# Detailed SOLUTIONS

## Previous Years' CBSE Board Questions

1. (a): Given, −3 is a zero of quadratic polynomial $(k-1)x^2 + kx + 1$.

$\therefore \quad (k-1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow \quad 9k - 9 - 3k + 1 = 0 \Rightarrow 6k - 8 = 0$

$\Rightarrow \quad k = 8/6 \Rightarrow k = \frac{4}{3}$

2. (a) : Since, the polynomial has two zeroes only. So, the degree of the polynomial is 2.

**Key Points**

- The degree of the polynomial is the highest power of the variable in a polynomial equation.

3. (b) : Given, 2 is a zero of the polynomial $p(x) = x^2 + 3x + k$.

$\therefore \quad p(2) = 0 \Rightarrow (2)^2 + 3(2) + k = 0 \Rightarrow 4 + 6 + k = 0$

$\Rightarrow \quad 10 + k = 0 \Rightarrow k = -10$

4. (b):

Here, $y = p(x)$ touches the $x$-axis at one point.

So, number of zeros is one.

5. (a): Consider option (a)

$3x(3x-5) = 0$

$\Rightarrow \quad 3x = 0$ or $3x - 5 = 0 \Rightarrow x = 0$ or $x = \frac{5}{3}$

6. (a): We have, $p(x) = x^2 + x - 1$, $\alpha + \beta = -1$ and $\alpha \cdot \beta = -1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{-1} = 1$

7. (d): Since, $\alpha, \beta$ are the zeroes of polynomial $x^2 - 1$

$\therefore \quad x^2 + 0x - 1 = 0$

$\therefore \quad$ Sum of zeroes, $(\alpha + \beta) = 0$

8. (d): Since, $\alpha, \beta$ are the zeroes of polynomial $p(x) = 4x^2 - 3x - 7$

$\therefore \quad$ Sum of zeroes, $(\alpha+\beta) = \frac{3}{4}$

and product of zeroes $(\alpha\beta) = \frac{-7}{4}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{3}{4}}{\frac{-7}{4}} = \frac{-3}{7}$

9. (a): Shape of curve $EFG$ is parabola.

**Answer Tips**

- A parabola is a curve where any point on the parabola is at equal distance from a fixed point.

10. (d): Given polynomial is $-(x^2 + 4x + 3)$.

Corresponding equation is $x^2 + 4x + 3 = 0$

$\Rightarrow \quad x^2 + x + 3x + 3 = 0 \Rightarrow (x+1)(x+3) = 0$

$\Rightarrow \quad x = -1, -3$

11. (c) : The polynomial having zeroes $\alpha, \beta$ is $k[x^2 - (\alpha+\beta)x + \alpha\beta]$, where $k$ is real.

Here $\alpha = -1$ and $\beta = 2$

$\therefore \quad$ Required polynomial $= k[x^2 - (-1+2)x + (-1)\times(2)]$

$= [x^2 - x - 2] \quad$ (for $k = 1$)

12. (a): Given curve cuts the $x$-axis at four distinct points. So, number of zeroes will be 4.

**Commonly Made Mistake**

- Remember the difference between $x$-axis and $y$-axis.

13. (b): The distance between point $C$ and $G$ is 6 units.

14. (a) : Let $\alpha, \beta$ be the zeroes of required polynomial $p(x)$.

Given, $\alpha + \beta = -5$ and $\alpha\beta = 6$

$\therefore \quad p(x) = k[x^2 - (-5)x + 6] = k[x^2 + 5x + 6]$

Thus, one of the polynomial which satisfy the given condition is $x^2 + 5x + 6$.

**Concept Applied**

- Quadratic polynomial $= k[x^2 -$ (sum of zeroes) $x +$ product of zeroes ]

15. Given, $\alpha$ and $\beta$ are the zeros of the polynomial $f(x) = x^2 - x - 4$.

$\therefore \quad \alpha + \beta = 1$ and $\alpha\beta = -4$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta = \frac{1}{-4} - (-4) = \frac{-1}{4} + 4 = \frac{15}{4}$

**Concept Applied**

- For the quadratic polynomial $ax^2 + bx + c$ sum of roots $= \frac{-b}{a}$

  Product of roots $= \frac{c}{a}$

16. Given, polynomial is $f(x) = x^2 + 3x + k$

Since, 2 is zero of the polynomial $f(x)$.

