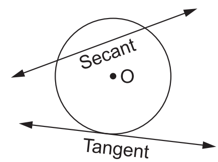
8 Circles

CIRCLE *A circle is a collection of all points in a plane which are at a constant distance from a fixed point. The constant distance is called the radius and the fixed point is called the centre of the circle.*

SECANT *A line which intersects a circle in two distinct points is called a secant to the circle.*

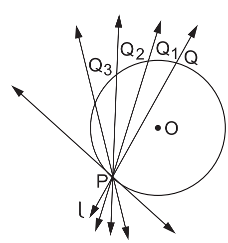


TANGENT *A line meeting a circle only in one point is called a tangent to the circle at that point.*

The point at which the tangent line intersects the circle is called the point of contact.

NOTE *The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.*

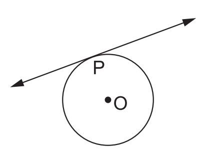
Consider a line l which is a secant to a circle with centre O. Let PQ be the corresponding chord of this secant. If we rotate the secant anticlockwise about the point P then point Q comes closer to the point P. Let Q_1, Q_2, Q_3 etc., be the positions of Q as the secant goes on rotating then we find these positions getting closer to P sequentially. Finally, as the position of Q coincides with P, the secant is reduced to a tangent at point P.



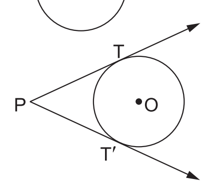
NUMBER OF TANGENTS TO A CIRCLE

(i) There is no tangent passing through a point lying inside the circle.

(ii) There is one and only one tangent passing through a point lying on a circle.



(iii) There are exactly two tangents through a point lying outside a circle. In the figure, PT and PT' are two tangents from a point P lying outside the circle.

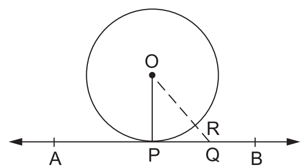


LENGTH OF A TANGENT *The length of the segment of the tangent from the external point P to the point of contact with the circle is called the length of the tangent from the point P to the circle.*

RESULTS ON TANGENTS

THEOREM 1 *The tangent at any point of a circle is perpendicular to the radius through the point of contact.* [CBSE 2007, '09, '11, '12, '13, '14, '15]

GIVEN A circle with centre O and a tangent AB at a point P of the circle.



TO PROVE $OP \perp AB$.

CONSTRUCTION Take a point Q, other than P, on AB. Join OQ.

PROOF Q is a point on the tangent AB, other than the point of contact P.

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

Then, $OR < OQ$ [a part is less than the whole]. … (i)

But, $OP = OR$ [radii of the same circle]. … (ii)

$\therefore$ $OP < OQ$ [from (i) and (ii)].

Thus, OP is shorter than any other line segment joining O to any point of AB, other than P.

In other words, OP is the shortest distance between the point O and the line AB.

But, the shortest distance between a point and a line is the perpendicular distance.

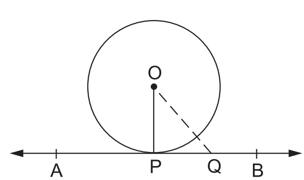
$\therefore$ $OP \perp AB$.

REMARKS (i) From this theorem we also conclude that at any point on a circle, one and only one tangent can be drawn to the circle.

(ii) The line containing the radius through the point of contact is called the 'normal' to the circle at the point of contact.

THEOREM 2 **(Converse of Theorem 1)** *A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.*

GIVEN A circle with centre O in which OP is a radius and AB is a line through P such that $OP \perp AB$.



TO PROVE AB is a tangent to the circle at the point P.

CONSTRUCTION Take a point Q, different from P, on AB. Join OQ.

PROOF We know that the perpendicular distance from a point to a line is the shortest distance between them.

$\therefore$ $OP \perp AB$ $\Rightarrow$ OP is the shortest distance from O to AB.

$\therefore$ $OP < OQ$.

$\therefore$ Q lies outside the circle [$\because$ OP is the radius and $OP < OQ$].

Thus, every point on AB, other than P, lies outside the circle.

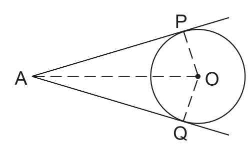
$\therefore$ AB meets the circle at the point P only.

Hence, AB is the tangent to the circle at the point P.

REMARK This theorem gives us a method of constructing a tangent at a point P on the circle. We first draw the radius OP and then draw $PT \perp OP$. Then, PT is the tangent at P.

THEOREM 3 *The lengths of tangents drawn from an external point to a circle are equal.* [CBSE 2007, '08, '08C, '09, '09C, '10, '11, '12, '13, '13C, '14, '15, '17]

GIVEN Two tangents AP and AQ are drawn from a point A to a circle with centre O.



TO PROVE $AP = AQ$.

CONSTRUCTION Join OP, OQ and OA.

PROOF AP is a tangent at P and OP is the radius through P.

$\therefore$ $OP \perp AP$.

Similarly, $OQ \perp AQ$.

In the right $\triangle$s OPA and OQA, we have:

$OP = OQ$ [radii of the same circle]

$OA = OA$ [common]

$\therefore$ $\triangle OPA \cong \triangle OQA$ [by RHS-congruence].

Hence, $AP = AQ$ [cpct].

REMARK The above theorem can also be proved by using the Pythagoras' theorem:

$$AP^2 = OA^2 - OP^2 = OA^2 - OQ^2 = AQ^2 \quad [\because OP = OQ = \text{radius}]$$

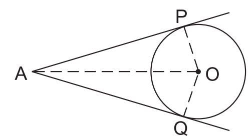
$\Rightarrow$ $AP = AQ$.

THEOREM 4 *If two tangents are drawn from an external point then*

(i) they subtend equal angles at the centre, and

(ii) they are equally inclined to the line segment joining the centre to that point.

GIVEN A circle with centre O and a point A outside it. Also, AP and AQ are the two tangents to the circle.



TO PROVE $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

PROOF In $\triangle AOP$ and $\triangle AOQ$, we have

$AP = AQ$ [tangents from an external point are equal]

$OP = OQ$ [radii of the same circle]

$OA = OA$ [common]

$\therefore$ $\triangle AOP \cong \triangle AOQ$ [by SSS-congruence].

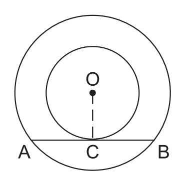
Hence, $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

REMARK Since $\angle OAP = \angle OAQ$, i.e., AO is the bisector of $\angle PAQ$, so the centre lies on the bisector of the angle between the two tangents.

THEOREM 5 *Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.*

[CBSE 2009, '12]

GIVEN Two circles with the same centre O and AB is a chord of the larger circle touching the smaller circle at C.



TO PROVE $AC = BC$.

CONSTRUCTION Join OC.

PROOF AB is a tangent to the smaller circle at the point C and OC is the radius through C.

$\therefore$ $OC \perp AB$.

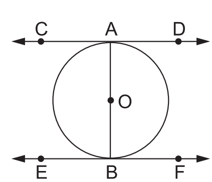
But, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ OC bisects AB [$\because$ AB is a chord of larger circle].

Hence, $AC = BC$.

THEOREM 6 *Prove that the tangents drawn at the ends of a diameter of a circle are parallel.* [CBSE 2011, '12, '14, '17]

GIVEN CD and EF are the tangents at the end points A and B of the diameter AB of a circle with centre O.



TO PROVE $CD \parallel EF$.

PROOF CD is the tangent to the circle at the point A.

$\therefore \quad CD \perp OA \Rightarrow \angle OAD = 90^\circ \Rightarrow \angle BAD = 90^\circ.$

EF is the tangent to the circle at the point *B*.

$\therefore \quad EF \perp OB \Rightarrow \angle OBE = 90^\circ \Rightarrow \angle ABE = 90^\circ.$

Thus, $\angle BAD = \angle ABE$ (each equal to 90°).

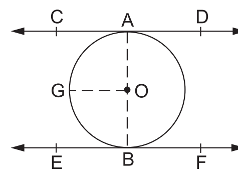
But these are alternate interior angles.

$\therefore \quad CD \parallel EF.$

<u>THEOREM 7</u> *Prove that the line segment joining the points of contact of two parallel tangents to a circle is a diameter of the circle.*

GIVEN *CD* and *EF* are two parallel tangents at the points *A* and *B* of a circle with centre *O*.

TO PROVE *AOB* is a diameter of the circle.



CONSTRUCTION Join *OA* and *OB*. Draw $OG \parallel CD$.

PROOF $OG \parallel CD$ and *AO* cuts them.

$\therefore \quad \angle CAO + \angle GOA = 180^\circ$

$\Rightarrow \quad \angle 90^\circ + \angle GOA = 180^\circ \quad [\because OA \perp CD]$

$\Rightarrow \quad \angle GOA = 90^\circ.$

Similarly, $\angle GOB = 90^\circ$.

$\therefore \quad \angle GOA + \angle GOB = 90^\circ + 90^\circ = 180^\circ$

$\Rightarrow$ *AOB* is a straight line.

Hence, *AOB* is a diameter of the circle with centre *O*.

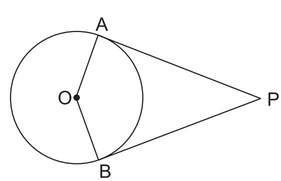
<u>THEOREM 8</u> *Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact to the centre.*

[CBSE 2008C, '14]

GIVEN *PA* and *PB* are the tangents drawn from a point *P* to a circle with centre *O*. Also, the line segments *OA* and *OB* are drawn.

TO PROVE $\angle APB + \angle AOB = 180^\circ$.

PROOF We know that the tangent to a circle is perpendicular to the radius through the point of contact.



$\therefore \quad PA \perp OA \Rightarrow \angle OAP = 90^\circ$, and

$\quad PB \perp OB \Rightarrow \angle OBP = 90^\circ.$

$\therefore \quad \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ. \quad \ldots$ (i)

But, we know that the sum of all the angles of a quadrilateral is 360°.

$\therefore \quad \angle OAP + \angle OBP + \angle APB + \angle AOB = 360°.$ ... (ii)

From (i) and (ii), we get

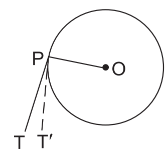
$\angle APB + \angle AOB = 180°.$

THEOREM 9 *Prove that there is one and only one tangent at any point on the circumference of a circle.*

PROOF Let P be a point on the circumference of a circle with centre O.

If possible, let PT and PT' be two tangents at a point P of the circle.

Now, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\therefore \quad OP \perp PT$ and similarly, $OP \perp PT'$

$\Rightarrow \quad \angle OPT = 90°$ and $\angle OPT' = 90°.$

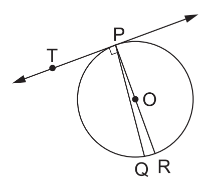
$\Rightarrow \quad \angle OPT = \angle OPT'$

This is possible only when PT and PT' coincide.

Hence, there is one and only one tangent at any point on the circumference of a circle.

THEOREM 10 *Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.*

PROOF Let PT be a tangent to a circle with centre O, where P is the point of contact.



Let $PQ \perp PT$, where Q lies on the circle, i.e., $\angle QPT = 90°$.

If possible, let PQ not pass through the centre O.

Join PO and produce it to meet the circle at R.

Then PO being the radius through the point of contact, we have

$PO \perp PT$ [$\because$ the tangent is perpendicular to the radius through the point of contact]

$\therefore \quad \angle OPT = 90° \Rightarrow \angle RPT = 90°.$

Thus, we have $\angle QPT = \angle RPT = 90°.$

This is possible only if P, Q and R are collinear.

But a straight line cuts a circle in at the most two points.

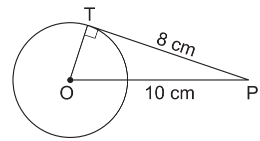
So, the points Q and R coincide.

Hence, PQ passes through the centre O, i.e., the perpendicular at the point of contact to the tangent passes through the centre.

SOLVED EXAMPLES

EXAMPLE 1 *From a point P,* 10 cm *away from the centre of a circle, a tangent PT of length* 8 cm *is drawn. Find the radius of the circle.* [CBSE 2002]

SOLUTION Let O be the centre of the given circle.



Then, $OP = 10$ cm. Also, $PT = 8$ cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through the point of contact T.

$\therefore \quad OT \perp PT$.

In the right $\triangle OTP$, we have

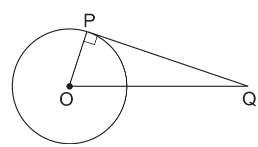
$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow \quad OT = \sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}$ cm $= \sqrt{36}$ cm $= 6$ cm.

Hence, the radius of the circle is 6 cm.

EXAMPLE 2 *A tangent PQ at a point P of a circle of radius* 5 cm *meets a line through the centre O at a point Q so that* $OQ = 13$ cm. *Find the length of PQ.* [CBSE 2010]

SOLUTION We have OP = radius = 5 cm, OQ = 13 cm.



PQ is a tangent at P and OP is the radius through the point of contact P.

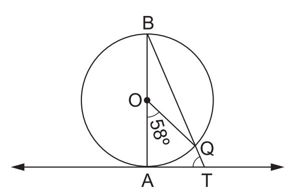
$\therefore \quad OP \perp PQ$.

In right $\triangle OPQ$, we have

$OQ^2 = OP^2 + PQ^2$ [by Pythagoras' theorem]

$\Rightarrow \quad PQ = \sqrt{OQ^2 - OP^2} = \sqrt{13^2 - 5^2}$ cm $= \sqrt{144}$ cm $= 12$ cm.

EXAMPLE 3 *In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If* $\angle AOQ = 58°$, *find* $\angle ATQ$. [CBSE 2015]



SOLUTION $\angle AOQ = 58°$

$\Rightarrow \quad \angle ABQ = \frac{1}{2} \angle AOQ = 29°$

[$\because$ the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow \quad \angle ABT = 29°.$

Now, AT is a tangent at A and OA is the radius through the point of contact A.

$\therefore \quad OA \perp AT$, i.e., $\angle OAT = 90° \Rightarrow \angle BAT = 90°$.

