cm

13. 36 m

HINTS TO SOME SELECTED QUESTIONS

1. (i) $\angle A = \angle Q$, $\angle B = \angle P$ and $\angle C = \angle R$.

$\therefore$ $\triangle ABC \sim \triangle QPR$ (AAA-similarity).

(ii) SAS-similarity is not satisfied as included angles are not equal.

(iii) $\triangle CAB \sim \triangle QRP$ (SAS-similarity), as $\frac{CA}{QR} = \frac{CB}{QP}$ and $\angle C = \angle Q$.

(iv) $FE = 2$ cm, $FD = 3$ cm, $ED = 2.5$ cm; $PQ = 4$ cm, $PR = 6$ cm, $QR = 5$ cm.

$\therefore$ $\triangle FED \sim \triangle PQR$ (SSS-similarity).

(v) $\angle B = 180^\circ - (\angle A + \angle C) = 180^\circ - (80^\circ + 70^\circ) = 30^\circ$.

$\therefore$ $\angle A = \angle M$ and $\angle B = \angle N$, and so $\triangle ABC \sim \triangle MNR$ (AA-similarity).

2. $\angle DOC = (180^\circ - 115^\circ) = 65^\circ$, $\angle DCO = 180^\circ - (70^\circ + 65^\circ) = 45^\circ$.

$\triangle ODC \sim \triangle OBA \Rightarrow \angle OAB = \angle DCO = 45^\circ$, $\angle OBA = \angle ODC = 70^\circ$.

3. $\triangle OAB \sim \triangle OCD \Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{BO}{DO} \Rightarrow \frac{OA}{3.5} = \frac{8}{5} = \frac{6.4}{DO}$.

   Find OA and DO.

4. $\angle ADE = \angle B, \angle A = \angle A$.

   $\triangle ADE \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{3.8}{(3.6+2.1)} = \frac{x}{4.2}$.

   Find x.

5. $\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{AB}{PQ}$.

7. $\triangle BDA \sim \triangle BAC \Rightarrow \frac{AD}{AC} = \frac{BA}{BC}$.

8. $\triangle ABC \sim \triangle BDC \Rightarrow \frac{AB}{BD} = \frac{BC}{DC}$.

9. In $\triangle DBA$ and $\triangle DCB$, we have

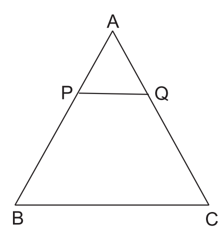
   $\angle BDA = \angle CDB$

   and $\angle DBA = \angle DCB$ [each $= 90^\circ - \angle A$].

   $\therefore$ $\triangle DBA \sim \triangle DCB$.

   $\therefore$ $\frac{BD}{CD} = \frac{AD}{BD} \Rightarrow CD = \frac{BD^2}{AD}$.

10. $\frac{AP}{AB} = \frac{2}{6} = \frac{1}{3}$ and $\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$.



   In $\triangle APQ$ and $\triangle ABC$, we have

   $\angle A = \angle A$ and $\frac{AP}{AB} = \frac{AQ}{AC}$.

   $\therefore$ $\triangle APQ - \triangle ABC$ [by SAS-similarity].

   $\frac{PQ}{BC} = \frac{AP}{AB} = \frac{1}{3} \Rightarrow BC = 3PQ$.

11. $\triangle AFD \sim \triangle EFB$, as $\angle AFD = \angle EFB$ (vert. opp. $\angle$s) and $\angle DAF = \angle BEF$ (alt. $\angle$s).

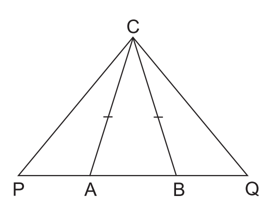
   $\therefore$ $\frac{AF}{EF} = \frac{FD}{FB}$.

12. $\left.\begin{array}{l}\angle BED = \angle ACB = 90^\circ \\ \angle EBD = \angle CAB = (90^\circ - \angle B)\end{array}\right\}; \therefore \triangle BED \sim \triangle ACB.$

14. $CA = CB \Rightarrow \angle CAB = \angle CBA$

   $\Rightarrow 180^\circ - \angle CAB = 180^\circ - \angle CBA$

   $\Rightarrow \angle CAP = \angle CBQ$.



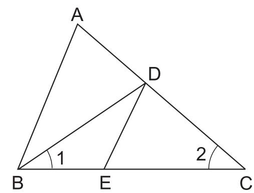
   Now, $AP \times BQ = AC^2 \Rightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

   $\Rightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ [$\because$ $AC = BC$].

   Thus, $\angle CAP = \angle CBQ$ and $\frac{AP}{AC} = \frac{BC}{BQ}$.

   $\therefore$ $\triangle ACP \sim \triangle BCQ$.

15. $\frac{AC}{BD} = \frac{CB}{CE} \Rightarrow \frac{AC}{CB} = \frac{BD}{CE}$.



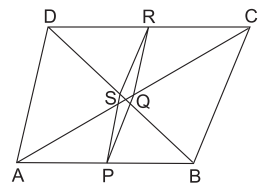
Also, $\angle 2 = \angle 1 \Rightarrow BD = DC$.

Thus, $\frac{AC}{CB} = \frac{DC}{CE} \Rightarrow \frac{AC}{DC} = \frac{CB}{CE}$

and $\angle ACB = \angle DCE = \angle C$.

$\therefore$ $\triangle ACB \sim \triangle DCE$ [SAS-similarity].

16. In $\triangle ABC$, P and Q are midpoints of AB and AC respectively.



$\therefore$ $PQ \parallel BC$ [by midpoint theorem].

And so, $\triangle APQ \sim \triangle ABC$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} \Rightarrow \frac{PQ}{BC} = \frac{1}{2} \Rightarrow PQ = \frac{1}{2}BC = \frac{1}{2}DA$$

$$\left[\because AP = \frac{1}{2}AB \text{ and } BC = DA\right].$$

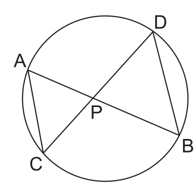
In $\triangle CDA$, $RQ \parallel DA$ and $RQ = \frac{1}{2}DA$.

In $\triangle BDA$, $SP \parallel DA$ and $SP = \frac{1}{2}DA$.

In $\triangle CDB$, $SR \parallel BC$ and $SR = \frac{1}{2}BC = \frac{1}{2}DA$.

$\therefore$ $SP \parallel RQ$ and $PQ \parallel SR$ and $PQ = RQ = SP = SR$.

17. In $\triangle PAC$ and $\triangle PDB$, we have



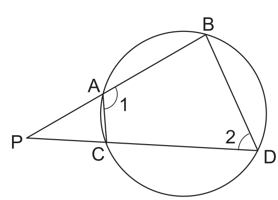
$\angle APC = \angle DPB$ [vertically opposite angles]

$\angle PAC = \angle PDB$ [angles in the same segment]

$\therefore$ $\triangle PAC \sim \triangle PDB$ [by AA-similarity].

And so, $\frac{PA}{PD} = \frac{PC}{PB} \Rightarrow PA \cdot PB = PC \cdot PD$.

18. Clearly, $ABDC$ is a cyclic quadrilateral.



$\therefore$ $\angle 1 + \angle 2 = 180^\circ$. … (i)

In $\triangle PAC$ and $\triangle PDB$, we have:

$\angle APC = \angle DPB$ [common]

$\angle PAC = \angle PDB$ [each equal to $(180^\circ - \angle 1)$]

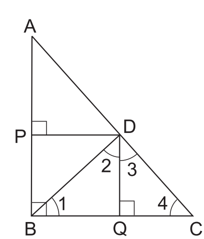
[**Note** $\angle PAC + \angle 1 = 180^\circ$ (linear pair)

and $\angle PDB + \angle 1 = 180^\circ$. {using (i)}

$\therefore$ $\triangle PAC \sim \triangle PDB$ [by AA-similarity]

And so, $\frac{PA}{PD} = \frac{PC}{PB} \Rightarrow PA \cdot PB = PC \cdot PD$.

19. (a) $AB \perp BC$ and $DP \perp AB \Rightarrow DP \parallel BC \Rightarrow DP \parallel BQ$



$BC \perp AB$ and $DQ \perp BC \Rightarrow DQ \parallel AB \Rightarrow DQ \parallel PB$

$\therefore$ $BQDP$ is a rectangle.

And so, $BQ = DP$ and $BP = DQ$

In rt. $\triangle BQD$: $\angle 1 + \angle 2 = 90^\circ$ … (i)

In rt. $\triangle DQC$: $\angle 3 = \angle 4 = 90^\circ$ ... (ii)

In rt. $\triangle BDC$: $\angle 2 + \angle 3 = 90^\circ$ ... (iii) $[\because BDC = 90^\circ]$

From (i) and (iii), we get $\angle 1 = \angle 3$.

From (ii) and (iii), we get $\angle 2 = \angle 4$.

$\therefore$ $\triangle BQD \sim \triangle DQC$ [by AA-similarity]

And so, $\frac{BQ}{DQ} = \frac{DQ}{QC} \Rightarrow DQ^2 = BQ \cdot QC \Rightarrow DQ^2 = DP \cdot QC$ $[\because BQ = DP]$

(b) By proving $\triangle PDA \sim \triangle PBD$ (in a similar way), we get

$\frac{PD}{PB} = \frac{AP}{DP} \Rightarrow DP^2 = BP \cdot AP \Rightarrow DP^2 = DQ \cdot AP$ $[\because BP = DQ]$.

RATIO OF THE AREAS OF TWO SIMILAR TRIANGLES

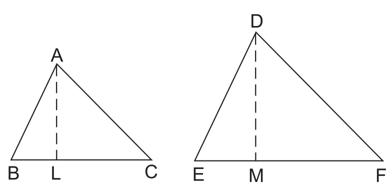
THEOREM 1 *The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.*

[CBSE 2002, '02C, '04, '05, '05C, '06, '06C, '07, '08, '08C, '09C, '10]

GIVEN $\triangle ABC \sim \triangle DEF$.

TO PROVE $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$.

CONSTRUCTION Draw $AL \perp BC$ and $DM \perp EF$.



PROOF Since $\triangle ABC \sim \triangle DEF$, it follows that they are equiangular and their sides are proportional.

$\therefore$ $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. ... (i)

Now, $\text{ar}(\triangle ABC) = \left(\frac{1}{2} \times BC \times AL\right)$

and $\text{ar}(\triangle DEF) = \left(\frac{1}{2} \times EF \times DM\right)$.

$$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM} = \frac{BC}{EF} \times \frac{AL}{DM}. \quad \text{... (ii)}$$

Also, $\frac{AL}{DM} = \frac{BC}{EF}$ ... (iii) [∵ in similar triangles, the ratio of the corres. sides is the same as the ratio of corresponding altitudes]

Using (iii) in (ii), we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{BC}{EF} \times \frac{BC}{EF}\right) = \frac{BC^2}{EF^2}.$$

Similarly, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2}$ and $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AC^2}{DF^2}$.

Hence, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$.

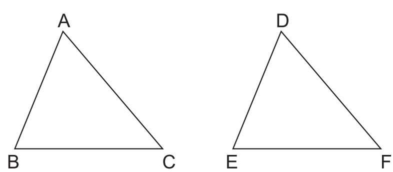
THEOREM 2 *If the areas of two similar triangles are equal then prove that the triangles are congruent.* [CBSE 2010]

GIVEN $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = \text{ar}(\triangle DEF)$.

TO PROVE $\triangle ABC \cong \triangle DEF$.

PROOF $\triangle ABC \sim \triangle DEF$ (given)

$$\Rightarrow \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}. \quad \text{... (i)}$$



Now, $\text{ar}(\triangle ABC) = \text{ar}(\triangle DEF)$ [given]

$$\Rightarrow \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = 1. \quad \text{... (ii)}$$

From (i) and (ii), we get

$$\frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = 1$$

$\Rightarrow$ $AB^2 = DE^2$, $AC^2 = DF^2$ and $BC^2 = EF^2$

$\Rightarrow$ $AB = DE$, $AC = DF$ and $BC = EF$

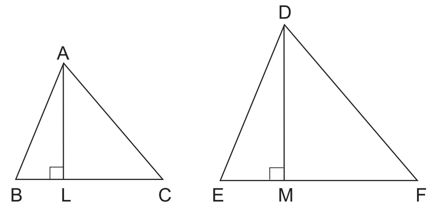
$\Rightarrow$ $\triangle ABC \cong \triangle DEF$ [by SSS-congruency].

THEOREM 3 *Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.*

GIVEN $\triangle ABC \sim \triangle DEF$, $AL \perp BC$ and $DM \perp EF$.

TO PROVE $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AL^2}{DM^2}$.

PROOF We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.



$\therefore \quad \dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AB^2}{DE^2}$. ... (i)

Now, in $\triangle ALB$ and $\triangle DME$, we have

$\angle ALB = \angle DME = 90^\circ$ and $\angle B = \angle E$ $\quad [\because \triangle ABC \sim \triangle DEF]$.

$\therefore \quad \triangle ALB \sim \triangle DME$ [by AA-similarity]

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AL}{DM}$

$\Rightarrow \quad \dfrac{AB^2}{DE^2} = \dfrac{AL^2}{DM^2}$. ... (ii)

From (i) and (ii), we get

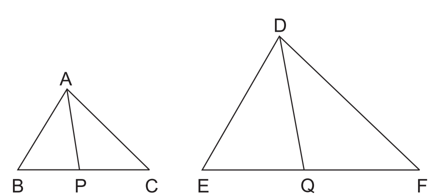
$$\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AL^2}{DM^2}.$$

THEOREM 4 *Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians.*

GIVEN $\triangle ABC \sim \triangle DEF$, AP and DQ are the medians of $\triangle ABC$ and $\triangle DEF$ respectively.

TO PROVE $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AP^2}{DQ^2}$.

PROOF We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



$\therefore \quad \dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AB^2}{DE^2}$. ... (i)

Now, $\triangle ABC \sim \triangle DEF$

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{2BP}{2EQ} = \dfrac{BP}{EQ}$

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{BP}{EQ}$ and $\angle B = \angle E \qquad [\because \ \triangle ABC \sim \triangle DEF]$

$\Rightarrow \quad \triangle APB \sim \triangle DQE$ [by SAS-similarity]

$\Rightarrow \quad \dfrac{AB}{DE} = \dfrac{AP}{DQ}$

$\Rightarrow \quad \dfrac{AB^2}{DE^2} = \dfrac{AP^2}{DQ^2}$. ... (ii)

From (i) and (ii), we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AP^2}{DQ^2}.$$

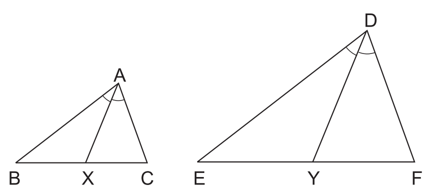
THEOREM 5 *Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding angle-bisector segments.*

GIVEN $\triangle ABC \sim \triangle DEF$ in which AX and DY are the bisectors of $\angle A$ and $\angle D$ respectively.

TO PROVE $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AX^2}{DY^2}$.

PROOF We know that ratio of the areas of two similar triangles is equal to ratio of the squares of their corresponding sides.

$\therefore \quad \dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{AB^2}{DE^2}$. ... (i)



$\triangle ABC \sim \triangle DEF$

$\Rightarrow \quad \angle A = \angle D$

$\Rightarrow \quad \dfrac{1}{2}\angle A = \dfrac{1}{2}\angle D$

$\Rightarrow \quad \angle BAX = \angle EDY$.

Now, in $\triangle ABX$ and $\triangle DEY$, we have

$\angle BAX = \angle EDY$ and $\angle B = \angle E \quad [\because \ \triangle ABC \sim \triangle DEF]$.

$\therefore \quad \triangle ABX \sim \triangle DEY$ [by AA-similarity]

$\dfrac{AB}{DE} = \dfrac{AX}{DY}$

$\Rightarrow \quad \dfrac{AB^2}{DE^2} = \dfrac{AX^2}{DY^2}$. ... (ii)

From (i) and (ii), we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AX^2}{DY^2}.$$

SOLVED EXAMPLES

EXAMPLE 1 *If the areas of two similar triangles are in the ratio* 25 : 64, *find the ratio of their corresponding sides.* [CBSE 2009]

SOLUTION Let $\triangle ABC$ and $\triangle DEF$ be similar. Then,

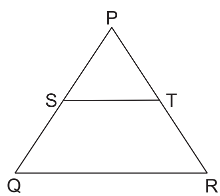
$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\Rightarrow \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \frac{25}{64} = \left(\frac{5}{8}\right)^2$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{8}.$$

Hence, the ratio of their corresponding sides is 5 : 8.

EXAMPLE 2 *In the adjoining figure, S and T are points on the sides PQ and PR respectively of* $\triangle PQR$ *such that* $PT = 2$ cm, $TR = 4$ cm *and ST is parallel to QR. Find the ratio of the areas of* $\triangle PST$ *and* $\triangle PQR$. [CBSE 2010]



SOLUTION $ST \parallel QR$ (given)

$\therefore$ $\angle PST = \angle PQR$ [corresponding angles]

and $\angle PTS = \angle PRQ$ [corresponding angles].

And so, $\triangle PST \sim \triangle PQR$ [by AA-similarity].

$$\therefore \frac{\text{ar}(\triangle PST)}{\text{ar}(\triangle PQR)} = \frac{PT^2}{PR^2} = \frac{PT^2}{(PT+TR)^2}$$

$$= \frac{2^2}{(2+4)^2} = \frac{4}{36} = \frac{1}{9}.$$

EXAMPLE 3 *The areas of two similar triangles* $\triangle ABC$ *and* $\triangle PQR$ *are* 25 cm^2 *and* 49 cm^2 *respectively. If* $QR = 9.8$ cm, *find BC.* [CBSE 2006]

SOLUTION We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \left(\frac{BC}{QR}\right)^2 = \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{25}{49} = \left(\frac{5}{7}\right)^2$$

$$\Rightarrow \frac{BC}{QR} = \frac{5}{7} \Rightarrow \frac{BC}{9.8} = \frac{5}{7}$$

$\Rightarrow \quad BC = \frac{5 \times 9.8}{7}$ cm $= 5 \times 1.4$ cm $= 7$ cm.

Hence, $BC = 7$ cm.

EXAMPLE 4 *The areas of two similar triangles are* 81 cm^2 *and* 49 cm^2 *respectively. If the altitude of the bigger triangle is* 4.5 cm, *find the corresponding altitude of the smaller triangle.* [CBSE 2002]

SOLUTION Let the given triangles be $\triangle ABC$ and $\triangle DEF$ such that $\text{ar}(\triangle ABC) = 81\text{ cm}^2$ and $\text{ar}(\triangle DEF) = 49\text{ cm}^2$. Let AL and DM be the corresponding altitudes of $\triangle ABC$ and $\triangle DEF$ respectively. Then, $AL = 4.5$ cm.