$\therefore \quad f(2) = 0$

$\Rightarrow \quad f(2) = (2)^2 + 3 \times 2 + k \Rightarrow 4 + 6 + k = 0 \Rightarrow k = -10$

17. Given, $\alpha$ and $\beta$ are zeros of $2x^2 - 5x - 4$

$\therefore \quad \alpha + \beta = 5/2$ and $\alpha\beta = -2$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5/2}{-2} = -5/4$

18. Given, $\alpha$ and $\beta$ are zeros of $-3x^2 + x - 5$

$\therefore \quad \alpha + \beta = 1/3$ and $\alpha\beta = 5/3$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{1/3}{5/3} = 1/5$

19. Let $\alpha, \beta$ be the zeroes of required polynomial $p(x)$.

Given, $\alpha + \beta = -3$ and $\alpha\beta = 2$

$\therefore \quad p(x) = k[x^2 - (-3)x + 2] = k[x^2 + 3x + 2]$

For $k = 1$, $p(x) = x^2 + 3x + 2$

Hence, one of the polynomial which satisfy the given condition is $x^2 + 3x + 2$.

20. We have, 3 and − 4 are the zeroes of quadratic polynomial. Now, sum of zeroes = 3 + (− 4) = − 1

Product of zeroes = 3(– 4) = – 12

∴ Required quadratic polynomial is

$k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$

$= k[x^2 - (-1)x + (-12)] = x^2 + x - 12$ (for $k = 1$)

21. Let one zero of the polynomial $p(x)$ be $\alpha$, then the other zero be $1/\alpha$.

For zeroes of polynomial $p(x) = 6x^2 + 37x - (k - 2)$

∴ Product of zeroes $= \alpha \times \frac{1}{\alpha} = \frac{-(k-2)}{6}$

$\Rightarrow \frac{-(k-2)}{6} = 1$

$\Rightarrow k - 2 = -6 \Rightarrow k = -4$

22. The given polynomial is $x^2 - p(x + 1) + c$

If $\alpha$ and $\beta$ are zeroes of given polynomial

∴ Sum of zeroes $= \alpha + \beta = -\frac{b}{a} = \frac{-(-p)}{1} = p$

and product of zeroes $= \alpha\beta = \frac{-p+c}{1} = -p + c$

Now, $(\alpha + 1)(\beta + 1) = 0$

$\Rightarrow \alpha\beta + \alpha + \beta + 1 = 0$

$\Rightarrow -p + c + p + 1 = 0 \Rightarrow c + 1 = 0 \therefore c = -1$

**Concept Applied**

- If $\alpha$ and $\beta$ are zeroes of quadratic polynomial $p(x) = ax^2 + bx + c$, then $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

23. Given, $\alpha$ and $\beta$ are zeroes of $4x^2 + 3x + 7$.

∴ $\alpha + \beta = -\frac{3}{4}$ and $\alpha\beta = \frac{7}{4}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-3}{4}}{\frac{7}{4}} = \frac{-3}{7}$

24. Let $\alpha, \beta$ are the zeroes of the polynomial $ax^2 + bx + c$.

∴ $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now we have to find quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-b/a}{c/a} = \frac{-b}{c}$ and $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c}$

Thus, the required polynomial is

$p(x) = k\left(x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c}\right) = \frac{k}{c}(cx^2 + bx + a)$

For $k = c$, $p(x) = cx^2 + bx + a$

**Key Points**

- A quadratic polynomial is a second-degree polynomial, where the value of the highest degree is equal to 2.

25. The given polynomial is $f(x) = x^2 - (k + 6)x + 2(2k - 1)$

Now, sum of zeroes $= \frac{k+6}{1}$ and product of zeroes $= \frac{2(2k-1)}{1}$

According to question,

Sum of zeroes $= \frac{1}{2}$(Product of zeroes) $\Rightarrow k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow k + 6 = 2k - 1 \Rightarrow k = 7$

26. Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$ or $\frac{21y^2 - 11y - 2}{3}$

$\Rightarrow f(y) = \frac{(3y-2)(7y+1)}{3}$

Zeros of polynomial is given by $f(y) = 0$

$\Rightarrow (3y - 2)(7y + 1) = 0 \Rightarrow$ Either $y = \frac{2}{3}$ or $y = \frac{-1}{7}$

Hence, $\frac{-1}{7}$ and $\frac{2}{3}$ are zeros of $f(y)$.