In $\triangle BAT$, we have

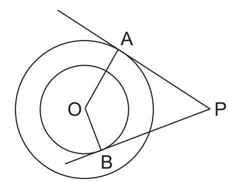
$$\angle BAT + \angle ABT + \angle ATB = 180°$$

$\Rightarrow \quad 90° + 29° + \angle ATB = 180° \Rightarrow \angle ATB = 61°.$

$\therefore \quad \angle ATQ = \angle ATB = 61°.$

EXAMPLE 4 *Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii* 8 cm *and* 5 cm *respectively, as shown in the figure. If* $AP = 15$ cm *then find the length of BP.*

[CBSE 2012]

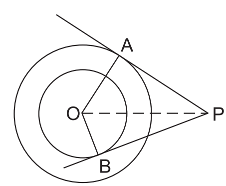


SOLUTION We have

$OA \perp AP$ and $OB \perp BP$

[∵ the tangent at any point of a circle is perpendicular to the radius through the point of contact].

Join OP.



In right $\triangle OAP$, we have

$OA = 8$ cm, $AP = 15$ cm.

$\therefore \quad OP^2 = OA^2 + AP^2$ [by Pythagoras' theorem]

$\Rightarrow \quad OP = \sqrt{OA^2 + AP^2} = \sqrt{8^2 + 15^2}$ cm $= \sqrt{289}$ cm $= 17$ cm.

In right $\triangle OBP$, we have

$OB = 5$ cm, $OP = 17$ cm.

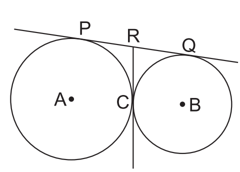
$\therefore \quad OP^2 = OB^2 + BP^2$ [by Pythagoras' theorem]

$\Rightarrow \quad BP = \sqrt{OP^2 - OB^2} = \sqrt{17^2 - 5^2}$ cm $= \sqrt{264}$ cm.

Thus, the length of $BP = \sqrt{264}$ cm $= 16.25$ cm (approx).

EXAMPLE 5 *In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.*

[CBSE 2013]



SOLUTION In the given figure, PR and CR are both tangents drawn to the same circle from an external point R.

$\therefore \quad PR = CR.$ … (i)

Also, QR and CR are both tangents drawn to the same circle (second circle) from an external point R.

$\therefore \quad QR = CR.$ … (ii)

From (i) and (ii), we get

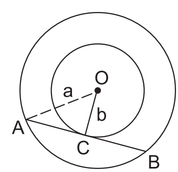
$PR = QR$ [each equal to CR].

$\therefore$ R is the midpoint of PQ,

i.e., the common tangent to the circles at C, bisects the common tangent at P and Q.

EXAMPLE 6 *Two concentric circles of radii a and b* $(a > b)$ *are given. Find the length of the chord of the larger circle which touches the smaller circle.* [CBSE 2015]

SOLUTION Let O be the common centre of the two circles and AB be the chord of the larger circle which touches the smaller circle at C.



Join OA and OC. Then, $OA = a$ and $OC = b$.

Now, $OC \perp AB$ and OC bisects AB

[$\because$ the chord of the larger circle touching the smaller circle, is bisected at the point of contact].

In right $\triangle ACO$, we have

$OA^2 = OC^2 + AC^2$ [by Pythagoras' theorem]

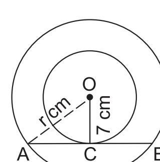
$\Rightarrow \quad AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}.$

$\therefore \quad AB = 2AC = 2\sqrt{a^2 - b^2}$ [$\because$ C is the midpoint of AB]

i.e., required length of the chord $AB = 2\sqrt{a^2 - b^2}$.

EXAMPLE 7 *Two concentric circles are of radii* 7 cm *and r* cm *respectively, where* $r > 7$. *A chord of the larger circle of length* 46 cm, *touches the smaller circle. Find the value of r.* [CBSE 2011]

SOLUTION Let O be the common centre of the two circles and AB be the chord of the larger circle which touches the smaller circle at C. Then, $AB = 46$ cm.



Join OA and OC. Then, $OA = r$ cm and $OC = 7$ cm.

Now, $OC \perp AB$ and OC bisects AB. [See Theorem 5.]

In right $\triangle ACO$, we have

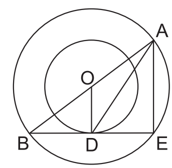
$$OA^2 = OC^2 + AC^2 \quad \text{[by Pythagoras' theorem]}$$

$$\Rightarrow \quad OA = \sqrt{OC^2 + AC^2} = \sqrt{OC^2 + \left(\frac{1}{2}AB\right)^2}$$

[$\because$ C is the midpoint of AB]

$$\Rightarrow \quad r \text{ cm} = \sqrt{7^2 + 23^2} \text{ cm} = \sqrt{578} \text{ cm} = 17\sqrt{2} \Rightarrow r = 17\sqrt{2} \text{ cm}.$$

EXAMPLE 8 *In the given figure, the radii of two concentric circles are* 13 cm *and* 8 cm. *AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D. Find the length of AD.* [HOTS] [CBSE 2014]



SOLUTION We have $\angle AEB = 90^\circ$ [angle in a semicircle].

Also, $OD \perp BE$ and OD bisects BE. [See Theorem 5.]

In right $\triangle OBD$, we have

$$OB^2 = OD^2 + BD^2 \quad \text{[by Pythagoras' theorem]}$$

$$\Rightarrow \quad BD = \sqrt{OB^2 - OD^2} = \sqrt{13^2 - 8^2} \text{ cm}$$

$$= \sqrt{105} \text{ cm} \qquad [\because \ OB = 13 \text{ cm}, OD = 8 \text{ cm}].$$

$$\therefore \quad BE = 2BD = 2\sqrt{105} \text{ cm} \qquad [\because \ D \text{ is the midpoint of } BE].$$

In right $\triangle AEB$, we have

$$AB^2 = AE^2 + BE^2 \quad \text{[by Pythagoras' theorem]}$$

$$\Rightarrow \quad AE = \sqrt{AB^2 - BE^2} = \sqrt{26^2 - (2\sqrt{105})^2} \text{ cm} = \sqrt{256} \text{ cm} = 16 \text{ cm}$$

$$[\because \ AB = \text{diameter} = 2 \times OB = 2 \times 13 \text{ cm} = 26 \text{ cm}].$$

In right $\triangle AED$, we have

$$AD^2 = AE^2 + DE^2 \quad \text{[by Pythagoras' theorem]}$$

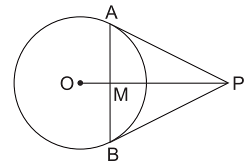
$$\Rightarrow \quad AD = \sqrt{AE^2 + DE^2} = \sqrt{16^2 + (\sqrt{105})^2} \text{ cm}$$

$$= 19 \text{ cm} \quad [\because \ DE = BD = \sqrt{105} \text{ cm}].$$

[**Note** We can also find AE by using midpoint theorem, since in $\triangle ABE$, O is the midpoint of AB and D is the midpoint of BE and so $OD \parallel AE$ and $AE = 2 \times OD = 16$ cm.]

EXAMPLE 9 *From a point P outside a circle with centre O, tangents PA and PB are drawn to the circle. Prove that OP is the right bisector of the line segment AB.* [CBSE 2015]

SOLUTION GIVEN PA and PB are tangents to a circle with centre O from an external point P.



TO PROVE OP is the right bisector of AB.

CONSTRUCTION Join AB. Let AB intersect OP at M.

PROOF In $\triangle MAP$ and $\triangle MBP$, we have

$PA = PB$ [$\because$ tangents to a circle from an external point are equal]

$MP = MP$ [common]

$\angle MPA = \angle MPB$ [$\because$ tangents from an external point are equally inclined to the line segment joining the centre to that point, i.e., $\angle OPA = \angle OPB$]

$\therefore$ $\triangle MAP \cong \triangle MBP$ [by SAS-congruence].

And so, $MA = MB$ [cpct]

and $\angle AMP = \angle BMP$ [cpct].

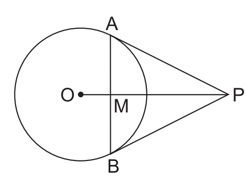
But, $\angle AMP + \angle BMP = 180°$ [linear pair]

$\therefore$ $\angle AMP = \angle BMP = 90°$.

Hence, OP is the right bisector of AB.

EXAMPLE 10 *Prove that the tangents at the extremities of any chord of a circle, make equal angles with the chord.* [CBSE 2014, '17]

SOLUTION GIVEN AB is any chord of a circle with centre O. Tangents at the extremities A and B of this chord meet at an external point P. Chord AB intersects the line segment OP at M.



TO PROVE $\angle MAP = \angle MBP$.

PROOF In $\triangle MAP$ and $\triangle MBP$, we have

$PA = PB$ [$\because$ tangents from an external point are equal]

$MP = MP$ [common]

$\angle MPA = \angle MPB$ [$\because$ $\angle OPA = \angle OPB$ since tangents from an external point are equally inclined to the line segment joining the point to the centre]

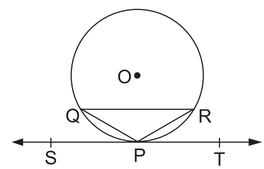
$\therefore \quad \triangle MAP \cong \triangle MBP$ [by SAS-congruence].

And so, $\angle MAP = \angle MBP$ [cpct].

EXAMPLE 11 *Prove that the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc.* [CBSE 2015]

SOLUTION GIVEN Point P is the midpoint of arc $\overparen{QR}$ of a circle with centre O. ST is the tangent to the circle at point P.

TO PROVE Chord $QR \parallel ST$.



PROOF P is the midpoint of $\overparen{QR}$

$\Rightarrow \quad \overparen{QP} = \overparen{PR}$

$\Rightarrow$ chord QP = chord PR

[$\because$ in a circle, if two arcs are equal, then their corresponding chords are equal]

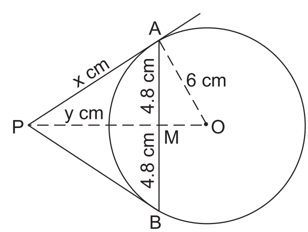
$\therefore \quad \angle PQR = \angle PRQ$

$\Rightarrow \quad \angle TPR = \angle PRQ$

[**Note** $\angle PQR = \angle TPR$, angles in alternate segments]

$\Rightarrow \quad QR \parallel ST$. [$\because$ $\angle TPR$ and $\angle PRQ$ are alternate int. $\angle$s]

EXAMPLE 12 *In the given figure, AB is a chord of length* 9.6 cm *of a circle with centre O and radius* 6 cm. *The tangents at A and B intersect at P. Find the length of PA.* [CBSE 2009C, '13C, '15]



SOLUTION GIVEN A circle with centre O and radius 6 cm. AB is a chord of length 9.6 cm. The tangents at A and B intersect at P.

TO FIND The length PA.

CONSTRUCTION Join OP and OA. Let OP and AB intersect at M.

Let $PA = x$ cm and $PM = y$ cm.

Now, $PA = PB$ [$\because$ tangents from an external point are equal]

and OP is the bisector of $\angle APB$

[$\because$ two tangents to a circle from an external point are equally inclined to the line segment joining the centre to that point].

Also, $OP \perp AB$ and OP bisects AB at M

[$\because$ OP is the right bisector of AB].

$\therefore \quad AM = MB = \dfrac{9.6}{2}$ cm = 4.8 cm.

In right $\triangle AMO$, we have $OA = 6$ cm and $AM = 4.8$ cm.

$\therefore \quad OM = \sqrt{OA^2 - AM^2} = \sqrt{6^2 - 4.8^2} = \sqrt{12.96} = 3.6$ cm.

In right $\triangle AMP$, we have

$AP^2 = PM^2 + AM^2 \Rightarrow x^2 = y^2 + (4.8)^2$

$\Rightarrow \quad x^2 = y^2 + 23.04.$ … (i)

In right $\triangle PAO$, we have

$OP^2 = PA^2 + OA^2$ [**Note** $\angle PAO = 90^\circ$, since AO is the radius at the point of contact]

$\Rightarrow \quad (y + 3.6)^2 = x^2 + 6^2$ [$\because$ $OP = PM + MO = (y + 3.6)$ cm]

$\Rightarrow \quad y^2 + 7.2y + 12.96 = x^2 + 36 \Rightarrow 7.2y = 46.08$ [using (i)]

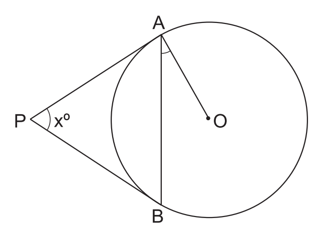
$\Rightarrow \quad y = 6.4$ cm.

Putting this value of y in (i), we get

$x^2 = (6.4)^2 + 23.04 = 40.96 + 23.04 = 64 \Rightarrow x = \sqrt{64} = 8.$

$\therefore \quad PA = 8$ cm.

EXAMPLE 13 *Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that* $\angle APB = 2\angle OAB$. [CBSE 2009, '14]



SOLUTION GIVEN A circle with centre O and PA, PB are the tangents on it from a point P outside it.

TO PROVE $\angle APB = 2\angle OAB$.

PROOF Let $\angle APB = x^\circ$.

We know that the tangents to a circle from an external point are equal. So, $PA = PB$.

Since the angles opposite to the equal sides of a triangle are equal, so

$PA = PB \Rightarrow \angle PBA = \angle PAB.$

Also, the sum of the angles of a triangle is 180°.

$\therefore \quad \angle APB + \angle PAB + \angle PBA = 180^\circ$

$\Rightarrow \quad x^\circ + 2\angle PAB = 180^\circ$ [$\because$ $\angle PBA = \angle PAB$]

$$\Rightarrow \quad \angle PAB = \frac{1}{2}(180^\circ - x^\circ) = \left(90^\circ - \frac{1}{2}x^\circ\right).$$

But, PA is a tangent and OA is the radius of the given circle.

