EXERCISE 2.2

1. Find the zeroes of each of the following polynomials and verify the relationship between zeroes and the coefficents in each case.

   (i) $6x^2 - 3 - 7x$ [NCERT] (ii) $t^2 - 15$ [NCERT]

   (iii) $3x^2 - x - 4$ [NCERT] (iv) $x^2 + 7x + 12$

   (v) $2x^2 - 9x - 45$ (vi) $x^2 - 20x + 96$

   (vii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ (viii) $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ [Exemplar Problems]

2. If one zero of the quadratic polynomial $2x^2 - 3x + p$ is 3, find its other zero. Also find the value of p.

3. Show that -1, 3 and 4 are the zeroes of the polynomial $x^3 - 6x^2 + 5x + 12$.

4. If 3 is a zero of the polynomial $x^2 + 2x + k$, find the value of k and the other zero.

5. If the zeroes of the polynomial $x^2 - \frac{5}{2}x + k$ are reciprocal of each other, find the value of k.

6. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then find the value of k. [Exemplar Problems]

7. Find the quadratic polynomial whose zeroes are 3 and -5. Verify the relation between the coefficients and the zeroes of the polynomial.

8. If $\frac{2}{3}$ and -3 are the zeroes of the quadratic polynomial $ax^2 + 7x + b$, then find the values of a and b.

9. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

   (i) $\frac{1}{4}$ and -1 [NCERT] (ii) 0 and $\sqrt{5}$ [NCERT]

   (iii) 1 and 1 [NCERT] (iv) 4 and 1 [NCERT]

   (v) $-2\sqrt{3}$ and -9 [Exemplar Problems] (vi) $\frac{21}{8}$ and $\frac{5}{16}$ [Exemplar Problems]

10. If the zeroes of the polynomial $x^2 - bx + c$ be in the ratio 2 : 3, prove that $6b^2 = 25c$.

11. If the zeroes of quadratic polynomial $px^2 + qx + r$ are in the ratio 3 : 4, show that $12q^2 = 49\ pr$.

12. Find the value of 'p' so that one zero of the quadratic polynomial $3x^2 + (1 + 4p)x + p^2 + 5$ may be one third of the other.

13. If α and β are the zeroes of the polynomial $5x^2 - 7x - 2$, find the sum of the reciprocals of the zeroes.

14. If α and β are the two zeroes of the polynomial $3x^2 - 17x + 24$, find

    (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

15. If α and β are the zeroes of the polynomial x^2-5x+6, form a quadratic polynomial whose zeroes are 2α and 2β.

16. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2+3x-10$, find the values of

(i) $\alpha^2\beta + \alpha\beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

17. If α and β are the zeroes of the quadratic polynomial $f(s) = s^2 - 6s + 5$, find the values of

(i) $\alpha^4\beta^3 + \alpha^3\beta^4$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

18. If p and q are the zeroes of the quadratic polynomial $f(y) = 6y^2 + y - 2$, find the value of

(i) $p^2 + q^2$ (ii) $\frac{1}{p} + \frac{1}{q} - pq$

19. If $p(x) = kx^2 + 6x + 9$ such that $\alpha^2 + \beta^2 = 18$, find the value of k.

20. If α and β are the zeroes of the quadratic polynomial $f(t) = 6t^2 + 11t - 10$, find the value of

(i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

21. If α, β are the zeros of the polynomial $3x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{19}{9}$, then find the value of k. [CBSE 2022]

22. If the sum of the cubes of zeroes of the quadratic polynomial $f(t) = t^2 - 8t + p$ is 224, find the value of p.

23. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $2\alpha + \beta$ and $2\beta + \alpha$.

24. If α and β are the zeroes of the quadratic polynomial $5x^2 - 6x + 1$, find a quadratic polynomial whose zeroes are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$.

25. If α and β are the zeroes of the quadratic polynomial $f(t) = 2t^2 - 5t - 3$, find the value of $6\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 3\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta$.

26. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4$, find a quadratic polynomial whose zeroes are $\frac{3\alpha}{\beta}$ and $\frac{3\beta}{\alpha}$.

27. Find the zeroes of the polynomial $4x^2 + 17x - 15$ and verify the relationship between the zeroes and the coefficients. [CBSE 2023]

28. If one root of the quadratic equation $x^2 + 12x - k = 0$ is thrice the other root, then find the value of k. [CBSE 2022]

