Progress Check-2

Sol. 1. (a) 3

For infinitely many solutions,

$$\frac{c}{6} = \frac{-1}{-2} = \frac{2}{4}$$

$$\Rightarrow \quad c = 3$$

Sol. 2. $-\frac{\sqrt{3}}{4}$ and $\frac{2}{\sqrt{3}}$.

$$4\sqrt{3}x^2 - 5x - 2\sqrt{3} = 0$$

$$4\sqrt{3}x^2 - 8x + 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}x - 2) + \sqrt{3}(3x - 2) = 0$$

$$\Rightarrow \quad (\sqrt{3}x - 2)(4x + \sqrt{3}) = 0$$

$$\Rightarrow \quad (\sqrt{3}x - 2) = 0, (4x + \sqrt{3}) = 0$$

$$\Rightarrow \quad x = \frac{2}{\sqrt{3}} \text{ and } x = -\frac{\sqrt{3}}{4}$$

$$\therefore \quad x = \frac{2}{\sqrt{3}}, \frac{-\sqrt{3}}{4}$$

Sol. 3. Given,

$$71x + 37y = 253 \quad \text{...(i)}$$

and

$$37x + 71y = 287 \quad \text{...(ii)}$$

On adding equations (i) and (ii), we get

$$108x + 108y = 540$$

$$\Rightarrow \quad 108\,(x + y) = 540$$

$$\Rightarrow \quad x + y = 5 \quad \text{...(iii)}$$

Also, subtracting equation (ii) from equation (i), we get

$$34x - 34y = -34$$

$$\Rightarrow \quad 34\,(x - y) = -34$$

$$\Rightarrow \quad x - y = -1 \quad \text{...(iv)}$$

On adding equations (iii) and (iv), we get

$$2x = 4$$

$$\Rightarrow \quad x = 2.$$

Substituting $x = 2$ in equation (iii), we get

$$2 + y = 5$$

$$\Rightarrow \quad y = 5 - 2 = 3$$

Thus, $x = 2$ and $y = 3$ is the required solution.

Sol. 4. Given, $$3\left(\frac{3x-1}{2x+3}\right) - 2\left(\frac{2x+3}{3x-1}\right) = 5$$

On putting $\frac{3x-1}{2x+3} = y$, the given equation becomes,

$$3y - \frac{2}{y} = 5$$

$$\Rightarrow \quad 3y^2 - 2 = 5y$$
$$\Rightarrow \quad 3y^2 - 5y - 2 = 0$$
$$\Rightarrow \quad 3y^2 - 6y + y - 2 = 0$$
$$\Rightarrow \quad 3y(y-2) + 1(y-2) = 0$$
$$\Rightarrow \quad (3y+1)(y-2) = 0$$
$$\Rightarrow \quad 3y + 1 = 0 \text{ or } y - 2 = 0$$
$$\Rightarrow \quad y = -\frac{1}{3} \text{ or } y = 2$$

Case I : $y = 2$

$$\Rightarrow \quad \frac{3x-1}{2x+3} = 2$$
$$\Rightarrow \quad 3x - 1 = 2(2x+3) \qquad \text{[By cross-multiplication]}$$
$$\Rightarrow \quad 3x - 1 = 4x + 6$$
$$\Rightarrow \quad 4x - 3x = -1 - 6$$
$$x = -7$$

Case II : $y = -\frac{1}{3}$

$$\Rightarrow \quad \frac{3x-1}{2x+3} = -\frac{1}{3}$$
$$\Rightarrow \quad 3(3x-1) = -1(2x+3) \qquad \text{[By cross-multiplication]}$$
$$\Rightarrow \quad 9x - 3 = -2x - 3$$
$$\Rightarrow \quad 9x + 2x = -3 + 3$$
$$\Rightarrow \quad 11x = 0$$
$$\Rightarrow \quad x = 0$$

Hence, $x = 0$ and -7 are the required roots of the given equation. **Ans.**

Sol. 5. Given, $$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\Rightarrow \quad \frac{1}{2a+b+2x} = \frac{bx + 2ax + ab}{2abx}$$
$$\Rightarrow \quad 2abx = (2a+b+2x)(bx+2ax+ab)$$
$$\Rightarrow \quad 2abx = 2abx + 4a^2x + 2a^2b + b^2x + 2abx + ab^2 + 2bx^2 + 4ax^2 + 2abx$$
$$\Rightarrow \quad (4ax^2 + 2bx^2) + (4abx + 4a^2x + b^2x) + (2a^2b + ab^2) = 0$$
$$\Rightarrow \quad 2x^2(2a+b) + x[4ab + 4a^2 + b^2] + ab(2a+b) = 0$$
$$\Rightarrow \quad 2x^2(2a+b) + x[4a^2 + 2ab + 2ab + b^2] + ab(2a+b) = 0$$
$$\Rightarrow \quad 2x^2(2a+b) + x[2a(2a+b) + b(2a+b)] + ab(2a+b) = 0$$
$$\Rightarrow \quad 2x^2(2a+b) + x[(2a+b)(2a+b)] + ab(2a+b) = 0$$
$$\Rightarrow \quad 2x^2(2a+b) + x(2a+b)^2 + ab(2a+b) = 0$$
$$\Rightarrow \quad (2a+b)[2x^2 + x(2a+b) + ab] = 0$$
$$\Rightarrow \quad 2x^2 + (2a+b)x + ab = 0$$
$$\Rightarrow \quad x = \frac{-(2a+b) \pm \sqrt{(2a+b)^2 - 4(2ab)}}{4}$$

