WORKSHEET-2

Sol. 1. (c) –1

Given quadratic equation is

$$3x^2 - 2x + c = 0$$

$$\text{Discriminant (D)} = B^2 - 4AC$$

Here, $A = 3, B = -2$ and $C = c$

$\therefore \quad D = (-2)^2 - 4 \times 3 \times c$

$\Rightarrow \quad 4 - 12c = 16$

$\Rightarrow \quad 12c = -12$

$\Rightarrow \quad c = -1.$

Sol. 2. 64

$$D = B^2 - 4AC$$
$$= (10)^2 - 4 \times 3\sqrt{3} \times \sqrt{3}$$
$$= 100 - 4 \times 3 \times 3$$
$$= 100 - 36 = 64$$

Sol. 3.

$$\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$$

$\Rightarrow \quad \sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$

$\Rightarrow \quad x(\sqrt{7}x - 13) + \sqrt{7}(\sqrt{7}x - 13) = 0$

$\Rightarrow \quad (x + \sqrt{7})(\sqrt{7}x - 13) = 0$

$\Rightarrow \quad x + \sqrt{7} = 0 \quad \text{or} \quad \sqrt{7}x - 13 = 0$

$\Rightarrow \quad x = -\sqrt{7} \quad \text{or} \quad x = \dfrac{13}{\sqrt{7}} = \dfrac{13}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{13\sqrt{7}}{7}$

So, $x = -\sqrt{7}$ or $x = \dfrac{13\sqrt{7}}{7}$ is the required solution of given equation.

Sol. 4. Given, $8x^2 - 14x - 15 = 0.$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$$a = 8, b = -14, c = -15$$

$\because \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 8 \times (-15)}}{2 \times 8}$

$\Rightarrow \quad x = \dfrac{14 \pm \sqrt{196 + 480}}{16}$

$\Rightarrow \quad x = \dfrac{14 \pm 26}{16}$

$\Rightarrow \quad x = \dfrac{14 + 26}{16} \quad \text{or} \quad x = \dfrac{14 - 26}{16}$

$\therefore \quad x = \dfrac{5}{2} \quad \text{or} \quad x = -\dfrac{3}{4}$

Sol. 5. Given,

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$$

On comparing with $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$a = 3$, $b = -8$ and $c = 2$.

Since, the equation is given by $ax^2 + bx + c = 0$

Hence, required equation is $3x^2 - 8x + 2 = 0$. **Ans.**

Sol. 6. (a) $$2x^2 - 9x + 4 = 0$$

$$\text{Sum of roots} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-9)}{2} = \frac{9}{2}$$

$$\text{Product of roots} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{4}{2} = 2$$ **Ans.**

OR

(b) $4x^2 - 5 = 0$

Here $a = 4$, $b = 0$, $c = -5$

$$\begin{aligned}\text{Discriminant (D)} &= b^2 - 4ac \\ &= 0 - 4 \times 4(-5) \\ &= 80 > 0\end{aligned}$$

Since, $D > 0$, therefore roots are real and distinct. **Ans.**

Sol. 7. We have,

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{5}{x+4}, x \neq -1, -2, -4.$$

$$\Rightarrow \frac{(x+2)+2(x+1)}{(x+1)(x+2)} = \frac{5}{x+4}$$

By cross-multiplication, we get

$$\begin{aligned}(x+4)(x+2) + 2(x+1)(x+4) &= 5(x+2)(x+1) \\ \Rightarrow 3x^2 + 16x + 16 &= 5x^2 + 15x + 10 \\ \Rightarrow 2x^2 - x - 6 &= 0 \\ \Rightarrow 2x^2 - 4x + 3x - 6 &= 0 \\ \Rightarrow 2x(x-2) + 3(x-2) &= 0 \\ \Rightarrow (x-2)(2x+3) &= 0 \\ \Rightarrow x &= 2 \text{ or } x = \frac{-3}{2}\end{aligned}$$

$\therefore x = 2$ and $x = \dfrac{-3}{2}$ are the required solution. **Ans.**

Sol. 8. We have,

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{b+a}{ab}$$

$$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)} = \frac{(a+b)}{ab}$$

$$\Rightarrow \quad \frac{-1}{x(a+b+x)} = \frac{1}{ab}$$ [On dividing both sides by $(a+b)$]

$$\Rightarrow \quad x(a+b+x) = -ab$$ [By cross-multiplication]

$$\Rightarrow \quad x^2 + ax + bx + ab = 0$$

$$\Rightarrow \quad x(x+a) + b(x+a) = 0$$

$$\Rightarrow \quad (x+a)(x+b) = 0$$

$$\Rightarrow \quad x = -a \text{ or } x = -b$$

Hence, $-a$ and $-b$ are the roots of given equation.

