VSA (1 mark)

7. If $xy = 180$ and HCF $(x, y) = 3$, then find the LCM (x, y). *(2020-21)* Ap

SA I (2 marks)

8. 3 bells ring at an interval of 4, 7 and 14 minutes. All three bells rang at 6 am, when the three bells will ring together next? *(2020-21)*

SA II (3 marks)

9. Given that $\sqrt{3}$ is irrational,prove that $5 + 2\sqrt{3}$ is irrational. *(2022-23)*

1.3 Revisiting Irrational Numbers

MCQ

10. If $a^2 = \frac{23}{25}$, then a is

(a) rational (b) irrational
(c) whole number (d) integer

(Term I, 2021-22) U

SA II (3 marks)

11. Prove that $2 - \sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational. *(2020-21)* Ap

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (c) : Total number of factors of a prime number is 2 *i.e.*, 1 and number itself.

2. (a): Least composite number = 4
Least prime number = 2
$\therefore$ HCF = 2, LCM = 4
$\therefore$ Required ratio = $\frac{2}{4}$ *i.e.*, 1:2

3. (b) : We know that,
HCF × LCM = Product of two numbers
$\Rightarrow 13 \times \text{LCM} = 39 \times 91 \Rightarrow \text{LCM} = \frac{39 \times 91}{13} = 273$

4. (a) : Given, HCF = 12
Let two numbers be $12a$ and $12b$
So, $12a \times 12b = 6336 \Rightarrow ab = 44$
We can write 44 as product of two numbers in these ways :
$ab = 1 \times 44 = 2 \times 22 = 4 \times 11$
Here, we will take $a = 1$ and $b = 44$; $a = 4$ and $b = 11$.
We do not take $ab = 2 \times 22$ because 2 and 22 are not co-prime to each other.
For $a = 1$ and $b = 44$, I[st] no. = $12a = 12$, 2[nd] no. = $12b = 528$
For $a = 4$ and $b = 11$, I[st] no. = $12a = 48$, 2[nd] no. = $12b = 132$
Hence, we get two pairs of numbers, (12, 528) and (48, 132).

Commonly Made Mistake

- Remember the difference between prime numbers and co-prime numbers.

5. (d) : For $n = 1, 2, 3, 4 ...$
$(12)^n$ cannot end with 0.

6. (b) : We have,

5	385
7	77
11	11
	1

$\therefore$ Prime factorisation of $385 = 5 \times 7 \times 11$

7. (c) : We have, $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$21 = 3 \times 7$
$15 = 3 \times 5$
$\therefore$ HCF (12, 21, 15) = 3 and
LCM (12, 21, 15) = $2^2 \times 3 \times 5 \times 7 = 420$

8. Let the other number be x.
As, HCF (a, b) × LCM $(a, b) = a \times b$
$\Rightarrow 13 \times 182 = 26 \times x \Rightarrow x = \frac{13 \times 182}{26} = 91$
Hence, other number is 91.

9. Let a and b be two number such that
LCM (a, b) = 9·HCF (a, b) ...(i)
and LCM (a, b) + HCF (a, b) = 500 ...(ii)
Using (i) in (ii), we get
9HCF (a, b) + HCF (a, b) = 500
$\Rightarrow$ 10 HCF (a, b) = 500 $\Rightarrow$ HCF (a, b) = 50

10. Since, HCF (a, b) × LCM $(a, b) = a \times b$
$\therefore$ HCF (336, 54) × LCM (336, 54) = 336 × 54
$\Rightarrow$ 6 × LCM(336, 54) = 18144
$\Rightarrow$ LCM (336, 54) = $\frac{18144}{6} = 3024$

Concept Applied

- Product of two numbers in equal to the product of their HCF and LCM.

11. We know that, HCF (a, b) × LCM $(a, b) = a \times b$
$\Rightarrow 5 \times 200 = ab \Rightarrow ab = 1000$

12. Smallest prime number = 2
Smallest composite number = 4
HCF (2, 4) = 2

Key Points

- A composite number is a natural number or a positive integer which has more than two factors.

