MATHEMATICS
WORKSHEET_130425
CHAPTER 01 A SQUARE AND A CUBE (Ganita Prakashan)
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : VIII **DURATION : 1½ hr**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. A locker is toggled an odd number of times. Based on the rules of the locker puzzle, which of the following statements must be true about the locker number?

(a) It is a prime number. (b) It has exactly three factors.

(c) It is a perfect square. (d) It is a perfect cube.

Ans. (c) It is a perfect square.

A locker remains open if it is toggled an odd number of times. The number of times a locker is toggled is the same as the number of factors of its number. The only numbers with an odd number of factors are perfect squares because they have one factor (the square root) which is paired with itself.

2. A number ends with 8 zeros. Can this number be a perfect square?

(a) Yes, because the number of zeros is even.

(b) Yes, if the non-zero part of the number is also a perfect square.

(c) No, because perfect squares cannot end with 8 zeros.

(d) Cannot be determined without more information.

Ans. (a) Yes, because the number of zeros is even.

A perfect square can only have an even number of zeros at the end. Since 8 is an even number, a number ending in 8 zeros can be a perfect square.

3. Given that x is a natural number and x^2 is a perfect square. What is the total number of factors of x^2 if x is a prime number?

(a) 2 (b) 3 (c) 4 (d) An even number

Ans. (b) 3

If a number x is a prime number, its factors are 1 and x. The number x^2 would be the square of a prime number. For example, if x is 2, x^2 is 4. The factors of 4 are 1, 2, and 4. There are three factors. Similarly, for any prime number p, the factors of p_2 are 1, p, and p^2, which is always 3 factors.

4. A number N has 12 factors. If N is a perfect square, which of the following statements about N is true?

(a) N cannot be a perfect square.

(b) N must have an odd number of factors.

(c) N must have an even number of factors.

(d) The number of factors of N is irrelevant to whether it is a perfect square.

Ans. (a) N cannot be a perfect square.

A key property of perfect squares is that they always have an odd number of factors. This is because all factors of a number, except for its square root, come in pairs. The square root is paired with itself, resulting in one "unpaired" factor. Since the number of factors of N is given as 12 (an even number), N cannot be a perfect square.

5. Without calculating, what is the units digit of 34^2?
(a) 4 (b) 6 (c) 8 (d) 2
Ans. (b) 6
To find the units digit of 34^2, we only need to look at the units digit of the base number, which is 4. The units digit of 4^2 is the units digit of 16, which is 6. This property for numbers ending in 4, like 14^2 = 196 and 74^2.

6. A number is obtained by multiplying a number by itself three times. What can be concluded about its prime factors?
(a) They must appear in pairs. (b) They must appear three times.
(c) They must appear an even number of times. (d) They must be prime numbers.
Ans. (b) They must appear three times.
Each prime factor of a number appears three times in the prime factorization of its cube.

7. Which of the following numbers cannot be a perfect cube?
(a) A number ending with 8. (b) A number ending with 7.
(c) A number ending with 2 zeroes. (d) A number ending with 9.
Ans. (c) A number ending with 2 zeroes.
A number that is a perfect cube and ends with zeros must have a number of zeros that is a multiple of 3. For example, 10^3 = 1000 (3 zeros), 20^3 = 8000 (3 zeros), 100^3 = 1,000,000 (6 zeros). Two zeros is not a multiple of 3.

8. Based on the properties of cubes, what can be said about the sum
21 + 23 + 25 + 27 + 29?
(a) The sum is 4^3. (b) The sum is a perfect square.
(c) The sum is 5^3. (d) The sum is 216.
Ans. (c) The sum is 5^3.
The pattern shows that the sum of a sequence of consecutive odd numbers results in a perfect cube. The sum 21 + 23 + 25 + 27 + 29 is the sum of the first five consecutive odd numbers after 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19. However, the example provided shows 21 + 23 + 25 + 27 + 29 = 125 = 5^3.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The number 1156 is a perfect square.
Reason (R): The prime factorization of 1156 can be grouped into two identical sets of factors.
Ans. (a) Both A and R are true, and R is the correct explanation of A.
If a number is a perfect square, one can look at its prime factorization and see if the factors can be grouped into two equal sets. It then asks to check if 1156 is a perfect square using prime factorization, which implies that this method works. Thus, the assertion is true, and the reason correctly explains it.