Sol. 9. Given, $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

Here, $A = c^2 - ab$, $B = -2(a^2 - bc)$ and $C = b^2 - ac$

For equal roots,

$$D = 0$$

$$\Rightarrow \quad B^2 - 4AC = 0$$

$$\Rightarrow \quad [-2(a^2 - bc)]^2 - 4[(c^2 - ab)(b^2 - ac)] = 0$$

$$\Rightarrow \quad 4[a^4 + b^2c^2 - 2a^2bc] - 4[c^2b^2 - ab^3 - ac^3 + a^2bc] = 0$$

$$\Rightarrow \quad 4[a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ab^3 + ac^3 - a^2bc] = 0$$

$$\Rightarrow \quad 4a[a^3 + b^3 + c^3 - 3abc] = 0$$

$$\Rightarrow \quad 4a = 0 \text{ or } a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow \quad a = 0 \text{ or } \quad a^3 + b^3 + c^3 = 3abc.$$

Hence Proved.

Sol. 10. Let, the average speed of flight be x km/hr.

Then, time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{2800}{x}$ hr

New average speed $= (x - 100)$ km/hr

Then, time taken $= \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{2800}{x-100} \text{ hr}$$

According to the question,

$$\frac{2800}{x-100} - \frac{2800}{x} = \frac{30}{60}$$

$$\Rightarrow \quad \frac{2800x - 2800(x-100)}{(x-100)x} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2800x - 2800x + 280000}{x^2 - 100x} = \frac{1}{2}$$

$$\Rightarrow \quad 280000 \times 2 = x^2 - 100x$$

$$\Rightarrow \quad x^2 - 100x - 560000 = 0$$

$$\Rightarrow \quad x^2 - 800x + 700x - 560000 = 0$$

$$\Rightarrow \quad x(x - 800) + 700(x - 800) = 0$$

$$\Rightarrow \quad x = 800$$ $[\because x \neq -700]$

Then, original duration of flight

$$= \frac{2800}{800}$$

$$= 3.5 \text{ hours.}$$

Sol. 11. Let, the required natural numbers be x and $(15-x)$.
Then,

$$\frac{1}{x}+\frac{1}{(15-x)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{(15-x)+x}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{5}{x(15-x)} = \frac{1}{10} \quad \text{[Dividing both sides by 3]}$$

$$\Rightarrow \quad 50 = x(15-x) \quad \text{[By cross-multiplication]}$$

$$\Rightarrow \quad 50 = 15x - x^2$$

$$\Rightarrow \quad x^2 - 15x + 50 = 0$$

$$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$$

$$\Rightarrow \quad x(x-10) - 5(x-10) = 0$$

$$\Rightarrow \quad (x-5)(x-10) = 0$$

$$\Rightarrow \quad x = 5 \text{ or } x = 10$$

Hence, 5 and 10 are the required numbers. **Ans.**

Sol. 12. Let, time taken by one pipe to fill the tank = x min.
Then, time taken by other pipe to fill the tank = $(x+5)$ min.
Together they fill the tank in $11\frac{1}{9}$ min = $\frac{100}{9}$ min.

Then, according to the question,

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{x+5} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{x+5+x}{x(x+5)} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{2x+5}{x^2+5x} = \frac{9}{100}$$

$$\Rightarrow \quad 100(2x+5) = 9(x^2+5x)$$

$$\Rightarrow \quad 200x + 500 = 9x^2 + 45x$$

$$\Rightarrow \quad 9x^2 - 155x - 500 = 0$$

$$\Rightarrow \quad 9x^2 - 180x + 25x - 500 = 0$$

$$\Rightarrow \quad 9x(x-20) + 25(x-20) = 0$$

$$\Rightarrow \quad (x-20)(9x+25) = 0$$

$$x = 20 \text{ or } x = \frac{-25}{9} \text{ [Rejected]}$$

So, one pipe fill the tank in 20 minutes.
And, other tap fill the tank in 20 + 5 = 25 minutes. **Ans.**