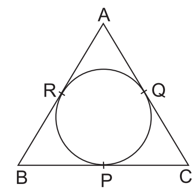
$\therefore \quad \angle OAB + \angle PAB = 90^\circ$

$$\Rightarrow \quad \angle OAB = 90^\circ - \left(90^\circ - \frac{1}{2}x^\circ\right) \Rightarrow \angle OAB = \frac{1}{2}x^\circ = \frac{1}{2}\angle APB$$

$\Rightarrow \quad \angle APB = 2\angle OAB.$

EXAMPLE 14 *In the given figure, the incircle of $\triangle ABC$ touches the sides BC, CA and AB at P, Q and R respectively. Prove that*

$$(AR + BP + CQ) = (AQ + BR + CP) = \frac{1}{2}\text{ (perimeter of } \triangle ABC).$$



SOLUTION We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore \quad AR = AQ, \quad \ldots \text{(i)} \quad$ [tangents from A]

$\quad BP = BR, \quad \ldots \text{(ii)} \quad$ [tangents from B]

$\quad CQ = CP. \quad \ldots \text{(iii)} \quad$ [tangents from C]

$\therefore \quad (AR + BP + CQ) = (AQ + BR + CP) = k$ (say).

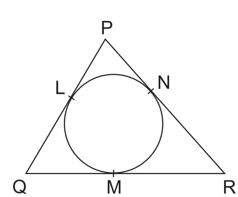
Perimeter of $\triangle ABC = (AB + BC + CA)$

$$= (AR + BR) + (BP + CP) + (CQ + AQ)$$
$$= (AR + BP + CQ) + (AQ + BR + CP)$$
$$= (k + k) = 2k$$

$$\Rightarrow \quad k = \frac{1}{2}\text{(perimeter of } \triangle ABC).$$

$$\therefore \quad (AR + BP + CQ) = (AQ + BR + CP) = \frac{1}{2}\text{(perimeter of } \triangle ABC).$$

EXAMPLE 15 *In the given figure, a circle is inscribed in a triangle PQR. If* $PQ = 10$ cm, $QR = 8$ cm *and* $PR = 12$ cm, *find the lengths of QM, RN and PL.* [CBSE 2012]



SOLUTION We know that the lengths of the tangents drawn from an external point to a circle are equal.

Let $PL = PN = x$; $QL = QM = y$; $RM = RN = z$.

Now, $PL + QL = PQ \Rightarrow x + y = 10,$ … (i)

$QM + RM = QR \Rightarrow y + z = 8,$ … (ii)

$RN + PN = PR \Rightarrow z + x = 12.$ … (iii)

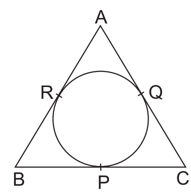
Subtracting (ii) from (iii), we get $x - y = 4$. … (iv)

Solving (i) and (iv), we get $x = 7$, $y = 3$.

Substituting $y = 3$ in (ii), we get $z = 5$.

$\therefore$ $QM = y = 3$ cm, $RN = z = 5$ cm, $PL = x = 7$ cm.

EXAMPLE 16 *A circle is inscribed in a $\triangle ABC$, touching BC, CA and AB at P, Q and R respectively, as shown in the given figure. If $AB = 10$ cm, $AQ = 7$ cm and $CQ = 5$ cm then find the length of BC.* [CBSE 2009C]



SOLUTION We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $AR = AQ = 7$ cm.

$BR = (AB - AR) = (10 - 7)$ cm $= 3$ cm.

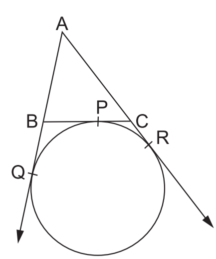
$\therefore$ $BP = BR = 3$ cm,

$CP = CQ = 5$ cm.

$\therefore$ $BC = (BP + CP) = (3 + 5)$ cm $= 8$ cm.

EXAMPLE 17 *A circle is touching the side BC of $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$).*

[CBSE 2001, '02, '06, '14, '17]



SOLUTION We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $AQ = AR,$ … (i) [tangents from A]

$BP = BQ,$ ... (ii) [tangents from B]

$CP = CR.$ ... (iii) [tangents from C]

Perimeter of $\triangle ABC$

$= AB + BC + AC$

$= AB + BP + CP + AC$

$= AB + BQ + CR + AC$ [using (ii) and (iii)]

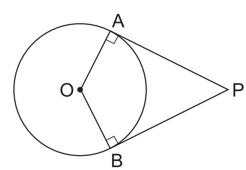
$= AQ + AR$

$= 2AQ$ [using (i)].

$\therefore \quad AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$).

EXAMPLE 18 *PA and PB are tangents to the circle with centre O from an external point P, touching the circle at A and B respectively. Show that the quadrilateral AOBP is cyclic.* [CBSE 2014]

SOLUTION GIVEN PA and PB are tangents to the circle with centre O from an external point P.



TO PROVE Quadrilateral $AOBP$ is cyclic.

PROOF We know that the tangent at any point of a circle is perpendicular to radius through the point of contact.

$\therefore \quad PA \perp OA$, i.e., $\angle OAP = 90^\circ$ ... (i)

and $PB \perp OB$, i.e., $\angle OBP = 90^\circ$ ... (ii)

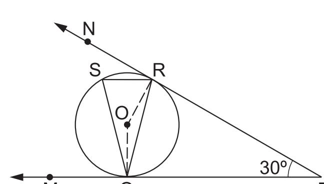
Now, the sum of all the angles of a quadrilateral is 360°.

$\therefore \quad \angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$

$\Rightarrow \quad \angle AOB + \angle APB = 180^\circ$ [using (i) and (ii)]

$\therefore$ quadrilateral $OAPB$ is cyclic [since both pairs of opposite angles have the sum 180°.].

EXAMPLE 19 *In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.*



SOLUTION Join OQ and OR. Also, produce PQ and PR to M and N respectively.

We know that the angle between two tangents from an external point is supplementary to the angle subtended by the radii at the points of contact.

$\therefore \quad \angle RPQ + \angle ROQ = 180^\circ$

$\Rightarrow \quad \angle ROQ = 180^\circ - \angle RPQ = 180^\circ - 30^\circ = 150^\circ$.

Now, $\angle RSQ = \frac{1}{2} \angle ROQ = \frac{1}{2} \times 150^\circ = 75^\circ$

[$\because$ the angle subtended by an arc at the centre is twice the angle subtended on the remaining part of the circle].

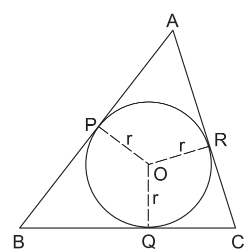
$\therefore \quad \angle SQM = \angle RSQ = 75^\circ$ [alternate int. ∠s; since $RS \parallel PQ$]

Also, $\angle PQR = \angle RSQ = 75^\circ$ [angles in alternate segments].

Now, $\angle SQM + \angle RQS + \angle PQR = 180^\circ$ [angles on a straight line]

$\therefore \quad \angle RQS = 180^\circ - (\angle SQM + \angle PQR) = 180^\circ - (75^\circ + 75^\circ) = 30^\circ$.

EXAMPLE 20 *In the given figure, the sides AB, BC and CA of a triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.*



Prove that

(a) $AB + CQ = AC + BQ$

(b) area $(\triangle ABC) = \frac{1}{2}$*(perimeter of* $\triangle ABC) \times r$. [CBSE 2013]

SOLUTION We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore \quad AP = AR,$ … (i) [tangents from A]

$BP = BQ,$ … (ii) [tangents from B]

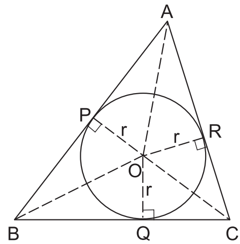
$CQ = CR.$ … (iii) [tangents from C]

(a) $AB + CQ = AP + BP + CQ$

$= AR + BQ + CR$ [using (i), (ii) and (iii)]

$= (AR + CR) + BQ = AC + BQ$.

(b) Join OA, OB and OC.



$$\text{Area}(\triangle ABC) = \text{area}(\triangle OAB) + \text{area}(\triangle OBC) + \text{area}(\triangle OCA)$$

$$= \left(\frac{1}{2}\times AB\times OP\right) + \left(\frac{1}{2}\times BC\times OQ\right) + \left(\frac{1}{2}\times CA\times OR\right)$$

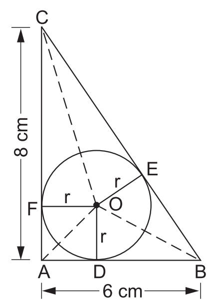
$$= \left(\frac{1}{2}\times AB\times r\right)+\left(\frac{1}{2}\times BC\times r\right)+\left(\frac{1}{2}\times CA\times r\right)$$

$$= \frac{1}{2}(AB+BC+CA)\times r$$

$$= \frac{1}{2}(\text{perimeter of } \triangle ABC)\times r.$$

EXAMPLE 21 *In the given figure, ABC is a right-angled triangle with AB* = 6 cm *and AC* = 8 cm. *A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.*

[CBSE 2006C, '13]



SOLUTION Join OA, OB and OC.

Draw $OD \perp AB$, $OE \perp BC$ and $OF \perp CA$.

Then, $OD = OE = OF = r$ cm.

$$\therefore \quad \text{ar}(\triangle ABC) = \frac{1}{2}\times AB\times AC = \left(\frac{1}{2}\times 6\times 8\right)\text{cm}^2 = 24\ \text{cm}^2.$$

Now, $\text{ar}(\triangle ABC) = \frac{1}{2}\times(\text{perimeter of } \triangle ABC)\times r$

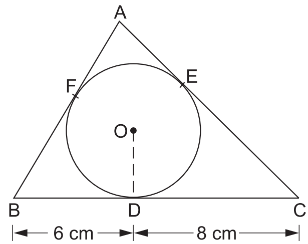
$$\Rightarrow \quad 24 = \frac{1}{2}\times(AB+BC+CA)\times r$$

$$\Rightarrow \quad 24 = \frac{1}{2}\times(6+10+8)\times r \Rightarrow r = 2$$

$$[\because\ BC^2 = AB^2 + AC^2 \Rightarrow BC = \sqrt{6^2+8^2} = 10].$$

Hence, the radius of the inscribed circle is 2 cm.

EXAMPLE 22 *A triangle ABC is drawn to circumscribe a circle of radius* 4 cm *such that the segments BD and DC into which BC is divided by the point of contact D are of lengths* 6 cm *and* 8 cm *respectively. Find the lengths of the sides AB and AC.* [HOTS] [CBSE 2014]



SOLUTION We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore \quad AE = AF = x$ cm (say), $BD = BF = 6$ cm, $CD = CE = 8$ cm.

And so, $AB = AF + BF = (x + 6)$ cm, $BC = BD + CD = 14$ cm,

$AC = CE + AE = (x + 8)$ cm.

Perimeter, $2s = AB + BC + AC = [(x+6) + 14 + (x+8)]$ cm

$= (2x + 28)$ cm

$\Rightarrow \quad s = (x + 14)$ cm.

$\therefore \quad \text{ar}(\triangle ABC) = \sqrt{s(s - AB)(s - BC)(s - AC)}$

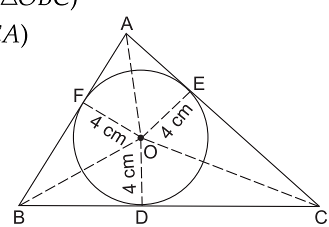
$= \sqrt{(x+14)\{(x+14)-(x+6)\}\{(x+14)-14\}\{(x+14)-(x+8)\}}\ \text{cm}^2$

$= \sqrt{48x(x+14)}\ \text{cm}^2$. ... (i)

Join OE and OF and also OA, OB and OC.

$\therefore \quad \text{ar}(\triangle ABC) = \text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OCA)$

$= \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times AC \times OE\right)$



$= \left[\frac{1}{2} \times (x+6) \times 4\right] + \left[\frac{1}{2} \times 14 \times 4\right] + \left[\frac{1}{2} \times (x+8) \times 4\right]$

$= 2[(x+6) + 14 + (x+8)] = 4(x+14)\ \text{cm}^2$. ... (ii)

From (i) and (ii), we get

$\sqrt{48x(x+14)} = 4(x+14)$

$\Rightarrow \quad 48x(x+14) = 16(x+14)^2$ [on squaring both sides]

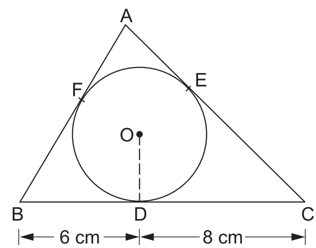
$\Rightarrow \quad 48x = 16(x+14) \Rightarrow x = \frac{16 \times 14}{32} = 7.$

$\therefore \quad AB = (x+6)$ cm $= (7+6)$ cm $= 13$ cm

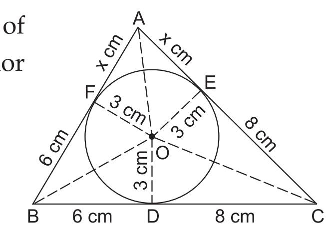
and $AC = (x+8)$ cm $= (7+8)$ cm $= 15$ cm.

EXAMPLE 23 *In the given figure, a triangle ABC is drawn to circumscribe a circle of radius* 3 cm, *such that the segments BD and DC into which BC is divided by the point of contact D are of lengths* 6 cm *and* 8 cm *respectively. Find the side AB, if the area of* $\triangle ABC$ *is* 63 cm^2.

[CBSE 2010, '11, '15]



SOLUTION We know that the lengths of tangents drawn from an exterior point to a circle are equal.



$\therefore$ $AE = AF = x$ cm (say);

$BD = BF = 6$ cm;

$CD = CE = 8$ cm.

And so, $AB = AF + BF = (x + 6)$ cm; $BC = BD + CD = 14$ cm;

$CA = CE + AE = (x + 8)$ cm.

Join OE and OF and also OA, OB and OC.

$\therefore$ $\text{ar}(\triangle ABC) = \text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OCA)$

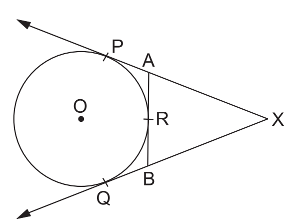
$\Rightarrow$ $63 = \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times CA \times OE\right)$

$\Rightarrow$ $63 = \left\{\frac{1}{2} \times (x + 6) \times 3\right\} + \left(\frac{1}{2} \times 14 \times 3\right) + \left\{\frac{1}{2} \times (x + 8) \times 3\right\}$

$\Rightarrow$ $63 = \frac{3}{2} \times (2x + 28) \Rightarrow x = 7.$

$\therefore$ $AB = (x + 6)$ cm $= (7 + 6)$ cm $= 13$ cm.