10. **Assertion (A):** The number 500 is not a perfect cube.
Reason (R): The prime factorization of 500 is 2 x 2 x 5 x 5 x 5, and the factors cannot be split into three identical groups.
Ans. (a) Both A and R are true, and R is the correct explanation of A.
The assertion states that 500 is not a perfect cube. The reason explains that this is because its prime factors (2 x 2 x 5 x 5 x 5) cannot be split into three identical groups.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the square root of 1296 by the prime factorisation method.
Ans. We write 1296 as product of prime factors.
$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
By forming pairs of equal factors, we have:
$1296 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3)$
Taking out one factor from each pair, we get the square root.
Therefore, $\sqrt{1296} = 2 \times 2 \times 3 \times 3 = 36$.

12. Find the cube root of 2197 by the prime factorisation method.
Ans. Resolving 2197 into prime factors, we get:
$2197 = 13 \times 13 \times 13$
Grouping the factors in triplets of the same factors, we get:
$2197 = (13 \times 13 \times 13)$
Taking one factor from the triplet, we get 13 which is the cube root of 2197.
Therefore, the cube root of 2197 is 13.

13. Find the smallest square number which is exactly divisible by 6, 8, and 15.
Ans. The smallest number divisible by 6, 8, and 15 is their LCM.
LCM of 6, 8, and 15 = 120
Now, resolving 120 into prime factors, we get:
$120 = 2 \times 2 \times 2 \times 3 \times 5$
In order to get a perfect square, each factor of 120 must be paired.
Grouping the factors into pairs, we see that 2, 3, and 5 are unpaired.
So, we need to make pairs of 2, 3, and 5. Therefore, 120 should be multiplied by $2 \times 3 \times 5 = 30$.
Thus, the required square number is $120 \times 30 = 3600$.

14. What is the smallest number by which 384 must be multiplied to get a perfect cube?
Ans. Resolving 384 into prime factors, we get:
$384 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
Grouping the factors in triplets, we see that one factor of 2 and one factor of 3 are left over.
To form complete triplets, we need two more factors of 2 and two more factors of 3.
Therefore, we must multiply 384 by $2 \times 2 \times 3 \times 3 = 36$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the smallest number by which 1575 must be divided so that it becomes a perfect square. Also, find the square root of the quotient.
Ans. First, resolve 1575 into prime factors:
$1575 = 3 \times 3 \times 5 \times 5 \times 7$
Grouping the factors into pairs, we see that the factor 7 is unpaired.
To make the number a perfect square, we must divide 1575 by 7.
The quotient is $1575 \div 7 = 225$.
Now, find the square root of the quotient:
$225 = (3 \times 3) \times (5 \times 5)$
The square root of 225 is $3 \times 5 = 15$.

16. Find the smallest number which when multiplied with 1600 will make the product a perfect cube. Find the cube root of the product.
Ans. First, resolve 1600 into prime factors:
$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

Grouping the factors in triplets, we see that two factors of 5 are left over. To form a perfect cube, we need one more 5.
Therefore, the smallest number to multiply by is 5.
The product is $1600 \times 5 = 8000$.
Now, find the cube root of the product:
$8000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$
The cube root of 8000 is $2 \times 2 \times 5 = 20$.

17. Find the smallest number by which 2048 must be divided so that the quotient has a cube root. Also, find the cube root of the quotient.
Ans. First, resolve 2048 into prime factors:
$2048 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
Grouping the factors in triplets, we see that two factors of 2 are left over.
To make the number a perfect cube, we must divide 2048 by their product, $2 \times 2 = 4$.
The quotient is $2048 \div 4 = 512$.
Now, find the cube root of the quotient:
$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
The cube root of 512 is $2 \times 2 \times 2 = 8$.

SECTION – D

Questions 18 carry 5 marks each.

