CHAPTER

04

Quadratic Equations

WORKSHEET-1

Sol. 1. (a) real and unequal

Given, $x - \frac{18}{x} = 6$

or $x^2 - 6x - 18 = 0$

Thus,
$$\begin{aligned} D &= b^2 - 4ac \\ &= (-6)^2 - 4 \times 1 \times (-18) \\ &= 36 + 72 = 108 > 0 \end{aligned}$$

Thus, the roots are real and unequal.

Sol. 2. $3x^2 - k\sqrt{3x} + 4 = 0$

For equal roots,

$$\begin{aligned} D &= 0 \\ \Rightarrow \quad b^2 - 4ac &= 0 \\ \Rightarrow \quad (-k\sqrt{3})^2 - 4 \times 3 \times 4 &= 0 \\ \Rightarrow \quad k^2 \times 3 - 48 &= 0 \\ \Rightarrow \quad 3k^2 &= 48 \\ \Rightarrow \quad k^2 &= \frac{48}{3} \\ \Rightarrow \quad k^2 &= 16 \\ \therefore \quad k &= \pm 4 \end{aligned}$$

Sol. 3. $\frac{1}{2}$

If $x = 3$ is a root of equation $x^2 - 2ax - 6 = 0$

$$\begin{aligned} \therefore \quad (3)^2 - 2 \times a \times 3 - 6 &= 0 \\ \Rightarrow \quad (3)^2 - 6a - 6 &= 0 \\ \Rightarrow \quad 9 - 6 - 6a &= 0 \\ 3 - 6a &= 0 \\ 6a &= 3 \\ a &= \frac{3}{6} = \frac{1}{2} \end{aligned}$$

Sol. 4. The given equation may be written as $\frac{14}{x+3} - \frac{5}{x+1} = 1$

$$\Rightarrow \quad \frac{14(x+1) - 5(x+3)}{(x+3)(x+1)} = 1$$

$$\Rightarrow \quad \frac{14x+14-5x-15}{x^3+3x+x+3} = 1$$

$$\Rightarrow \quad \frac{9x-1}{x^2+4x+3} = 1$$

$$\begin{aligned}
\Rightarrow \quad 9x-1 &= x^2+4x+3 \\
\Rightarrow \quad x^2-5x+4 &= 0 \\
\Rightarrow \quad x^2-4x-x+4 &= 0 \\
\Rightarrow \quad x(x-4)-1(x-4) &= 0 \\
\Rightarrow \quad (x-4)(x-1) &= 0 \\
\Rightarrow \quad x-4 &= 0 \quad \text{or} \quad x-1=0 \\
\Rightarrow \quad x &= 4 \quad \text{or} \quad x=1
\end{aligned}$$

Hence, 4 and 1 are the roots of the given equation. **Ans.**

Sol. 5. We have,

$$\begin{aligned}
\sqrt{2x+9}+x &= 13 \\
\sqrt{2x+9} &= 13-x
\end{aligned}$$

On squaring both sides, we get

$$\begin{aligned}
(\sqrt{2x+9})^2 &= (13-x)^2 \\
\Rightarrow \quad 2x+9 &= 169+x^2-26x \\
\Rightarrow \quad x^2-28x+160 &= 0 \\
\Rightarrow \quad x^2-20x-8x+160 &= 0 \\
\Rightarrow \quad x(x-20)-8(x-20) &= 0 \\
\Rightarrow \quad (x-8)(x-20) &= 0 \\
\Rightarrow \quad x &= 20 \text{ or } x = 8. \\
\therefore \quad x &= 8
\end{aligned}$$

[As $x = 20$ doesn't satisfy the given equation.] **Ans.**

Sol. 6.

$$\frac{x-2}{x-3}+\frac{x-4}{x-5} = \frac{10}{3}$$

$$\frac{(x-2)(x-5)(x-4)(x-3)}{(x-3)(x-5)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2-2x-5x+10+x^2-4x-3x+12}{x^2-5x-3x+15} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{2x^2-14x+22}{x^2-8x+15} = \frac{10}{3}$$

$$\begin{aligned}
\Rightarrow \quad 6x^2-42x+66 &= 10x^2-80x+150 \\
\Rightarrow \quad 4x^2-38x+84 &= 0 \\
\Rightarrow \quad 2x^2-19x+42 &= 0
\end{aligned}$$

Here, $a = 2, b = -19, c = 42$

Applying quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2-4\times 2\times 42}}{2\times 2}$$

$$x = \frac{19 \pm \sqrt{361-336}}{4}$$

$$x = \frac{19 \pm \sqrt{25}}{4} \Rightarrow \frac{19 \pm 5}{4}$$

$$x = \frac{19+5}{4}, \frac{19-5}{4}$$

$$x = \frac{24}{4}, \frac{14}{4}$$

As roots are 6, $\frac{7}{2}$

and quadratic equation is $2x^2 - 19x + 42 = 0$

Sol. 7. Given, $ad \neq bc$.