13. The prime factor of $6^n = (2 \times 3)^n = 2^n \times 3^n$.
Therefore prime factorisation of 6^n does not contain any prime factor 5. Hence, 6^n can never ends with the digit 0 for any natural number.

14. Let the other number be x.
As, HCF $(a, b) \times$ LCM $(a, b) = a \times b$

$\therefore \quad 27 \times 162 = 54x \Rightarrow x = \frac{27 \times 162}{54} = 81$

Hence, other number is 81.

15. Let the other number be x.
We know that, HCF $(a, b) \times$ LCM $(a, b) = a \times b$

$\therefore \quad 27 \times 2079 = 297 \times x$

$\Rightarrow \quad x = \frac{2079 \times 27}{297} = 189$

Hence, other number is 189.

16. Given, least number which when divided by 12, 16 and 24 leaves remainder 7 in each case

$\therefore$ least number = LCM(12, 16, 24) + 7 = 48 + 7 = 55

17. Let the two numbers be $2x$ and $3x$.
LCM of $2x$ and $3x = 6x$, HCF$(2x, 3x) = x$

Now, $6x = 180$ [Given]

$\Rightarrow \quad x = \frac{180}{6} = 30$

$\therefore$ HCF $(2x, 3x) = x = 30$

18. We have, $2 \times 3 \times 5 + 5$ and $5 \times 7 \times 11 + 7 \times 5$.
We can write these numbers as :
$2 \times 3 \times 5 + 5 = 5(2 \times 3 + 1)$
$= 1 \times 5 \times 7$
and $5 \times 7 \times 11 + 7 \times 5 = 5 \times 7(11 + 1)$
$= 5 \times 7 \times 12 = 1 \times 5 \times 7 \times 12$
Since, on simplifying, we find that both the numbers have more than two factors. So, these are composite numbers.

19. Since, HCF (65, 117) = 13
Given HCF (65, 117) = $65n - 117$
$\Rightarrow \quad 13 = 65n - 117$
$\Rightarrow \quad 65n = 13 + 117 \Rightarrow n = 2$.

20. Prime factorisation of 612 and 1314 are
$612 = 2 \times 2 \times 3 \times 3 \times 17$
$1314 = 2 \times 3 \times 3 \times 73$
$\therefore$ HCF (612, 1314) = $2 \times 3 \times 3 = 18$

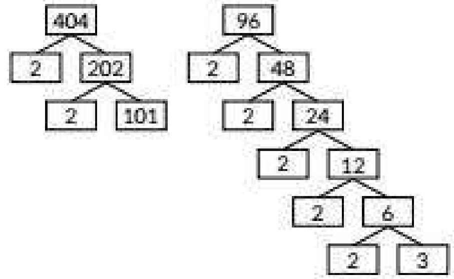
21. $5050 = 2 \times 5 \times 5 \times 101 = 2 \times 5^2 \times 101$
Yes, it is unique.

22. Prime factorisation of 231 and 396 are
$231 = 3 \times 7 \times 11$
$396 = 2 \times 2 \times 3 \times 3 \times 11$
HCF (231, 396) = $3 \times 11 = 33 \neq 1$
Hence, the two numbers are not co-prime.

Concept Applied

- Every natural number greater than 1 can be written as a product of prime numbers and that up to rearrangement of the factors, the product is unique.

23. Using the factor tree method, we have



$\Rightarrow$ $404 = 2 \times 2 \times 101$ and $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\therefore$ HCF of 404 and 96 = $2 \times 2 = 4$

LCM of 404 and 96 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 101 = 9696$

Also $404 \times 96 = 38784$

LCM $\times$ HCF = $9696 \times 4 = 38784$

Thus, HCF $\times$ LCM = Product of two numbers.