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

$$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AL^2}{DM^2}$$

$$\Rightarrow \quad \left(\frac{AL}{DM}\right)^2 = \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{81}{49} = \left(\frac{9}{7}\right)^2$$

$$\Rightarrow \quad \frac{AL}{DM} = \frac{9}{7} \Rightarrow \frac{4.5}{DM} = \frac{9}{7}$$

$$\Rightarrow \quad DM = \frac{4.5 \times 7}{9}\text{ cm} = \frac{7}{2}\text{ cm} = 3.5\text{ cm}.$$

Hence, the altitude of the smaller triangle is 3.5 cm.

EXAMPLE 5 *The areas of two similar triangles are* 121 cm^2 *and* 64 cm^2 *respectively. If the median of first triangle is* 12.1 cm, *find the corresponding median of the other.* [CBSE 2001]

SOLUTION Let $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = 121\text{ cm}^2$ and $\text{ar}(\triangle DEF) = 64\text{ cm}^2$.

Let AP and DQ be the corresponding medians of $\triangle ABC$ and $\triangle DEF$ respectively such that $AP = 12.1$ cm.

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians.

$$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \quad \left(\frac{AP}{DQ}\right)^2 = \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{121}{64} = \left(\frac{11}{8}\right)^2$$

$$\Rightarrow \quad \frac{AP}{DQ} = \frac{11}{8} \Rightarrow \frac{12.1}{DQ} = \frac{11}{8} \qquad [\because AP = 12.1\text{ cm}]$$

$$\Rightarrow \quad DQ = \frac{(12.1 \times 8)}{11} \text{ cm} = (1.1 \times 8) \text{ cm} = 8.8 \text{ cm}.$$

Hence, the corresponding median is 8.8 cm.

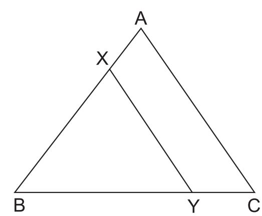
EXAMPLE 6 *$\triangle ABC \sim \triangle DEF$ in which AX and DY are the bisectors of $\angle A$ and $\angle D$ respectively. If $AX = 6.5$ cm and $DY = 5.2$ cm, find the ratio of the areas of $\triangle ABC$ and $\triangle DEF$.*

SOLUTION We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding angle-bisector segments.

$$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AX^2}{DY^2} = \frac{(6.5)^2}{(5.2)^2} = \left(\frac{6.5}{5.2}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}.$$

$$\therefore \quad \text{ar}(\triangle ABC) : \text{ar}(\triangle DEF) = 25 : 16.$$

EXAMPLE 7 *In the given figure, the line segment XY is parallel to side AC of $\triangle ABC$ and it divides the triangle into two parts of equal area. Prove that $AX : AB = (2 - \sqrt{2}) : 2$.*



SOLUTION Since $XY \parallel AC$, we have

$\angle A = \angle BXY$ and $\angle C = \angle BYX$ [corres. $\angle$s].

$\therefore \quad \triangle ABC \sim \triangle XBY$

$$\Rightarrow \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle XBY)} = \frac{AB^2}{XB^2}. \qquad \ldots \text{(i)}$$

But, $\text{ar}(\triangle ABC) = 2 \times \text{ar}(\triangle XBY)$ [given]

$$\Rightarrow \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle XBY)} = 2. \qquad \ldots \text{(ii)}$$

From (i) and (ii), we get

$$\frac{AB^2}{XB^2} = 2 \Rightarrow \left(\frac{AB}{XB}\right)^2 = 2$$

$$\Rightarrow \quad \frac{AB}{XB} = \sqrt{2} \Rightarrow AB = \sqrt{2}(XB)$$

$$\Rightarrow \quad AB = \sqrt{2}(AB - AX)$$

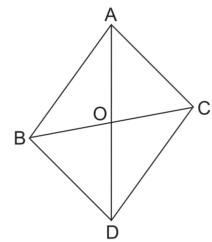
$$\Rightarrow \quad \sqrt{2}AX = (\sqrt{2} - 1)AB$$

$$\Rightarrow \quad \frac{AX}{AB} = \frac{(\sqrt{2} - 1)}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(2 - \sqrt{2})}{2}.$$

Hence, $AX : AB = (2 - \sqrt{2}) : 2$.

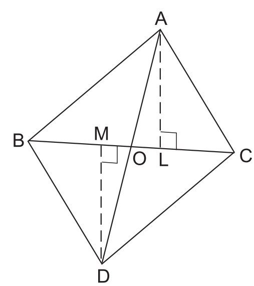
EXAMPLE 8 *In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD intersects BC at O, prove that*

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}\cdot$$ [CBSE 2000, '05, '07C]



SOLUTION GIVEN $\triangle ABC$ and $\triangle DBC$ are on the same base BC and AD intersects BC at O.

TO PROVE $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \dfrac{AO}{DO}$.



CONSTRUCTION Draw $AL \perp BC$ and $DM \perp BC$.

PROOF In $\triangle ALO$ and $\triangle DMO$, we have

$$\angle ALO = \angle DMO = 90^\circ$$

and $\angle AOL = \angle DOM$ (vert. opp. $\angle$s).

$\therefore$ $\triangle ALO \sim \triangle DMO$ [by AA-similarity]

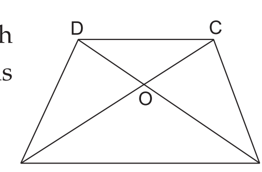
$$\Rightarrow \quad \frac{AL}{DM} = \frac{AO}{DO}\cdot \qquad \dots \text{(i)}$$

$$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2}\times BC \times AL}{\frac{1}{2}\times BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO} \qquad \text{[using (i)]}$$

Hence, $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \dfrac{AO}{DO}$.

EXAMPLE 9 *ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2DC$. If the diagonals of the trapezium intersect each other at a point O, find the ratio of the areas of $\triangle AOB$ and $\triangle COD$.*

SOLUTION GIVEN A trapezium $ABCD$ in which $AB \parallel DC$ and $AB = 2DC$. Its diagonals intersect each other at the point O.



TO FIND $\dfrac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)}$.

METHOD OF SOLUTION In $\triangle AOB$ and $\triangle COD$, we have

$$\angle AOB = \angle COD \quad \text{[vert. opp. } \angle\text{s]}$$

$\angle OAB = \angle OCD$ [alt. int. $\angle$s]

$\therefore$ $\triangle AOB \sim \triangle COD$ [by AA-similarity].

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

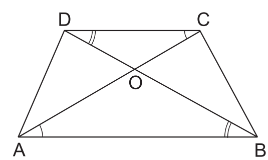
$$\therefore \quad \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{AB^2}{DC^2} = \frac{(2 \times DC)^2}{DC^2} \qquad [\because AB = 2DC]$$

$$= \frac{4 \times DC^2}{DC^2} = \frac{4}{1}.$$

Hence, $\text{ar}(\triangle AOB) : \text{ar}(\triangle COD) = 4 : 1$.

EXAMPLE 10 *In a trapezium ABCD, O is the point of intersection of AC and BD, $AB \parallel CD$ and $AB = 2 \times CD$. If the area of $\triangle AOB = 84 \text{ cm}^2$, find the area of $\triangle COD$.* [CBSE 2005, '09]

SOLUTION In $\triangle AOB$ and $\triangle COD$, we have



$\angle OAB = \angle OCD$ [alt. int. $\angle$s]

and $\angle OBA = \angle ODC$ [alt. int. $\angle$s].

$\therefore$ $\triangle AOB \sim \triangle COD$ [by AA-similarity]

$$\Rightarrow \quad \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} \qquad [\because AB = 2 \times CD]$$

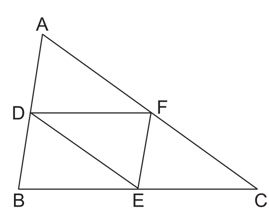
$$= \frac{4 \times CD^2}{CD^2} = 4$$

$$\Rightarrow \quad \text{ar}(\triangle COD) = \frac{1}{4} \times \text{ar}(\triangle AOB) = \left(\frac{1}{4} \times 84\right) \text{cm}^2 = 21 \text{ cm}^2.$$

Hence, the area of $\triangle COD$ is 21 cm^2.

EXAMPLE 11 *D, E and F are respectively the midpoints of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.*

SOLUTION In $\triangle ABC$, D and F are the midpoints of sides AB and CA respectively.



$\therefore$ $DF \parallel BC$ [by midpoint theorem]

$\Rightarrow$ $DF \parallel BE$.

Similarly, $EF \parallel BD$.

$\therefore$ $BEFD$ is a parallelogram

$\Rightarrow$ $\angle B = \angle EFD$, $EF = BD = \frac{1}{2}AB$

and $DF = BE = \frac{1}{2}BC$.

Also, $ECFD$ is a parallelogram

$\Rightarrow \quad \angle EDF = \angle C$.

Now, in $\triangle DEF$ and $\triangle CAB$, we have

$\angle EFD = \angle B$

and $\angle EDF = \angle C$.

$\therefore \quad \triangle DEF \sim \triangle CAB$ [by AA-similarity].

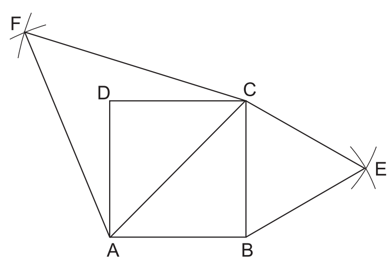
And so, $\dfrac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \dfrac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle CAB)} = \dfrac{DF^2}{BC^2} = \dfrac{\left(\frac{1}{2}BC\right)^2}{BC^2} = \dfrac{1}{4}$.

EXAMPLE 12 *Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of an equilateral triangle described on one of its diagonals.* [CBSE 2005C]

SOLUTION GIVEN A square $ABCD$ and equilateral $\triangle$s BCE and ACF have been described on side BC and diagonal AC respectively.

TO PROVE $\text{ar}(\triangle BCE) = \frac{1}{2}\text{ar}(\triangle ACF)$.

PROOF Since each of the $\triangle BCE$ and $\triangle ACF$ is an equilateral triangle, so each angle of each one of them is 60°. So, the triangles are equiangular, and hence similar.



$\therefore \quad \triangle BCE \sim \triangle ACF$.

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

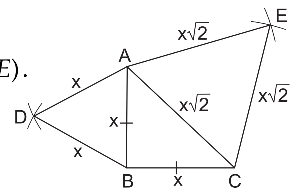
$$\therefore \quad \frac{\text{ar}(\triangle BCE)}{\text{ar}(\triangle ACF)} = \frac{BC^2}{AC^2} = \frac{BC^2}{2(BC)^2} \quad [\because AC = \sqrt{2}BC]$$

$$= \frac{1}{2}.$$

Hence, $\text{ar}(\triangle BCE) = \frac{1}{2} \times \text{ar}(\triangle ACF)$.

EXAMPLE 13 *Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.* [CBSE 2006]

SOLUTION GIVEN A $\triangle ABC$ in which $\angle ABC = 90^\circ$ and $AB = BC$. $\triangle ABD$ and $\triangle CAE$ are equilateral triangles.



TO PROVE $\text{ar}(\triangle ABD) = \frac{1}{2} \times \text{ar}(\triangle CAE)$.

PROOF Let $AB = BC = x$ units.

$\therefore$ hyp. $CA = \sqrt{x^2 + x^2} = x\sqrt{2}$ units.

Each of the $\triangle$s ABD and CAE being equilateral, each angle of each one of them is 60°.

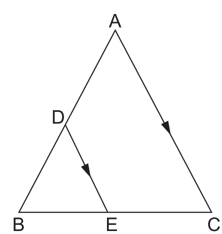
$\therefore$ $\triangle ABD \sim \triangle CAE$ [by AAA-similarity].

But, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}.$$

Hence, $\text{ar}(\triangle ABD) = \frac{1}{2} \times \text{ar}(\triangle CAE)$.

EXAMPLE 14 *If D is a point on the side AB of $\triangle ABC$ such that $AD : DB = 3 : 2$ and E is a point on BC such that $DE \parallel AC$, find the ratio of the areas of $\triangle ABC$ and $\triangle DBE$.* [CBSE 2008C]



SOLUTION Let $AD = 3x$ cm and $DB = 2x$ cm.

Then,

$$AB = (AD + DB) = (3x + 2x) \text{ cm} = 5x \text{ cm}.$$

In $\triangle ABC$ and $\triangle DBE$, we have

$\angle CAB = \angle EDB$ [corresponding $\angle$s]

and $\angle ACB = \angle DEB$ [corresponding $\angle$s].

$\therefore$ $\triangle ABC \sim \triangle DBE$ [by AA-similarity]

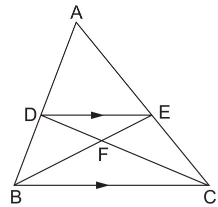
$$\Rightarrow \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBE)} = \frac{(AB)^2}{(DB)^2} = \frac{(5x)^2}{(2x)^2} = \frac{25x^2}{4x^2} = \frac{25}{4}$$

$\Rightarrow$ $\text{ar}(\triangle ABC) : \text{ar}(\triangle DBE) = 25 : 4$.

EXAMPLE 15 *In the given figure, $DE \parallel BC$ and $AD : DB = 5 : 4$.*

Find the ratio $ar(\triangle DFE) : ar(\triangle CFB)$.

[CBSE 2000]



SOLUTION Let $AD = 5x$ cm and $DB = 4x$ cm. Then,

$$AB = AD + DB = 5x \text{ cm} + 4x \text{ cm} = 9x \text{ cm}.$$

In $\triangle ADE$ and $\triangle ABC$, we have

$\angle ADE = \angle ABC$ [corres. ∠s]

$\angle AED = \angle ACB$ [corres. ∠s].

$\therefore$ $\triangle ADE \sim \triangle ABC$ [by AA-similarity]

$$\Rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{5x}{9x} = \frac{5}{9}. \quad \ldots \text{(i)}$$

In $\triangle DFE$ and $\triangle CFB$, we have

$\angle EDF = \angle BCF$ [alt. int. ∠s]

and $\angle DEF = \angle CBF$ [alt. int. ∠s].

$\therefore$ $\triangle DFE \sim \triangle CFB$

$$\Rightarrow \frac{ar(\triangle DFE)}{ar(\triangle CFB)} = \frac{DE^2}{CB^2} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

$\Rightarrow$ $ar(\triangle DFE) : ar(\triangle CFB) = 25 : 81$.

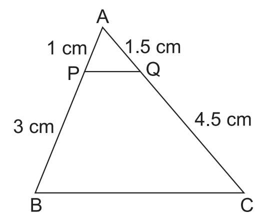
EXERCISE 7C

1. $\triangle ABC \sim \triangle DEF$ and their areas are respectively 64 cm^2 and 121 cm^2. If $EF = 15.4$ cm, find BC.
2. The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If $BC = 4.5$ cm, find the length of QR. [CBSE 2004]
3. $\triangle ABC \sim \triangle PQR$ and $ar(\triangle ABC) = 4ar(\triangle PQR)$. If $BC = 12$ cm, find QR.
4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.
5. $\triangle ABC \sim \triangle DEF$ and their areas are respectively 100 cm^2 and 49 cm^2. If the altitude of $\triangle ABC$ is 5 cm, find the corresponding altitude of $\triangle DEF$. [CBSE 2002]
6. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

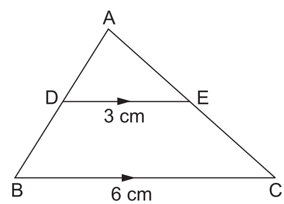
7. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the first triangle is 6.3 cm, find the corresponding altitude of the other. [CBSE 2001]

8. The areas of two similar triangles are 100 cm^2 and 64 cm^2 respectively. If a median of the smaller triangle is 5.6 cm, find the corresponding median of the other.

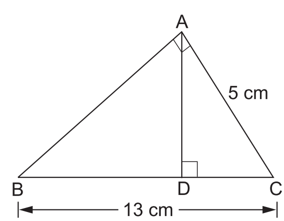
9. In the given figure, ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm, $QC = 4.5$ cm, prove that area of $\triangle APQ$ is $\frac{1}{16}$ of the area of $\triangle ABC$. [CBSE 2005]



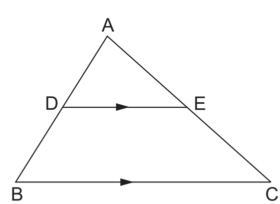
10. In the given figure, $DE \parallel BC$. If $DE = 3$ cm, $BC = 6$ cm and $\text{ar}(\triangle ADE) = 15\text{ cm}^2$, find the area of $\triangle ABC$.



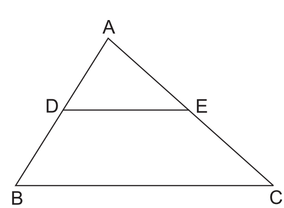
11. $\triangle ABC$ is right-angled at A and $AD \perp BC$. If $BC = 13$ cm and $AC = 5$ cm, find the ratio of the areas of $\triangle ABC$ and $\triangle ADC$. [CBSE 2000C]



12. In the given figure, $DE \parallel BC$ and $DE : BC = 3 : 5$. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium $BCED$.



13. In $\triangle ABC$, D and E are the midpoints of AB and AC respectively. Find the ratio of the areas of $\triangle ADE$ and $\triangle ABC$.



ANSWERS (EXERCISE 7C)

1. 11.2 cm **2.** 6 cm **3.** 6 cm **4.** 22 cm **5.** 3.5 cm **6.** 4 : 9
7. 4.9 cm **8.** 7 cm **10.** 60 cm^2 **11.** 169 : 25 **12.** 9 : 16 **13.** 1 : 4

HINTS TO SOME SELECTED QUESTIONS

6. Required ratio $= \frac{(6)^2}{(9)^2} = \frac{36}{81} = \frac{4}{9} = 4:9.$

9. $\frac{AP}{AB} = \frac{1}{(1+3)} = \frac{1}{4}, \frac{AQ}{AC} = \frac{1.5}{(1.5+4.5)} = \frac{1.5}{6} = \frac{1}{4}.$

$\therefore \quad \triangle APQ \sim \triangle ABC$ [by SAS-similarity]

$\Rightarrow \quad \frac{ar(\triangle APQ)}{ar(\triangle ABC)} = \frac{AP^2}{AB^2} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\Rightarrow \quad ar(\triangle APQ) = \frac{1}{16} \cdot ar(\triangle ABC).$

10. $\frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{DE^2}{BC^2} = \frac{3^2}{6^2} = \frac{9}{36} = \frac{1}{4}$ [$\because \triangle ADE \sim \triangle ABC$]

$\Rightarrow \quad ar(\triangle ABC) = 4 \times ar(\triangle ADE) = (4 \times 15)\ cm^2 = 60\ cm^2.$

11. In $\triangle BAC$ and $\triangle ADC$, we have

$\angle BAC = \angle ADC = 90°$

and $\angle ACB = \angle DCA = \angle C.$

$\therefore \quad \triangle BAC \sim \triangle ADC.$ [by AA-similarity].