EXAMPLE 24 *In the given figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that* $XA + AR = XB + BR$. [CBSE 2014]



SOLUTION We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ $XP = XQ$, ... (i) [tangents from X]

$AP = AR$, ... (ii) [tangents from A]

$BR = BQ$. ... (iii) [tangents from B]

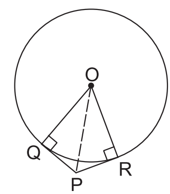
Now, $XP = XQ \Rightarrow XA + AP = XB + BQ$

$\Rightarrow \quad XA + AR = XB + BR$ [using (ii) and (iii)].

EXAMPLE 25 *If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that* $\angle QPR = 120^\circ$, *prove that* $2PQ = PO$.

[HOTS] [CBSE 2014]

SOLUTION In $\triangle OPQ$, we have



$\angle PQO = 90^\circ$ [$\because$ the tangent at any point is perpendicular to the radius through the point of contact]

and $\angle QPO = \frac{1}{2} \times \angle QPR = \frac{1}{2} \times 120^\circ = 60^\circ$.

[$\because$ the two tangents drawn from an external point are equally inclined to the line segment joining the centre to that point and so $\angle QPO = \angle RPO$]

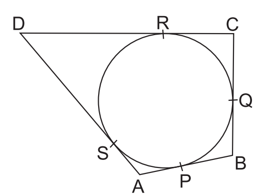
In right $\triangle OPQ$, we have

$$\cos(\angle QPO) = \frac{PQ}{PO}$$

$$\Rightarrow \quad \cos 60^\circ = \frac{PQ}{PO} \Rightarrow \frac{1}{2} = \frac{PQ}{PO} \Rightarrow 2PQ = PO.$$

EXAMPLE 26 *A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that* $AB + CD = AD + BC$.

[CBSE 2008, '08C, '09, '12, '13, '14, '17]



SOLUTION We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore \quad AP = AS,$ ... (i) [tangents from A]

$BP = BQ,$ ... (ii) [tangents from B]

$CR = CQ,$ ... (iii) [tangents from C]

$DR = DS.$ ... (iv) [tangents from D]

$\therefore \quad AB + CD = (AP + BP) + (CR + DR)$

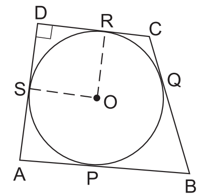
$= (AS + BQ) + (CQ + DS)$ [using (i), (ii), (iii), (iv)]

$= (AS + DS) + (BQ + CQ)$

$= AD + BC.$

Hence, $AB + CD = AD + BC$.

EXAMPLE 27 *In the given figure, ABCD is a quadrilateral such that $\angle D = 90°$. A circle with centre O and radius r, touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If* $BC = 40$ cm, $CD = 25$ cm *and* $BP = 28$ cm, *find r.*



SOLUTION It is given that $\angle D = 90°$.

Also, $\angle ORD = \angle OSD = 90°$. [∵ tangent at a point is perpendicular to the radius through the point of contact]

∴ $\angle ROS = 180° - \angle D = 90°$. [∵ angle between the tangents from an external point is supplementary to the angle subtended by the line segments joining the points of contact to the centre]

And, $OR = OS = r$.

∴ $ROSD$ is a square and so $OR = DR$, i.e., $r = DR$. … (i)

We know that the lengths of tangents drawn from an exterior point to a circle are equal.

∴ $BP = BQ$ and $CQ = CR$.

Now, $CQ = BC - BQ = BC - BP = (40 - 28)$ cm = 12 cm.

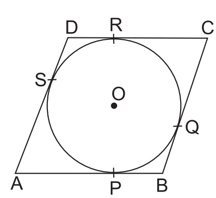
And so, $DR = CD - CR = CD - CQ = (25 - 12)$ cm = 13 cm.

∴ $r = DR = 13$ cm [using (i)].

EXAMPLE 28 *Prove that the parallelogram circumscribing a circle is a rhombus.*

[CBSE 2008, '09, '10, '12, '13, '14]

SOLUTION GIVEN A parallelogram $ABCD$ circumscribes a circle with centre O.



TO PROVE $AB = BC = CD = AD$.

PROOF We know that the lengths of tangents drawn from an exterior point to a circle are equal.

∴ $AP = AS$, … (i) [tangents from A]
$BP = BQ$, … (ii) [tangents from B]
$CR = CQ$, … (iii) [tangents from C]
$DR = DS$. … (iv) [tangents from D]

$\therefore \quad AB + CD = AP + BP + CR + DR$

$= AS + BQ + CQ + DS$ [from (i), (ii), (iii) and (iv)]

$= (AS + DS) + (BQ + CQ)$

$= AD + BC.$

Thus, $AB + CD = AD + BC$

$\Rightarrow \quad 2AB = 2AD \quad$ [$\because$ opposite sides of a ||gm are equal]

$\Rightarrow \quad AB = AD.$

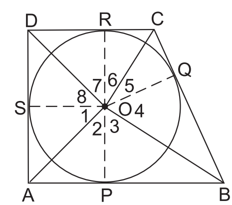
$\therefore \quad CD = AB = AD = BC.$

Hence, $ABCD$ is a rhombus.

EXAMPLE 29 *Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.*

[CBSE 2012, '13C, '14, '17]

SOLUTION GIVEN A quad. $ABCD$ circumscribes a circle with centre O.



TO PROVE $\angle AOB + \angle COD = 180^\circ$,

and $\angle AOD + \angle BOC = 180^\circ$.

CONSTRUCTION Join OP, OQ, OR and OS.

PROOF We know that the tangents drawn from an external point of a circle subtend equal angles at the centre.

$\therefore \quad \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$.

And, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

[$\angle$s at a point]

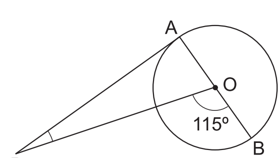
$\Rightarrow \quad 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ$, and

$2(\angle 1 + \angle 8) + 2(\angle 4 + \angle 5) = 360^\circ$

$\Rightarrow \quad \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180^\circ$ and $\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$

$\Rightarrow \quad \angle AOB + \angle COD = 180^\circ$ and $\angle AOD + \angle BOC = 180^\circ$.

EXAMPLE 30 *In the given figure, PA is a tangent from an external point P to a circle with centre O. If* $\angle POB = 115^\circ$, *find* $\angle APO$. [CBSE 2009C]



SOLUTION We know that the tangent at a point to a circle is perpendicular to the radius passing through the point of contact.

$\therefore \quad \angle OAP = 90°.$

Now, $\angle AOP + \angle BOP = 180°$

$\Rightarrow \quad \angle AOP = 180° - \angle BOP = 180° - 115° = 65°.$

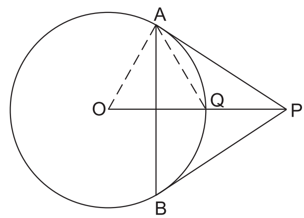
Now, $\angle OAP + \angle AOP + \angle APO = 180°$

[sum of angles of a triangle is 180°]

$\Rightarrow \quad \angle APO = 180° - (\angle OAP + \angle AOP) = 180° - (90° + 65°) = 25°.$

EXAMPLE 31 *From a point P, two tangents PA and PB are drawn to a circle C(O, r). If OP = 2r, show that △APB is equilateral.*

[CBSE 2008, '11, '12]



SOLUTION Let OP meet the circle at Q.

Join OA and AQ.

Clearly, $OA \perp AP \Rightarrow \angle OAP = 90°$ [radius through the point of contact is perpendicular to the tangent].

Now, $OQ = QP = r$.

Thus, Q is the midpoint of the hypotenuse OP of $\triangle OAP$.

So, Q is equidistant from O, A and P.

$\therefore \quad QA = OQ = QP = r$

$\Rightarrow \quad OA = OQ = QA = r$

$\Rightarrow \quad \triangle AOQ$ is equilateral

$\Rightarrow \quad \angle AOQ = 60°$ [$\because$ each angle of an equilateral triangle is 60°]

$\Rightarrow \quad \angle AOP = 60°$

$\Rightarrow \quad \angle APO = 30° \qquad [\because \angle AOP + \angle OAP + \angle APO = 180°]$

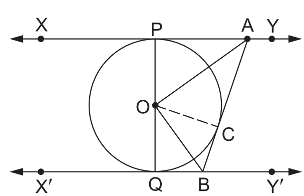
$\Rightarrow \quad \angle APB = 2\angle APO = 60°.$

Also, $PA = PB \Rightarrow \angle PAB = \angle PBA = 60°.$

Hence, $\triangle PAB$ is an equilateral triangle.

EXAMPLE 32 *In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X′Y′ at B. Prove that* $\angle AOB = 90°$.

[HOTS] [CBSE 2012, '13, '14, '17]



SOLUTION In quad. $APQB$, we have

$$\angle APO = 90^\circ \text{ and } \angle BQO = 90^\circ$$

[∵ tangent at any point is perpendicular to the radius through the point of contact].

Now, $\angle APO + \angle BQO + \angle QBC + \angle PAC = 360^\circ$

$\Rightarrow \quad \angle PAC + \angle QBC = 360^\circ - (\angle APO + \angle BQO) = 180^\circ$. ... (i)

We have

$$\angle CAO = \frac{1}{2}\angle PAC \text{ and } \angle CBO = \frac{1}{2}\angle QBC$$

[∵ tangents from an external point are equally inclined to the line segment joining the centre to that point].

$\therefore \quad \angle CAO + \angle CBO = \frac{1}{2}(\angle PAC + \angle QBC) = \frac{1}{2} \times 180^\circ = 90^\circ$. ... (ii)

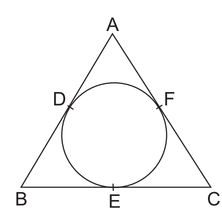
In $\triangle AOB$, we have

$$\angle CAO + \angle AOB + \angle CBO = 180^\circ$$

$\Rightarrow \quad \angle AOB = 180^\circ - (\angle CAO + \angle CBO) = 90^\circ$ [using (ii)].

EXAMPLE 33 *The incircle of an isosceles triangle ABC, with AB = AC, touches the sides AB, BC, CA at D, E and F respectively. Prove that E bisects BC.* [CBSE 2008, '12, '13C, '14]

SOLUTION We know that the tangents drawn from an external point to a circle are equal.



$\therefore \quad AD = AF$, ... (i) [tangents from A]

$BD = BE$, ... (ii) [tangents from B]

$CE = CF$. ... (iii) [tangents from C]

Now, $AB = AC$ [given]

$\Rightarrow \quad AD + BD = AF + CF$

$\Rightarrow \quad BD = CF$

$\Rightarrow \quad BE = CE$ [using (ii) and (iii)]

$\Rightarrow \quad E$ bisects BC.

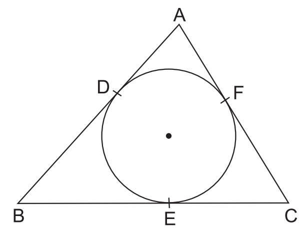
EXERCISE 8A

1. A point P is at a distance of 29 cm from the centre of a circle of radius 20 cm. Find the length of the tangent drawn from P to the circle. [CBSE 2017]

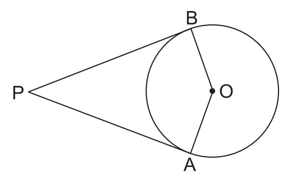
2. A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

3. Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE 2011]

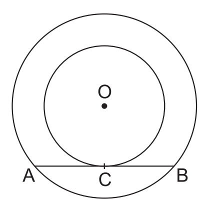
4. In the given figure, a circle inscribed in a triangle ABC, touches the sides AB, BC and AC at points D, E and F respectively. If $AB = 12$ cm, $BC = 8$ cm and $AC = 10$ cm, find the lengths of AD, BE and CF. [CBSE 2013]



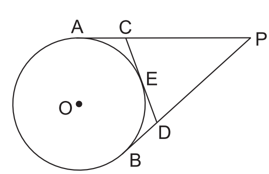
5. In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.



6. In the given figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that $AC = CB$.

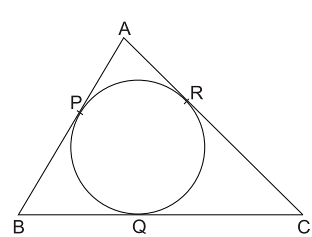


7. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14$ cm, find the perimeter of $\triangle PCD$. [CBSE 2002]

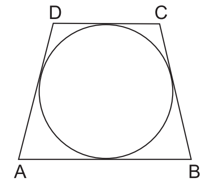


8. A circle is inscribed in a $\triangle ABC$ touching AB, BC and AC at P, Q and R respectively. If $AB = 10$ cm, $AR = 7$ cm and $CR = 5$ cm, find the length of BC.

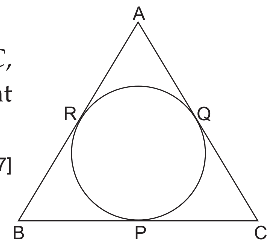
[CBSE 2009C]



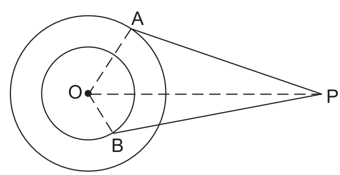
9. In the given figure, a circle touches all the four sides of a quadrilateral $ABCD$ whose three sides are $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD.



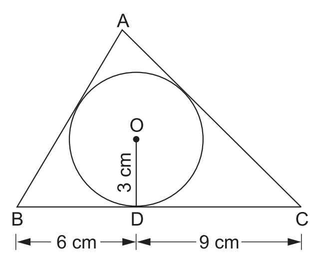
10. In the given figure, an isosceles triangle ABC, with $AB = AC$, circumscribes a circle. Prove that the point of contact P bisects the base BC.
[CBSE 2012, '17]



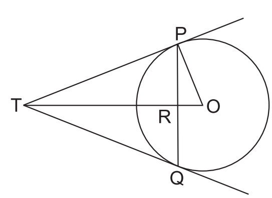
11. In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If $PA = 10$ cm, find the length of PB up to one place of decimal.



12. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 6 cm and 9 cm respectively. If the area of $\triangle ABC = 54\text{ cm}^2$ then find the lengths of sides AB and AC.
[CBSE 2011, '15]

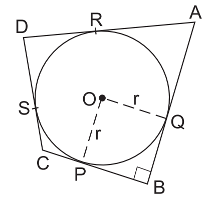


13. PQ is a chord of length 4.8 cm of a circle of radius 3 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP.
[CBSE 2013C]

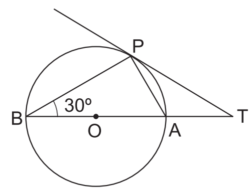


14. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. [CBSE 2014]

15. In the given figure, a circle with centre O, is inscribed in a quadrilateral $ABCD$ such that it touches the side BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm then find the radius of the circle. [CBSE 2008, '13]



16. In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, prove that $BA : AT = 2 : 1$. [HOTS] [CBSE 2015]



ANSWERS (EXERCISE 8A)

1. 21 cm **2.** 7 cm **3.** 12 cm

4. $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm **7.** 28 cm **8.** $BC = 8$ cm

9. $AD = 3$ cm **11.** 10.9 cm **12.** $AB = 10$ cm, $AC = 12$ cm

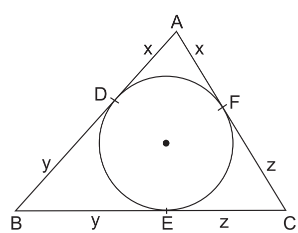
13. $TP = 4$ cm **15.** 11 cm

HINTS TO SOME SELECTED QUESTIONS

4. $x + y = 12$, $y + z = 8$, $z + x = 10$.

Solving, we get $x = 7$, $y = 5$, $z = 3$.

$\therefore$ $AD = x = 7$ cm, $BE = y = 5$ cm, $CF = z = 3$ cm.

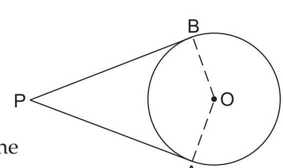


5. $OA \perp AP$ and $OB \perp BP$

$\Rightarrow$ $\angle OAP = 90^\circ$ and $\angle OBP = 90^\circ$

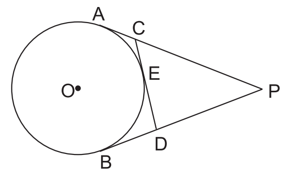
$\Rightarrow$ $\angle OAP + \angle OBP = 180^\circ$

$\Rightarrow$ quad. $AOBP$ is cyclic [$\because$ a quad. is cyclic if the sum of a pair of opposite angles is 180°].



7. $PA = PB, CA = CE, DB = DE$

[$\because$ tangents from an external point are equal].



Perimeter of $\triangle PCD = PC + CD + PD$

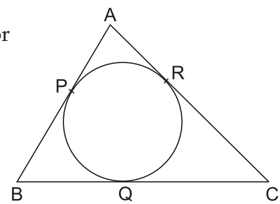
$= PC + CE + DE + PD$

$= PC + CA + DB + PD$

$= PA + PB = 2PA$ [$\because$ $PB = PA$]

$= (2 \times 14)$ cm $= 28$ cm.

8. We know that the tangents to a circle from an exterior point are equal.



$\therefore$ $AP = AR = 7$ cm;

$BQ = BP = AB - AP = (10 - 7)$ cm $= 3$ cm;

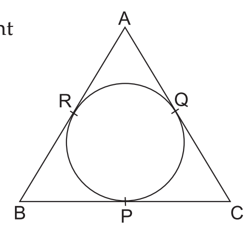
$CQ = CR = 5$ cm.

And so, $BC = BQ + CQ = (3 + 5)$ cm $= 8$ cm.

9. When a qudrilateral $ABCD$ is drawn to circumscribe a circle then $AB + CD = AD + BC$.

$\therefore$ $AD = AB + CD - BC = (6 + 4 - 7)$ cm $= 3$ cm.

10. We know that the tangents to a circle from an external point are equal.



$\therefore$ $AQ = AR, BP = BR, CP = CQ$.

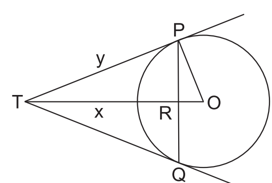
Now, $AB = AC \Rightarrow AR + BR = AQ + CQ$

$\Rightarrow$ $AR + BP = AR + CP \Rightarrow BP = CP$

$\Rightarrow$ P bisects the base BC.

13. R is the midpoint of PQ, i.e., $PR = QR = \left(\frac{1}{2} \times 4.8\right)$ cm $= 2.4$ cm.

Also, $TO \perp PQ$.



In right $\triangle PRO$, we have

$PO^2 = PR^2 + RO^2 \Rightarrow RO = \sqrt{PO^2 - PR^2}$

$= \sqrt{3^2 - 2.4^2} = \sqrt{3.24} = 1.8.$

Let $TR = x$ and $TP = y$.

In right $\triangle PTR$, we have $TP^2 = TR^2 + PR^2 \Rightarrow y^2 = x^2 + (2.4)^2$. ... (i)

In right $\triangle OTP$, we have

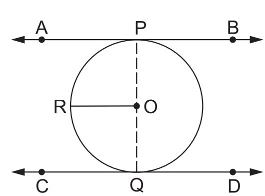
$TO^2 = TP^2 + PO^2 \Rightarrow (TR + RO)^2 = TP^2 + PO^2$

$\Rightarrow$ $(x + 1.8)^2 = y^2 + 3^2 \Rightarrow x^2 + 3.6x + 3.24 = y^2 + 9.$ ... (ii)

Solving (i) and (ii), we get $x = 3.2, y = 4$.

$\therefore$ $TP = 4$ cm.

14. Let APB and CQD be two parallel tangents to a circle with centre O.



Join OP and OQ. Draw $RO \parallel AB$.

Now, $\angle APO + \angle ROP = 180^\circ$ [co-interior angles]

$\Rightarrow$ $\angle ROP = 90^\circ$ [$\because$ $\angle APO = 90^\circ$].

Similarly, $\angle CQO + \angle ROQ = 180^\circ$ [co-interior angles]

$\Rightarrow \quad \angle ROQ = 90^\circ \quad [\because \angle CQO = 90^\circ]$.

$\therefore \quad \angle POQ = \angle ROP + \angle ROQ = 180^\circ$.

Hence, POQ is a straight line passing through O.

15. $POQB$ is a square, since $\angle PBQ = \angle BQO = \angle BPO = 90^\circ$, $OP = OQ = r$.

$\therefore \quad r = OQ = QB$.

We know that tangents to a circle from an exterior point are equal.

$\therefore \quad DS = DR,\ AR = AQ$.

Now, $AR = AD - DR = AD - DS = (23 - 5)$ cm $= 18$ cm

and $r = QB = AB - AQ = AB - AR = (29 - 18)$ cm $= 11$ cm.

16. $\angle APB = 90^\circ$ [angle in a semicircle]

$\therefore \quad \angle PAB = 90^\circ - \angle PBA = 90^\circ - 30^\circ = 60^\circ \quad [\because \angle PBA = \angle PBT = 30^\circ]$.

$\angle PAT + \angle PAB = 180^\circ$ [linear pair]

$\Rightarrow \quad \angle PAT = 180^\circ - \angle PAB = 180^\circ - 60^\circ = 120^\circ$.

$\angle APT = \angle PBA = 30^\circ$ [angles in alternate segments].

In $\triangle PAT$, we have

$\angle APT + \angle PAT + \angle PTA = 180^\circ$

$\Rightarrow \quad \angle PTA = 180^\circ - (\angle APT + \angle PAT) = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$.

Now, $\angle APT = \angle PTA = 30^\circ \Rightarrow AT = AP$ ... (i)

[sides opposite equal angles are equal]

In right $\triangle APB$, we have

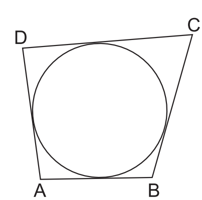
$$\cos(\angle PAB) = \frac{AP}{BA} \Rightarrow \cos 60^\circ = \frac{AP}{BA} \Rightarrow BA = 2AP. \quad \text{... (ii)}$$

From (i) and (ii), we get $BA : AT = 2AP : AP = 2 : 1$.

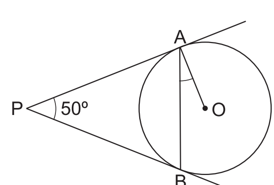
EXERCISE 8B

Very-Short-Answer Questions

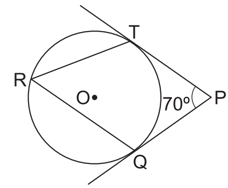
1. In the adjoining figure, a circle touches all the four sides of a quadrilateral $ABCD$ whose sides are $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm. Find the length of side AD. [CBSE 2011]



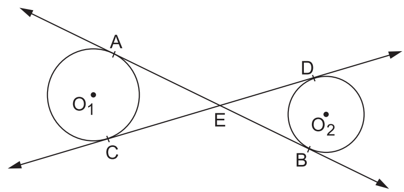
2. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50^\circ$ then what is the measure of $\angle OAB$. [CBSE 2015]



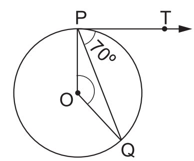
3. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$. [CBSE 2015]



4. In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that $AB = CD$. [CBSE 2014]

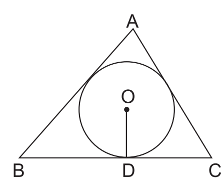


5. If PT is a tangent to a circle with centre O and PQ is a chord of the circle such that $\angle QPT = 70^\circ$, then find the measure of $\angle POQ$.



Short-Answer Questions

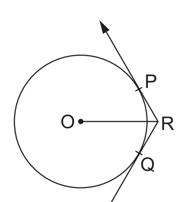
6. In the given figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D, are of lengths 4 cm and 3 cm respectively. If the area of $\triangle ABC = 21\text{ cm}^2$ then find the lengths of sides AB and AC. [CBSE 2011]



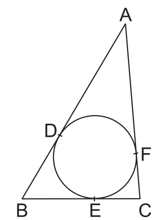
7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle (in cm) which touches the smaller circle. [CBSE 2012, '14]

8. Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

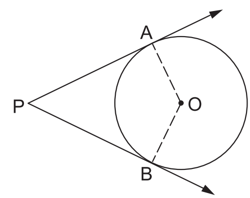
9. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$. [CBSE 2015]



10. In the given figure, a circle inscribed in a triangle ABC touches the sides AB, BC and CA at points D, E and F respectively. If $AB = 14$ cm, $BC = 8$ cm and $CA = 12$ cm. Find the lengths AD, BE and CF. [CBSE 2013]

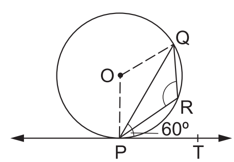


11. In the given figure, O is the centre of the circle. PA and PB are tangents. Show that $AOBP$ is a cyclic quadrilateral. [CBSE 2014]

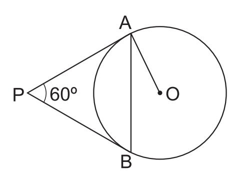


12. In two concentric circles, a chord of length 8 cm of the larger circle touches the smaller circle. If the radius of the larger circle is 5 cm then find the radius of the smaller circle. [CBSE 2013C]

13. In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$. [HOTS] [CBSE 2015]



14. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 60^\circ$ then find the measure of $\angle OAB$.



15. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60° then find the length of OP. [CBSE 2017]

ANSWERS (EXERCISE 8B)

1. $AD = 5$ cm **2.** 25° **3.** 55° **5.** 140°

6. $AB = 7.5$ cm, $AC = 6.5$ cm **7.** 8 cm

10. $AD = 9$ cm, $BE = 5$ cm, $CF = 3$ cm **12.** 3 cm **13.** 120°

14. 30° **15.** $a\sqrt{3}$

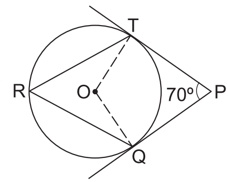
HINTS TO SOME SELECTED QUESTIONS

1. When a quadrilateral $ABCD$ circumscribes a circle then $AB + CD = AD + BC$.

$\therefore \quad AD = AB + CD - BC = (6 + 8 - 9)$ cm $= 5$ cm.

2. $\angle APB = 2\angle OAB \Rightarrow \angle OAB = \frac{1}{2} \times \angle APB = \frac{50^\circ}{2} = 25^\circ$. [See Solved Example 13.]

3. Join OT and OQ.



Then, $\angle TOQ + \angle TPQ = 180^\circ \Rightarrow \angle TOQ = 180^\circ - 70^\circ = 110^\circ$.
[See Theorem 8.]

Now, $\angle TRQ = \frac{1}{2} \times \angle TOQ = \frac{1}{2} \times 110^\circ = 55^\circ$

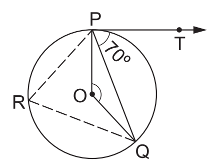
[∵ the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle].

4. We know that the tangents to a circle from an external point are equal.

∴ $AE = CE, BE = DE$

$\Rightarrow$ $AE + BE = CE + DE \Rightarrow AB = CD$.

5. Mark a point R in the alternate segment.



Join RP and RQ.

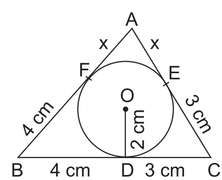
Then, $\angle PRQ = \angle QPT = 70^\circ$

[angles in the alternate segments].

∴ $\angle POQ = 2\angle PRQ = 2 \times 70^\circ = 140^\circ$.

[∵ angle subtended by an arc on the centre is double the angle subtended at any point on the remaining part of the circle.]

6. Mark the points of contact E and F of tangents AC and AB respectively.



We know that the tangents to a circle from an external point are equal.

∴ $AE = AF = x$ cm (say);

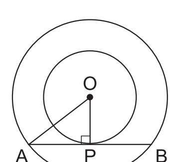
$BD = BF = 4$ cm; $CD = CE = 3$ cm.

Now, $\text{ar}(\triangle ABC) = \frac{1}{2} \times (\text{perimeter of } \triangle ABC) \times r$

$\Rightarrow$ $21 = \frac{1}{2} \times \{(x+4) + (4+3) + (3+x)\} \times 2 \Rightarrow x = 3.5$.