24. Number of soldiers in an army contingent
$= 678 = 2 \times 3 \times 113$
Number of members in an army band = $36 = 2 \times 2 \times 3 \times 3$
The maximum number of columns such that two groups can march in same number of columns is HCF of 678 and 36.
HCF (678, 36) = $2 \times 3 = 6$
So, the maximum number of columns they can march is 6.

Key Points

- HCF (Highest Common Factor) is also called GCD (Greatest Common Divisor)

25. The prime factorisation of 40, 42, 45 are
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$
$42 = 2 \times 3 \times 7$
$45 = 3 \times 3 \times 5 = 3^2 \times 5$
$\therefore$ LCM (40, 42, 45) = $2^3 \times 3^2 \times 5^1 \times 7^1$
$= 8 \times 9 \times 5 \times 7 = 2520$
$\therefore$ Required distance = 2520 cm or 0.0252 km.

Answer Tips

- 1 km = 100000 cm

26. Number of gulab jamuns = $396 = 2 \times 2 \times 3 \times 3 \times 11$
Number of ras-gullas = $342 = 2 \times 3 \times 3 \times 19$
HCF (396, 342) = $2 \times 3 \times 3 = 18$
So, shopkeeper will put 18 sweets in each box such that number of boxes are least.

27. It is given that the required number when divides 125, 162, 259 leaves the remainder 5, 6, 7 respectively. This means that $125 - 5 = 120$, $162 - 6 = 156$, $259 - 7 = 252$ are divisible by the required number.
The required number is HCF of all these numbers.
The prime factorisation of 120, 156, 252 are
$120 = 2 \times 2 \times 2 \times 3 \times 5$
$156 = 2 \times 2 \times 3 \times 13$; $252 = 2 \times 2 \times 3 \times 3 \times 7$
HCF (120, 156, 252) = $2 \times 2 \times 3 = 12$
Hence, the required number is 12.

Answer Tips

- The LCM is the value of two numbers that is evenly divisible by the given numbers.

28. Suppose $5+2\sqrt{7}$ is a rational number.

$\therefore$ We can find two integers $a, b (b \neq 0)$ such that

$5+2\sqrt{7}=\frac{a}{b}$, where a and b are co-prime.

$\Rightarrow 2\sqrt{7}=\frac{a}{b}-5 \Rightarrow \sqrt{7}=\frac{1}{2}\left[\frac{a}{b}-5\right]$

$\Rightarrow \sqrt{7}$ is a rational number

[$\because a, b$ are integers, so $\frac{1}{2}\left(\frac{a}{b}-5\right)$ is a rational number]

But this contradicts the fact that $\sqrt{7}$ is an irrational number. Hence, our assumption is wrong. Thus, $5+2\sqrt{7}$ is an irrational number.

29. Suppose $\frac{3+\sqrt{7}}{5}$ is rational number.

$\therefore$ We can find two integers p and q ($q \neq 0$) such that $\frac{3+\sqrt{7}}{5}=\frac{p}{q}$, where p and q are co-prime.

$\Rightarrow \quad 3+\sqrt{7}=\frac{5p}{q}$

$\Rightarrow \quad \sqrt{7}=\frac{5p}{q}-3 \Rightarrow \sqrt{7}$ is a rational number.

[$\because$ p and q are integers, so $\frac{5p}{q}-3$ is a rational number]

But this contradicts the fact that $\sqrt{7}$ is an irrational number.

Hence, our supposition is wrong.

Thus, $\frac{3+\sqrt{7}}{5}$ is an irrational number.

30. Let $(5 + 3\sqrt{2})$ is rational.

Then, $5 + 3\sqrt{2}=\frac{a}{b}$, where $a, b (\neq 0)$ are coprime numbers

$\therefore \quad 3\sqrt{2}=\frac{a}{b}-5 \quad \Rightarrow \quad \sqrt{2}=\frac{a-5b}{3b}$

$\Rightarrow \quad \sqrt{2}$ is rational number.