$\therefore \quad \frac{ar(\triangle ABC)}{ar(\triangle ADC)} = \frac{ar(\triangle BAC)}{ar(\triangle ADC)} = \frac{BC^2}{AC^2}.$

12. $\triangle ADE \sim \triangle ABC.$

$\therefore \quad \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}.$

Let $ar(\triangle ADE) = 9x$ sq units.

Then, $ar(\triangle ABC) = 25x$ sq units.

$\therefore \quad ar(\text{trap. } BCED) = ar(\triangle ABC) - ar(\triangle ADE)$

$= (25x - 9x)$ sq units $= (16x)$ sq units.

$\therefore \quad \frac{ar(\triangle ADE)}{ar(\text{trap. } BCED)} = \frac{9x}{16x} = \frac{9}{16}.$

13. Clearly, $DE \parallel BC.$

$\therefore \quad \triangle ADE \sim \triangle ABC.$

$\therefore \quad \frac{ar(\triangle ADE)}{ar(\triangle ABC)} = \frac{AD^2}{AB^2} = \frac{AD^2}{(2AD)^2} = \frac{1}{4}.$

....

PYTHAGORAS' THEOREM

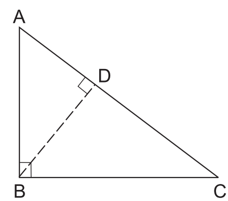
We have proved earlier in this chapter that:

"If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other."

Now, we shall use this theorem to prove the Pythagoras' theorem.

THEOREM 1 **(Pythagoras' theorem)** *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

[CBSE 2001, '02, '03, '04, '04C, '05, '06, '06C, '07, '07C, '09]



GIVEN A $\triangle ABC$ in which $\angle ABC = 90°$.

TO PROVE $AC^2 = AB^2 + BC^2$.

CONSTRUCTION Draw $BD \perp AC$.

PROOF In $\triangle ADB$ and $\triangle ABC$, we have

$\angle A = \angle A$ (common)

$\angle ADB = \angle ABC$ [each equal to 90°]

$\therefore \quad \triangle ADB \sim \triangle ABC$ [by AA-similarity]

$\Rightarrow \quad \frac{AD}{AB} = \frac{AB}{AC}$

$\Rightarrow \quad AD \times AC = AB^2$. … (i)

In $\triangle BDC$ and $\triangle ABC$, we have

$\angle C = \angle C$ (common)

$\angle BDC = \angle ABC$ [each equal to 90°]

$\therefore \quad \triangle BDC \sim \triangle ABC$ [by AA-similarity]

$\Rightarrow \quad \frac{DC}{BC} = \frac{BC}{AC}$

$\Rightarrow \quad DC \times AC = BC^2$. … (ii)

From (i) and (ii), we get

$AD \times AC + DC \times AC = AB^2 + BC^2$

$\Rightarrow \quad (AD + DC) \times AC = AB^2 + BC^2$

$\Rightarrow \quad AC \times AC = AB^2 + BC^2 \Rightarrow AC^2 = AB^2 + BC^2$.

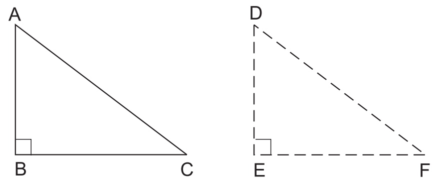
THEOREM 2 **(Converse of Pythagoras' theorem)** *In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.*

[CBSE 2001, '03, '05C, '06, '06C, '07, '09, '09C]

GIVEN A $\triangle ABC$ in which $AC^2 = AB^2 + BC^2$.

TO PROVE $\angle B = 90°$.

CONSTRUCTION Draw a $\triangle DEF$ such that $DE = AB$, $EF = BC$ and $\angle E = 90°$.



PROOF In $\triangle DEF$, we have $\angle E = 90°$.

So, by Pythagoras' theorem, we have

$$DF^2 = DE^2 + EF^2$$

$\Rightarrow$ $DF^2 = AB^2 + BC^2$. ... (i) [$\because$ $DE = AB$ and $EF = BC$]

But, $AC^2 = AB^2 + BC^2$. ... (ii) [given]

From (i) and (ii), we get $AC^2 = DF^2 \Rightarrow AC = DF$.

Now, in $\triangle ABC$ and $\triangle DEF$, we have

$$AB = DE, BC = EF \text{ and } AC = DF.$$

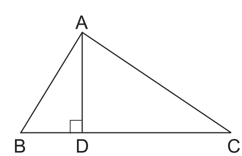
$\therefore$ $\triangle ABC \cong \triangle DEF$.

Hence, $\angle B = \angle E = 90°$.

SOME IMPORTANT RESULTS BASED UPON PYTHAGORAS' THEOREM

THEOREM 1 *In a $\triangle ABC$, AD is perpendicular to BC. Prove that* $(AB^2 + CD^2) = (AC^2 + BD^2)$.

[CBSE 2003, '05C, '09]



GIVEN A $\triangle ABC$ in which $AD \perp BC$.

TO PROVE $(AB^2 + CD^2) = (AC^2 + BD^2)$.

PROOF From right $\triangle ADB$, we have

$AB^2 = AD^2 + BD^2$ [by Pythagoras' theorem]

$\Rightarrow$ $(AB^2 - BD^2) = AD^2$. ... (i)

From right $\triangle ADC$, we have

$$AC^2 = AD^2 + CD^2$$

$\Rightarrow$ $(AC^2 - CD^2) = AD^2$. ... (ii)

From (i) and (ii), we get $(AB^2 - BD^2) = (AC^2 - CD^2)$.

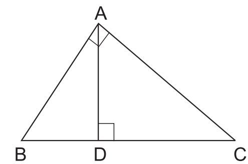
Hence, $(AB^2 + CD^2) = (AC^2 + BD^2)$.

<u>THEOREM 2</u> *Given a $\triangle ABC$ in which $\angle A = 90°$ and $AD \perp BC$. Prove that*

$$AD^2 = BD \cdot CD.$$

[CBSE 2004C, '06, '09]

GIVEN A $\triangle ABC$ in which $\angle A = 90°$ and $AD \perp BC$.



TO PROVE $AD^2 = BD \cdot CD$.

PROOF In $\triangle BAC$, $\angle A = 90°$.

$\therefore \quad BC^2 = AB^2 + AC^2$ ... (i)

[by Pythagoras' theorem].

In $\triangle ADB$, $\angle ADB = 90°$.

$\therefore \quad AB^2 = AD^2 + BD^2$. ... (ii) [by Pythagoras' theorem]

In $\triangle ADC$, $\angle ADC = 90°$.

$\therefore \quad AC^2 = AD^2 + CD^2$. ... (iii) [by Pythagoras' theorem]

From (ii) and (iii), we get

$(AB^2 + AC^2) = 2AD^2 + BD^2 + CD^2$

$\Rightarrow \quad BC^2 = 2AD^2 + BD^2 + CD^2$ [using (i)]

$\Rightarrow \quad (BD + CD)^2 = 2AD^2 + BD^2 + CD^2$

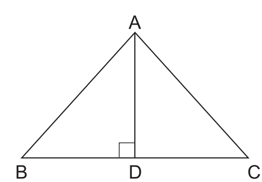
$\Rightarrow \quad BD^2 + CD^2 + 2BD \cdot CD = 2AD^2 + BD^2 + CD^2$

$\Rightarrow \quad 2AD^2 = 2BD \cdot CD \Rightarrow AD^2 = BD \cdot CD$.

Hence, $AD^2 = BD \cdot CD$.

<u>THEOREM 3</u> *In $\triangle ABC$, $AD \perp BC$ such that $AD^2 = BD \cdot CD$. Prove that $\triangle ABC$ is right-angled at A.* [CBSE 2006]

GIVEN A $\triangle ABC$ in which $AD \perp BC$ and $AD^2 = BD \cdot CD$.



TO PROVE $\angle A = 90°$.

PROOF In right $\triangle ADB$, $\angle ADB = 90°$.

$\therefore \quad AB^2 = AD^2 + BD^2$. ... (i)

[by Pythagoras' theorem]

In right $\triangle ADC$, $\angle ADC = 90°$.

$\therefore \quad AC^2 = AD^2 + CD^2$. ... (ii) [by Pythagoras' theorem]

Adding (i) and (ii), we get

$(AB^2 + AC^2) = BD^2 + CD^2 + 2AD^2$

$= BD^2 + CD^2 + 2BD \cdot CD \quad [\because AD^2 = BD \cdot CD]$

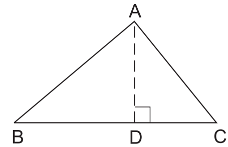
$= (BD + CD)^2 = BC^2$.

Thus, $(AB^2 + AC^2) = BC^2$.

Hence, $\triangle ABC$ is right-angled at A.

<u>THEOREM 4</u> *In a $\triangle ABC$, $\angle ABC < 90^\circ$ (i.e., $\angle B$ is acute) and $AD \perp BC$. Prove that*

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD.$$



GIVEN A $\triangle ABC$ in which $\angle ABC < 90^\circ$ and $AD \perp BC$.

TO PROVE $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.

PROOF In $\triangle ADB$, $\angle ADB = 90^\circ$.

$\therefore \quad AB^2 = AD^2 + BD^2$. … (i) [by Pythagoras' theorem]

In $\triangle ADC$, $\angle ADC = 90^\circ$.

$\therefore \quad AC^2 = AD^2 + CD^2$ [by Pythagoras' theorem]

$= AD^2 + (BC - BD)^2$

$= AD^2 + BC^2 + BD^2 - 2BC \cdot BD$

$= (AD^2 + BD^2) + BC^2 - 2BC \cdot BD$

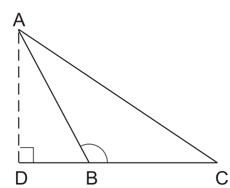
$= AB^2 + BC^2 - 2BC \cdot BD$ [using (i)].

Hence, $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.

Note BD is known as the projection of AB on BC. So, this theorem is stated as:

"In an acute-angled triangle, the square of the side opposite to an acute angle is equal to the sum of the squares of the other two sides minus twice the product of one side and the projection of the other on the first."

<u>THEOREM 5</u> *In a $\triangle ABC$, $\angle ABC > 90^\circ$ (i.e., $\angle B$ is obtuse) and $AD \perp (CB$ produced). Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.*



GIVEN A $\triangle ABC$ in which $\angle ABC > 90^\circ$ and $AD \perp (CB$ produced).

TO PROVE $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

PROOF In $\triangle ADB$, $\angle ADB = 90^\circ$.

$\therefore \quad AB^2 = AD^2 + BD^2$. … (i) [by Pythagoras' theorem]

In $\triangle ADC$, $\angle ADC = 90^\circ$.

$\therefore \quad AC^2 = AD^2 + CD^2$ [by Pythagoras' theorem]

$= AD^2 + (BC + BD)^2$ [$\because$ $CD = BC + BD$]

$= AD^2 + BC^2 + BD^2 + 2BC \cdot BD$

$= (AD^2 + BD^2) + BC^2 + 2BC \cdot BD$

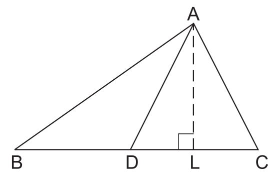
$= AB^2 + BC^2 + 2BC \cdot BD$ [using (i)].

Note BD is the projection of AB on BC. So, this theorem is stated as:

"In an obtuse-angled triangle, the square of the side opposite to the obtuse angle is equal to the sum of the squares of the other two sides plus twice the product of one side and the projection of the other on the first."

THEOREM 6 *In $\triangle ABC$, if AD is the median, then prove that*

$$(AB^2 + AC^2) = 2(AD^2 + BD^2).$$



GIVEN A $\triangle ABC$ in which AD is the median.

TO PROVE $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

CONSTRUCTION Draw $AL \perp BC$.

PROOF In $\triangle ALD$, $\angle ALD = 90°$.

$\therefore$ $\angle ADL < 90°$ and therefore,

$\angle ADB > 90°$.

Thus, in $\triangle ADB$, $\angle ADB > 90°$ and $AL \perp (BD$ produced).

$\therefore$ $AB^2 = AD^2 + BD^2 + 2BD \cdot DL$. ... (i)

In $\triangle ADC$, $\angle ADC < 90°$ and $AL \perp DC$.

$\therefore$ $AC^2 = AD^2 + CD^2 - 2CD \cdot DL$

$\Rightarrow$ $AC^2 = AD^2 + BD^2 - 2BD \cdot DL$. ... (ii) [$\because$ $CD = BD$]

Adding (i) and (ii), we get $(AB^2 + AC^2) = 2(AD^2 + BD^2)$.

Note This theorem can be stated as:

"In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side."

SOLVED EXAMPLES

EXAMPLE 1 *Sides of some triangles are given below. Determine which of them are right triangles.*

(i) 8 cm, 15 cm, 17 cm *(ii)* 9 cm, 11 cm, 6 cm

(iii) $(2a - 1)$ cm, $2\sqrt{2a}$ cm *and* $(2a + 1)$ cm

SOLUTION For the given triangle to be right-angled, the sum of the squares of the two smaller sides must be equal to the square of the largest side.

(i) Let $a = 8$ cm, $b = 15$ cm and $c = 17$ cm. Then,

$$(a^2 + b^2) = \{(8)^2 + (15)^2\} \text{ cm}^2 = (64 + 225) \text{ cm}^2 = 289 \text{ cm}^2$$

and $c^2 = (17)^2 \text{ cm}^2 = 289 \text{ cm}^2$.

$\therefore$ $(a^2 + b^2) = c^2$.

Hence, the given triangle is right-angled.

(ii) Let $a = 9$ cm, $b = 6$ cm and $c = 11$ cm. Then,

$$(a^2 + b^2) = \{(9)^2 + (6)^2\} \text{ cm}^2 = (81 + 36) \text{ cm}^2 = 117 \text{ cm}^2$$

and $c^2 = (11)^2 \text{ cm}^2 = 121 \text{ cm}^2$.

$\therefore \quad (a^2 + b^2) \neq c^2$.

Hence, the given triangle is not right-angled.

(iii) Let $p = (2a - 1)$ cm, $q = 2\sqrt{2a}$ cm and $r = (2a + 1)$ cm. Then,

$$(p^2 + q^2) = (2a - 1)^2 \text{ cm}^2 + (2\sqrt{2a})^2 \text{ cm}^2$$
$$= \{(4a^2 + 1 - 4a) + 8a\} \text{ cm}^2 = (4a^2 + 4a + 1) \text{ cm}^2$$
$$= (2a + 1)^2 \text{ cm}^2 = r^2.$$

$\therefore \quad (p^2 + q^2) = r^2$.

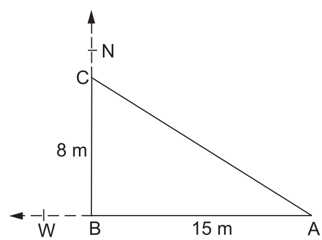
Hence, the given triangle is right-angled.

EXAMPLE 2 *A man goes* 15 m *due west and then* 8 m *due north. How far is he from the starting point?*

SOLUTION Starting from A, let the man go from A to B and then from B to C, as shown in the figure. Then,

$AB = 15$ m, $BC = 8$ m and $\angle ABC = 90°$.

From right $\triangle ABC$, we have



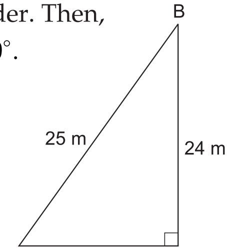
$$AC^2 = AB^2 + BC^2$$
$$= \{(15)^2 + (8)^2\} \text{ m}^2$$
$$= (225 + 64) \text{ m}^2$$
$$= 289 \text{ m}^2$$

$\therefore \quad AC = \sqrt{289}$ m $= 17$ m.

Hence, the man is 17 m away from the starting position.

EXAMPLE 3 *A ladder* 25 m *long just reaches the top of a building* 24 m *high from the ground. Find the distance of the foot of the ladder from the building.*

SOLUTION Let AB be the building and CB be the ladder. Then,



$AB = 24$ m, $CB = 25$ m and $\angle CAB = 90°$.

By Pythagoras' theorem, we have

$$CB^2 = AB^2 + AC^2$$

$\Rightarrow \quad AC^2 = CB^2 - AB^2 = [(25)^2 - (24)^2] \text{ m}^2$

$$= (625 - 576) \text{ m}^2 = 49 \text{ m}^2$$

$\Rightarrow \quad AC = \sqrt{49}$ m $= 7$ m.

Hence, the distance of the foot of the ladder from the building is 7 m.

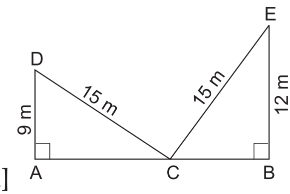
EXAMPLE 4 *A ladder* 15 m *long reaches a window which is* 9 m *above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window* 12 m *high. Find the width of the street.*

SOLUTION Let AB be the street and let C be the foot of the ladder. Let D and E be the given windows such that $AD = 9$ m and $BE = 12$ m.

Then, CD and CE are the two positions of the ladder.

Clearly, $\angle CAD = 90^\circ$, $\angle CBE = 90^\circ$

and $CD = CE = 15$ m.



From right $\triangle CAD$, we have

$$CD^2 = AC^2 + AD^2 \quad \text{[by Pythagoras' theorem]}$$

$$\Rightarrow \quad AC^2 = CD^2 - AD^2$$

$$= [(15)^2 - (9)^2] \text{ m}^2 = (225 - 81) \text{ m}^2 = 144 \text{ m}^2.$$

$$\Rightarrow \quad AC = \sqrt{144} \text{ m} = 12 \text{ m}.$$

From right $\triangle CBE$, we have

$$CE^2 = CB^2 + BE^2 \quad \text{[by Pythagoras' theorem]}$$

$$\Rightarrow \quad CB^2 = CE^2 - BE^2$$

$$= [(15)^2 - 12)^2] \text{ m}^2 = (225 - 144) \text{ m}^2 = 81 \text{ m}^2$$

$$\Rightarrow \quad CB = \sqrt{81} \text{ m} = 9 \text{ m}.$$

Width of the street $= AC + CB = 12 \text{ m} + 9 \text{ m} = 21$ m.