∴ $AB = (x+4)$ cm $= 7.5$ cm; $AC = (x+3)$ cm $= 6.5$ cm.

7. Let AB be the chord, P be the point where it touches the smaller circle and O be the common centre.



AB is tangent to smaller circle and so, $OP \perp AB$.

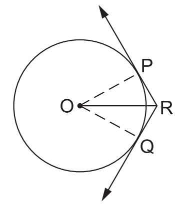
∴ P is the midpoint of AB, i.e., $AP = PB$

[∵ a perpendicular from the centre on any chord bisects the chord].

Now, $OP = 3$ cm, $OA = 5$ cm $\Rightarrow AP = \sqrt{OA^2 - OP^2} = \sqrt{5^2 - 3^2} = 4$.

∴ $AB = 2AP = 8$ cm.

9. Join OP and OQ.



$\angle PRQ = 120^\circ \Rightarrow \angle PRO = \angle QRO = \frac{1}{2} \times \angle PRQ = 60^\circ$.

[$\because$ two tangents from an external point are equally inclined to the line segment joining the centre to that point.

And so, $\angle PRO = \angle QRO = \frac{1}{2} \angle PRQ$.]

In right $\triangle OPR$, we have

$$\cos \angle PRO = \frac{PR}{OR} \Rightarrow \cos 60^\circ = \frac{PR}{OR} \Rightarrow \frac{1}{2} = \frac{PR}{OR} \Rightarrow PR = \frac{1}{2} \times OR.$$

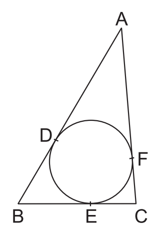
Similarly, $RQ = \frac{1}{2} \times OR$.

$\therefore \quad PR + RQ = OR$.

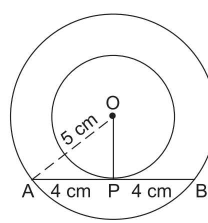
10. $x + y = 14$, $y + z = 8$, $z + x = 12$.

Solving, $x = 9$, $y = 5$, $z = 3$.

$\therefore \quad AD = x = 9$ cm, $BE = y = 5$ cm, $CF = z = 3$ cm.



12. Let AB be the chord, P be the point where it touches the smaller circle and O be the common centre.



Then, AB is the tangent to smaller circle at P.

$\therefore \quad OP \perp AB$.

And so, P is the midpoint of AB,

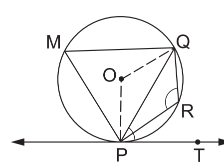
i.e., $AP = PB = \frac{1}{2}AB = \left(\frac{1}{2} \times 8\right)$ cm $= 4$ cm

[$\because$ perpendicular from the centre on any chord bisects the chord].

Now, $OA = 5$ cm, $AP = 4$ cm $\Rightarrow OP = \sqrt{OA^2 - AP^2} = \sqrt{5^2 - 4^2} = 3$.

$\therefore \quad$ radius of smaller circle $= OP = 3$ cm.

13. Mark a point M in the alternate segment.



Join MP and MQ.

Then, $\angle PMQ = \angle QPT = 60^\circ$

[angles in the alternate segments].

Now, $\angle PMQ + \angle PRQ = 180^\circ$

[$\because$ $PMQR$ is a cyclic quadrilateral]

$\Rightarrow \quad \angle PRQ = 180^\circ - \angle PMQ = 180^\circ - 60^\circ = 120^\circ$.

14. $PA = PB$ [$\because$ tangents from an external point are equal]

$\Rightarrow \quad \angle PAB = \angle PBA = \frac{1}{2}\times(180^\circ - \angle APB) = \frac{1}{2}\times(180^\circ - 60^\circ) = 60^\circ.$

Now, $\angle PAO = 90^\circ$ [$\because$ radius through the point of contact is perpendicular to the tangent]

$\Rightarrow \quad \angle OAB + \angle PAB = 90^\circ \Rightarrow \angle OAB = 90^\circ - \angle PAB = 90^\circ - 60^\circ = 30^\circ.$

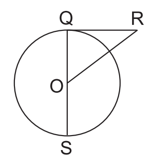
MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

1. The number of tangents that can be drawn from an external point to a circle is [CBSE 2011, '12]

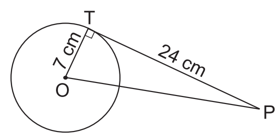
   (a) 1 (b) 2 (c) 3 (d) 4

2. In the given figure, RQ is a tangent to the circle with centre O. If $SQ = 6$ cm and $QR = 4$ cm, then OR is equal to [CBSE 2014]



   (a) 2.5 cm (b) 3 cm
   (c) 5 cm (d) 8 cm

3. In a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then length $OP = ?$



   (a) 30 cm (b) 28 cm (c) 25 cm (d) 18 cm

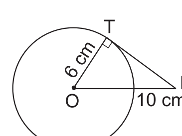
4. Which of the following pairs of lines in a circle cannot be parallel? [CBSE 2011]

   (a) Two chords (b) A chord and a tangent
   (c) Two tangents (d) Two diameters

5. The chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is [CBSE 2014]

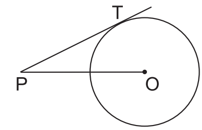
   (a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$ (c) $10\sqrt{2}$ (d) $10\sqrt{3}$

6. In the given figure, PT is a tangent to the circle with centre O. If $OT = 6$ cm and $OP = 10$ cm, then the length of tangent PT is



   (a) 8 cm (b) 10 cm
   (c) 12 cm (d) 16 cm

7. In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then, the radius of the circle is [CBSE 2011, '12]

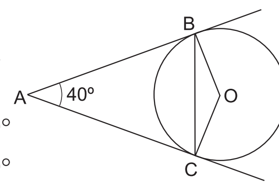


(a) 10 cm (b) 12 cm
(c) 13 cm (d) 15 cm

8. PQ is a tangent to a circle with centre O at the point P. If $\triangle OPQ$ is an isosceles triangle, then $\angle OQP$ is equal to [CBSE 2014]

(a) 30° (b) 45° (c) 60° (d) 90°

9. In the given figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 40°$. Then, $\angle BOC$ is equal to [CBSE 2011, '14]

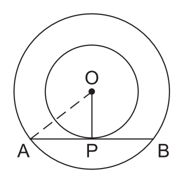


(a) 80° (b) 100°
(c) 120° (d) 140°

10. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents to the circle drawn from A and B is [CBSE 2013C]

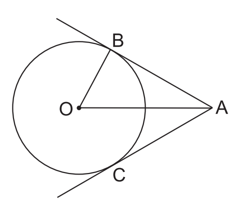
(a) 30° (b) 60° (c) 90° (d) 120°

11. In the given figure, O is the centre of two concentric circles of radii 6 cm and 10 cm. AB is a chord of outer circle which touches the inner circle. The length of chord AB is



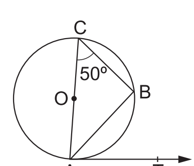
(a) 8 cm (b) 14 cm
(c) 16 cm (d) $\sqrt{136}$ cm

12. In the given figure, AB and AC are tangents to a circle with centre O and radius 8 cm. If $OA = 17$ cm, then the length of AC (in cm) is [CBSE 2012]



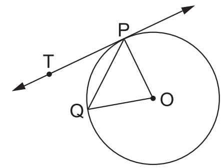
(a) 9 (b) 15
(c) $\sqrt{353}$ (d) 25

13. In the given figure, O is the centre of a circle, AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A then $\angle BAT = ?$



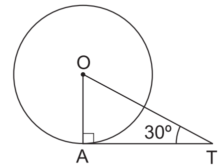
(a) 40° (b) 50°
(c) 60° (d) 65°

14. In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If $\angle POQ = 70^\circ$, then $\angle TPQ$ is equal to [CBSE 2011]



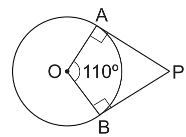
(a) 35° (b) 45°
(c) 55° (d) 70°

15. In the given figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then, $AT = ?$



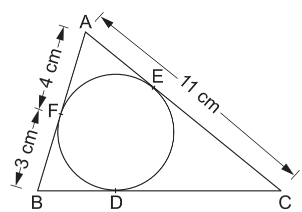
(a) 4 cm (b) 2 cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

16. If PA and PB are two tangents to a circle with centre O such that $\angle AOB = 110^\circ$ then $\angle APB$ is equal to [CBSE 2011, '14]



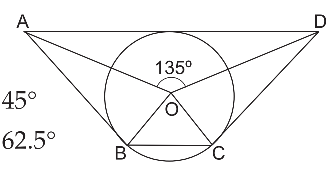
(a) 55° (b) 60°
(c) 70° (d) 90°

17. In the given figure, the length of BC is [CBSE 2012, '14]



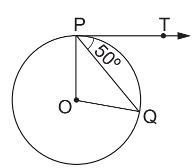
(a) 7 cm
(b) 10 cm
(c) 14 cm
(d) 15 cm

18. In the given figure, if $\angle AOD = 135^\circ$ then $\angle BOC$ is equal to [CBSE 2013C]



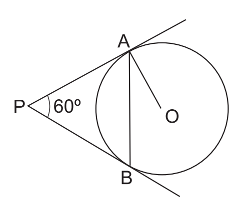
(a) 25° (b) 45°
(c) 52.5° (d) 62.5°

19. In the given figure, O is the centre of a circle and PT is the tangent to the circle. If PQ is a chord such that $\angle QPT = 50^\circ$ then $\angle POQ = ?$



(a) 100° (b) 90°
(c) 80° (d) 75°

20. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 60^\circ$ then $\angle OAB$ is [CBSE 2011]

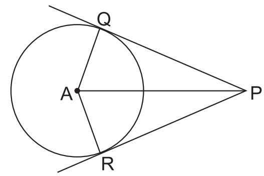


(a) 15° (b) 30°
(c) 60° (d) 90°

21. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm then the length of each tangent is

(a) 3 cm (b) $\frac{3\sqrt{3}}{2}$ cm (c) $3\sqrt{3}$ cm (d) 6 cm

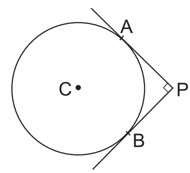
22. In the given figure, PQ and PR are tangents to a circle with centre A. If $\angle QPA = 27^\circ$ then $\angle QAR$ equals [CBSE 2012]



(a) 63° (b) 117°

(c) 126° (d) 153°

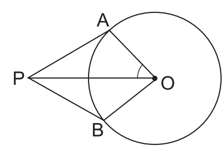
23. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then the length of each tangent is [CBSE 2013]



(a) 3 cm (b) 4 cm

(c) 5 cm (d) 6 cm

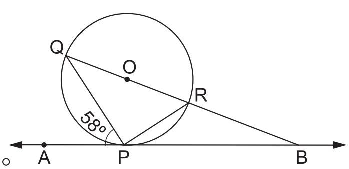
24. If PA and PB are two tangents to a circle with centre O such that $\angle APB = 80^\circ$. Then, $\angle AOP = ?$



(a) 40° (b) 50°

(c) 60° (d) 70°

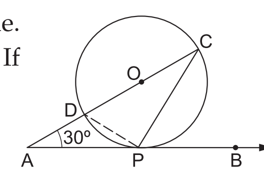
25. In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If $\angle APQ = 58^\circ$ then the measure of $\angle PQB$ is [CBSE 2014]



(a) 32° (b) 58°

(c) 122° (d) 132°

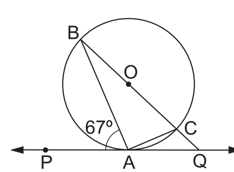
26. In the given figure, O is the centre of the circle. AB is the tangent to the circle at the point P. If $\angle PAO = 30^\circ$ then $\angle CPB + \angle ACP$ is equal to



(a) 60° (b) 90°

(c) 120° (d) 150°

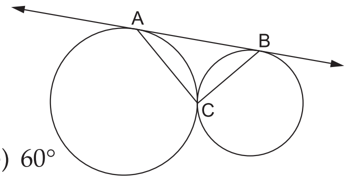
27. In the given figure, PQ is a tangent to a circle with centre O. A is the point of contact. If $\angle PAB = 67^\circ$, then the measure of $\angle AQB$ is



(a) 73° (b) 64°

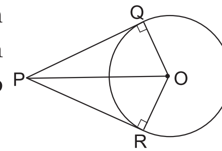
(c) 53° (d) 44°

28. In the given figure, two circles touch each other at C and AB is a tangent to both the circles. The measure of $\angle ACB$ is [HOTS] [CBSE 2013C]



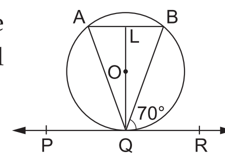
(a) 45° (b) 60°
(c) 90° (d) 120°

29. O is the centre of a circle of radius 5 cm. At a distance of 13 cm from O, a point P is taken. From this point, two tangents PQ and PR are drawn to the circle. Then, the area of quad. $PQOR$ is



(a) 60 cm^2 (b) 32.5 cm^2
(c) 65 cm^2 (d) 30 cm^2

30. In the given figure, PQR is a tangent to the circle at Q, whose centre is O and AB is a chord parallel to PR such that $\angle BQR = 70^\circ$. Then, $\angle AQB = ?$

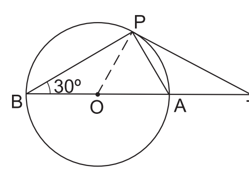


(a) 20° (b) 35°
(c) 40° (d) 45°

31. The length of the tangent from an external point P to a circle of radius 5 cm is 10 cm. The distance of the point from the centre of the circle is
(a) 8 cm (b) $\sqrt{104}$ cm (c) 12 cm (d) $\sqrt{125}$ cm

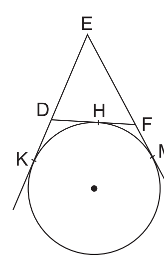
[CBSE 2013C]

32. In the given figure, O is the centre of a circle, BOA is its diameter and the tangent at the point P meets BA extended at T. If $\angle PBO = 30^\circ$ then $\angle PTA = ?$



(a) 60° (b) 30°
(c) 15° (d) 45°

33. In the given figure, a circle touches the side DF of $\triangle EDF$ at H and touches ED and EF produced at K and M respectively. If $EK = 9$ cm then the perimeter of $\triangle EDF$ is [CBSE 2012]