Sol. 13. The given equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots.
Here, $A = 1 + m^2$, $B = 2mc$, $C = c^2 - a^2$.
For equal roots,

$$D = 0$$

$$B^2 - 4AC = 0$$

$$\Rightarrow \quad (2mc)^2 - 4(1+m^2)(c^2-a^2) = 0$$

$$\Rightarrow \quad 4m^2c^2 - 4(c^2 + m^2c^2 - a^2 - m^2a^2) = 0$$

$$\Rightarrow \quad 4[m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2] = 0$$
$$\Rightarrow \quad -c^2 + a^2(1 + m^2) = 0$$
$$\Rightarrow \quad c^2 = a^2(1 + m^2).$$

Hence Proved

Sol. 14. Let, the denominator of the fraction be x.

Then, numerator is $x - 3$ and fraction is $\frac{x-3}{x}$.

New fraction is, $\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$

According to the question,

$$\Rightarrow \quad \frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$
$$\Rightarrow \quad \frac{(x-3)(x+2)+x(x-1)}{x(x+2)} = \frac{29}{20}$$
$$\Rightarrow \quad 20(x^2 - 3x + 2x - 6 + x^2 - x) = 29(x^2 + 2x)$$
$$\Rightarrow \quad 40x^2 - 40x - 120 = 29x^2 + 58x$$
$$\Rightarrow \quad 11x^2 - 98x - 120 = 0$$
$$\Rightarrow \quad 11x^2 - 110x + 12x - 120 = 0$$
$$\Rightarrow \quad 11x(x - 10) + 12(x - 10) = 0$$
$$\Rightarrow \quad (11x + 12)(x - 10) = 0$$
$$x = 10 \text{ or } \frac{-12}{11} \text{ (neglecting)}$$

Hence, the fraction is $\frac{10-3}{10}$ *i.e.,* $\frac{7}{10}$.

Sol. 15. Let, the tens digit be x and the units digit be y.

Thus, the given number is $(10x + y)$ and its reverse is $(10y + x)$.

So, $$xy = 20$$
$$\Rightarrow \quad x = \frac{20}{y}$$

and $$10x + y + 9 = 10y + x$$
$$\Rightarrow \quad 9x - 9y = -9$$
$$\Rightarrow \quad x - y = -1$$
$$\Rightarrow \quad \frac{20}{y} - y = -1 \quad \text{[From equation (i)]}$$
$$\Rightarrow \quad 20 - y^2 = -y$$
$$\Rightarrow \quad y^2 - y - 20 = 0$$
$$\Rightarrow \quad y^2 - 5y + 4y - 20 = 0$$
$$\Rightarrow \quad y(y - 5) + 4(y - 5) = 0$$
$$\Rightarrow \quad (y - 5)(y + 4) = 0$$
$$\Rightarrow \quad y = 5 \quad \text{or} \quad y = -4$$

So, $$x = \frac{20}{5} \quad \text{or} \quad x = \frac{20}{-4}$$

$\Rightarrow \qquad x = 4 \quad$ or $\quad x = -5$ (Neglected)

Thus, the number is 45. **Ans.**

Sol. 16. (i) (b) $(3x + 3)m$

The length of the grassland is 3m more than twice the length of the flowerbed. Thus it will be $2x + 3$. Now the total length of field is $2x + 3 + x = 3x + 3$.

(ii) (a) $(8x + 6)\ m$

$$\text{Perimeter} = 2(3x + 3 + x)$$
$$= 2(4x + 3) = (8x + 6)$$

(iii) (c) We have

$$A = (3x + 3) \times x$$
$$1260 = 3x^2 + 3x$$
$$420 = x^2 + x$$
$$x^2 + x - 420 = 0$$
$$x^2 + 21x - 20x - 420 = 0$$
$$x(x + 21) - 20(x + 21) = 0$$
$$(x + 21)(x - 20) = 0$$

Thus, $x = 20$ is only possible value.

(iv) (d) 860 m^2

Area of grassland,

$$A_g = (2x + 3)\ x$$
$$= (2 \times 20 + 3)\ 20 = 860\ m^2$$

(v) (a) $\frac{20}{43}$

Area of flowerbed,

$$A_f = x^2 = 20^2 = 400\ m^2$$
$$\text{Ratio} = \frac{400}{860} = \frac{20}{43}$$

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