[$\because$ a, b are integers $\therefore$ $\frac{a-5b}{3b}$ is rational]

But this contradicts the fact that $\sqrt{2}$ is irrational.

Hence, $5+3\sqrt{2}$ an also irrational number.

Commonly Made Mistake

- Remember the difference between rational and irrational numbers.

31. There are infinite irrational numbers between $\sqrt{2}$ and $\sqrt{3}$. Examples are $\sqrt{2.1}$ and $\sqrt{2.3}$.

32. Let us assume that $\sqrt{3}$ is a rational number.

Then $\sqrt{3}=\frac{a}{b}$; where a and b ($\neq 0$) are co-prime positive integers.

Squaring on both sides, we get

$3=\frac{a^2}{b^2} \Rightarrow a^2 = 3b^2 \Rightarrow 3$ divides a^2

$\Rightarrow \quad 3$ divides a ...(i)

$\Rightarrow \quad a = 3c$, where c is an integer

Again, squaring on both sides, we get

$a^2 = 9c^2$

$\Rightarrow \quad 3b^2 = 9c^2 \Rightarrow b^2 = 3c^2 \Rightarrow 3$ divides b^2

$\Rightarrow \quad 3$ divides b ...(ii)

From (i) and (ii), we get 3 divides both a and b.

$\Rightarrow \quad a$ and b are not co-prime integers.

This contradicts the fact that a and b are co-primes.

Hence, $\sqrt{3}$ is an irrational number.

33. Let us assume that $\sqrt{5}$ is a rational number.

Then $\sqrt{5}=\frac{a}{b}$; where a and b ($\neq 0$) are co-prime integers.

Squaring on both sides, we get

$5=\frac{a^2}{b^2} \Rightarrow a^2 = 5b^2$

$\Rightarrow \quad 5$ divides a^2

$\Rightarrow \quad 5$ divides a ...(i)

$\Rightarrow \quad a = 5c$, where c is an integer

Again, squaring on both sides, we get

$a^2 = 25c^2$

$\Rightarrow \quad 5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

$\Rightarrow \quad 5$ divides b^2

$\Rightarrow \quad 5$ divides b ...(ii)

From (i) and (ii), we get 5 divides both a and b.

$\Rightarrow \quad a$ and b are not co-prime integers.

Hence, our supposition is wrong.

Thus, $\sqrt{5}$ is an irrational number.

Concept Applied

- Irrational number is a number which can not be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

34. Let us assume $\sqrt{2}$ be a rational number.

Then, $\sqrt{2}=\frac{p}{q}$, where p, q ($q \neq 0$) are integers and co-prime.

On squaring both sides, we get

$2=\frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$ ...(i)

$\Rightarrow \quad 2$ divides $p^2 \Rightarrow 2$ divides p ...(ii)

So, $p = 2a$, where a is some integer.

Again squaring on both sides, we get

$p^2 = 4a^2 \Rightarrow 2q^2 = 4a^2$ (using (i))

$\Rightarrow \quad q^2 = 2a^2$

$\Rightarrow \quad 2$ divides $q^2 \Rightarrow 2$ divides q ...(iii)

From (ii) and (iii), we get

2 divides both p and q.

$\therefore$ p and q are not co-prime integers.
Hence, our assumption is wrong.
Thus, $\sqrt{2}$ is an irrational number.

Concept Applied

- Co-prime numbers are pairs of numbers that do not have any common factor other than 1.

35. Suppose $2+5\sqrt{3}$ is a rational number.

$\therefore$ We can find two integers a, b ($b \neq 0$) such that $2+5\sqrt{3}=\frac{a}{b}$, where a and b are co-prime integers.

$\Rightarrow$ $5\sqrt{3}=\frac{a}{b}-2 \Rightarrow \sqrt{3}=\frac{1}{5}\left[\frac{a}{b}-2\right]$

$\Rightarrow$ $\sqrt{3}$ is a rational number.