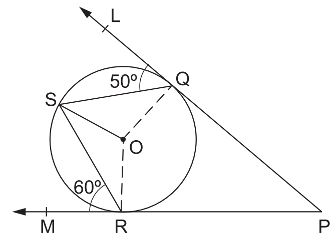
(a) 9 cm (b) 12 cm
(c) 13.5 cm (d) 18 cm

34. To draw a pair of tangents to a circle, which are inclined to each other at an angle of 45°, we have to draw tangents at the end points of those

two radii, the angle between which is [CBSE 2011]

(a) 105° (b) 135° (c) 140° (d) 145°

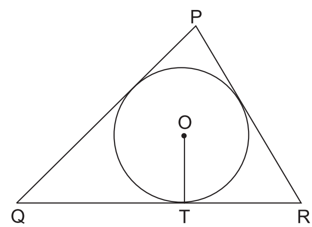
35. In the given figure, O is the centre of a circle; PQL and PRM are the tangents at the points Q and R respectively and S is a point on the circle such that $\angle SQL = 50°$ and $\angle SRM = 60°$. Then, $\angle QSR = ?$



(a) 40° (b) 50°

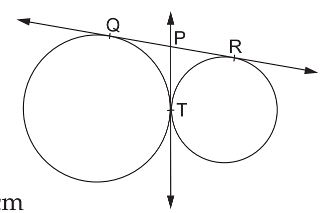
(c) 60° (d) 70°

36. In the given figure, a triangle PQR is drawn to circumscribe a circle of radius 6 cm such that the segments QT and TR into which QR is divided by the point of contact T, are of lengths 12 cm and 9 cm respectively. If the area of $\triangle PQR = 189\ \text{cm}^2$ then the length of side PQ is [CBSE 2011]



(a) 17.5 cm (b) 20 cm (c) 22.5 cm (d) 25 cm

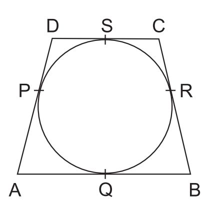
37. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If $PT = 3.8$ cm then the length of QR is [CBSE 2014]



(a) 1.9 cm (b) 3.8 cm

(c) 5.7 cm (d) 7.6 cm

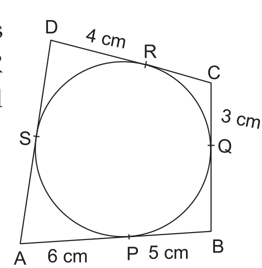
38. In the given figure, quad. $ABCD$ is circumscribed, touching the circle at P, Q, R and S. If $AP = 5$ cm, $BC = 7$ cm and $CS = 3$ cm. Then, the length $AB = ?$



(a) 9 cm (b) 10 cm

(c) 12 cm (d) 8 cm

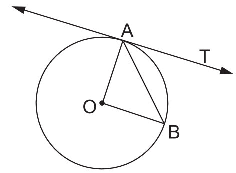
39. In the given figure, quad. $ABCD$ is circumscribed, touching the circle at P, Q, R and S. If $AP = 6$ cm, $BP = 5$ cm, $CQ = 3$ cm and $DR = 4$ cm then perimeter of quad. $ABCD$ is



(a) 18 cm (b) 27 cm

(c) 36 cm (d) 32 cm

40. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100^\circ$ then $\angle BAT$ is equal to [CBSE 2011]

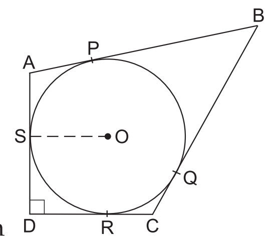


(a) 40° (b) 50°
(c) 90° (d) 100°

41. In a right triangle ABC, right-angled at B, $BC = 12$ cm and $AB = 5$ cm. The radius of the circle inscribed in the triangle is [CBSE 2014]

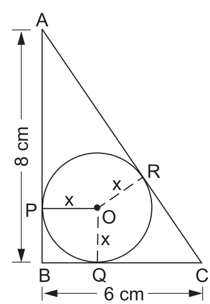
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

42. In the given figure, a circle is inscribed in a quadrilateral $ABCD$ touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius of the circle is 10 cm, $BC = 38$ cm, $PB = 27$ cm and $AD \perp CD$ then the length of CD is [CBSE 2013]



(a) 11 cm (b) 15 cm
(c) 20 cm (d) 21 cm

43. In the given figure, $\triangle ABC$ is right-angled at B such that $BC = 6$ cm and $AB = 8$ cm. A circle with centre O has been inscribed inside the triangle. $OP \perp AB$, $OQ \perp BC$ and $OR \perp AC$. If $OP = OQ = OR = x$ cm then $x = ?$

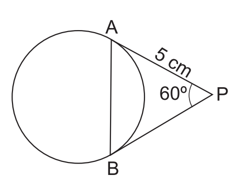


(a) 2 cm (b) 2.5 cm
(c) 3 cm (d) 3.5 cm

44. Quadrilateral $ABCD$ is circumscribed to a circle. If $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm then the length of AD is [CBSE 2012]

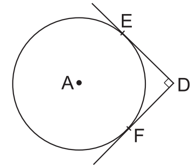
(a) 3 cm (b) 4 cm
(c) 6 cm (d) 7 cm

45. In the given figure, PA and PB are tangents to the given circle such that $PA = 5$ cm and $\angle APB = 60^\circ$. The length of chord AB is



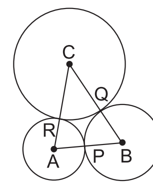
(a) $5\sqrt{2}$ cm (b) 5 cm
(c) $5\sqrt{3}$ cm (d) 7.5 cm

46. In the given figure, DE and DF are tangents from an external point D to a circle with centre A. If $DE = 5$ cm and $DE \perp DF$ then the radius of the circle is [CBSE 2013]



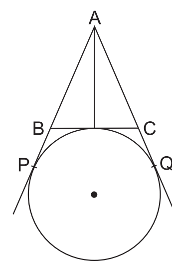
(a) 3 cm (b) 4 cm
(c) 5 cm (d) 6 cm

47. In the given figure, three circles with centres A, B, C respectively touch each other externally. If $AB = 5$ cm, $BC = 7$ cm and $CA = 6$ cm then the radius of the circle with centre A is



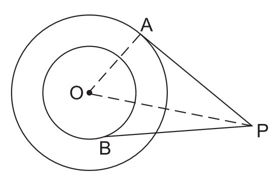
(a) 1.5 cm (b) 2 cm
(c) 2.5 cm (d) 3 cm

48. In the given figure, AP, AQ and BC are tangents to the circle. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm then the length of AP is [CBSE 2012]



(a) 15 cm (b) 10 cm
(c) 9 cm (d) 7.5 cm

49. In the given figure, O is the centre of two concentric circles of radii 5 cm and 3 cm. From an external point P tangents PA and PB are drawn to these circles. If $PA = 12$ cm then PB is equal to



(a) $5\sqrt{2}$ cm (b) $3\sqrt{5}$ cm
(c) $4\sqrt{10}$ cm (d) $5\sqrt{10}$ cm

True/False Type

50. Which of the following statements is not true?
   (a) If a point P lies inside a circle, no tangent can be drawn to the circle, passing through P.
   (b) If a point P lies on the circle, then one and only one tangent can be drawn to the circle at P.
   (c) If a point P lies outside the circle, then only two tangents can be drawn to the circle from P.
   (d) A circle can have more than two parallel tangents, parallel to a given line.

51. Which of the following statements is not true?
   (a) A tangent to a circle intersects the circle exactly at one point.
   (b) The point common to the circle and its tangent is called the point of contact.
   (c) The tangent at any point of a circle is perpendicular to the radius of the circle through the point of contact.
   (d) A straight line can meet a circle at one point only.
52. Which of the following statements is not true?
   (a) A line which intersects a circle in two points, is called a secant of the circle.
   (b) A line intersecting a circle at one point only, is called a tangent to the circle.
   (c) The point at which a line touches the circle, is called the point of contact.
   (d) A tangent to the circle can be drawn from a point inside the circle.

Assertion-and-Reason Type

Each question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer, use the following code:

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

53.

Assertion (A)	Reason (R)
At a point P of a circle with centre O and radius 12 cm, a tangent PQ of length 16 cm is drawn. Then, OQ = 20 cm.	The tangent at any point of a circle is perpendicular to the radius through the point of contact.

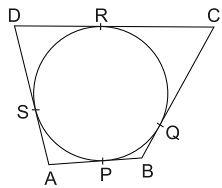
The correct answer is (a)/(b)/(c)/(d).

54.

Assertion (A)	Reason (R)
If two tangents are drawn to a circle from an external point then they subtend equal angles at the centre.	A parallelogram circumscribing a circle is a rhombus.

The correct answer is (a)/(b)/(c)/(d).

55.

Assertion (A)	Reason (R)
In the given figure, a quad. *ABCD* is drawn to circumscribe a given circle, as shown. Then, $AB + BC = AD + DC$. 	In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

The correct answer is (a)/(b)/(c)/(d).

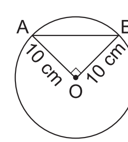
ANSWERS (MCQ)

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (d)
10. (d)	**11.** (c)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (b)
19. (a)	**20.** (b)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (d)
28. (c)	**29.** (a)	**30.** (c)	**31.** (d)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (d)	**36.** (c)
37. (d)	**38.** (a)	**39.** (c)	**40.** (b)	**41.** (b)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (b)
46. (c)	**47.** (b)	**48.** (d)	**49.** (c)	**50.** (d)	**51.** (d)	**52.** (d)	**53.** (a)	**54.** (b)
55. (d)								

HINTS TO SOME SELECTED QUESTIONS

4. Every diameter passes through the centre and so no two diameters of a circle can be parallel.

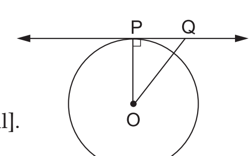
5. $AB = \sqrt{OA^2 + OB^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ cm.



8. We have $OP \perp PQ$, i.e., $\angle OPQ = 90°$.

$\triangle OPQ$ is isosceles $\Rightarrow OP = PQ \Rightarrow \angle OQP = \angle POQ = 45°$

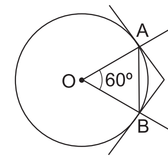
[$\because$ in a triangle, angles opposite equal sides are equal].



9. *ABOC* is a cyclic quadrilateral [see Solved Example 18].

$\therefore \quad \angle BAC + \angle BOC = 180° \Rightarrow \angle BOC = 180° - \angle BAC = 180° - 40° = 140°$.

10. Let P be the point of intersection of tangents at A and B.

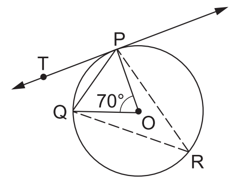


Then, $OAPB$ is a cyclic quadrilateral. [see Solved Example 18]

$\therefore \quad \angle AOB + \angle APB = 180^\circ \Rightarrow \angle APB = 180^\circ - 60^\circ = 120^\circ.$

13. $\angle BAT = \angle ACB = 50^\circ$ [angles in alternate segments].

14. Mark a point R in the alternate segment.



Join PR and QR.

Then, $\angle PRQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 70^\circ = 35^\circ.$

Now, $\angle TPQ = \angle PRQ = 35^\circ$ [angles in alternate segments].

15. In right ΔOAT (having right angle at A), we have

$$\cos(\angle OTA) = \frac{AT}{OT} \Rightarrow \cos 30^\circ = \frac{AT}{4} \Rightarrow AT = \left(\frac{\sqrt{3}}{2} \times 4\right) \text{cm} = 2\sqrt{3} \text{ cm}.$$

17. We have $AF = AE$, $BD = BF$, $CD = CE$. [$\because$ tangents from an external point are equal]

$\therefore \quad CE = AC - AE = AC - AF = (11 - 4)$ cm $= 7$ cm;

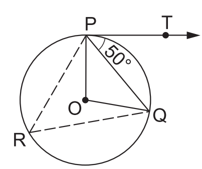
$BD = BF = 3$ cm; $CD = CE = 7$ cm.

And so, $BC = BD + CD = (3 + 7)$ cm $= 10$ cm.

18. $\angle AOD + \angle BOC = 180^\circ$ [see Solved Example 29]

$\Rightarrow \quad \angle BOC = 180^\circ - \angle AOD = 180^\circ - 135^\circ = 45^\circ.$

19. Mark a point R in the alternate segment.



Join PR and QR.

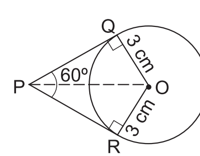
Then, $\angle PRQ = \angle QPT = 50^\circ$ [angles in alternate segments].

Now, $\angle POQ = 2\angle PRQ = 100^\circ.$

[$\because$ angle at the centre is double the angle on the circle]

20. $\angle APB = 2\angle OAB \Rightarrow \angle OAB = \frac{1}{2}\angle APB = 30^\circ.$ [See Solved Example 13.]

21. Let PQ and PR be tangents to a circle with centre O from a point P such that $\angle QPR = 60^\circ$.



Then, $\angle QPO = \angle RPO = \frac{1}{2}\angle QPR = 30^\circ$

[$\because$ tangents from an external point are equally inclined to the line joining the point and the centre].

In right ΔOPQ, we have

$$\tan(\angle OPQ) = \frac{OQ}{PQ} \Rightarrow \tan 30^\circ = \frac{3}{PQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{PQ} \Rightarrow PQ = 3\sqrt{3} \text{ cm}.$$

Now, $PR = PQ = 3\sqrt{3}$ cm.

22. $\angle QPR = 2\angle QPA = 2 \times 27^\circ = 54^\circ$ [$\because$ AP bisects $\angle QPR$].

Now, $QPRA$ is a cyclic quadrilateral [see Solved Example 18]

$\Rightarrow \quad \angle QAR + \angle QPR = 180^\circ$

$\Rightarrow \quad \angle QAR = 180^\circ - 54^\circ = 126^\circ$.

23. Join AC and BC. Then, $ACBP$ is a square.

$\therefore \quad PA = PB = AC =$ radius of the circle = 4 cm.

25. $\angle QPR = 90^\circ$ [angle in a semicircle]

and $\angle QRP = \angle APQ = 58^\circ$ [angles in alternate segments].

