MATHEMATICS
WORKSHEET_120626
CHAPTER 01 A SQUARE AND A CUBE (Ganita Prakashan)
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : VIII **DURATION : 1½ hr**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Which of the following numbers will definitely not be a perfect square?
(a) 1936 (b) 2500 (c) 1729 (d) 1600
Ans. (c) 1729
1729 is not a perfect square (it lies between $41^2 = 1681$ and $42^2 = 1764$).

2. The number of lockers that remain open in the "locker puzzle" with 225 lockers is:
(a) 15 (b) 225 (c) 30 (d) 16
Ans. (a) 15
Open lockers = perfect squares $\leq 225 \rightarrow 1^2,\ldots,15^2 \Rightarrow 15$.

3. Which of the following square numbers will end with the digit 1?
(a) 15^2 (b) 18^2 (c) 21^2 (d) 32^2
Ans. A number ending in 1 or 9 can give a square ending in 1; $21^2 = 441$ is an example. (Option (c) corresponds to a number whose square ends with 1.)

4. The smallest number by which 882 must be multiplied to make it a perfect square is:
(a) 2 (b) 3 (c) 7 (d) 11
Ans. (a) 2
$882=2 \times 3^2 \times 7^2$. Only 2 is unpaired → multiply by 2 → square.

5. Which of the following is not a perfect cube?
(a) 2197 (b) 2744 (c) 4913 (d) 2500
Ans. (d) 2500
$2197 = 13^3$, $2744 = 14^3$, $4913 = 17^3$. 2500 is not a perfect cube (lies between $13^3 = 2197$ and $14^3 = 2744$).

6. The value of 126^2, if $125^2 = 15625$, is:
(a) 15625 + 126 (b) 15625 + 251 (c) 15625 + 253 (d) 15625 + 252
Ans. (b) 15625 + 251
$(n + 1)^2 = n^2 + 2n + 1$. For n = 125, add $2 \times 125 + 1 = 251$.

7. The cube root of 3375 is:
(a) 12 (b) 15 (c) 18 (d) 25
Ans. (b) 15
$3375 = 3^3 \times 5^3 = (3 \times 5)^3 = 15^3$.

8. Which of the following numbers is not a perfect cube?

(a) 1331 (b) 512 (c) 1000 (d) 2000

Ans. (d) 2000

$1331 = 11^3$, $512 = 8^3$, $1000 = 10^3$. 2000 is not a perfect cube (between $12^3 = 1728$ and $13^3 = 2197$).

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. Assertion (A): A square number always has an odd number of factors.

Reason (R): Square numbers have one unpaired factor which is repeated.

Ans. (a) Both A and R true; R correctly explains A.

Squares have a middle factor $\sqrt{n}$ unpaired → odd number of factors.

10. Assertion (A): No cube can end with exactly two zeros.

Reason (R): Cube numbers always contain factors grouped in triplets.

Ans. (a) Both A and R true; R correctly explains A.

Trailing zeros require factors 2 and 5 in triplets for a cube → trailing zeros in a cube are multiples of 3, so exactly two zeros is impossible.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the smallest square number which is exactly divisible by 12, 15, and 20.

Ans. The smallest number divisible by 12, 15, and 20 is their LCM.

LCM of 12, 15, and 20 = 60

Now, resolving 60 into prime factors, we get:$60 = 2 \times 2 \times 3 \times 5$

In order to get a perfect square, each factor of 60 must be paired.

Grouping the factors into pairs, we get 3 and 5 are left out.

So, we need to make pairs of 3 and 5.

Therefore, 60 should be multiplied by $3 \times 5 = 15$.

Thus, the required square number is $60 \times 15 = 900$.

12. Find the square root of 2401 by prime factorisation method.

Ans. We write 2401 as product of prime factors:

$2401 = 7 \times 7 \times 7 \times 7$

By forming pairs of equal factors, we have:

$2401 = (7 \times 7) \times (7 \times 7)$

Taking out one factor from each pair, we get the square root.

Therefore, $\sqrt{2401} = 7 \times 7 = 49$.