$$\Rightarrow \quad x = \frac{-(2a+b) \pm \sqrt{4a^2+4ab+b^2-8ab}}{4}$$

$$\Rightarrow \quad x = \frac{-(2a+b) \pm \sqrt{4a^2+b^2-4ab}}{4}$$

$$\Rightarrow \quad x = \frac{-(2a+b) \pm \sqrt{(2a-b)^2}}{4}$$

$$\Rightarrow \quad x = \frac{-(2a+b) \pm (2a-b)}{4}$$

$$\Rightarrow \quad x = \frac{-(2a+b)+2a-b}{4} \quad \text{and} \quad x = \frac{-(2a+b)-(2a-b)}{4}$$

(taking positive sign) (taking negative sign)

$$\Rightarrow \quad x = -\frac{2b}{4} \quad \text{and} \quad x = -\frac{4a}{4}$$

$$\Rightarrow \quad x = -\frac{b}{2} \quad \text{and} \quad x = -a.$$

So, $x = -\frac{b}{2}$ or $-a$

Sol. 6. Let the speed of the stream be x km/h.
The speed of the boat upstream $= (18-x)$ km/h
The speed of the boat downstream $= (18+x)$ km/h.

The time taken to go upstream $= \frac{\text{Distance}}{\text{Speed}} = \frac{24}{18-x}$ hours

The time taken to go downstream $= \frac{\text{Distance}}{\text{Speed}} = \frac{24}{18+x}$ hours

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24\,(18+x) - 24\,(18-x) = (18-x)\,(18+x)$$

$$x^2 + 48x - 324 = 0$$

$$x = 6 \text{ or } -54$$

Since x is the speed of the stream, it cannot be negative.
Therefore, $x = 6$ gives the speed of the stream = 6 km/h.

OR

Let the time taken by the smaller pipe to fill the tank $= x$ hr.
Time taken by the larger pipe $= (x-10)$ hr.

Part of the tank filled by smaller pipe in 1 hour $= \frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour $= \frac{1}{x-10}$

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Part of the tank filled by both the pipes in 1 hour $= \frac{8}{75}$

Therefore, $$\frac{1}{x}+\frac{1}{x-10} = \frac{8}{75}$$

$$8x^2-230x+750 = 0$$

$$x = 25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours, as the time taken by the larger pipe will become negative, which is logically not possible.

Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and $25-10 = 15$ hours, respectively.

Sol. 7. Given, $$0.4x-1.5y = 6.5$$

and $$0.3x+0.2y = 0.9$$

On multiplying given equations by 10, we get

$$4x-15y = 65 \quad ...(i)$$

and $$3x+2y = 9 \quad ...(ii)$$

Now, on multiplying equation (i) with 2 and equation (ii) with 15, we get

$$8x-30y = 130 \quad ...(iii)$$

and $$45x+30y = 135 \quad ...(iv)$$

On adding equations (iii) and (iv), we get

$$53x = 265$$

$$\Rightarrow \quad x = 5$$

On substituting $x = 5$ in equation (ii), we get

$$3\times 5+2y = 9$$

$$\Rightarrow \quad 15+2y = 9$$

$$\Rightarrow \quad 2y = -15+9 = -6$$

$$\Rightarrow \quad y = -3$$

Thus, $x = 5$ and $y = -3$. **Ans.**

Sol. 8. Given, $(a-b)x^2+(b-c)x+(c-a) = 0$

On comparing the given equation with $Ax^2+Bx+c = 0$, we get

$$A = a-b, B = (b-c) \text{ and } C = (c-a)$$

Since, the roots are equal

$$\therefore \quad D = 0$$

$$\Rightarrow \quad B^2-4AC = 0$$

$$\Rightarrow \quad (b-c)^2-4(a-b)(c-a) = 0$$

$$\Rightarrow \quad (b-c)^2-4(ac-a^2-bc+ab) = 0$$

$$\Rightarrow \quad b^2+c^2-2bc-4ac+4a^2+4bc-4ba = 0$$

$$\Rightarrow \quad 4a^2+b^2+c^2+2bc-4ac-4ab = 0$$

$$\Rightarrow \quad (b+c-2a)^2 = 0$$

$$\Rightarrow \quad b+c-2a = 0$$

$$\therefore \quad b+c = 2a.$$

Hence Proved.