EXAMPLE 5 *Two poles of heights* 6 *metres and* 11 *metres stand vertically on a plane ground. If the distance between their feet is* 12 *metres, find the distance between their tops.* [CBSE 2002]

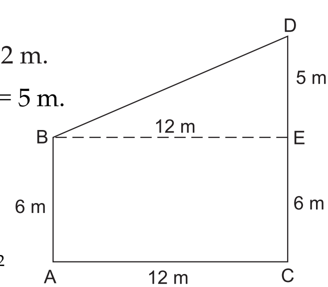
SOLUTION Let AB and CD be the given vertical poles. Then,

$$AB = 6 \text{ m}, CD = 11 \text{ m and } AC = 12 \text{ m}.$$

Draw $BE \parallel AC$. Then,

$$CE = AB = 6 \text{ m}, BE = AC = 12 \text{ m}.$$

$$\therefore \quad DE = CD - CE = 11 \text{ m} - 6 \text{ m} = 5 \text{ m}.$$



In right $\triangle BED$, we have:

$$BD^2 = BE^2 + DE^2$$

$$= \{(12)^2 + (5)^2\} \text{ m}^2$$

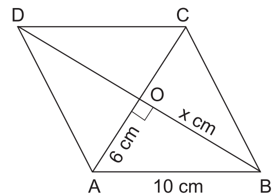
$$= (144 + 25) \text{ m}^2 = 169 \text{ m}^2$$

$\Rightarrow \quad BD = \sqrt{169}$ m = 13 m.

Hence, the distance between the tops of the poles = 13 m.

EXAMPLE 6 *In a rhombus of side* 10 cm, *one of the diagonals is* 12 cm *long. Find the length of the second diagonal.* [CBSE 2001]

SOLUTION Let $ABCD$ be the given rhombus whose diagonals intersect at O. Then, $AB = 10$ cm.



Let $AC = 12$ cm and $BD = 2x$ cm.

We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore \quad OA = \frac{1}{2}AC = 6$ cm, $OB = \frac{1}{2}BD = x$ cm, and $\angle AOB = 90^\circ$.

From right $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad OB^2 = AB^2 - OA^2$$

$$= \{(10)^2 - (6)^2\} \text{ cm}^2 = (100 - 36) \text{ cm}^2 = 64 \text{ cm}^2$$

$$\Rightarrow \quad x^2 = 64 \Rightarrow x = \sqrt{64} = 8.$$

$\therefore \quad OB = 8$ cm.

$\therefore \quad BD = 2 \times OB = 2 \times 8$ cm = 16 cm.

Hence, the length of the second diagonal is 16 cm.

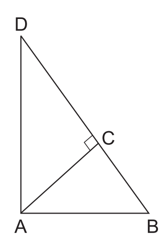
EXAMPLE 7 $\triangle ABD$ *is a right triangle in which* $\angle A = 90^\circ$ *and* $AC \perp BD$.

Prove that:

(i) $AB^2 = BC \cdot BD$ *(ii)* $AC^2 = BC \cdot DC$ *(iii)* $AD^2 = BD \cdot CD$

SOLUTION We know that:

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.



$\therefore \quad \triangle ABC \sim \triangle DBA$

$\triangle ABC \sim \triangle DAC$

$\triangle DBA \sim \triangle DAC$.

(i) $\triangle ABC \sim \triangle DBA$

$$\Rightarrow \frac{AB}{DB} = \frac{BC}{BA} \Rightarrow \frac{AB}{BD} = \frac{BC}{AB} \Rightarrow AB^2 = BC \cdot BD.$$

(ii) $\triangle ABC \sim \triangle DAC$

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC} \Rightarrow AC^2 = BC \cdot DC.$$

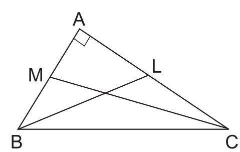
(iii) $\triangle DBA \sim \triangle DAC$

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD} \Rightarrow AD^2 = BD \cdot CD.$$

EXAMPLE 8 *BL and CM are medians of a $\triangle ABC$, right-angled at A. Prove that*

$$4(BL^2 + CM^2) = 5BC^2.$$ [CBSE 2006C, '10]

SOLUTION GIVEN A $\triangle ABC$ in which BL and CM are medians and $\angle A = 90°$.



TO PROVE $4(BL^2 + CM^2) = 5BC^2$.

PROOF In $\triangle BAC$, $\angle A = 90°$.

$\therefore \quad BC^2 = AB^2 + AC^2$. ... (i)

[by Pythagoras' theorem]

In $\triangle BAL$, $\angle A = 90°$.

$\therefore \quad BL^2 = AL^2 + AB^2$ [by Pythagoras' theorem]

$\Rightarrow \quad BL^2 = \left(\frac{1}{2}AC\right)^2 + AB^2 \Rightarrow BL^2 = \frac{1}{4}AC^2 + AB^2$

$\Rightarrow \quad 4BL^2 = AC^2 + 4AB^2$. ... (ii)

In $\triangle CAM$, $\angle A = 90°$.

$\therefore \quad CM^2 = AM^2 + AC^2$

$\Rightarrow \quad CM^2 = \left(\frac{1}{2}AB\right)^2 + AC^2 \Rightarrow CM^2 = \frac{1}{4}AB^2 + AC^2$

$\Rightarrow \quad 4CM^2 = AB^2 + 4AC^2$. ... (iii)

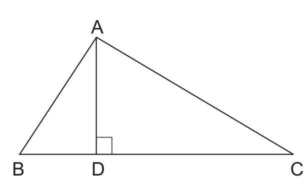
On adding (ii) and (iii), we get $4(BL^2 + CM^2) = 5(AB^2 + AC^2)$.

Hence, $4(BL^2 + CM^2) = 5BC^2$ [using (i)].

EXAMPLE 9 *In the given figure, the perpendicular from A on side BC of a $\triangle ABC$, intersects BC at D such that $DB = 3CD$. Prove that*

$$2AB^2 = 2AC^2 + BC^2.$$

[CBSE 2003, '05, '09]



SOLUTION GIVEN A $\triangle ABC$ in which $AD \perp BC$ and $BD = 3CD$.

TO PROVE $2AB^2 = 2AC^2 + BC^2$.

PROOF We have $BD = 3CD$.

$\therefore \quad BC = BD + CD = 3CD + CD = 4CD$

$\Rightarrow \quad CD = \frac{1}{4}BC.$ … (i)

In $\triangle ADB$, $\angle ADB = 90^\circ$.

$\therefore \quad AB^2 = AD^2 + BD^2.$ … (ii) [by Pythagoras' theorem]

In $\triangle ADC$, $\angle ADC = 90^\circ$.

$\therefore \quad AC^2 = AD^2 + CD^2.$ … (iii) [by Pythagoras' theorem]

On subtracting (iii) from (ii), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

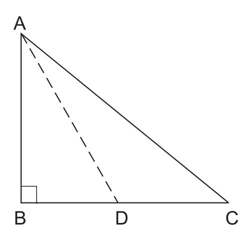
$$= [(3CD)^2 - (CD^2)] = 8CD^2 \quad [\because BD = 3CD]$$

$$= 8 \times \frac{1}{16}BC^2 = \frac{1}{2}BC^2 \quad \text{[using (i)]}.$$

$\therefore \quad 2AB^2 - 2AC^2 = BC^2$.

Hence, $2AB^2 = 2AC^2 + BC^2$.

EXAMPLE 10 *In $\triangle ABC$, $\angle B = 90^\circ$ and D is the midpoint of BC. Prove that* $AC^2 = AD^2 + 3CD^2$.



SOLUTION GIVEN A $\triangle ABC$ in which $\angle B = 90^\circ$ and D is the midpoint of BC.

TO PROVE $AC^2 = AD^2 + 3CD^2$.

CONSTRUCTION Join AD.

PROOF In $\triangle ABC$, $\angle B = 90^\circ$.

$\therefore \quad AC^2 = AB^2 + BC^2.$ … (i) [by Pythagoras' theorem]

In $\triangle ABD$, $\angle B = 90^\circ$.

$\therefore \quad AD^2 = AB^2 + BD^2$ … (ii) [by Pythagoras' theorem]

$\Rightarrow \quad AB^2 = AD^2 - BD^2$.

$\therefore \quad AC^2 = (AD^2 - BD^2) + BC^2$ [using (i)]

$\Rightarrow \quad AC^2 = AD^2 - CD^2 + (2CD)^2$ [$\because BD = CD$ and $BC = 2CD$]

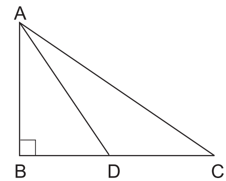
$\Rightarrow \quad AC^2 = AD^2 + 3CD^2$.

Hence, $AC^2 = AD^2 + 3CD^2$.

<u>EXAMPLE 11</u> *$\triangle ABC$ is right-angled at B and D is the midpoint of BC. Prove that* $AC^2 = (4AD^2 - 3AB^2)$. [CBSE 2008C, '10]

SOLUTION GIVEN A $\triangle ABC$ in which $\angle B = 90^\circ$ and D is the midpoint of BC.

TO PROVE $AC^2 = (4AD^2 - 3AB^2)$.



PROOF In $\triangle ABC$, $\angle B = 90^\circ$.

$\therefore \quad AC^2 = AB^2 + BC^2$ [by Pythagoras' theorem]

$= AB^2 + (2BD)^2$ [$\because$ $BC = 2BD$]

$= AB^2 + 4BD^2$

$= AB^2 + 4(AD^2 - AB^2)$ [$\because$ $AB^2 + BD^2 = AD^2$]

$= (4AD^2 - 3AB^2)$.

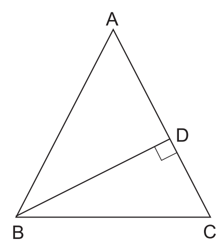
Hence, $AC^2 = (4AD^2 - 3AB^2)$.

<u>EXAMPLE 12</u> *In an isosceles $\triangle ABC$, $AB = AC$ and $BD \perp AC$. Prove that*

$(BD^2 - CD^2) = 2CD \cdot AD$.

SOLUTION GIVEN A $\triangle ABC$ in which $AB = AC$ and $BD \perp AC$.

TO PROVE $(BD^2 - CD^2) = 2CD \cdot AD$.



PROOF From right $\triangle ADB$, we have

$AB^2 = AD^2 + BD^2$ [by Pythagoras' theorem]

$\Rightarrow \quad AC^2 = AD^2 + BD^2$ [$\because$ $AB = AC$]

$\Rightarrow \quad (CD + AD)^2 = AD^2 + BD^2$ [$\because$ $AC = CD + AD$]

$\Rightarrow \quad CD^2 + AD^2 + 2CD \cdot AD = AD^2 + BD^2$

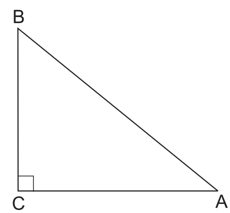
$\Rightarrow \quad (BD^2 - CD^2) = 2CD \cdot AD$.

Hence, $(BD^2 - CD^2) = 2CD \cdot AD$.

<u>EXAMPLE 13</u> *$\triangle ABC$ is an isosceles triangle, right-angled at C. Prove that*

$AB^2 = 2AC^2$.

SOLUTION $\triangle ABC$ is an isosceles triangle, right-angled at C



$\Rightarrow \quad BC = AC$ ... (i)

Now, by Pythagoras' theorem,

$AB^2 = BC^2 + AC^2$

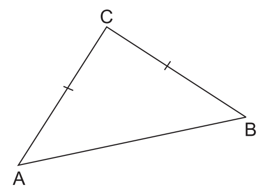
$\Rightarrow \quad AB^2 = AC^2 + AC^2$ [using (i)]

$\Rightarrow \quad AB^2 = 2AC^2.$

<u>EXAMPLE 14</u> *$\triangle ABC$ is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that $\triangle ABC$ is a right triangle.* [CBSE 2000C]

SOLUTION In $\triangle ABC$, we have

$AC = BC.$ ... (i) (given)



Now, $AB^2 = 2AC^2$ (given)

$\Rightarrow \quad AB^2 = AC^2 + AC^2$

$\Rightarrow \quad AB^2 = AC^2 + BC^2$ [using (i)]

$\Rightarrow \quad \angle C = 90^\circ$

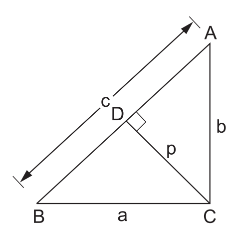
[by converse of Pythagoras' theorem]

Hence, $\triangle ABC$ is a right triangle.

<u>EXAMPLE 15</u> *$\triangle ABC$ is a right triangle in which $\angle C = 90^\circ$ and $CD \perp AB$. If $BC = a$, $CA = b$, $AB = c$ and $CD = p$ then prove that*

(i) $cp = ab$ *(ii)* $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}.$ [CBSE 1997C, '98, '99, '02]

SOLUTION (i) We have



$\text{ar}(\triangle ABC) = \frac{1}{2} \times AB \times CD = \frac{1}{2}cp$

[taking AB as base]

and $\text{ar}(\triangle ABC) = \frac{1}{2} \times BC \times AC = \frac{1}{2}ab$

[taking BC as base].

$\therefore \quad \frac{1}{2}cp = \frac{1}{2}ab \Rightarrow cp = ab.$

Hence, $cp = ab$.

(ii) $cp = ab \Rightarrow \dfrac{1}{p} = \dfrac{c}{ab}.$

$\therefore \quad \dfrac{1}{p^2} = \dfrac{c^2}{a^2b^2} = \dfrac{b^2 + a^2}{a^2b^2}$ $[\because AB^2 = AC^2 + BC^2]$

$= \left(\dfrac{b^2}{a^2b^2} + \dfrac{a^2}{a^2b^2}\right) = \left(\dfrac{1}{a^2} + \dfrac{1}{b^2}\right).$

Hence, $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}.$

EXAMPLE 16 *In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.*

[CBSE 2002, '07C]

SOLUTION GIVEN A $\triangle ABC$ in which $AB = BC = CA$ and $AD \perp BC$.

TO PROVE $3AB^2 = 4AD^2$.

PROOF In $\triangle ADB$ and $\triangle ADC$, we have

$AB = AC$ (given), $\angle B = \angle C = 60°$

and $\angle ADB = \angle ADC = 90°$.

$\therefore \quad \triangle ADB \cong \triangle ADC$ [AAS-congruence]

$\therefore \quad BD = DC = \frac{1}{2}BC$.

From right $\triangle ADB$, we have

$$AB^2 = AD^2 + BD^2 \quad \text{[by Pythagoras' theorem]}$$

$$= AD^2 + \left(\frac{1}{2}BC\right)^2 = AD^2 + \frac{1}{4}BC^2$$

$\Rightarrow \quad 4AB^2 = 4AD^2 + BC^2$

$\Rightarrow \quad 3AB^2 = 4AD^2 \quad [\because\ BC = AB]$.

Hence, $3AB^2 = 4AD^2$.

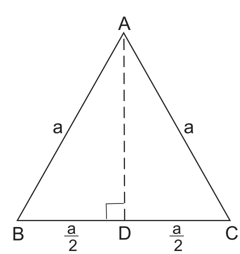
EXAMPLE 17 *In an equilateral triangle with side a, prove that*

(i) altitude $= \frac{\sqrt{3}}{2}a$ *(ii) area* $= \frac{\sqrt{3}}{4}a^2$. [CBSE 1997, '99, '01C, '02C]

SOLUTION Let $\triangle ABC$ be an equilateral triangle with side a.

Then, $AB = AC = BC = a$.

Draw $AD \perp BC$.



In $\triangle ADB$ and $\triangle ADC$, we have

$AB = AC$ (given), $\angle B = \angle C = 60°$

and $\angle ADB = \angle ADC = 90°$.

$\therefore \quad \triangle ADB \cong \triangle ADC$.

$\therefore \quad BD = DC = \frac{a}{2}\cdot$

(i) From right $\triangle ADB$, we have

$$AB^2 = AD^2 + BD^2 \text{ [by Pythagoras' theorem]}$$

$$\Rightarrow \quad AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}}{2}a.$$

Hence, altitude $= \frac{\sqrt{3}}{2}a$.

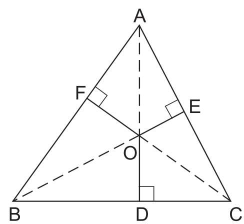
(ii) Area of $\triangle ABC = \frac{1}{2} \times \text{base} \times \text{altitude} = \left(\frac{1}{2} \times BC \times AD\right)$

$$= \left(\frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a\right) \quad \text{[using (i)]}$$

$$= \left(\frac{\sqrt{3}}{4}a^2\right) \text{ sq units.}$$

Hence, area$(\triangle ABC) = \left(\frac{\sqrt{3}}{4}a^2\right)$ sq units.

EXAMPLE 18 *O is a point in the interior of $\triangle ABC$, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$, as shown in the figure.*



Prove that:

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$

SOLUTION (i) Using Pythagoras' theorem for each of the right triangles namely $\triangle OFA$, $\triangle ODB$ and $\triangle OEC$, we get

$$OA^2 = OF^2 + AF^2 \quad \ldots \text{(i)}$$

$$OB^2 = OD^2 + BD^2 \quad \ldots \text{(ii)}$$

$$OC^2 = OE^2 + CE^2 \quad \ldots \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$OA^2 + OB^2 + OC^2$$
$$= OD^2 + OE^2 + OF^2 + AF^2 + BD^2 + CE^2.$$

Hence, $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$
$$= AF^2 + BD^2 + CE^2.$$

(ii) Using Pythagoras' theorem for each of the right triangles, namely $\triangle ODB$ and $\triangle ODC$, we get

$OB^2 = OD^2 + BD^2$ and $OC^2 = OD^2 + CD^2$.

$$\therefore \quad OB^2 - OC^2 = BD^2 - CD^2 \quad \ldots \text{(iv)}$$

Similarly, we have

$$OC^2 - OA^2 = CE^2 - AE^2 \quad \ldots \text{(v)}$$

and $OA^2 - OB^2 = AF^2 - BF^2$. … (vi)

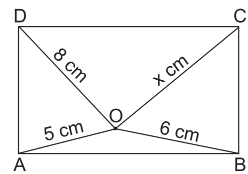
Adding the corresponding sides of (iv), (v) and (vi), we get

$$AF^2 + BD^2 + CE^2 - AE^2 - BF^2 - CD^2 = 0.$$

Hence, $AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$.

EXAMPLE 19 *O is any point inside a rectangle ABCD. Prove that* $OB^2 + OD^2 = OA^2 + OC^2$. [CBSE 2006C]

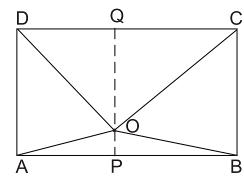
DEDUCTION *In the given figure, O is a point inside a rectangle ABCD such that* $OB = 6$ cm, $OD = 8$ cm *and* $OA = 5$ cm, *find the length of OC.* [CBSE 2009C]



SOLUTION GIVEN O is a point inside a rectangle $ABCD$.

TO PROVE $OB^2 + OD^2 = OA^2 + OC^2$.

CONSTRUCTION Through O, draw $POQ \parallel BC$ so that P lies on AB and Q lies on DC.



PROOF we have

$POQ \parallel BC \Rightarrow PQ \perp AB$ and $QP \perp DC$

$\Rightarrow$ $\angle BPQ = 90^\circ$ and $\angle CQP = 90^\circ$.

$\therefore$ $BPQC$ and $APQD$ are both rectangles.