29. Find the value of 'p' for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. [CBSE 2022]

30. If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2. [CBSE 202

ANSWERS 2.2

1. (i) $-\frac{1}{3}, \frac{3}{2}$ (ii) $\pm\sqrt{15}$ (iii) $-1, \frac{4}{3}$ (iv) $-3, -4$
   (v) $-3, \frac{15}{2}$ (vi) 8, 12 (vii) $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ (viii) $\sqrt{2}, \frac{1}{2}$
2. $-\frac{3}{2}$, $p = -9$ 4. $k = -15$, other zero $= -5$ 5. $k = 1$
6. $\frac{4}{3}$ 7. $x^2 + 2x - 15$ 8. $a = 3, b = -6$
9. (i) $4x^2 - x - 4$ (ii) $x^2 + \sqrt{5}$ (iii) $x^2 - x + 1$ (iv) $x^2 - 4x + 1$
   (v) $x^2 + 2\sqrt{3}x - 9$ (vi) $16x^2 - 42x + 5$
12. $p = \frac{79}{8}$ 13. $\frac{-7}{2}$
14. (i) $\frac{145}{9}$ (ii) $\frac{1241}{27}$ (iii) $\frac{17}{24}$ 15. $x^2 - 10x - 24$
16. (i) 30 (ii) $\frac{-29}{10}$ 17. (i) 750 (ii) $\frac{126}{5}$
18. (i) $\frac{25}{36}$ (ii) $\frac{5}{6}$ 19. 1, – 2
20. (i) $-\frac{241}{60}$ (ii) $\frac{133}{30}$ 21. 2 22. 12
23. $x^2 - 9x + 20$ 24. $5x^2 - 126x + 25$ 25. –45 26. $x^2 + 6x + 9$
27. Zeroes are –5 and $\frac{3}{4}$ 28. $k = -27$ 29. $p = 3$ 30. $x^2 - 29x + 100 = 0$

HINTS TO SELECTED PROBLEMS

1. (vii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (4x - \sqrt{3})(\sqrt{3}x + 2)$

$\Rightarrow$ Zeroes are $\frac{\sqrt{3}}{4}$ and $\frac{-2}{\sqrt{3}}$

(viii) $2s^2 - s - 2\sqrt{2}s + \sqrt{2} = s(2s - 1) - \sqrt{2}(2s - 1)$

$= (s - \sqrt{2})(2s - 1)$

$\Rightarrow$ Zeroes are $\sqrt{2}$ and $\frac{1}{2}$

5. Two zeroes are α and $\frac{1}{\alpha}$

$\therefore$ Product of zeroes $= \alpha \cdot \frac{1}{\alpha} = 1 = \frac{k}{1} \Rightarrow k = 1$

6. Let $f(x) = (k-1)x^2 + kx + 1$, then $f(-3) = 0$

7. $\alpha = 3$ and $\beta = -5$

One polynomial can be $(x-3)(x+5) = x^2 + 2x - 15$

10. $\alpha : \beta = 2x : 3x \Rightarrow 5x = b$ and $6x^2 = c$

$x = \frac{b}{5} \Rightarrow 6 \cdot \frac{b^2}{25} = c$ or $6b^2 = 25c$

11. $\alpha : \beta = 3x : 4x \Rightarrow 7x = \frac{-q}{p}$ and $12x^2 = \frac{r}{p}$

$x = \frac{-q}{7p} \Rightarrow x^2 = \frac{q^2}{49p^2}$

$12\left(\frac{q^2}{49p^2}\right) = \frac{r}{p} \Rightarrow 12q^2 = 49pr$

12. Let zeroes be α and $\frac{\alpha}{3}$.

Then, $\alpha + \frac{\alpha}{3} = \frac{-(1+4p)}{3} \Rightarrow \alpha = \frac{-(1+4p)}{4}$ ....(i)

and $\alpha\left(\frac{\alpha}{3}\right) = \frac{p^2+5}{3} \Rightarrow \frac{(1+4p)^2}{16} = p^2 + 5$ ($\because$ using value of α form (i))

14. (ii) $\alpha + \beta = \frac{17}{3}$ and $\alpha\beta = \frac{24}{3} = 8$

Now $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(\frac{17}{3}\right)^3 - 3(8)\left(\frac{17}{3}\right) = \frac{1241}{27}$

21. Here $\alpha + \beta = \frac{-5}{3}$ and $\alpha\beta = \frac{k}{3}$

$\alpha^2 + \beta^2 + \alpha\beta = \frac{19}{9} \Rightarrow (\alpha+\beta)^2 - \alpha\beta = \frac{19}{9} \Rightarrow \left(\frac{-5}{3}\right)^2 - \left(\frac{k}{3}\right) = \frac{19}{9}$

22. Here $\alpha + \beta = 8$ and $\alpha\beta = P$

Now, $\alpha^3 + \beta^3 = 224 \Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 224$

$\Rightarrow (8)^3 - 3(p)(8) = 224$

23. Here, $\alpha + \beta = 3$ and $\alpha\beta = 2$

Let S be sum and P be product of zeroes of new quadratic polynomial.

Now, $S = (2\alpha + \beta) + (2\beta + \alpha) = 3(\alpha + \beta) = 3(3) = 9$

$P = (2\alpha + \beta)(2\beta + \alpha) = 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta = 5\alpha\beta + 2[(\alpha+\beta)^2 - 2\alpha\beta]$

$= 5(2) + 2[(3)^2 - 2(2)] = 20$