13. Find the cube root of 2744, by the prime factorisation method.

Ans. Resolving 2744 into prime factors, we get:

$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

Grouping the factors in triplets of same factors, we get:

$2744 = (2 \times 2 \times 2) \times (7 \times 7 \times 7)$

Taking one factor from each triplet, we get $2 \times 7 = 14$ which is the cube root of 2744.

Therefore, the cube root of 2744 is 14.

14. Find the smallest number by which 200 must be multiplied so that the product becomes a perfect cube.

Ans. Resolving 200 into prime factors, we get:

$200 = 2\times 2 \times 2 \times 5 \times 5$

Grouping the factors in triplets of same factors, we see that the factors of 5 are not in a complete triplet.
To form a complete triplet, we need one more 5.
Therefore, we must multiply 200 by 5 so that the product becomes a perfect cube.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the smallest number by which 405 must be divided to obtain a perfect square. Also, find the square root of the quotient.
Ans. First, resolve 405 into prime factors:
$405 = 3 \times 3 \times 3 \times 3 \times 5$
Grouping the factors into pairs, we see that the factor 5 is unpaired.
To make the number a perfect square, we must divide 405 by 5.
The quotient is $405 \div 5 = 81$.
Now, find the square root of the quotient:
$81 = (3 \times 3) \times (3 \times 3)$
The square root of $\sqrt{81}$ is $3 \times 3 = 9$.

16. Find the smallest number by which 1352 must be multiplied so that the product has a cube root. Also, find the cube root of the product.
Ans. First, resolve 1352 into prime factors:
$1352 = 2 \times 2 \times 2 \times 13 \times 13$
Grouping the factors in triplets, we see that the factor 13 is not in a triplet.
To form a perfect cube, we need one more 13.
Therefore, the smallest number to multiply by is 13.
The product is $1352 \times 13 = 17576$.
Now, find the cube root of the product:
$17576 = (2 \times 2 \times 2) \times (13 \times 13 \times 13)$
The cube root of 17576 is $2 \times 13 = 26$.

17. Find the smallest number by which 1536 must be divided so that the quotient has a cube root. Also, find the cube root of the quotient.
Ans. First, resolve 1536 into prime factors:
$1536 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
Grouping the factors in triplets, we see that the factor 3 is left out.
To make the number a perfect cube, we must divide 1536 by the leftover factor, which is 3.
The quotient is $1536 \div 3 = 512$.
Now, find the cube root of the quotient:
$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
The cube root of 512 is $2 \times 2 \times 2 = 8$.

SECTION – D

Questions 18 carry 5 marks each.

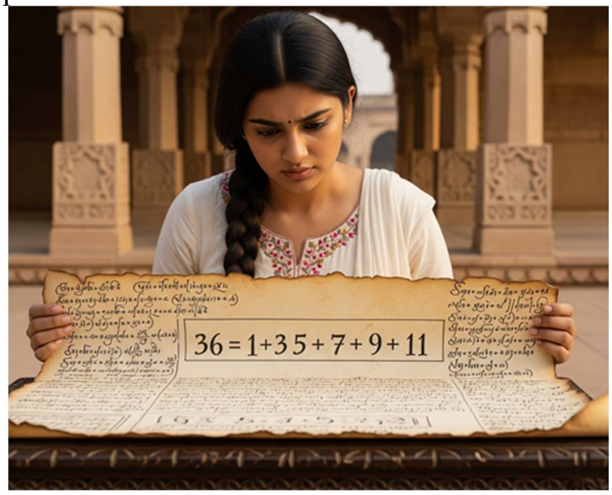
18. A school has 1475 students. What is the minimum number of students that need to be added so that the total number of students can be arranged in a perfect square grid?
Ans. To find the number of students to be added, we first need to find the prime factors of 1475.
$1475 = 5 \times 5 \times 59$
Grouping the factors into pairs, we see that the factor 59 is left unpaired. To make the number a perfect square, we would need to multiply 1475 by 59.
This would give us a perfect square, but the question asks for the smallest number of students to add.
So, we need to find the smallest perfect square greater than 1475.
We can find the square root of 1475 to estimate.
$\sqrt{1475} \approx 38.39$
The next integer is 39. So, the next perfect square is 392.

392 = 39 × 39 = 1521.
The number of students to be added is the difference between the next perfect square and the current number of students.
Number of students to add = 1521 − 1475 = 46.
Thus, the minimum number of students to add is 46.