18. A group of students can be arranged into rows of 18, 24, or 30 without any student being left out. What is the minimum number of students if they can also be arranged in a perfect square grid?
Ans. The total number of students must be a multiple of 18, 24, and 30. The minimum such number is their LCM.
LCM of 18, 24, and 30 = 360
To form a perfect square grid, the number of students must be a perfect square.
Resolving 360 into prime factors, we get:
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
To make it a perfect square, the factors 2 and 5 are unpaired.
So, we must multiply 360 by $2 \times 5 = 10$.
The minimum number of students is $360 \times 10 = 3600$.

SECTION – E (Case Study Based Question)

Question 19 to 20 carry 4 mark

19. Case Study – 1 (Locker Puzzle Extended)
In a palace, 400 lockers are arranged. The same process is followed as in Queen Ratnamanjuri's puzzle: Person 1 toggles every locker, Person 2 toggles every 2nd, Person 3 toggles every 3rd, and so on, until all 400 have had their turn.
Raghav observed that only lockers numbered with perfect squares would remain open. He wanted to find patterns in their distribution.

Answer the following:

(a) How many lockers remain open?
(b) Write the numbers of the first five lockers that remain open.
(c) Find the largest locker number that remains open.
(d) Explain why locker number 289 is open.
Ans. (a) There are 20 perfect square numbers from 1 to 400.
Therefore, 20 lockers remain open.
(b) The lockers that remain open are those with perfect square numbers.
The numbers of the first five lockers that remain open are 1, 4, 9, 16, and 25.
(c) The total number of lockers is 400. The lockers that remain open are the perfect squares. The largest perfect square less than or equal to 400 is 202 = 400.
Therefore, the largest locker number that remains open is 400.
(d) Since 289 is a perfect square, therefore, locker number 289 remains open.

20. Case Study – 2 (Cube Roots in Real Life)

An artisan was tasked with crafting wooden dice for a new board game. Starting with a large wooden cube of 10,648 cm^3, he planned to cut it into smaller cubical dice, each with an edge of 7 cm. He was curious about how many dice he could make and if there would be any wood left over from the original cube. This calculation was a critical first step in his design, as it would determine the final number of dice for the game.

Answer the following:
(a) Verify whether 10648 is a perfect cube.
(b) Find the cube root of 10648.
(c) How many dice of side 7 cm can be cut out from the big cube?
(d) If each die is painted at ₹2 per cm^2 of surface area, find the cost of painting one die.
Ans. (a) First, we find the prime factors of 10648: $10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$
Grouping the factors in triplets, we get: $10648 = (2 \times 2 \times 2) \times (11 \times 11 \times 11)$
Since all prime factors appear in groups of three, 10648 is a perfect cube.
(b) From the prime factorization in part (a), we have: $10648 = (2 \times 2 \times 2) \times (11 \times 11 \times 11)$
To find the cube root, we take one factor from each triplet: $\sqrt[3]{10648} = 2 \times 11 = 22$
The cube root of 10648 is 22. This means the big wooden cube has an edge length of 22 cm.
(c) The volume of the large wooden cube is given as 10,648 cm^3.
The volume of one small die is calculated using the formula $V = s^3$, where s is the side length.
Volume of one die = 7 cm × 7 cm × 7 cm = 343 cm^3.
To find the number of dice that can be cut, we divide the volume of the large cube by the volume of one small die.
Number of dice = Volume of large cube ÷ Volume of one die
Number of dice = 10648 ÷ 343 = 31.04...
Since the number of dice must be a whole number, only 31 complete dice can be cut from the big cube. There will be wood left over.
(d) First, we need to find the total surface area of one die. A cube has 6 faces, and the area of each face is s^2, where s is the side length.

Surface area of one die = $6 \times s^2$
Surface area of one die = $6 \times (7\text{ cm})^2 = 6 \times 49\text{ cm}^2 = 294\text{ cm}^2$.
The cost of painting is ₹2 per cm^2.
Total cost of painting one die = $294\text{ cm}^2 \times$ ₹$2/\text{cm}^2$ = ₹588.
The cost of painting one die is ₹588.