In $\triangle PQR$, we have $\angle PQR + \angle QRP + \angle QPR = 180^\circ \Rightarrow \angle PQR = 32^\circ$.

$\therefore \quad \angle PQB = \angle PQR = 32^\circ$.

26. $\angle DPC = 90^\circ$ [angle in a semicircle]

and $\angle DPA = \angle DCP$ [angles in alternate segments].

Now, $\angle CPB + \angle DPA + \angle DPC = 180^\circ$

$\Rightarrow \quad \angle CPB + \angle DPA = 90^\circ \Rightarrow \angle CPB + \angle DCP = 90^\circ$

$\Rightarrow \quad \angle CPB + \angle ACP = 90^\circ$.

27. $\angle BAC = 90^\circ$ [angle in a semicircle].

$\angle PAB + \angle BAC + \angle CAQ = 180^\circ \Rightarrow \angle CAQ = 180^\circ - (90^\circ + 67^\circ) = 23^\circ$.

$\angle ACB = \angle PAB = 67^\circ$ [angles in alternate segments]

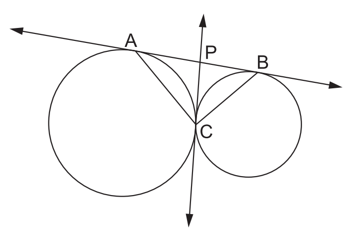
$\angle ACB + \angle ACQ = 180^\circ$ [linear pair]

$\Rightarrow \quad \angle ACQ = 180^\circ - 67^\circ = 113^\circ$.

Now, $\angle CAQ + \angle ACQ + \angle AQC = 180^\circ$ [in $\triangle ACQ$]

$\Rightarrow \quad \angle AQC = 180^\circ - (23^\circ + 113^\circ) = 44^\circ \Rightarrow \angle AQB = 44^\circ$.

28. Draw a tangent to the circles at point C. Let it meet AB at P.



Then, $PA = PC$ and $PB = PC$.

$PA = PC \Rightarrow \angle PAC = \angle PCA$

$PB = PC \Rightarrow \angle PBC = \angle PCB$

$\therefore \quad \angle PAC + \angle PBC = \angle PCA + \angle PCB = \angle ACB$

$\Rightarrow \quad \angle PAC + \angle PBC + \angle ACB = 2\angle ACB$

$\Rightarrow \quad 180^\circ = 2\angle ACB \Rightarrow \angle ACB = 90^\circ$.

29. Clearly, $OQ = OR = 5$ cm, $\angle OQP = \angle ORP = 90^\circ$ and $OP = 13$ cm.

$\therefore \quad PQ^2 = OP^2 - OQ^2 = (13)^2 - (5)^2 = 169 - 25 = 144 \Rightarrow PQ = \sqrt{144} = 12$ cm.

$\therefore \quad \text{ar}(\triangle OQP) = \frac{1}{2} \times PQ \times OQ = \left(\frac{1}{2} \times 12 \times 5\right) \text{cm}^2 = 30 \text{ cm}^2$.

Similarly, $\text{ar}(\triangle ORP) = 30 \text{ cm}^2$.

$\therefore \quad$ ar(quad. $PQOR$) $= (30 + 30) \text{ cm}^2 = 60 \text{ cm}^2$.

30. $\angle QAB = \angle BQR = 70^\circ$ [angles in alternate segments]

$\angle ABQ = \angle BQR = 70^\circ$ [alternate int. angles].

In $\triangle AQB$, we have

$$\angle AQB = 180^\circ - (\angle QAB + \angle ABQ) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ.$$

32. $\angle APB = 90^\circ$ [angle in a semicircle].

$\therefore$ $\angle PAB = 90^\circ - \angle PBA = 90^\circ - 30^\circ = 60^\circ$.

$\angle PAT + \angle PAB = 180^\circ$ [linear pair]

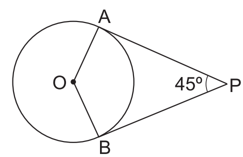
$\Rightarrow$ $\angle PAT = 180^\circ - \angle PAB = 180^\circ - 60^\circ = 120^\circ$.

$\angle APT = \angle PBA = 30^\circ$ [angles in alternate segments].

Now, $\angle APT + \angle PAT + \angle PTA = 180^\circ$ [in $\triangle PAT$]

$\Rightarrow$ $\angle PTA = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$.

34. Let PA and PB be the desired tangents to a circle with centre O from an exterior point P.



Then, $\angle APB = 45^\circ$.

$BOAP$ must be a cyclic quadrilateral.

$\therefore$ $\angle AOB + \angle APB = 180^\circ \Rightarrow \angle AOB = 180^\circ - 45^\circ = 135^\circ$.

So, the angle between the two radii must be 135°.

35. Since PQL is a tangent and OQ is a radius, so $\angle OQL = 90^\circ$.

$\therefore$ $\angle OQS = 90^\circ - 50^\circ = 40^\circ$.

Now, $OQ = OS \Rightarrow \angle OSQ = \angle OQS = 40^\circ$.

Similarly, $\angle ORS = 90^\circ - 60^\circ = 30^\circ$.

And, $OR = OS \Rightarrow \angle OSR = \angle ORS = 30^\circ$.

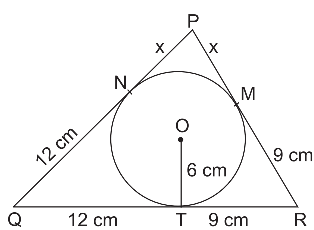
$\therefore$ $\angle QSR = \angle OSQ + \angle OSR = 40^\circ + 30^\circ = 70^\circ$.

36. $\text{ar}(\triangle PQR) = \frac{1}{2} \times (\text{perimeter of } \triangle PQR) \times r$

$\Rightarrow$ $189 = \frac{1}{2} \times \{(x+12) + (12+9) + (9+x)\} \times 6$

$\Rightarrow$ $x = 10.5$

$\therefore$ $PQ = (x+12)$ cm $= (10.5+12)$ cm $= 22.5$ cm.



37. $PQ = PT = 3.8$ cm, $PR = PT = 3.8$ cm,

$QR = PQ + PR = (3.8 + 3.8)$ cm $= 7.6$ cm.

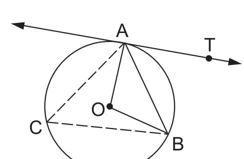
38. Since the lengths of tangents drawn from an external point to a circle are equal, we have $AQ = AP = 5$ cm.

$CR = CS = 3$ cm and $BR = (BC - CR) = (7-3)$ cm $= 4$ cm.

$BQ = BR = 4$ cm.

$\therefore$ $AB = (BR + BQ) = (5+4)$ cm $= 9$ cm.

40. Mark a point C in the alternate segment. Join AC and BC.



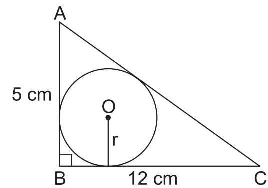
$\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 100^\circ = 50^\circ$

[$\because$ angle at the centre is double the angle on the circle].

Now, $\angle BAT = \angle ACB = 50^\circ$ [angles in alternate segments].

41. In right $\triangle ABC$, we have

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{5^2 + 12^2} \text{ cm} = 13 \text{ cm}.$$



$$\text{ar}(\triangle ABC) = \frac{1}{2} \times (\text{perimeter of } \triangle ABC) \times r$$

$$\Rightarrow \quad \frac{1}{2} \times 5 \times 12 = \frac{1}{2} \times (5 + 12 + 13) \times r \Rightarrow r = 2 \text{ cm}.$$

$$\left[\because \text{ ar}(\triangle ABC) = \frac{1}{2} \times BC \times AB\right]$$

42. $BQ = PB = 27$ cm; $CQ = BC - BQ = (38 - 27)$ cm $= 11$ cm; $CR = CQ = 11$ cm.

Join OR. Then, $SDRO$ is a square.

$\therefore \quad DR = SO =$ radius $= 10$ cm.

And so, $CD = DR + CR = (10 + 11)$ cm $= 21$ cm.

43. $AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow \quad AC = \sqrt{100}$ cm $= 10$ cm.

$CR = CQ = (BC - BQ) = (6 - x)$ cm.

$AR = AP = (AB - BP) = (8 - x)$ cm.

$AC = (AR + CR) = [(8 - x) + (6 - x)]$ cm $= (14 - 2x)$ cm

$\Rightarrow \quad (14 - 2x) = 10 \Rightarrow 2x = 4 \Rightarrow x = 2$ cm.

44. $AB + CD = AD + BC$ [see Solved Example 26]

$\Rightarrow \quad AD = AB + CD - BC = (6 + 4 - 7)$ cm $= 3$ cm.

45. $PA = PB \Rightarrow \angle PBA = \angle PAB = x^\circ$ (say)

Then, $x^\circ + x^\circ + 60^\circ = 180 \Rightarrow x = 60^\circ$.

$\therefore \quad \triangle APB$ is equilateral and so, $AB = PA = 5$ cm.

46. Join AE and AF. Then, $AFDE$ is a square.

$\therefore \quad$ radius of circle $= AE = DE = 5$ cm.

47. Let the radii of the three circles be x, y, z respectively. Then,

$$x + y = 5,\ y + z = 7 \text{ and } z + x = 6 \Rightarrow 2(x + y + z) = 18 \Rightarrow x + y + z = 9.$$

$\therefore \quad x = (x + y + z) - (y + z) = (9 - 7) = 2$ cm.

49. Join OB. Then, $OA = 5$ cm, $OB = 3$ cm.

$OP = \sqrt{OA^2 + PA^2} = \sqrt{5^2 + 12^2} = 13$ cm.

$PB = \sqrt{OP^2 - OB^2} = \sqrt{13^2 - 3^2} = \sqrt{160}$ cm $= 4\sqrt{10}$ cm.

53. $OQ = \sqrt{OP^2 + PQ^2} = \sqrt{12^2 + 16^2} = 20$ cm.

$\therefore \quad$ A is true. Also, R is true and is a correct explanation of A.

Hence, the correct answer is (a).

54. A and R are both true (see Theorem 4(i) and Solved Example 28) but R is not a correct explanation of A.

Hence, the correct answer is (b).

55. A is false. Correct relation is $AB + CD = AD + BC$.

R is true (see Theorem 5).

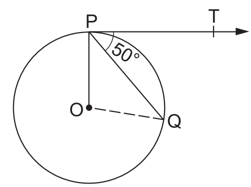
Hence, the correct answer is (d).

....

TEST YOURSELF

MCQ

1. In the given figure, O is the centre of a circle, PQ is a chord and the tangent PT at P makes an angle of 50° with PQ. Then, $\angle POQ = ?$



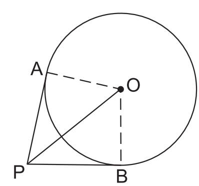
(a) 130° (b) 100°

(c) 90° (d) 75°

2. If the angle between two radii of a circle is 130° then the angle between the tangents at the ends of the radii is

(a) 65° (b) 40° (c) 50° (d) 90°

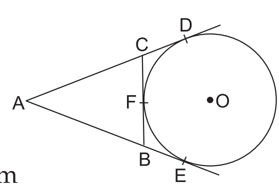
3. If tangents PA and PB from a point P to a circle with centre O are drawn so that $\angle APB = 80°$ then $\angle POA = ?$



(a) 40° (b) 50°

(c) 80° (d) 60°

4. In the given figure, AD and AE are the tangents to a circle with centre O and BC touches the circle at F. If $AE = 5$ cm then perimeter of $\triangle ABC$ is

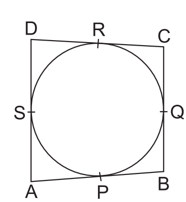


(a) 15 cm (b) 10 cm

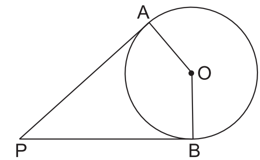
(c) 22.5 cm (d) 20 cm

Short-Answer Questions

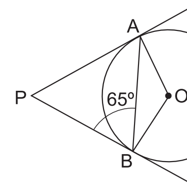
5. In the given figure, a quadrilateral $ABCD$ is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If $AB = x$ cm, $BC = 7$ cm, $CR = 3$ cm and $AS = 5$ cm, find x.



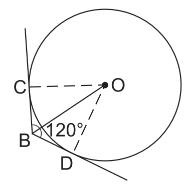
6. In the given figure, PA and PB are the tangents to a circle with centre O. Show that the points A, O, B, P are concyclic.



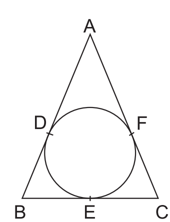
7. In the given figure, PA and PB are two tangents from an external point P to a circle with centre O. If $\angle PBA = 65°$, find $\angle OAB$ and $\angle APB$.



8. Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$. Prove that $OB = 2BC$.



9. Fill in the blanks.
   (i) A line intersecting a circle in two distinct points is called a ....
   (ii) A circle can have ...... parallel tangents at the most.
   (iii) The common point of a tangent to a circle and the circle is called the ....
   (iv) A circle can have ...... tangents.
10. Prove that the lengths of two tangents drawn from an external point to a circle are equal.
11. Prove that the tangents drawn at the ends of the diameter of a circle are parallel.
12. In the given figure, if $AB = AC$, prove that $BE = CE$.

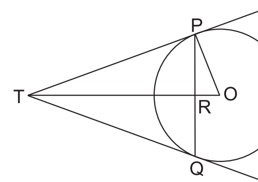


13. If two tangents are drawn to a circle from an external point, show that they subtend equal angles at the centre.
14. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

15. Prove that the parallelogram circumscribing a circle, is a rhombus.
16. Two concentric circles are of radii 5 cm and 3 cm respectively. Find the length of the chord of the larger circle which touches the smaller circle.

Long-Answer Questions

17. A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal.
18. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
19. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.
20. *PQ* is a chord of length 16 cm of a circle of radius 10 cm. The tangents at *P* and *Q* intersect at a point *T* as shown in the figure. Find the length of *TP*.



ANSWERS (TEST YOURSELF)

1. (b) **2.** (c) **3.** (b) **4.** (b) **5.** 9 cm

7. $\angle OAB = 25^\circ, \angle APB = 50^\circ$

9. (i) secant (ii) two (iii) point of contact (iv) infinitely many

16. 8 cm **20.** 10.7 cm (approx.)

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