SECTION – E (Case Study Based Question)

Question 19 to 20 carry 4 mark

19. Case Study – 1 (Squares and Odd Numbers)
Riya was reading about ancient Indian mathematicians. She discovered that square numbers can be expressed as the sum of consecutive odd numbers. She tried with 36:
36 = 1 + 3 + 5 + 7 + 9 + 11.
She wondered if she could find other squares this way. She also noticed that the number of odd numbers added equals the square root of the number.



Answer the following:
(a) Express 49 as the sum of consecutive odd numbers.
(b) How many odd numbers are required to express 121 as a sum?
(c) Find whether 50 can be expressed as the sum of consecutive odd numbers.
(d) If 2025 is a perfect square, how many odd numbers will be required to express it?
Ans. (a) The square root of 49 is 7. According to the pattern Riya discovered, 49 can be expressed as the sum of the first 7 consecutive odd numbers.
1 + 3 + 5 + 7 + 9 + 11 + 13 = 49.
Therefore, 49 is the sum of the first 7 consecutive odd numbers.
(b) The property states that the number of consecutive odd numbers required to express a perfect square as a sum is equal to its square root.
The number is 121. Its square root is $\sqrt{121} = 11$.
Therefore, 11 odd numbers are required to express 121 as a sum.
(c) For a number to be expressed as the sum of consecutive odd numbers starting from 1, it must be a perfect square.
To check if 50 is a perfect square, we can try to find its square root.
7 × 7 = 49
8 × 8 = 64
Since 50 lies between two consecutive perfect squares (49 and 64), it is not a perfect square.
Therefore, 50 cannot be expressed as the sum of consecutive odd numbers.
(d) The number of consecutive odd numbers required to express a perfect square as a sum is equal to its square root.
We need to find the square root of 2025.
We can use the prime factorization method:
2025 = 3 × 3 × 3 × 3 × 5 × 5
Group the factors into pairs: 2025 = (3 × 3) × (3 × 3) × (5 × 5)
Taking one factor from each pair gives the square root: $\sqrt{2025} = 3 \times 3 \times 5 = 45$.

Therefore, 45 odd numbers will be required to express 2025 as a sum.

20. Case Study – 2 (Cubes and Taxicab Numbers)
During a math club discussion, Arjun told the famous story of Hardy and Ramanujan. Hardy thought 1729 was dull, but Ramanujan replied it was special since $1729 = 1^3 + 12^3 = 9^3 + 10^3$. This made Arjun curious about cubes and their properties. He started exploring other such numbers called “taxicab numbers.”

Answer the following:
(a) Find the units digit of the cube of 25.
(b) What is the smallest number by which 250 must be multiplied to make it a perfect cube?
(c) What is the cube root of 2744?
(d) Can a perfect cube end with the digits 00? Justify your answer.
Ans. (a) The units digit of the cube of a number is determined by the units digit of the original number.
The units digit of 25 is 5.
When we cube 5, we get 53 = 5 × 5 × 5 = 125.
The units digit of 125 is 5.
Therefore, the units digit of the cube of 25 is 5.
(b) First, we find the prime factors of 250.
$250 = 2 \times 5 \times 5 \times 5$
To be a perfect cube, each prime factor must appear in a group of three.
Here, the factor 2 is not in a triplet. To complete the triplet for 2, we need two more 2s.
Therefore, the smallest number to multiply by is $2 \times 2 = 4$.
(c) To find the cube root, we use the prime factorization method.
$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$
Grouping the factors in triplets: $2744 = (2 \times 2 \times 2) \times (7 \times 7 \times 7)$
Taking one factor from each triplet gives the cube root: $\sqrt[3]{2744} = 2 \times 7 = 14$.
(d) No, a perfect cube cannot end with exactly two zeros.
For a number to have a cube ending in 00, its cube root must end in 0. The cube of a number ending in 0 (e.g., 10, 20, 30...) will always end in three zeros (e.g., 103 =1000, 203 = 8000, 303 = 27000).
Since a perfect cube must have a number of trailing zeros that is a multiple of 3, a number ending in only two zeros (like 100 or 500) cannot be a perfect cube.
Therefore, a perfect cube cannot end with the digits 00.