∴ Sum of zeros $\frac{-1}{7} + \frac{2}{3} = \frac{-3+14}{21} = \frac{11}{21}$

$= -\frac{(\text{Coefficient of } y)}{(\text{Coefficient of } y^2)}$

and product of zeros $= \left(\frac{-1}{7}\right)\left(\frac{2}{3}\right) = \frac{-2}{21} = \frac{\text{Constant term}}{\text{Coefficient of } y}$

Hence, the relationship between zeros and coefficients of the polynomial is verified.

27. Let the zeros of the quadratic polynomial be $\alpha$ and $\beta$.

Given, $\alpha + \beta = -1$ and $\alpha \cdot \beta = -20$

Then, the quadratic polynomial will be

$f(x) = x^2 - (\alpha + \beta)x + \alpha \cdot \beta$

$\Rightarrow f(x) = x^2 - (-1)x + (-20) \Rightarrow f(x) = x^2 + x - 20$

To find the zeros of the polynomial, we put $f(x) = 0$

$\Rightarrow x^2 + x - 20 = 0 \Rightarrow x^2 + 5x - 4x - 20 = 0$

$\Rightarrow x(x + 5) - 4(x + 5) = 0 \Rightarrow x = 4, -5$

∴ $x = 4$ and $-5$ are the zero of the polynomial so obtained.

28. Given, $\alpha$ and $\beta$ are the zeroes of the polynomial $4x^2 - x - 4$.

∴ $\alpha + \beta = \frac{1}{4}$ and $\alpha\beta = \frac{-4}{4} = -1$

Let $S$ and $P$ denote respectively the sum and product of the zeroes of the polynomial whose zeroes are $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$.

Then, $S = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{\alpha+\beta}{2\alpha\beta} = \frac{\frac{1}{4}}{2(-1)} = \frac{-1}{8}$

$P = \left(\frac{1}{2\alpha}\right)\left(\frac{1}{2\beta}\right) = \frac{1}{4\alpha\beta} = \frac{1}{4(-1)} = \frac{-1}{4}$

Thus, the required polynomial is $k(x^2 - Sx + P)$

$= k\left(x^2 - \left(-\frac{1}{8}\right)x + \left(-\frac{1}{4}\right)\right)$

$= 8x^2 + x - 2$ (for $k = 8$)

29. Given, $\alpha$ and $\beta$ are the zeroes of the polynomial $6x^2 - 7x + 2$.

∴ $\alpha + \beta = \frac{7}{6}$ and $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

Now, we have to find quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Now, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{7}{6}}{\frac{1}{3}}=\frac{7}{2}$ and $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}=\frac{1}{\frac{1}{3}}=3$

Thus, the required polynomial is

$p(x) = k\left(x^2-\frac{7}{2}x+3\right)$

For $k = 2$, $p(x) = 2x^2 - 7x + 6$.

30. Let $f(x) = 6x^2 - 3 - 7x$ or $6x^2 - 7x - 3$
$= 6x^2 - 9x + 2x - 3 = (3x + 1)(2x - 3)$
Zeroes of polynomial is given by $f(x) = 0$

$\Rightarrow$ $(3x + 1)(2x - 3) = 0 \Rightarrow$ either $x=-\frac{1}{3}$ or $x=\frac{3}{2}$

Hence, $-\frac{1}{3}$ and $\frac{3}{2}$ are zeroes of $f(x)$.

$\therefore$ Sum of zeroes $=-\frac{1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}$

$=\frac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

and product of zeroes $=\left(-\frac{1}{3}\right)\left(\frac{3}{2}\right)=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, the relationship between zeroes and coefficients of the polynomial is verified.

**Concept Applied**

- Coefficients of the polynomial are numbers that come before a term.

## CBSE Sample Questions

1. (b): Given polynomial, $f(x) = px^2 - 2x + 3p$
$\alpha, \beta$ are the roots of $px^2 - 2x + 3p = 0$

So, $\alpha+\beta=\frac{2}{p}$ and $\alpha\cdot\beta=3$

Given, $\alpha + \beta = \alpha\beta$

$\Rightarrow$ $\frac{2}{p}=3$ $\Rightarrow$ $p=\frac{2}{3}$ **(1)**

2. (b): We have, sum of zeroes $= 2+\frac{1}{2}=\frac{-5}{p}$

$\Rightarrow \frac{5}{2}=\frac{-5}{p} \Rightarrow p=-2$

Also, product of zeroes $= 2\times\frac{1}{2}=\frac{r}{p}$

$\Rightarrow \frac{r}{p}=1 \Rightarrow r=p=-2$ **(1)**

3. (c): Initially, at $t = 0$, Annie's height above water level is 48 feet
So, at $t = 0$, $h(0) = 48$ feet
$\therefore$ $h(0) = -16(0)^2 + 8(0) + k = 48$
$\Rightarrow$ $k = 48$ **(1)**