[$\because$ a, b are integers, so $\frac{1}{5}\left[\frac{a}{b}-2\right]$ is a rational number.]

But this contradicts the fact that $\sqrt{3}$ is an irrational number.
Hence, our assumption is wrong.
Thus, $2+5\sqrt{3}$ is an irrational number.

Key Points

- $\sqrt{p}$ is irrational if $p > 0$ add not a perfect square, *i.e.*, $\sqrt{2}$ is irrational but $\sqrt{4}=2$ is rational.

36. Irrational number is a number which can not be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

First, we prove that $\sqrt{5}$ is an irrational number.

Let us assume that $\sqrt{5}$ is a rational number.

Then $\sqrt{5}=\frac{a}{b}$; where a and b ($\neq 0$) are co-prime integers.

Squaring on both sides, we get

$5=\frac{a^2}{b^2} \Rightarrow a^2=5b^2 \Rightarrow 5$ divides a^2

$\Rightarrow$ 5 divides a ...(i)

$\Rightarrow$ $a=5c$, where c is an integer

Again, squaring on both sides, we get

$a^2=25c^2$

$\Rightarrow$ $5b^2=25c^2 \Rightarrow b^2=5c^2 \Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b ...(ii)

From (i) and (ii), we get 5 divides both a and b.

$\Rightarrow$ a and b are not co-prime integers.

This contradicts the fact that a and b are co-primes.

Hence, $\sqrt{5}$ is an irrational number.

Now, to prove $3+2\sqrt{5}$ is an irrational number.

Suppose $3+2\sqrt{5}$ is a rational number

$\therefore$ We can find two integers a, $b(b \neq 0)$ such that $3+2\sqrt{5}=\frac{a}{b}$ (where a and b are co-prime)

$\Rightarrow$ $2\sqrt{5}=\frac{a}{b}-3 \Rightarrow \sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$

$\Rightarrow$ $\sqrt{5}$ is a rational number

$\left[\begin{array}{l}\because a, b \text{ are integers,} \\ \therefore \frac{1}{2}\left(\frac{a}{b}-3\right) \text{ is a rational number}\end{array}\right]$

But this contradicts the fact that $\sqrt{5}$ is an irrational number.
Hence, our assumption is wrong.
Thus, $3+2\sqrt{5}$ is an irrational number.

37. First we prove that $\sqrt{5}$ is an irrational number.

Let us assume that $\sqrt{5}$ is a rational number.

Then $\sqrt{5}=\frac{a}{b}$; where a and b ($\neq 0$) are co-prime integers.

Squaring on both sides, we get

$5=\frac{a^2}{b^2} \Rightarrow a^2=5b^2 \Rightarrow 5$ divides a^2

$\Rightarrow$ 5 divides a ...(i)

$\Rightarrow$ $a=5c$, where c is an integer

Again, squaring on both sides, we get

$a^2=25c^2$

$\Rightarrow$ $5b^2=25c^2 \Rightarrow b^2=5c^2$

$\Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b ...(ii)

From (i) and (ii), we get 5 divides both a and b.

$\Rightarrow$ a and b are not co-prime integers.

This contradicts the fact that a and b are co-primes.

Hence, $\sqrt{5}$ is an irrational number.

Now, to prove that $2+\sqrt{5}$ is an irrational number.

Suppose $2+\sqrt{5}$ is a rational number.

$\therefore$ We can find two integers a, b ($b \neq 0$) such that $2+\sqrt{5}=\frac{a}{b}$ (where a, b are co-prime)

$\Rightarrow$ $\sqrt{5}=\frac{a}{b}-2$

$\Rightarrow$ $\sqrt{5}$ is a rational number as a, b are integers and so, $\frac{a}{b}-2$ is rational number.

But this contradicts the fact that $\sqrt{5}$ is an irrational number.
Hence our assumption is wrong.
Thus, $2+\sqrt{5}$ is an irrational number.