$\therefore$ $BP = CQ$ [opposite sides of a rectangle]

$DQ = AP$ [opposite sides of a rectangle]

From right $\triangle OPB$, we have $OB^2 = OP^2 + BP^2$.

From right $\triangle OQD$, we have $OD^2 = OQ^2 + DQ^2$.

From right $\triangle OPA$, we have $OA^2 = OP^2 + AP^2$.

From right $\triangle OQC$, we have $OC^2 = OQ^2 + CQ^2$.

$\therefore$ $OB^2 + OD^2 = OP^2 + OQ^2 + BP^2 + DQ^2$

$= OP^2 + OQ^2 + CQ^2 + AP^2$

[$\because$ $BP = CQ$ and $DQ = AP$]

$= (OP^2 + AP^2) + (OQ^2 + CQ^2)$

$= OA^2 + OC^2$.

Hence, $OB^2 + OD^2 = OA^2 + OC^2$.

DEDUCTION Let $OC = x$ cm. Then,

$$OB^2 + OD^2 = OA^2 + OC^2$$

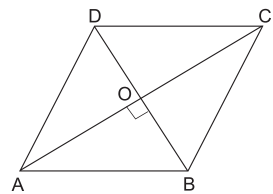
$\Rightarrow$ $6^2 + 8^2 = 5^2 + x^2$

$\Rightarrow$ $x^2 = 36 + 64 - 25 = 75$

$\Rightarrow$ $x = \sqrt{75} = 5\sqrt{3} = (5 \times 1.732) = 8.66 \Rightarrow OC = 8.66$ cm.

EXAMPLE 20 *Prove that the sum of the squares on the sides of a rhombus is equal to the sum of the squares on its diagonals.* [CBSE 2005, '06, '08C]

SOLUTION GIVEN A rhombus $ABCD$ whose diagonals AC and BD intersect at O.



TO PROVE $(AB^2 + BC^2 + CD^2 + DA^2) = (AC^2 + BD^2)$.

PROOF We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore$ $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$,

$$OA = \frac{1}{2}AC \text{ and } OB = \frac{1}{2}BD.$$

From right $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2 \text{ [by Pythagoras' theorem]}$$

$$= \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = \frac{1}{4}(AC^2 + BD^2)$$

$\Rightarrow$ $4AB^2 = (AC^2 + BD^2)$. ... (i)

Similarly, we have:

$4BC^2 = (AC^2 + BD^2)$ ... (ii)

$4CD^2 = (AC^2 + BD^2)$ ... (iii)

$4DA^2 = (AC^2 + BD^2)$ ... (iv)

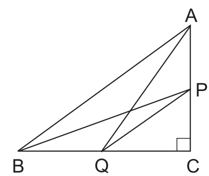
On adding (i), (ii), (iii) and (iv), we get

$$(AB^2 + BC^2 + CD^2 + DA^2) = (AC^2 + BD^2).$$

REMARK In a rhombus $ABCD$, we have $AB = BC = CD = DA$, so the above result may be given as $4AB^2 = (AC^2 + BD^2)$.

EXAMPLE 21 *P and Q are points on the sides CA and CB of a $\triangle ABC$, right-angled at C. Prove that* $(AQ^2 + BP^2) = (AB^2 + PQ^2)$. [CBSE 2007, '08]

SOLUTION GIVEN A $\triangle ABC$ in which $\angle C = 90°$. P and Q are points on CA and CB respectively.



TO PROVE $(AQ^2 + BP^2) = (AB^2 + PQ^2)$.

PROOF From right $\triangle ACQ$, we have

$AQ^2 = (AC^2 + CQ^2)$. ... (i) [by Pythagoras' theorem]

From right $\triangle BCP$, we have

$BP^2 = (BC^2 + CP^2)$. ... (ii) [by Pythagoras' theorem]

From right $\triangle ACB$, we have

$AB^2 = AC^2 + BC^2$. ... (iii) [by Pythagoras' theorem]

From right $\triangle PCQ$, we have

$PQ^2 = (CQ^2 + CP^2)$. ... (iv) [by Pythagoras' theorem]

From (i) and (ii), we get

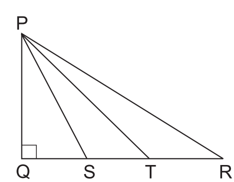
$$(AQ^2 + BP^2) = (AC^2 + BC^2) + (CQ^2 + CP^2)$$
$$= (AB^2 + PQ^2) \text{ [using (iii) and (iv)].}$$

Hence, $(AQ^2 + BP^2) = (AB^2 + PQ^2)$.

EXAMPLE 22 *In the figure given below, $\triangle PQR$ is right-angled at Q and the points S and T trisect the side QR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.*

[CBSE 2006C]

SOLUTION GIVEN A $\triangle PQR$ in which $\angle PQR = 90°$, S and T are the points of trisection of QR.



TO PROVE $8PT^2 = 3PR^2 + 5PS^2$.

PROOF Let $QS = ST = TR = x$. Then,

$QS = x$, $QT = 2x$ and $QR = 3x$.

From right triangles PQS, PQT and PQR, by Pythagoras' theorem, we have

$PS^2 = PQ^2 + QS^2$, $PT^2 = PQ^2 + QT^2$ and $PR^2 = PQ^2 + QR^2$.

$\therefore$ $3PR^2 + 5PS^2 - 8PT^2$

$$= 3(PQ^2 + QR^2) + 5(PQ^2 + QS^2) - 8(PQ^2 + QT^2)$$
$$= 3QR^2 + 5QS^2 - 8QT^2$$
$$= 3 \times (3x)^2 + 5(x)^2 - 8 \times (2x)^2$$

[$\because$ $QR = 3x$, $QS = x$ and $QT = 2x$]

$$= (27x^2 + 5x^2 - 32x^2) = 0.$$

Thus, $3PR^2 + 5PS^2 - 8PT^2 = 0$.

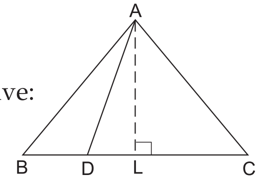
Hence, $8PT^2 = 3PR^2 + 5PS^2$.

EXAMPLE 23 *In an isosceles* $\triangle ABC$, $AB = AC$ *and* D *is a point on* BC. *Prove that* $AB^2 - AD^2 = BD \cdot CD$.

SOLUTION GIVEN A $\triangle ABC$ in which $AB = AC$ and D is a point on BC.

TO PROVE $(AB^2 - AD^2) = BD \cdot CD$.

CONSTRUCTION Draw $AL \perp BC$.



PROOF In right $\triangle$s ALB and ALC, we have:

hyp. AB = hyp. AC (given)

$AL = AL$ (common)

$\therefore$ $\triangle ALB \cong \triangle ALC$ [by RHS-congruence]

$\therefore$ $BL = CL$.

From right $\triangle$s ALB and ALD, by Pythagoras' theorem, we have:

$AB^2 = AL^2 + BL^2$ … (i)

$AD^2 = AL^2 + DL^2$ … (ii)

$\therefore$ $AB^2 - AD^2 = BL^2 - DL^2$

$= (BL - DL)(BL + DL)$

$= BD \cdot (CL + DL)$ [$\because$ $BL = CL$]

$= BD \cdot (CL + DL)$

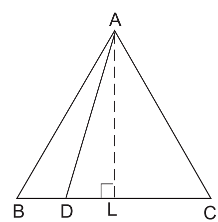
[$\because$ $BL - DL = BD$ and $BL = CL$]

$= BD \cdot CD$.

Hence, $AB^2 - AD^2 = BD \cdot CD$.

EXAMPLE 24 *In an equilateral* $\triangle ABC$, D *is a point on side* BC *such that* $BD = \frac{1}{3}BC$. *Prove that* $9AD^2 = 7AB^2$.

SOLUTION GIVEN A $\triangle ABC$ in which $AB = BC = CA$ and D is a point on BC such that $BD = \frac{1}{3}BC$.



TO PROVE $9AD^2 = 7AB^2$.

CONSTRUCTION Draw $AL \perp BC$.

PROOF In right triangles ALB and ALC, we have

$AB = AC$ (given) and $AL = AL$ (common).

$\therefore \quad \triangle ALB \cong \triangle ALC$ [by RHS axiom]

So, $BL = CL$.

Thus, $BD = \frac{1}{3}BC$ and $BL = \frac{1}{2}BC$.

In $\triangle ALB$, $\angle ALB = 90°$.

$\therefore \quad AB^2 = AL^2 + BL^2$. ... (i) [by Pythagoras' theorem]

In $\triangle ALD$, $\angle ALD = 90°$.

$\therefore \quad AD^2 = AL^2 + DL^2$ [by Pythagoras' theorem]

$= AL^2 + (BL - BD)^2$

$= AL^2 + BL^2 + BD^2 - 2BL \cdot BD$

$= (AL^2 + BL^2) + BD^2 - 2BL \cdot BD$

$= AB^2 + BD^2 - 2BL \cdot BD$ [using (i)]

$= BC^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{1}{2}BC\right) \cdot \frac{1}{3}BC$

$\left[\because AB = BC, BD = \frac{1}{3}BC \text{ and } BL = \frac{1}{2}BC\right]$

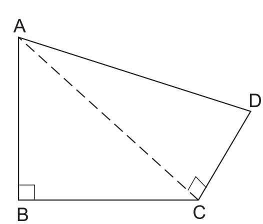
$= BC^2 + \frac{1}{9}BC^2 - \frac{1}{3}BC^2$

$= \frac{7}{9}BC^2 = \frac{7}{9}AB^2$ [$\because BC = AB$].

Hence, $9AD^2 = 7AB^2$.

EXAMPLE 25 *In a quadrilateral* $ABCD$, $\angle B = 90°$. *If* $AD^2 = AB^2 + BC^2 + CD^2$, *prove that* $\angle ACD = 90°$.

SOLUTION GIVEN A quad. $ABCD$ in which $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$.



TO PROVE $\angle ACD = 90°$.

CONSTRUCTION Join AC.

PROOF In $\triangle ABC$, $\angle B = 90°$.

$\therefore \quad AC^2 = AB^2 + BC^2$. ... (i) [by Pythagoras' theorem]

Now, $AD^2 = AB^2 + BC^2 + CD^2$ (given)

$\Rightarrow \quad AD^2 = AC^2 + CD^2$ [using (i)].

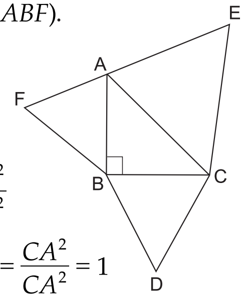
Thus, in $\triangle ACD$, we have $AD^2 = AC^2 + CD^2$.

Hence, $\angle ACD = 90^\circ$ [by converse of Pythagoras' theorem].

EXAMPLE 26 *Equilateral triangles are drawn on the sides of a right triangle. Prove that the area of the triangle on the hypotenuse is equal to the sum of the areas of the triangles on the other two sides.* [CBSE 2002]

SOLUTION GIVEN A $\triangle ABC$ in which $\angle B = 90^\circ$. Equilateral triangles $\triangle BCD$, $\triangle CAE$ and $\triangle ABF$ are drawn on the sides BC, CA and AB respectively.

TO PROVE $\text{ar}(\triangle CAE) = \text{ar}(\triangle BCD) + \text{ar}(\triangle ABF)$.



PROOF $\triangle BCD$, $\triangle CAE$ and $\triangle ABF$ are equiangular and hence similar.

We have

$$\frac{\text{ar}(\triangle BCD)}{\text{ar}(\triangle CAE)} + \frac{\text{ar}(\triangle ABF)}{\text{ar}(\triangle CAE)} = \frac{BC^2}{CA^2} + \frac{AB^2}{CA^2}$$

$$\Rightarrow \quad \frac{\text{ar}(\triangle BCD) + \text{ar}(\triangle ABF)}{\text{ar}(\triangle CAE)} = \frac{BC^2 + AB^2}{CA^2} = \frac{CA^2}{CA^2} = 1$$

[by Pythagoras' theorem]

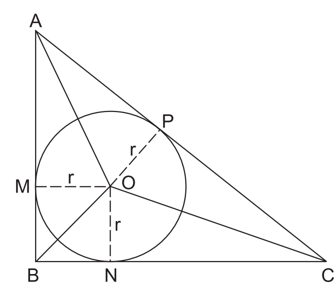
$$\Rightarrow \quad \text{ar}(\triangle BCD) + \text{ar}(\triangle ABF) = \text{ar}(\triangle CAE).$$

EXAMPLE 27 *Given a right-angled $\triangle ABC$. The lengths of the sides containing the right angle are* 6 cm *and* 8 cm. *A circle is inscribed in $\triangle ABC$. Find the radius of the circle.*

SOLUTION In $\triangle ABC$, we have

$\angle B = 90^\circ$, $AB = 6$ cm

and $BC = 8$ cm.



A circle is inscribed in $\triangle ABC$. Let O be its centre and M, N and P be the points where it touches the sides AB, BC and CA respectively. Then,

$$OM \perp AB, ON \perp BC, OP \perp CA.$$

Let r cm be the radius of the circle.

Then, $OM = ON = OP = r$ cm.

Now, $AB^2 + BC^2 = CA^2$ [by Pythagoras' theorem]

$$\Rightarrow \quad 6^2 \text{ cm}^2 + 8^2 \text{ cm}^2 = CA^2 \Rightarrow CA = 10 \text{ cm}.$$

Now, $\text{ar}(\triangle ABC) = \text{ar}(\triangle AOB) + \text{ar}(\triangle BOC) + \text{ar}(\triangle COA)$

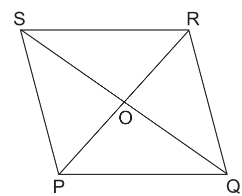
$\Rightarrow \quad \frac{1}{2} \times AB \times BC = \left(\frac{1}{2} \times AB \times OM\right) + \left(\frac{1}{2} \times BC \times ON\right) + \left(\frac{1}{2} \times CA \times OP\right)$

$\Rightarrow \quad \frac{1}{2} \times 6 \times 8 = \left(\frac{1}{2} \times 6 \times r\right) + \left(\frac{1}{2} \times 8 \times r\right) + \left(\frac{1}{2} \times 10 \times r\right)$

$\Rightarrow \quad r = 2 \Rightarrow$ radius = 2 cm.

EXAMPLE 28 *Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.*

SOLUTION GIVEN A parallelogram $PQRS$.

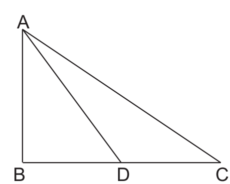


TO PROVE $PR^2 + QS^2 = PQ^2 + QR^2 + RS^2 + SP^2$.

PROOF We have proved earlier that if AD is a median of a $\triangle ABC$, then

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

i.e., $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$. … (i)



Now, let O be the point of intersection of the diagonal PR and QS.

The diagonals of a parallelogram bisect each other.

$\therefore$ O is the midpoint of PR as well as QS.

Applying result (i) to $\triangle PQR$ and $\triangle RSP$, we get

$PQ^2 + QR^2 = 2OQ^2 + \frac{1}{2}PR^2$ … (ii)

and $RS^2 + SP^2 = 2OS^2 + \frac{1}{2}PR^2$. … (iii)

Adding (ii) and (iii), we get

$$PQ^2 + QR^2 + RS^2 + SP^2 = 2\left(\frac{1}{2}QS\right)^2 + 2\left(\frac{1}{2}QS\right)^2 + PR^2$$

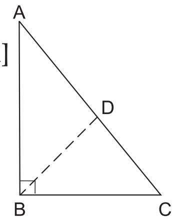
$\Rightarrow \quad PQ^2 + QR^2 + RS^2 + SP^2 = PR^2 + QS^2$.

EXAMPLE 29 *Given a* $\triangle ABC$ *in which* $\angle B = 90^\circ$ and $AB = \sqrt{3}BC$. *Prove that* $\angle C = 60^\circ$.

SOLUTION Let D be the midpoint of the hypotenuse AC.

Join BD.

We have



$AC^2 = AB^2 + BC^2$ [by Pythagoras' theorem]

$\Rightarrow\quad AC^2 = (\sqrt{3}\,BC)^2 + BC^2$

[$\because$ $AB = \sqrt{3}\,BC$ (given)]

$\Rightarrow\quad AC^2 = 4BC^2 \Rightarrow AC = 2BC$

$\Rightarrow\quad 2CD = 2BC$ [$\because$ D is the midpoint of AC]

$\Rightarrow\quad CD = BC.$ ... (i)

Also, we know that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

$\therefore\quad BD = CD$ ... (ii)

From (i) and (ii), we get

$BC = BD = CD.$

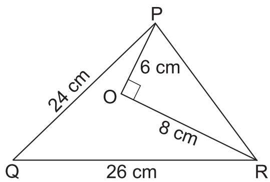
$\therefore\quad \triangle BCD$ is equilateral and hence $\angle C = 60°$.

EXERCISE 7D

1. The sides of certain triangles are given below. Determine which of them are right triangles.
   (i) 9 cm, 16 cm, 18 cm
   (ii) 7 cm, 24 cm, 25 cm
   (iii) 1.4 cm, 4.8 cm, 5 cm
   (iv) 1.6 cm, 3.8 cm, 4 cm
   (v) $(a-1)$ cm, $2\sqrt{a}$ cm, $(a+1)$ cm
2. A man goes 80 m due east and then 150 m due north. How far is he from the starting point?
3. A man goes 10 m due south and then 24 m due west. How far is he from the starting point?
4. A 13-m-long ladder reaches a window of a building 12 m above the ground. Determine the distance of the foot of the ladder from the building.
5. A ladder is placed in such a way that its foot is at a distance of 15 m from a wall and its top reaches a window 20 m above the ground. Find the length of the ladder.
6. Two vertical poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

7. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

8. In the given figure, O is a point inside a $\triangle PQR$ such that $\angle POR = 90^\circ$, $OP = 6$ cm and $OR = 8$ cm. If $PQ = 24$ cm and $QR = 26$ cm, prove that $\triangle PQR$ is right-angled. [CBSE 2006, '09C]



9. $\triangle ABC$ is an isosceles triangle with $AB = AC = 13$ cm. The length of altitude from A on BC is 5 cm. Find BC. [CBSE 2000C]

10. Find the length of altitude AD of an isosceles $\triangle ABC$ in which $AB = AC = 2a$ units and $BC = a$ units.

11. $\triangle ABC$ is an equilateral triangle of side $2a$ units. Find each of its altitudes.

12. Find the height of an equilateral triangle of side 12 cm.

13. Find the length of a diagonal of a rectangle whose adjacent sides are 30 cm and 16 cm.

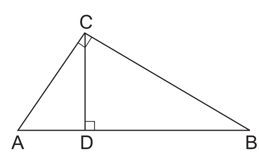
14. Find the length of each side of a rhombus whose diagonals are 24 cm and 10 cm long.