$(a^2 + b^2)x^2 + 2(ac + bd)x + c^2 + d^2 = 0$

$$\begin{aligned}
\text{Discriminant, } D &= B^2 - 4AC \\
&= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) \\
&= 4[a^2c^2 + b^2d^2 + 2abcd] - 4[a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2] \\
&= 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - b^2c^2 - a^2d^2 - b^2d^2] \\
&= 4[-a^2d^2 - b^2c^2 + 2abcd] \\
&= -4[a^2d^2 + b^2c^2 - 2abcd] \\
&= -4[ad - bc]^2
\end{aligned}$$

$\because$ D is negative.

Hence, given equation has no real roots. **Hence Proved.**

Sol. 8. Given, Hypotenuse of triangle = 13 cm

Let, the base of the right angle triangle be x cm.

Then the perpendicular/altitude of this triangle be $(x - 7)$ cm. By Pythagoras theorem,

$$\begin{aligned}
H^2 &= P^2 + B^2 \\
(13)^2 &= (x - 7)^2 + (x)^2 \\
169 &= x^2 + x^2 + 49 - 14x \\
2x^2 - 14x - 120 &= 0 \\
2x^2 - 24x + 10x - 120 &= 0 \\
2x(x - 12) + 10(x - 12) &= 0 \\
(x - 12)(2x + 10) &= 0 \\
x &= 12 \quad \text{or } x = \frac{-10}{2} \\
x &= 12 \quad \text{or } x = -5
\end{aligned}$$

(Neglecting, since length can not be negative)

$\therefore$ Base of the triangle is 12 cm and altitude of the triangle is $12 - 7 = 5$ cm. **Ans.**

Sol. 9. The given equations are

$$ay^2 + ay + 3 = 0 \quad \text{and} \quad y^2 + y + b = 0$$

Substituting $y = 1$ in both the equations, we get

$\Rightarrow \quad a(1)^2 + a(1) + 3 = 0 \quad$ and $\quad (1)^2 + (1) + b = 0$

$\Rightarrow \quad 2a + 3 = 0 \quad$ and $\quad 2 + b = 0$

$\Rightarrow \quad a = -\frac{3}{2} \quad$ and $\quad b = -2$

Thus, $\quad ab = \left(-\frac{3}{2}\right)(-2) = 3$ **Ans.**

Sol. 10. Given, $\quad (3p + 1)x^2 + 4x + 2 = 0$

$\Rightarrow \quad x^2 + \frac{4}{(3p+1)}x + \frac{2}{3p+1} = 0$

Put $$3p + 1 = 0$$

Thus, $$p = \frac{-1}{3}$$

Hence, $(3p + 1)x^2 + 4x + 2 = 0$ will not be a quadratic equation for $p = \frac{-1}{3}$. **Ans.**

Sol. 11. Let x be an positive odd number, then $x + 2$ will be its consecutive odd number.

Thus,
$$x^2 + (x + 2)^2 = 514$$
$$\Rightarrow \quad x^2 + x^2 + 4 + 4x = 514$$
$$\Rightarrow \quad 2x^2 + 4x - 510 = 0$$
$$\Rightarrow \quad x^2 + 2x - 255 = 0$$
$$\Rightarrow \quad x^2 + 17x - 15x - 255 = 0$$
$$\Rightarrow \quad x(x + 17) - 15(x + 17) = 0$$
$$\Rightarrow \quad (x + 17)(x - 15) = 0$$
$$\Rightarrow \quad x = -17 \text{ or } x = 15$$

As number is positive then,
$$x = 15.$$

Then two positive odd numbers are 15 and $15 + 2 = 17$. **Ans.**

Sol. 12. Given equation is :
$$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$$

Here, $a = 3k + 1$, $b = 2(k + 1)$ and $c = 1$

For real and equal roots
$$b^2 - 4ac = 0$$
$$\Rightarrow \quad [2(k + 1)]^2 - 4(3k + 1)(1) = 0$$
$$\Rightarrow \quad 4[k^2 + 1 + 2k] - 12k - 4 = 0$$
$$\Rightarrow \quad 4k^2 + 8k + 4 - 12k - 4 = 0$$
$$\Rightarrow \quad 4k^2 - 4k = 0$$
$$\Rightarrow \quad 4k(k - 1) = 0$$
$$\Rightarrow \quad k = 0 \text{ or } k = 1$$ **Ans.**

Sol. 13. Let the side of square $= x$ cm.

then length of rectangle $= 3x$ cm,

and width of rectangle $= (x - 4)$ cm.