4. (b): When Annie touches the pool,
her height = 0 feet
*i.e.*, $-16t^2 + 8t + 48 = 0$
$\Rightarrow$ $2t^2 - t - 6 = 0$ $\Rightarrow$ $2t^2 - 4t + 3t - 6 = 0$
$\Rightarrow$ $2t(t - 2) + 3(t - 2) = 0$
$\Rightarrow$ $(2t + 3)(t - 2) = 0$ $\Rightarrow$ $t = 2$ or $t = -3/2$ **(1)**
Since time cannot be negative, so $t = 2$ seconds.

5. (d): $\because$ $t = -1$ and $t = 2$ are the two zeroes of the polynomial $p(t)$, then
$p(t) = k(t - (-1))(t - 2)$
$= k(t + 1)(t - 2) = k(t^2 - t - 2)$
When $t = 0$ (initially), $h(0) = 48$ feet
$\Rightarrow$ $p(0) = k(0^2 - 0 - 2) = 48 \Rightarrow -2k = 48$ $\Rightarrow$ $k = -24$
So the polynomial is
$p(t) = -24(t^2 - t - 2) = -24t^2 + 24t + 48$ **(1)**

6. (c): A polynomial $q(t)$ with sum of zeroes as 1 and the product of zeroes as $-6$ is given by
$q(t) = k(t^2 - (\text{sum of zeroes})t + \text{product of zeroes})$
$= k(t^2 - 1t + (-6)) = k(t^2 - t - 6)$
When $t = 0$ (initially), $q(0) = 48$ feet
$\Rightarrow$ $q(0) = k(0^2 - 1(0) - 6) = 48$
$\Rightarrow$ $-6k = 48$ $\Rightarrow$ $k = -8$
$\therefore$ Required polynomial $q(t) = -8(t^2 - t - 6)$
$= -8t^2 + 8t + 48$ **(1)**

7. (a): When the zeroes are negative of each other, then sum of the zeroes = 0

$\Rightarrow -\frac{k-3}{-12}=0 \Rightarrow \frac{k-3}{12}=0 \Rightarrow k-3=0 \Rightarrow k=3$ **(1)**

8. (i) (b): Let $p(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$
$= x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$
Clearly, $p(x) = 0$ when $x = 4, -2$
So, $(4, -2)$ are zeroes of polynomial.

(ii) (a): Intersects $x$-axis

(iii) (c): parabola

(iv) (b): Since sum of zeroes is 0 and one zero is 6.
$\therefore$ The other zero is $-6$.
So, the required equation of curve is $(x + 6)(x - 6)$
$= x^2 - 36$

(v) (c): Let $f(x) = 0$
$\Rightarrow$ $(x - 2)^2 + 4 = 0 \Rightarrow (x - 2)^2 = -4$ which is not possible. **(1 × 4)**

9. Given polynomial is $3x^2 - kx + 6$
Clearly, sum of zeroes, $\alpha + \beta = k/3$
$\Rightarrow$ $3 = k/3 \Rightarrow k = 9$ **(1)**

10. Given, $5-3\sqrt{2}$ and $5+3\sqrt{2}$ are the zeroes of a quadratic polynomial.

$\therefore$ Sum of zeroes $= 5-3\sqrt{2}+5+3\sqrt{2}=10$ **(1/2)**

Product of zeroes $= (5-3\sqrt{2})(5+3\sqrt{2})=7$ **(1/2)**

$\therefore$ Required polynomial is $k[x^2 - 10x + 7]$
or $x^2 - 10x + 7$ (For $k = 1$) **(1)**

11. Let $\alpha$ and $\beta$ be the zeros of the polynomial $2x^2 - 5x - 3$
Then sum of roots $\alpha + \beta = 5/2$ **(1/2)**
And product of roots $\alpha\beta = -3/2$. **(1/2)**
According to question,
$2\alpha$ and $2\beta$ will be the zeros of $x^2 + px + q$
Then $2\alpha + 2\beta = -p$
$\Rightarrow$ $2(\alpha + \beta) = -p$
$\Rightarrow$ $2 \times 5/2 = -p$
$\Rightarrow$ $p = -5$ **(1)**
And $2\alpha \times 2\beta = q$
$4\alpha\beta = q$
So $q = 4 \times -3/2 = -6$ **(1)**