15. In $\triangle ABC$, D is the midpoint of BC and $AE \perp BC$. If $AC > AB$, show that

$$AB^2 = AD^2 - BC \cdot DE + \frac{1}{4}BC^2.$$ [CBSE 2006C]

16. In the given figure, $\angle ACB = 90^\circ$ and $CD \perp AB$.

Prove that $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$.



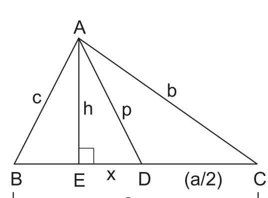
17. In the given figure, D is the midpoint of side BC and $AE \perp BC$. If $BC = a$, $AC = b$, $AB = c$, $ED = x$, $AD = p$ and $AE = h$, prove that

(i) $b^2 = p^2 + ax + \dfrac{a^2}{4}$

(ii) $c^2 = p^2 - ax + \dfrac{a^2}{4}$

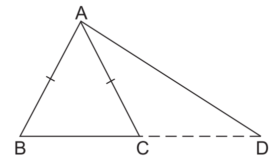
(iii) $(b^2 + c^2) = 2p^2 + \dfrac{1}{2}a^2$

(iv) $(b^2 - c^2) = 2ax$

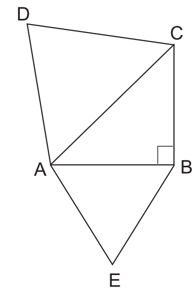


18. In $\triangle ABC$, $AB = AC$. Side BC is produced to D. Prove that

$$(AD^2 - AC^2) = BD \cdot CD.$$



19. ABC is an isosceles triangle, right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of $\triangle ABE$ and $\triangle ACD$. [CBSE 2002]



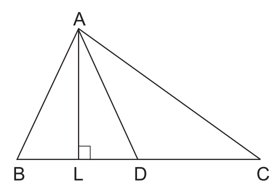
20. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

21. In a $\triangle ABC$, AD is a median and $AL \perp BC$. Prove that:

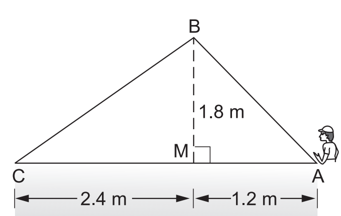
(a) $AC^2 = AD^2 + BC \cdot DL + \left(\frac{BC}{2}\right)^2$

(b) $AB^2 = AD^2 - BC \cdot DL + \left(\frac{BC}{2}\right)^2$

(c) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



22. Naman is doing fly-fishing in a stream. The tip of his fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away from him and 2.4 m from the point directly under the tip of the rod. Assuming that the string (from the tip of his rod to the fly) is taut, how much string does he have out (see the adjoining figure)? If he pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from him after 12 seconds?

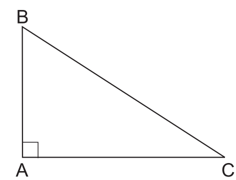


ANSWERS (EXERCISE 7D)

1. (ii), (iii), (v) **2.** 170 m **3.** 26 m **4.** 5 m **5.** 25 m
6. 13 m **7.** $6\sqrt{7}$ m **9.** 24 m **10.** $\frac{a\sqrt{15}}{2}$ units
11. $a\sqrt{3}$ units **12.** $6\sqrt{3}$ cm **13.** 34 m **14.** 13 cm **19.** 1 : 2
20. $300\sqrt{61}$ km **22.** 2.8 m (approx.)

HINTS TO SOME SELECTED QUESTIONS

7. Let AB be the vertical pole and C be the position of the stake so that BC is a taut wire.



In rt. $\triangle ABC$, $\angle A = 90°$, $AB = 18$ m, $BC = 24$ m.

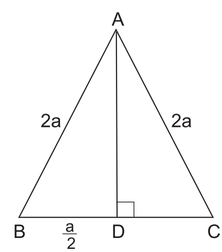
Now, $AC^2 + AB^2 = BC^2 \Rightarrow AC^2 = (24^2 - 18^2)\ m^2 = 252\ m^2$

$\Rightarrow AC = \sqrt{252}$ m $= 6\sqrt{7}$ m.

8. $PR = \sqrt{OP^2 + OR^2} = \sqrt{6^2 + 8^2} = 10.$

Now, $24^2 + 10^2 = 26^2$, i.e., $PQ^2 + PR^2 = QR^2 \Rightarrow \angle QPR = 90°$.

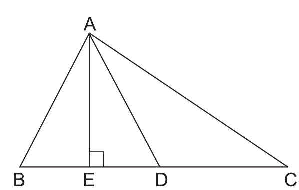
10. In $\triangle ABC$, $AB = AC = 2a$ and $BC = a$.



Draw $AD \perp BC$. Then, D is the midpoint of BC and so $BD = \frac{a}{2}\cdot$

$AD^2 = AB^2 - BD^2 = 4a^2 - \frac{a^2}{4} = \frac{15a^2}{4} \Rightarrow AD = \frac{a\sqrt{15}}{2}\cdot$

15. In $\triangle AEB$, $\angle AEB = 90°$.



$\therefore\quad AB^2 = AE^2 + BE^2$ … (i)

In $\triangle AED$, $\angle AED = 90°$.

$\therefore\quad AD^2 = (AE^2 + DE^2)$

$\Rightarrow\quad AE^2 = (AD^2 - DE^2).$

$\therefore\quad AB^2 = (AD^2 - DE^2) + BE^2$ [using (i)]

$= (AD^2 - DE^2) + (BD - DE)^2$

$= (AD^2 - DE^2) + \left(\frac{1}{2}BC - DE\right)^2$

$= AD^2 + \frac{1}{4}BC^2 - BC \cdot DE.$

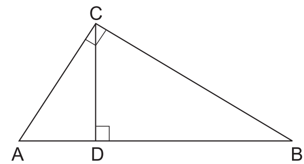
16. $\triangle ABC \sim \triangle CBD$

$\Rightarrow\quad \frac{BC}{BD} = \frac{AB}{CB} \Rightarrow BC^2 = AB \cdot BD.$ … (i)

$\triangle ABC \sim \triangle ACD$

$\Rightarrow \quad \dfrac{AC}{AD} = \dfrac{AB}{AC} \Rightarrow AC^2 = AB \cdot AD. \quad \dots \text{(ii)}$

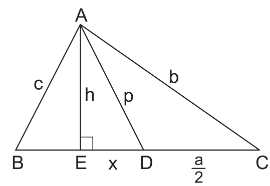
Dividing (i) by (ii), we get $\dfrac{BC^2}{AC^2} = \dfrac{BD}{AD}$.



17. (i) $AC^2 = AE^2 + EC^2$

$\Rightarrow \quad b^2 = h^2 + \left(x + \dfrac{a}{2}\right)^2 = (h^2 + x^2) + ax + \dfrac{a^2}{4}$

$= p^2 + ax + \dfrac{a^2}{4}$.



(ii) $AB^2 = AE^2 + BE^2$

$\Rightarrow \quad c^2 = h^2 + \left(\dfrac{a}{2} - x\right)^2 = (h^2 + x^2) - ax + \dfrac{a^2}{4}$

$= p^2 - ax + \dfrac{a^2}{4}$.

(iii) Add (i) and (ii).

(iv) Subtract (ii) from (i).

18. Draw $AE \perp BC$. Then, $BE = CE$.

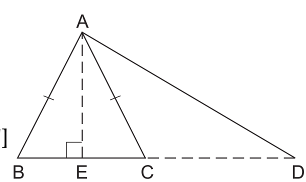
$AD^2 = AE^2 + DE^2$ and $AC^2 = AE^2 + CE^2$

$\Rightarrow \quad (AD^2 - AC^2) = DE^2 - CE^2$

$= (DE + CE)(DE - CE)$

$= (DE + BE)(DE - CE) \quad [\because CE = BE]$

$= BD \cdot CD$.



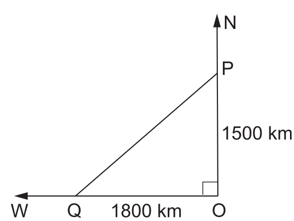
19. $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle ACD)} = \dfrac{AB^2}{AC^2} = \dfrac{AB^2}{2AB^2} = \dfrac{1}{2} \quad [\because AC^2 = AB^2 + BC^2 = 2AB^2]$.

20. Distance covered by first plane in $1\frac{1}{2}$ hours $= OP = 1500$ km (north).

Distance covered by second plane in $1\frac{1}{2}$ hours $= OQ = 1800$ km (west).

Distance between the two planes after $1\frac{1}{2}$ hours

$= PQ = \sqrt{OP^2 + OQ^2} = 300\sqrt{61}$ km.



21. (a) In right $\triangle ACL$, $AC^2 = AL^2 + LC^2$. $\dots$ (i)

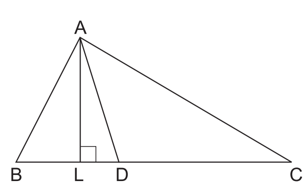
In right $\triangle ALD$, $AL^2 = AD^2 - DL^2$. $\dots$ (ii)

$\therefore \quad AC^2 = (AD^2 - DL^2) + LC^2$ [from (i) and (ii)]

$= (AD^2 - DL^2) + (DL + DC)^2$

$= (AD^2 - DL^2) + \left(DL + \dfrac{1}{2}BC\right)^2$

$= AD^2 + BC \cdot DL + \dfrac{1}{4}BC^2 = AD^2 + BC \cdot DL + \left(\dfrac{BC}{2}\right)^2$.



(b) In right $\triangle ABL$, $AB^2 = AL^2 + LB^2$. ... (iii)

In right $\triangle ALD$, $AL^2 = AD^2 - DL^2$.. ... (iv)

$\therefore \quad AB^2 = (AD^2 - DL^2) + LB^2$ [from (iii) and (iv)]

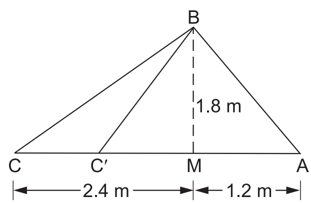
$= (AD^2 - DL^2) + (BD - DL)^2$

$= (AD^2 - DL^2) + \left(\frac{1}{2}BC - DL\right)^2 = AD^2 - BC \cdot DL + \left(\frac{BC}{2}\right)^2.$

(c) Adding the results of (a) and (b), we get

$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2.$$

22. Let of the string that was out of the rod



$= BC = \sqrt{BM^2 + CM^2} = \sqrt{(1.8)^2 + (2.4)^2}$ m

$= \sqrt{3.24 + 5.76}$ m $= \sqrt{9}$ m $= 3$ m.

He pulls the string at a rate of 5 cm per second.

$\therefore$ length of string pulled in 12 s

$= (5 \times 12)$ cm $= 60$ cm $= 0.6$ m.

So, after 12 s, we have $BC' = (3 - 0.6)$ m $= 2.4$ m and $BM = 1.8$ m.

$\therefore \quad C'M = \sqrt{(BC')^2 - BM^2} = \sqrt{(2.4)^2 - (1.8)^2}$ m $= \sqrt{2.52}$ m ≈ 1.6 m.

Horizontal distance of the fly from him after 12 s $= C'A$

$= C'M + MA = (1.6 + 1.2)$ m

$= 2.8$ m.

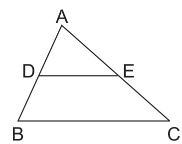
....

EXERCISE 7E

Very-Short-Answer and Short-Answer Questions:

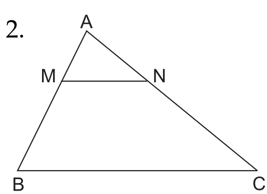
1. State the two properties which are necessary for given two triangles to be similar.
2. State the basic proportionality theorem.
3. State the converse of Thales' theorem.
4. State the midpoint theorem.
5. State the AAA-similarity criterion.
6. State the AA-similarity criterion.
7. State the SSS-criterion for similarity of triangles.
8. State the SAS-similarity criterion.
9. State Pythagoras' theorem.
10. State the converse of Pythagoras' theorem.
11. If D, E and F are respectively the midpoints of sides AB, BC and CA of $\triangle ABC$ then what is the ratio of the areas of $\triangle DEF$ and $\triangle ABC$?

12. Two triangles ABC and PQR are such that $AB = 3$ cm, $AC = 6$ cm, $\angle A = 70^\circ$, $PR = 9$ cm, $\angle P = 70^\circ$ and $PQ = 4.5$ cm. Show that $\triangle ABC \sim \triangle PQR$ and state the similarity criterion.
13. If $\triangle ABC \sim \triangle DEF$ such that $2AB = DE$ and $BC = 6$ cm, find EF.
14. In the given figure, $DE \parallel BC$ such that $AD = x$ cm, $DB = (3x + 4)$ cm, $AE = (x + 3)$ cm and $EC = (3x + 19)$ cm. Find the value of x.



15. A ladder 10 m long reaches the window of a house 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.
16. Find the length of the altitude of an equilateral triangle of side $2a$ cm.
17. $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = 64 \text{ cm}^2$ and $\text{ar}(\triangle DEF) = 169 \text{ cm}^2$. If $BC = 4$ cm, find EF.
18. In a trapezium $ABCD$, it is given that $AB \parallel CD$ and $AB = 2CD$. Its diagonals AC and BD intersect at the point O such that $\text{ar}(\triangle AOB) = 84 \text{ cm}^2$. Find $\text{ar}(\triangle COD)$.
19. The corresponding sides of two similar triangles are in the ratio 2 : 3. If the area of the smaller triangle is 48 cm^2, find the area of the larger triangle.
20. In an equilateral triangle with side a, prove that area $= \dfrac{\sqrt{3}}{4}a^2$.
21. Find the length of each side of a rhombus whose diagonals are 24 cm and 10 cm long.
22. Two triangles DEF and GHK are such that $\angle D = 48^\circ$ and $\angle H = 57^\circ$. If $\triangle DEF \sim \triangle GHK$ then find the measure of $\angle F$.
23. In the given figure, $MN \parallel BC$ and $AM : MB = 1 : 2$.

    Find $\dfrac{\text{area}(\triangle AMN)}{\text{area}(\triangle ABC)}$.

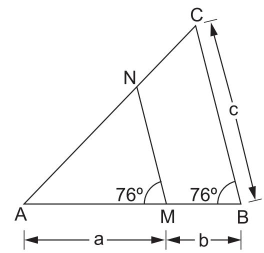


24. In triangles BMP and CNR it is given that $PB = 5$ cm, $MP = 6$ cm, $BM = 9$ cm and $NR = 9$ cm. If $\triangle BMP \sim \triangle CNR$ then find the perimeter of $\triangle CNR$.
25. Each of the equal sides of an isosceles triangle is 25 cm. Find the length of its altitude if the base is 14 cm.

26. A man goes 12 m due south and then 35 m due west. How far is he from the starting point?

27. If the lengths of the sides *BC*, *CA* and *AB* of a $\triangle ABC$ are *a*, *b* and *c* respectively and *AD* is the bisector of $\angle A$ then find the lengths of *BD* and *DC*.

28. In the given figure, $\angle AMN = \angle MBC = 76°$. If *p*, *q* and *r* are the lengths of *AM*, *MB* and *BC* respectively then express the length of *MN* in terms of *p*, *q* and *r*.



29. The lengths of the diagonals of a rhombus are 40 cm and 42 cm. Find the length of each side of the rhombus.

For each of the following statements state whether true (T) or false (F):

30. (i) Two circles with different radii are similar.

(ii) Any two rectangles are similar.

(iii) If two triangles are similar then their corresponding angles are equal and their corresponding sides are equal.

(iv) The length of the line segment joining the midpoints of any two sides of a triangle is equal to half the length of the third side.

(v) In a $\triangle ABC$, $AB = 6$ cm, $\angle A = 45°$ and $AC = 8$ cm and in a $\triangle DEF$, $DF = 9$ cm, $\angle D = 45°$ and $DE = 12$ cm, then $\triangle ABC \sim \triangle DEF$.

(vi) The polygon formed by joining the midpoints of the sides of a quadrilateral is a rhombus.

(vii) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding angle-bisector segments.

(viii) The ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding medians.

(ix) If *O* is any point inside a rectangle *ABCD* then $OA^2 + OC^2 = OB^2 + OD^2$.

(x) The sum of the squares on the sides of a rhombus is equal to the sum of the squares on its diagonals.

ANSWERS (EXERCISE 7E)

11. 1 : 4 **12.** SAS-similarity **13.** 12 cm **14.** $x = 2$ **15.** 6 m

16. $\sqrt{3}a$ cm **17.** 6.5 cm **18.** 21 cm^2 **19.** 108 cm^2 **21.** 13 cm

22. 75° **23.** $\frac{1}{9}$ **24.** 30 cm **25.** 24 cm **26.** 37 cm

27. $BD = \frac{ac}{b+c}; DC = \frac{ab}{b+c}$ **28.** $MN = \frac{ac}{a+b}$ **29.** 29 cm

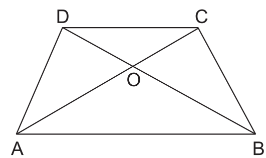
30. (i) T (ii) F (iii) F (iv) T (v) F (vi) F (vii) T (viii) T (ix) T (x) T

HINTS TO SOME SELECTED QUESTIONS

14. $\frac{AD}{DB} = \frac{AE}{EC}$ [by Thales' theorem]

17. $\frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \frac{AB^2}{CD^2} = \left(\frac{AB}{CD}\right)^2 = \frac{4}{1}$ [$\because \triangle AOB \sim \triangle COD$]

$\Rightarrow \quad \text{ar}(\triangle COD) = \frac{1}{4} \times \text{ar}(\triangle AOB)$.



22. $\angle D = 48°, \angle E = \angle H = 57°$ [$\because \triangle DEF \sim \triangle GHK$]

$\therefore \quad \angle F = 180° - (\angle D + \angle E) = 180° - (48° + 57°) = 75°$.

23. $MN \parallel BC \Rightarrow \triangle AMN \sim \triangle ABC$.

$\therefore \quad \frac{\text{area}(\triangle AMN)}{\text{area}(\triangle ABC)} = \frac{AM^2}{AB^2} = \left(\frac{AM}{AM+MB}\right)^2 = \left(\frac{x}{x+2x}\right)^2 = \frac{1}{9}$

[$AM : MB = 1 : 2$, so, let $AM = x$, then $MB = 2x$].

24. $\frac{\text{Perimeter of } \triangle BMP}{\text{Perimeter of } \triangle CNR} = \frac{MP}{NR}$ [$\because \triangle BMP \sim \triangle CNR$]

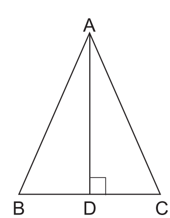
$\Rightarrow \quad \text{perimeter of } \triangle CNR = \frac{NR}{MP} \times \text{perimeter of } \triangle BMP = \frac{NR}{MP} \times (PB + MP + BM)$

$= \frac{9}{6} \times (5 + 6 + 9)$ cm $= 30$ cm.