Now, according to question,

$$\text{Area of rectangle} = \text{Area of square}$$
$$3x(x - 4) = x^2$$
$$\Rightarrow \quad 3x^2 - 12x = x^2$$
$$\Rightarrow \quad 2x^2 - 12x = 0$$
$$\Rightarrow \quad 2x(x - 6) = 0$$
$$\Rightarrow \quad x = 0 \text{ or } x = 6$$

As side of square cannot be zero.

$\therefore$ Side of square $= 6$ cm **Ans.**

Length of rectangle $= 3x = 3 \times 6 = 18$ cm **Ans.**

Width of rectangle $= x - 4 = 6 - 4 = 2$ cm. **Ans.**

Sol. 14. The given equation is
$$\frac{a}{(ax - 1)} + \frac{b}{(bx - 1)} = a + b$$

$$\Rightarrow \quad \left\{\frac{a}{(ax-1)} - b\right\} + \left\{\frac{b}{(bx-1)} - a\right\} = 0$$

$$\Rightarrow \quad \left\{\frac{a - abx + b}{(ax-1)}\right\} + \left\{\frac{b - abx + a}{(bx-1)}\right\} = 0$$

$$\Rightarrow \quad (a - abx + b)\left[\frac{1}{(ax-1)} + \frac{1}{(bx-1)}\right] = 0$$

$$\Rightarrow \quad (a - abx + b)\left[\frac{bx - 1 + ax - 1}{(ax-1)(bx-1)}\right] = 0$$

$$\Rightarrow \quad (a - abx + b)\left[\frac{bx + ax - 2}{(ax-1)(bx-1)}\right] = 0$$

$$\Rightarrow \quad [(a + b - abx)]\,[(a+b)\,x - 2] = 0$$

$$\Rightarrow \quad [(a+b) - abx] = 0 \text{ or } [(a+b)x - 2] = 0$$

$$\Rightarrow \quad x = \frac{a+b}{ab} \quad \text{or} \quad x = \frac{2}{a+b}.$$

Sol. 15. (a) Let the required natural number be x.

According to question,

$$x + 12 = \frac{160}{x}$$

$$\Rightarrow \quad x^2 + 12x = 160$$

$$\Rightarrow \quad x^2 + 12x - 160 = 0$$

$$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$$

$$\Rightarrow \quad x(x+20) - 8\,(x+20) = 0$$

$$\Rightarrow \quad (x+20)\,(x-8) = 0$$

$$\Rightarrow \quad x + 20 = 0$$

$$\Rightarrow \quad x = -20 \qquad [x \neq -20 \text{ because it is not a natural number}]$$

$$\Rightarrow \quad (x - 8) = 0$$

$$\Rightarrow \quad x = 8$$

Hence, the required natural number is 8.

OR

(b) Compare quadratic equation $x^2 + 12x - k = 0$ with $ax^2 + bx + c = 0$

Then, $a = 1$, $b = 12$, $c = -k$

Let first root of given quadratic equation be α

According to question second root $= 3\alpha$

We know that

$$\text{Sum of roots} = -\frac{b}{a}$$

$$\alpha + 3\alpha = \frac{-b}{a}$$

$$\Rightarrow \quad 4\alpha = \frac{-12}{1}$$

$$\Rightarrow \quad \alpha = -3$$

$$\text{Product of roots} = \frac{c}{a}$$

$$\alpha \times 3\alpha = \frac{-k}{1}$$

$$3\alpha^2 = -k$$

Put the value of α

$$3 \times (-3)^2 = -k$$

$$k = -27$$

Ans.

Sol. 16. Let breadth of rectangular field be x m.

So, its length will be $(2x + 4)$ m

$\because$ Area of field $= 240 \text{ m}^2$

$\therefore$ $(2x + 4)x = 240$

$2x^2 + 4x - 240 = 0$

$\Rightarrow$ $x^2 + 2x - 120 = 0$

$\Rightarrow$ $x^2 + 12x - 10x - 120 = 0$

$x(x + 12) - 10(x + 12) = 0$

$\Rightarrow$ $(x + 12)(x - 10) = 0$

$\Rightarrow$ $x = -12$ (neglecting)

and $x = 10$

(i) Breadth of the field = 10 m

(ii) Length of the field = $(2 \times 10 + 4)$ m = 24 m

(iii) Distance covered by Meeta,

$$DB = \sqrt{AB^2 + AD^2}$$

$$= \sqrt{24^2 + 10^2}$$

$$= \sqrt{576 + 100}$$

$$= \sqrt{676} = 26 \text{ m}$$

Distance covered by Geeta $= DA + AB$

$= 10 + 24 = 34$ m

So, Meeta covers less distance.

Ans.