Concept Applied

- Rational number is a number which can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

CBSE Sample Questions

1. (c) : Given $a = p^3q^4$ and $b = p^2q^3$

$\therefore$ LCM $(a, b) = p^3q^4$ and HCF $(a, b) = p^2q^3$

Comparing the obtained LCM and HCF with the given LCM and HCF, we get

$m = 2, n = 3, r = 3$ and $s = 4$

$\therefore$ $(m + n)(r + s) = 5 \times 7 = 35$ **(1)**

2. (b): Product of two numbers is equal to Product of their HCF and LCM.

So, $5780 = 340 \times 17 = 5780$

HCF is always a factor of LCM.

So, both Assertion (A) and Reason (R) are true but reason is not correct explanation of Assertion (A). **(1)**

3. (b): Least composite number is 4 and the least prime number is 2.

LCM (4, 2) : HCF (4, 2) = 4 : 2 = 2 : 1 **(1)**

4. (c) : We know that, LCM × HCF = product of the numbers.

$\Rightarrow$ $36 \times 2 = 18 \times x \Rightarrow x = 4$ **(1)**

5. (c) : Since HCF = 81, two numbers can be taken as $81x$ and $81y$.

According to question, we have

$81x + 81y = 1215 \Rightarrow x + y = 15$

which gives four pairs as

(1, 14), (2, 13), (4, 11), (7, 8) **(1)**

6. (c) : LCM of two prime numbers = product of the numbers

$\Rightarrow$ $221 = p \times q$

Also, $221 = 13 \times 17$

So, $p = 17$ and $q = 13$ ($\because p > q$)

$\therefore$ $3p - q = 51 - 13 = 38$ **(1)**

7. Given HCF $(x, y) = 3$

$\Rightarrow$ (LCM) (3) = 180 **(1/2)**

[$\because$ HCF × LCM = Product of the numbers]

$\Rightarrow$ LCM = 60 **(1/2)**

8. Let us first write the prime factorisation of 4, 7 and 14, which is given below.

$4 = 2 \times 2$ **(1/2)**

$7 = 7 \times 1$ **(1/2)**

$14 = 2 \times 7$ **(1/2)**

$\therefore$ LCM (4, 7, 14) = $2 \times 2 \times 7 = 28$ **(1/2)**

Thus, the three bells will ring together again at 6 : 28 am.

9. Suppose $5 + 2\sqrt{3}$ is rational, then it must be in the form of p/q where p and q are co-prime integers and $q \neq 0$. That is,

$5 + 2\sqrt{3} = p/q$ **(1/2)**

So $\sqrt{3} = \dfrac{p - 5q}{2q}$ ...(i) **(1/2)**

Since p, q, 5 and 2 are integers and $q \neq 0$, R.H.S. of equation (i) is rational. But L.H.S. of (i) is $\sqrt{3}$ which is irrational. This contradicts the fact. **(1)**

This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5 + 2\sqrt{3}$ is irrational. **(1)**

10. (b): $a^2 = \dfrac{23}{25}$, then $a = \pm\dfrac{\sqrt{23}}{5}$, which is irrational. **(1)**

11. Let us suppose that $(2 - \sqrt{3})$ is rational. **(1/2)**

So, we can find co-prime numbers a and $b (b \neq 0)$ such that

$(2 - \sqrt{3}) = \dfrac{a}{b}$. **(1/2)**

$\Rightarrow$ $2 - \dfrac{a}{b} = \sqrt{3} \Rightarrow \sqrt{3} = \dfrac{2b - a}{b}$ **(1/2)**

$\Rightarrow$ $\sqrt{3}$ is a rational number.

[$\because$ a and b are integers $\dfrac{2b - a}{b}$ is a rational number]

But this contradicts the fact that $\sqrt{3}$ is irrational. **(1/2)**

So, our supposition is wrong. **(1/2)**

Hence, $2 - \sqrt{3}$ is irrational. **(1/2)**