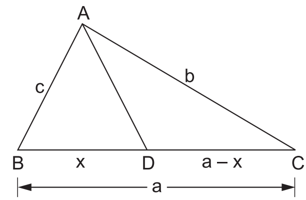
25. Let the given triangle be $\triangle ABC$ having $AB = AC = 25$ cm, base $BC = 14$ cm.

Let $AD \perp BC$. Then, D is the midpoint of BC.

Altitude $AD = \sqrt{AC^2 - DC^2} = \sqrt{25^2 - 7^2}$ cm $= 24$ cm.



27. Let $BD = x$. Then, $DC = BC - BD = a - x$.



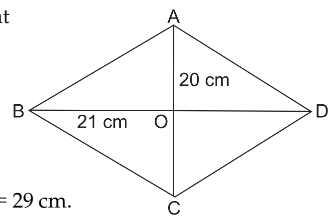
Now, $\frac{BD}{DC} = \frac{AB}{AC}$ [by angle-bisector theorem]

$\Rightarrow \quad \frac{x}{a-x} = \frac{c}{b} \Rightarrow x = \frac{ac}{b+c}$ and so, $a - x = \frac{ab}{b+c}$.

28. $\angle AMN = \angle MBC \Rightarrow MN \parallel BC \Rightarrow \triangle AMN \sim \triangle ABC$.

$\therefore \quad \frac{AM}{AB} = \frac{MN}{BC} \Rightarrow \frac{a}{a+b} = \frac{MN}{c} \Rightarrow MN = \frac{ac}{a+b}$.

29. The diagonals of a rhombus intersect at right angles and bisect each other.



In the figure,

$OA = \frac{40}{2}$ cm = 20 cm, $OB = \frac{42}{2}$ cm = 21 cm.

$AB = \sqrt{OA^2 + OB^2} = \sqrt{20^2 + 21^2}$ cm $= \sqrt{841}$ cm = 29 cm.

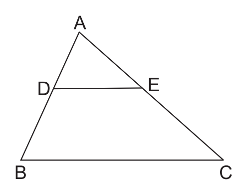
30. (ii) Two rectangles are similar only if their corresponding sides are proportional.

(iii) If two triangles are similar, then

(a) their corresponding angles are equal

(b) their corresponding sides are proportional (but not necessarily equal)

(iv) Let D and E be the midpoints of sides AB and AC respectively of a $\triangle ABC$.



Then, by the midpoint theorem,

$DE \parallel BC \Rightarrow \triangle ADE \sim \triangle ABC$.

$\therefore \quad \frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{1}{2} = \frac{DE}{BC} \Rightarrow DE = \frac{1}{2}BC$.

(v) We have $\frac{AB}{DE} = \frac{6}{12} = \frac{1}{2}, \ \frac{AC}{DF} = \frac{8}{9}$.

Clearly, $\frac{AB}{DE} \neq \frac{AC}{DF}$ and so $\triangle ABC$ is not similar to $\triangle DEF$.

Note Here $\triangle ABC \sim \triangle DFE$.

(vi) The polygon formed by joining the midpoints of the sides of a quadrilateral is a parallelogram (not necessarily a rhombus).

(vii) The ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides which is the same as the ratio of the corresponding medians.

....

MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. A man goes 24 m due west and then 10 m due north. How far is he from the starting point?

   (a) 34 m (b) 17 m (c) 26 m (d) 28 m

2. Two poles of height 13 m and 7 m respectively stand vertically on a plane ground at a distance of 8 m from each other. The distance between their tops is

   (a) 9 m (b) 10 m (c) 11 m (d) 12 m

3. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high?

   (a) 2.4 m (b) 1.35 m (c) 1.5 m (d) 13.5 m

4. A vertical pole 6 m long casts a shadow of length 3.6 m on the ground. What is the height of a tower which casts a shadow of length 18 m at the same time?

   (a) 10.8 m (b) 28.8 m (c) 32.4 m (d) 30 m

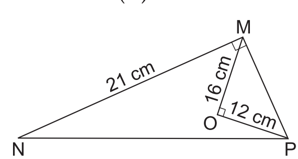
5. The shadow of a 5-m-long stick is 2 m long. At the same time the length of the shadow of a 12.5-m-high tree (in m) is [CBSE 2011]

   (a) 3.0 (b) 3.5 (c) 4.5 (d) 5.0

6. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of the ladder from the building?

   (a) 7 m (b) 14 m (c) 21 m (d) 24.5 m

7. In the given figure, O is a point inside a $\triangle MNP$ such that $\angle MOP = 90^\circ$, $OM = 16$ cm and $OP = 12$ cm. If $MN = 21$ cm and $\angle NMP = 90^\circ$ then $NP = ?$



   (a) 25 cm (b) 29 cm (c) 33 cm (d) 35 cm

8. The hypotenuse of a right triangle is 25 cm. The other two sides are such that one is 5 cm longer than the other. The lengths of these sides are

   (a) 10 cm, 15 cm (b) 15 cm, 20 cm

   (c) 12 cm, 17 cm (d) 13 cm, 18 cm

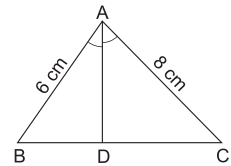
9. The height of an equilateral triangle having each side 12 cm, is

   (a) $6\sqrt{2}$ cm (b) $6\sqrt{3}$ cm (c) $3\sqrt{6}$ cm (d) $6\sqrt{6}$ cm

10. $\triangle ABC$ is an isosceles triangle with $AB = AC = 13$ cm and the length of altitude from A on BC is 5 cm. Then, $BC = ?$

(a) 12 cm (b) 16 cm (c) 18 cm (d) 24 cm

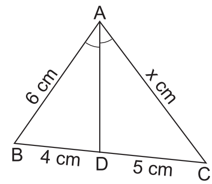
11. In a $\triangle ABC$ it is given that $AB = 6$ cm, $AC = 8$ cm and AD is the bisector of $\angle A$. Then, $BD : DC = ?$



(a) $3 : 4$ (b) $9 : 16$

(c) $4 : 3$ (d) $\sqrt{3} : 2$

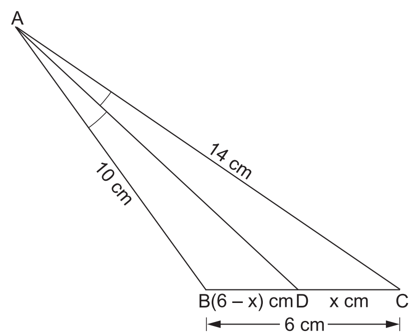
12. In a $\triangle ABC$ it is given that AD is the internal bisector of $\angle A$. If $BD = 4$ cm, $DC = 5$ cm and $AB = 6$ cm, then $AC = ?$



(a) 4.5 cm (b) 8 cm

(c) 9 cm (d) 7.5 cm

13. In a $\triangle ABC$, it is given that AD is the internal bisector of $\angle A$. If $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm, then $CD = ?$



(a) 4.8 cm

(b) 3.5 cm

(c) 7 cm

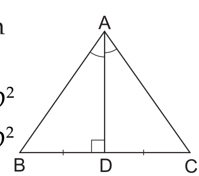
(d) 10.5 cm

14. In a triangle, the perpendicular from the vertex to the base bisects the base. The triangle is

(a) right-angled (b) isosceles

(c) scalene (d) obtuse-angled

15. In an equilateral triangle ABC, if $AD \perp BC$ then which of the following is true?



(a) $2AB^2 = 3AD^2$ (b) $4AB^2 = 3AD^2$

(c) $3AB^2 = 4AD^2$ (d) $3AB^2 = 2AD^2$

16. In a rhombus of side 10 cm, one of the diagonals is 12 cm long. The length of the second diagonal is

(a) 20 cm (b) 18 cm (c) 16 cm (d) 22 cm

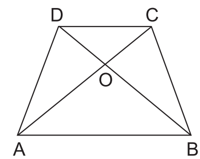
17. The lengths of the diagonals of a rhombus are 24 cm and 10 cm. The length of each side of the rhombus is

(a) 12 cm (b) 13 cm (c) 14 cm (d) 17 cm

18. If the diagonals of a quadrilateral divide each other proportionally then it is a

(a) parallelogram (b) trapezium
(c) rectangle (d) square

19. In the given figure, $ABCD$ is a trapezium whose diagonals AC and BD intersect at O such that $OA = (3x - 1)$ cm, $OB = (2x + 1)$ cm, $OC = (5x - 3)$ cm and $OD = (6x - 5)$ cm. Then, $x = ?$



(a) 2 (b) 3 (c) 2.5 (d) 4

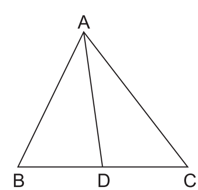
20. The line segments joining the midpoints of the adjacent sides of a quadrilateral form

(a) a parallelogram (b) a rectangle
(c) a square (d) a rhombus

21. If the bisector of an angle of a triangle bisects the opposite side then the triangle is

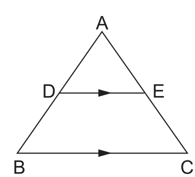
(a) scalene (b) equilateral
(c) isosceles (d) right-angled

22. In $\triangle ABC$ it is given that $\frac{AB}{AC} = \frac{BD}{DC}\cdot$ If $\angle B = 70^\circ$ and $\angle C = 50^\circ$ then $\angle BAD = ?$



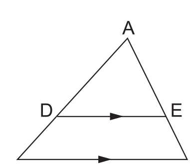
(a) 30° (b) 40°
(c) 45° (d) 50°

23. In $\triangle ABC$, $DE \parallel BC$ so that $AD = 2.4$ cm, $AE = 3.2$ cm and $EC = 4.8$ cm. Then, $AB = ?$



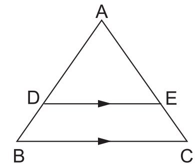
(a) 3.6 cm (b) 6 cm
(c) 6.4 cm (d) 7.2 cm

24. In a $\triangle ABC$, if DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that $AB = 7.2$ cm, $AC = 6.4$ cm and $AD = 4.5$ cm. Then, $AE = ?$



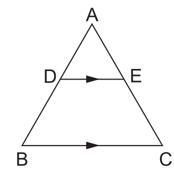
(a) 5.4 cm (b) 4 cm
(c) 3.6 cm (d) 3.2 cm

25. In $\triangle ABC$, $DE \parallel BC$ so that $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x + 4)$ cm and $EC = 3x$ cm. Then, we have



(a) $x = 3$ (b) $x = 5$
(c) $x = 4$ (d) $x = 2.5$

26. In $\triangle ABC$, $DE \parallel BC$ such that $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm then $AE = ?$



(a) 4.2 cm (b) 3.1 cm
(c) 2.8 cm (d) 2.1 cm

27. $\triangle ABC \sim \triangle DEF$ and the perimeters of $\triangle ABC$ and $\triangle DEF$ are 30 cm and 18 cm respectively. If $BC = 9$ cm then $EF = ?$

(a) 6.3 cm (b) 5.4 cm (c) 7.2 cm (d) 4.5 cm

28. $\triangle ABC \sim \triangle DEF$ such that $AB = 9.1$ cm and $DE = 6.5$ cm. If the perimeter of $\triangle DEF$ is 25 cm, what is the perimeter of $\triangle ABC$?

(a) 35 cm (b) 28 cm (c) 42 cm (d) 40 cm

29. In $\triangle ABC$, it is given that $AB = 9$ cm, $BC = 6$ cm and $CA = 7.5$ cm. Also, $\triangle DEF$ is given such that $EF = 8$ cm and $\triangle DEF \sim \triangle ABC$. Then, perimeter of $\triangle DEF$ is

(a) 22.5 cm (b) 25 cm (c) 27 cm (d) 30 cm

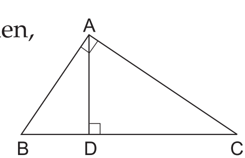
30. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. Ratio of the areas of triangles ABC and BDE is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

31. It is given that $\triangle ABC \sim \triangle DFE$. If $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm then which of the following is true?

(a) $DE = 12$ cm, $\angle F = 50^\circ$ (b) $DE = 12$ cm, $\angle F = 100^\circ$
(c) $EF = 12$ cm, $\angle D = 100^\circ$ (d) $EF = 12$ cm, $\angle D = 30^\circ$

32. In the given figure, $\angle BAC = 90^\circ$ and $AD \perp BC$. Then,



(a) $BC \cdot CD = BC^2$
(b) $AB \cdot AC = BC^2$
(c) $BD \cdot CD = AD^2$
(d) $AB \cdot AC = AD^2$

33. In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm. Then, $\angle B$ is

(a) 45° (b) 60° (c) 90° (d) 120°

34. In $\triangle ABC$ and $\triangle DEF$, it is given that $\frac{AB}{DE} = \frac{BC}{FD}$ then

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

35. In $\triangle DEF$ and $\triangle PQR$, it is given that $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

36. If $\triangle ABC \sim \triangle EDF$ and $\triangle ABC$ is not similar to $\triangle DEF$ then which of the following is not true?

(a) $BC \cdot EF = AC \cdot FD$ (b) $AB \cdot EF = AC \cdot DE$
(c) $BC \cdot DE = AB \cdot EF$ (d) $BC \cdot DE = AB \cdot FD$

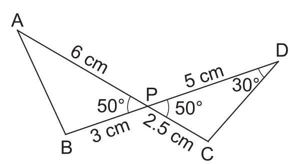
37. In $\triangle ABC$ and $\triangle DEF$, it is given that $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$, then the two triangles are

(a) congruent but not similar (b) similar but not congruent
(c) neither congruent nor similar (d) similar as well as congruent

38. If in $\triangle ABC$ and $\triangle PQR$, we have $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then

(a) $\triangle PQR \sim \triangle CAB$ (b) $\triangle PQR \sim \triangle ABC$
(c) $\triangle CBA \sim \triangle PQR$ (d) $\triangle BCA \sim \triangle PQR$

39. In the given figure, two line segments AC and BD intersect each other at the point P such that $PA = 6$ cm, $PB = 3$ cm, $PC = 2.5$ cm, $PD = 5$ cm, $\angle APB = 50^\circ$ and $\angle CDP = 30^\circ$ then $\angle PBA = ?$



(a) 50° (b) 30° (c) 60° (d) 100°

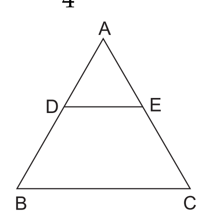
40. Corresponding sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(a) 2 : 3 (b) 4 : 9 (c) 9 : 4 (d) 16 : 81

41. It is given that $\triangle ABC \sim \triangle PQR$ and $\frac{BC}{QR} = \frac{2}{3}$ then $\frac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle ABC)} = ?$

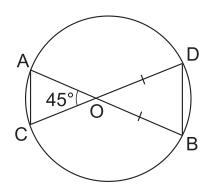
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

42. In an equilateral $\triangle ABC$, D is the midpoint of AB and E is the midpoint of AC. Then, $\text{ar}(\triangle ABC) : \text{ar}(\triangle ADE) = ?$



(a) 2 : 1 (b) 4 : 1
(c) 1 : 2 (d) 1 : 4

43. In $\triangle ABC$ and $\triangle DEF$, we have $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then $ar(\triangle ABC) : ar(\triangle DEF) = ?$
    (a) 5 : 7 (b) 25 : 49 (c) 49 : 25 (d) 125 : 343
44. $\triangle ABC \sim \triangle DEF$ such that $ar(\triangle ABC) = 36\text{ cm}^2$ and $ar(\triangle DEF) = 49\text{ cm}^2$. Then, the ratio of their corresponding sides is
    (a) 36 : 49 (b) 6 : 7 (c) 7 : 6 (d) $\sqrt{6} : \sqrt{7}$
45. Two isosceles triangles have their corresponding angles equal and their areas are in the ratio 25 : 36. The ratio of their corresponding heights is
    (a) 25 : 36 (b) 36 : 25 (c) 5 : 6 (d) 6 : 5
46. The line segments joining the midpoints of the sides of a triangle form four triangles, each of which is
    (a) congruent to the original triangle
    (b) similar to the original triangle
    (c) an isosceles triangle
    (d) an equilateral triangle
47. If $\triangle ABC \sim \triangle QRP$, $\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{9}{4}$, $AB = 18$ cm and $BC = 15$ cm then $PR = ?$
    (a) 8 cm (b) 10 cm (c) 12 cm (d) $\frac{20}{3}$ cm
48. In the given figure, O is the point of intersection of two chords AB and CD such that $OB = OD$ and $\angle AOC = 45°$. Then, $\triangle OAC$ and $\triangle ODB$ are



    (a) equilateral and similar
    (b) equilateral but not similar
    (c) isosceles and similar
    (d) isosceles but not similar
49. In an isosceles $\triangle ABC$, if $AC = BC$ and $AB^2 = 2AC^2$ then $\angle C = ?$
    (a) 30° (b) 45° (c) 60° (d) 90°
50. In $\triangle ABC$, if $AB = 16$ cm, $BC = 12$ cm and $AC = 20$ cm, then $\triangle ABC$ is
    (a) acute-angled (b) right-angled
    (c) obtuse-angled (d) not possible

True/False Type

51. Which of the following is a true statement?
    (a) Two similar triangles are always congruent.
    (b) Two figures are similar if they have the same shape and size.

(c) Two triangles are similar if their corresponding sides are proportional.

(d) Two polygons are similar if their corresponding sides are proportional.

52. Which of the following is a false statement?

(a) If the areas of two similar triangles are equal then the triangles are congruent.

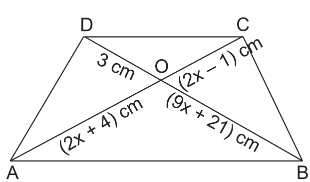
(b) The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

(c) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding medians.

(d) The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

Matching of columns

53. Match the following columns:

Column I	Column II
(a) In a given $\triangle ABC, DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm then $AE = \ldots\ldots$ cm.	(p) 6
(b) If $\triangle ABC \sim \triangle DEF$ such that $2AB = 3DE$ and $BC = 6$ cm then $EF = \ldots\ldots$ cm.	(q) 4
(c) If $\triangle ABC \sim \triangle PQR$ such that $\text{ar}(\triangle ABC) : \text{ar}(\triangle PQR) = 9 : 16$ and $BC = 4.5$ cm then $QR = \ldots\ldots$ cm.	(r) 3
(d) In the given figure, $AB \parallel CD$ and $OA = (2x + 4)$ cm, $OB = (9x - 21)$ cm, $OC = (2x - 1)$ cm and $OD = 3$ cm. Then $x = ?$ 	(s) 2.1

The correct answer is

(a) —……, (b) —……, (c) —……, (d) —……

54. Match the following columns:

Column I	Column II
(a) A man goes 10 m due east and then 20 m due north. His distance from the starting point is …… m.	(p) $25\sqrt{3}$
(b) In an equilateral triangle with each side 10 cm, the altitude is …… cm.	(q) $5\sqrt{3}$
(c) The area of an equilateral triangle having each side 10 cm is …… cm².	(r) $10\sqrt{5}$
(d) The length of diagonal of a rectangle having length 8 m and breadth 6 m is …… m.	(s) 10

The correct answer is

(a) —……, (b) —……,
(c) —……, (d) —……

ANSWERS (MCQ)

1. (c) **2.** (b) **3.** (c) **4.** (d) **5.** (d) **6.** (a) **7.** (b) **8.** (b) **9.** (b)
10. (d) **11.** (a) **12.** (d) **13.** (b) **14.** (b) **15.** (c) **16.** (c) **17.** (b) **18.** (b)
19. (a) **20.** (a) **21.** (c) **22.** (a) **23.** (b) **24.** (b) **25.** (c) **26.** (d) **27.** (b)
28. (a) **29.** (d) **30.** (d) **31.** (b) **32.** (c) **33.** (c) **34.** (c) **35.** (b) **36.** (c)
37. (b) **38.** (a) **39.** (d) **40.** (d) **41.** (d) **42.** (b) **43.** (b) **44.** (b) **45.** (c)
46. (b) **47.** (b) **48.** (c) **49.** (d) **50.** (b) **51.** (c) **52.** (b)
53. (a)–(s), (b)–(q), (c)–(p), (d)–(r) **54.** (a)–(r), (b)–(q), (c)–(p), (d)–(s)

HINTS TO SOME SELECTED QUESTIONS

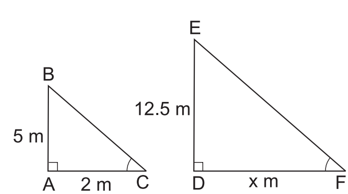
5. Let AB be the stick and AC be its shadow.

Let DE be the tree and DF be its shadow.

$\triangle ABC \sim \triangle DEF$.

$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{5}{12.5} = \frac{2}{x}$

$\Rightarrow \quad x = \left(\frac{12.5 \times 2}{5}\right) = 5.$

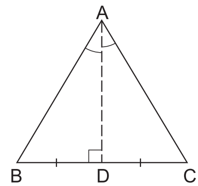


11. $\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$ [by angle-bisector theorem]

15. $AB^2 = BD^2 + AD^2$

$$= \left(\frac{1}{2}AB\right)^2 + AD^2 = \frac{1}{4}AB^2 + AD^2$$

$\Rightarrow \quad \frac{3}{4}AB^2 = AD^2 \Rightarrow 3AB^2 = 4AD^2.$



19. We know that the diagonals of a trapezium divide each other proportionally.

$\therefore \quad \frac{OA}{OC} = \frac{OB}{OD} \Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$

$\Rightarrow \quad (3x-1)(6x-5) = (5x-3)(2x+1)$

$\Rightarrow \quad 18x^2 - 21x + 5 = 10x^2 - x - 3 \Rightarrow 8x^2 - 20x + 8 = 0$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0 \Rightarrow 2x^2 - 4x - x + 2 = 0 \Rightarrow 2x(x-2) - (x-2) = 0$

$\Rightarrow \quad (x-2)(2x-1) = 0 \Rightarrow x = 2 \text{ or } x = \frac{1}{2}\cdot$

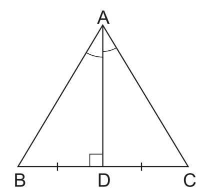
But, $x = \frac{1}{2}$ gives $(6x-5) < 0$ and the distance cannot be negative.

$\therefore \quad x = 2.$

21. Let AD be the bisector of $\angle A$ of $\triangle ABC$ such that $BD = DC$.

Then, $\frac{AB}{AC} = \frac{BD}{DC} = 1 \Rightarrow AB = AC.$

So, the given triangle is isosceles.



22. $\angle A = 180° - (70° + 50°) = 60°.$

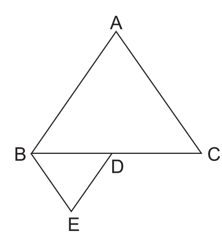
Since $\frac{BD}{DC} = \frac{AB}{AC}$, it means AD is the bisector of $\angle A$.

$\therefore \quad \angle BAD = \left(\frac{1}{2} \times 60°\right) = 30°.$

30. $\triangle ABC \sim \triangle BDE \quad [\because$ both are equilateral].

Also, $BD = \frac{1}{2}BC = \frac{1}{2}AB \quad [\because D$ is the midpoint of BC].

Now, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle BDE)} = \frac{AB^2}{BD^2} = \frac{AB^2}{\left(\frac{1}{2}AB\right)^2} = \frac{4}{1}\cdot$



31. $\angle B = 180° - (30° + 50°) = 100°.$

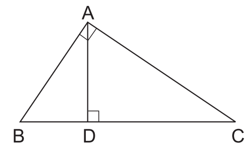
Since $\triangle ABC \sim \triangle DFE$, we have $\angle D = \angle A = 30°$, $\angle F = \angle B = 100°$ and $\angle E = \angle C = 50°$. Let $DE = x$ cm. Then,

$$\frac{AB}{DF} = \frac{AC}{DE} \Rightarrow \frac{5}{7.5} = \frac{8}{x}$$

$\therefore \quad 5x = 8 \times 7.5 \Rightarrow x = \frac{8 \times 7.5}{5} = 12.$

Hence, $DE = 12$ cm and $\angle F = 100°$.

32. In $\triangle DBA$ and $\triangle DAC$, we have



$\angle ADB = \angle CDA = 90°$, $\angle ABD = \angle CAD = 90° - \angle C$

and $\angle BAD = \angle ACD = 90° - \angle B$.

$\therefore$ $\triangle DBA \sim \triangle DAC \Rightarrow \dfrac{BD}{AD} = \dfrac{AD}{CD}$

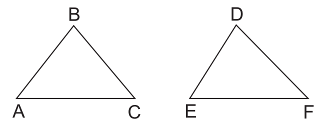
$\Rightarrow BD \cdot CD = AD^2$.

33. In $\triangle ABC$, AC is the longest side.

$AB^2 + BC^2 = \{(6\sqrt{3})^2 + 6^2\}\text{ cm}^2 = (108 + 36)\text{ cm}^2 = 144\text{ cm}^2 = (12\text{ cm})^2 = AC^2$.

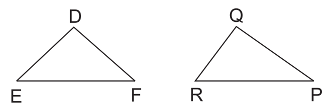
$\therefore$ by the converse of Pythagoras' theorem, we have $\angle B = 90°$.

34. Clearly, $B \leftrightarrow D$, $A \leftrightarrow E$ and $C \leftrightarrow F$



$\therefore$ $\angle B = \angle D$.

35. $\angle D = \angle Q$, $\angle E = \angle R$ and $\angle F = \angle P$.



$\therefore$ $\triangle DEF \sim \triangle QRP \Rightarrow \dfrac{DE}{QR} = \dfrac{DF}{QP} = \dfrac{EF}{RP}$.

$\therefore$ $\dfrac{DE}{PQ} = \dfrac{EF}{RP}$ is not true.

36. Since $\triangle ABC \sim \triangle EDF$ and $\triangle ABC$ is not similar to $\triangle DEF$, so

$\dfrac{AB}{DE} \neq \dfrac{BC}{EF} \Rightarrow BC \cdot DE = AB \cdot EF$ is not true.

37. $\triangle ABC \sim \triangle DEF$ [by AA-similarity].

But $\triangle ABC$ and $\triangle DEF$ are not congruent since their corresponding sides AB and DE are not equal.

38. $\dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{CA}{PQ} \Rightarrow B \leftrightarrow R$, $A \leftrightarrow Q$ and $C \leftrightarrow P$.

$\therefore$ $\triangle PQR \sim \triangle CAB$.

41. $\dfrac{\text{ar}(\triangle PQR)}{\text{ar}(\triangle ABC)} = \dfrac{QR^2}{BC^2} = \left(\dfrac{QR}{BC}\right)^2 = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$.

42. Clearly, $\dfrac{AD}{AB} = \dfrac{AE}{AC} = \dfrac{1}{2}$ and $\angle A = \angle A$ (common).

$\therefore$ $\triangle ABC \sim \triangle ADE$ [by SAS-similarity].

$\therefore$ $\text{ar}(\triangle ABC) : \text{ar}(\triangle ADE) = \left(\dfrac{AB}{AD}\right)^2 = \left(\dfrac{2}{1}\right)^2 = \dfrac{4}{1} = 4:1$.

44. $\left(\dfrac{AB}{DE}\right)^2 = \dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \dfrac{36}{49} = \left(\dfrac{6}{7}\right)^2 \Rightarrow \dfrac{AB}{DE} = \dfrac{6}{7}$.

$\therefore$ the ratio of the corresponding sides is 6 : 7.

45. The two triangles have corresponding angles equal and so they are similar.

$\therefore$ the ratio of their areas is equal to the ratio of the squares of their corresponding sides but the ratio of their corresponding sides is equal to the ratio of their corresponding altitudes (or heights).

So, the ratio of their areas is equal to the ratio of the squares of their heights.

Now, ratio of their areas $= \frac{25}{36} = \frac{5^2}{6^2}$

$\Rightarrow$ ratio of their heights $= \frac{5}{6}$.

46. $\text{ar}(\triangle QRP) = \text{ar}(\triangle PQR)$.

$\therefore \quad \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle QRP)} = \frac{9}{4} = \frac{3^2}{2^2} \Rightarrow \frac{BC}{RP} = \frac{3}{2} \quad [\because \triangle ABC \sim \triangle QRP]$

$\Rightarrow PR = \frac{2}{3} \times BC = \frac{2}{3} \times 15 \text{ cm} = 10 \text{ cm}.$

48. In $\triangle OAC$ and $\triangle ODB$, we have

$\angle AOC = \angle DOB$ (ver. opp. $\angle$s) and $\angle OAC = \angle ODB$ ($\angle$s in the same segment).

$\therefore \quad \triangle OAC \sim \triangle ODB \Rightarrow \frac{OC}{OB} = \frac{OA}{OD} = \frac{AC}{BD}$

$\therefore \quad \frac{OC}{OA} = \frac{OB}{OD} = 1 \Rightarrow OC = OA \quad [\because OB = OD \text{ (given)}].$

Clearly, $OA \neq OD$.

$\therefore \quad \frac{OA}{OD} \neq 1 \Rightarrow \frac{AC}{BD} \neq 1 \Rightarrow AC \neq BD.$

$\therefore$ $\triangle OAC$ and $\triangle ODB$ are isosceles and similar.

49. $AB^2 = 2AC^2 = AC^2 + AC^2 = BC^2 + AC^2 \quad [\because AC = BC]$.

$\therefore$ by converse of Pythagoras' theorem, we have $\angle C = 90°$.

50. AC is the longest side of $\triangle ABC$.

$AB^2 + BC^2 = 16^2 + 12^2 = 256 + 144 = 400 = 20^2 = AC^2$.

$\therefore \quad \angle B = 90°$ [by the converse of Pythagoras' theorem].

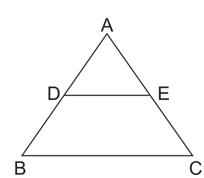
51. (a) Two similar triangles need not be congruent.

(b) Similar figures need not be of the same size.

(c) It is clearly true.

(d) Two polygons are similar only when their corresponding angles are equal and their corresponding sides are proportional.

53. (a) Let $AE = x$ cm. Then, $EC = (5.6 - x)$ cm.



$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{5} = \frac{x}{5.6 - x}$

$\therefore \quad 3(5.6 - x) = 5x \Rightarrow 8x = 3 \times 5.6$

$\therefore \quad x = \frac{3 \times 5.6}{8} = \frac{16.8}{8} = 2.1.$

(b) $\frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{3}{2} = \frac{6}{x} \Rightarrow 3x = 12 \Rightarrow x = 4.$

(c) $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC^2}{QR^2} \Rightarrow \frac{9}{16} = \frac{BC^2}{QR^2} \Rightarrow \left(\frac{3}{4}\right)^2 = \left(\frac{BC}{QR}\right)^2$

$\Rightarrow \quad \frac{BC}{QR} = \frac{3}{4} \Rightarrow QR = \frac{4}{3} \times BC = \left(\frac{4}{3} \times 4.5\right) \text{ cm} = 6 \text{ cm}.$

(d) $\Delta OAB \sim \Delta OCD \Rightarrow \frac{OA}{OC} = \frac{OB}{OD} \Rightarrow \frac{2x+4}{2x-1} = \frac{9x-21}{3}$

$\Rightarrow \quad 6x + 12 = 18x^2 - 51x + 21 \Rightarrow 18x^2 - 57x + 9 = 0$

$\Rightarrow \quad 6x^2 - 19x + 3 = 0 \Rightarrow (x-3)(6x-1) = 0 \Rightarrow x = 3$ or $x = \frac{1}{6}$.

But, $x = \frac{1}{6}$ makes $(2x - 1) < 0$. So, we reject it.

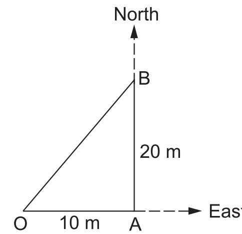
$\therefore \quad x = 3.$

The correct answer is: (a)–(s), (b)–(q), (c)–(p), (d)–(r).

54. (a) Let $OA = 10$ m and $AB = 20$ m. Then,

$OB^2 = OA^2 + AB^2 = \{(10)^2 + (20)^2\}\ m^2 = 500\ m^2$

$\Rightarrow \quad OB = \sqrt{500}\ m = \sqrt{5 \times 100}\ m = 10\sqrt{5}\ m.$



(b) Altitude $= \frac{\sqrt{3}}{2}a = \left(\frac{\sqrt{3}}{2} \times 10\right)$ cm $= 5\sqrt{3}$ cm.

(c) Area $= \frac{\sqrt{3}}{4}a^2 = \left(\frac{\sqrt{3}}{4} \times 10 \times 10\right)$ cm$^2 = 25\sqrt{3}$ cm^2.

(d) $d^2 = (8^2 + 6^2)\ m^2 = (64 + 36)\ m^2 = 100\ m^2 \Rightarrow d = \sqrt{100}\ m = 10\ m.$

Then, the correct answer is: (a)–(r), (b)–(q), (c)–(p), (d)–(s).

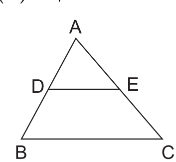
TEST YOURSELF

MCQ

1. $\Delta ABC \sim \Delta DEF$ and their perimeters are 32 cm and 24 cm respectively. If $AB = 10$ cm then $DE = ?$

   (a) 8 cm (b) 7.5 cm (c) 15 cm (d) $5\sqrt{3}$ cm

2. In the given figure, $DE \parallel BC$. If $DE = 5$ cm, $BC = 8$ cm and $AD = 3.5$ cm then $AB = ?$



   (a) 5.6 cm (b) 4.8 cm

   (c) 5.2 cm (d) 6.4 cm

3. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their feet is 12 m then the distance between their tops is

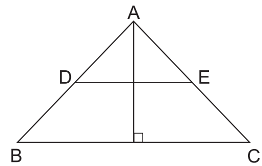
   (a) 12 m (b) 13 m (c) 14 m (d) 15 m

4. The areas of two similar triangles are 25 cm² and 36 cm² respectively. If the altitude of the first triangle is 3.5 cm then the corresponding altitude of the other triangle is

   (a) 5.6 cm (b) 6.3 cm (c) 4.2 cm (d) 7 cm

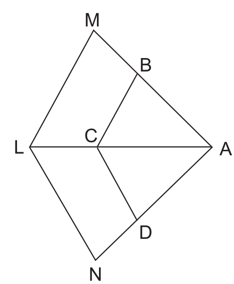
Short-Answer Questions

5. If $\triangle ABC \sim \triangle DEF$ such that $2AB = DE$ and $BC = 6$ cm, find EF.
6. In the given figure, $DE \parallel BC$ such that $AD = x$ cm, $DB = (3x+4)$ cm, $AE = (x+3)$ cm and $EC = (3x+19)$ cm. Find the value of x.



7. A ladder 10 m long reaches the window of a house 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.
8. Find the length of the altitude of an equilateral triangle of side $2a$ cm.
9. $\triangle ABC \sim \triangle DEF$ such that $\text{ar}(\triangle ABC) = 64\ \text{cm}^2$ and $\text{ar}(\triangle DEF) = 169\ \text{cm}^2$. If $BC = 4$ cm, find EF.
10. In a trapezium $ABCD$, it is given that $AB \parallel CD$ and $AB = 2CD$. Its diagonals AC and BD intersect at the point O such that $\text{ar}(\triangle AOB) = 84\ \text{cm}^2$. Find $\text{ar}(\triangle COD)$.
11. The corresponding sides of two similar triangles are in the ratio 2 : 3. If the area of the smaller triangle is 48 cm^2, find the area of the larger triangle.
12. In the given figure, $LM \parallel CB$ and $LN \parallel CD$.

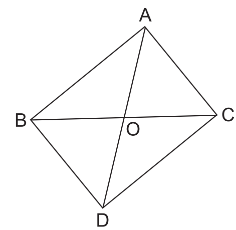
    Prove that $\dfrac{AM}{AB} = \dfrac{AN}{AD}$.



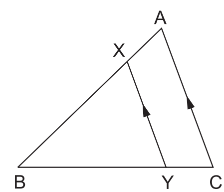
13. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
14. In an equilateral triangle with side a, prove that area $= \dfrac{\sqrt{3}}{4}a^2$.
15. Find the length of each side of a rhombus whose diagonals are 24 cm and 10 cm long.
16. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Long-Answer Questions

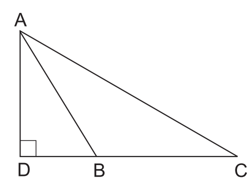
17. In the given figure, $\triangle ABC$ and $\triangle DBC$ have the same base BC. If AD and BC intersect at O, prove that $\dfrac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \dfrac{AO}{DO}$.



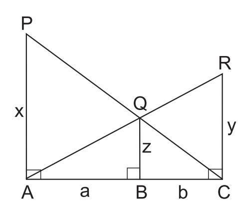
18. In the given figure, $XY \parallel AC$ and XY divides $\triangle ABC$ into two regions, equal in area. Show that $\dfrac{AX}{AB} = \dfrac{(2-\sqrt{2})}{2}$.



19. In the given figure, $\triangle ABC$ is an obtuse triangle, obtuse-angled at B. If $AD \perp CB$ (produced) prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.



20. In the given figure, each one of PA, QB and RC is perpendicular to AC. If $AP = x$, $QB = z$, $RC = y$, $AB = a$ and $BC = b$, show that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$.



ANSWERS (TEST YOURSELF)

1. (b) **2.** (a) **3.** (b) **4.** (c) **5.** 12 cm **6.** $x = 2$ **7.** 6 m
8. $\sqrt{3}a$ cm **9.** 6.5 cm **10.** 21 cm^2 **11.** 108 cm^2 **15.** 13 cm

✿