Sol. 9. Let the cost of 1 pencil be ₹ x and the cost of 1 pen be ₹ y.

According to the question,

$$5x+7y = 50 \quad ...(i)$$

$$7x+5y = 46 \quad ...(ii)$$

For equation (i),

$$x = \frac{50-7y}{5}$$

x	3	10	-4
y	5	0	10

The points are (3, 5), (10, 0) and (– 4, 10)

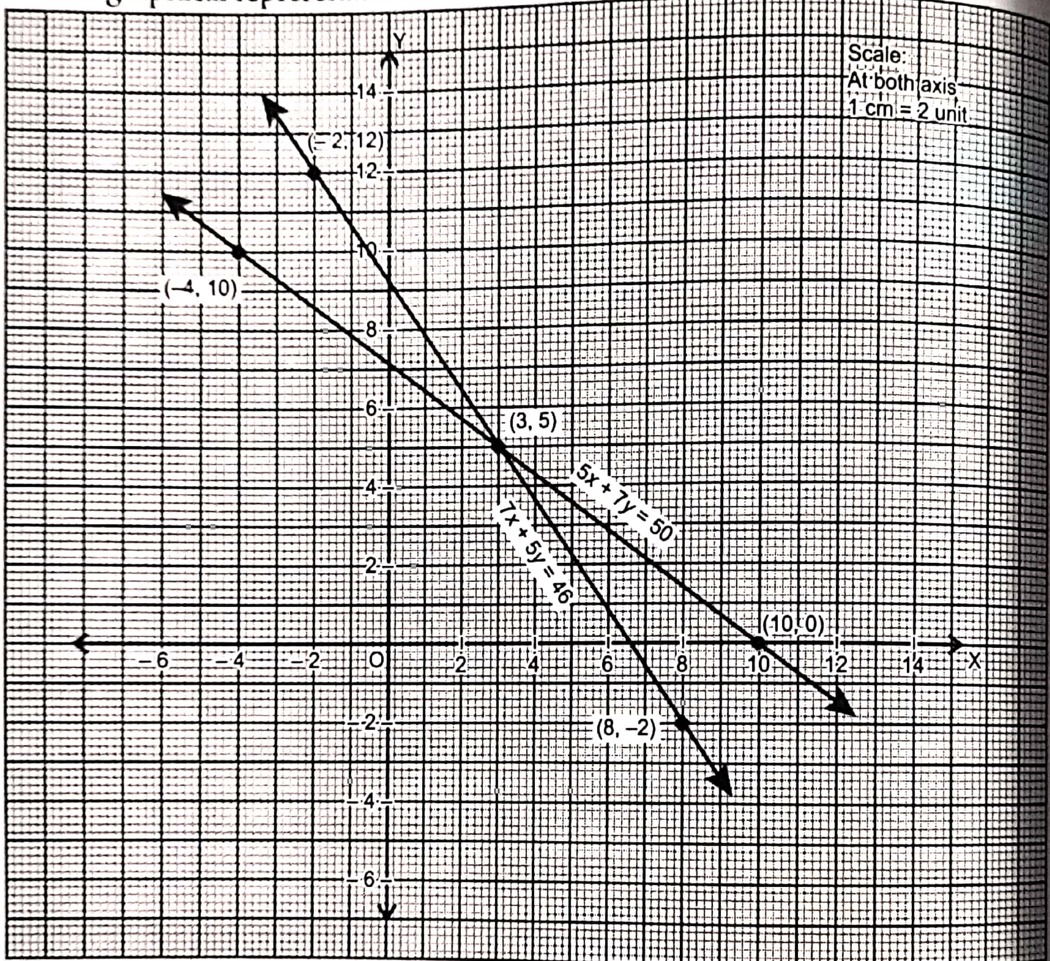
For equation (ii)

$$x = \frac{46-5y}{7}$$

x	8	3	-2
y	-2	5	12

The points are (8, – 2), (3, 5) and (– 2, 12)

Hence the graphical representation is as follows :



From the graph, it can be observed that the lines intersect each other at point (3, 5). Therefore, the cost of pencil and a pen are ₹ 3 and ₹ 5 respectively.

Sol. 10. Let, XOX′ and YOY′ be the X-axis and Y-axis respectively.

Now, $x - y + 1 = 0$

$\Rightarrow$ $x = y - 1$

If $y = 1$ then $x = 0$

If $y = 2$ then $x = 1$

If $y = 3$ then $x = 2$

If $y = 4$ then $x = 3$

x	0	1	2	3
y	1	2	3	4

Now, $3x + 2y - 12 = 0$

$\Rightarrow$ $2y = 12 - 3x$

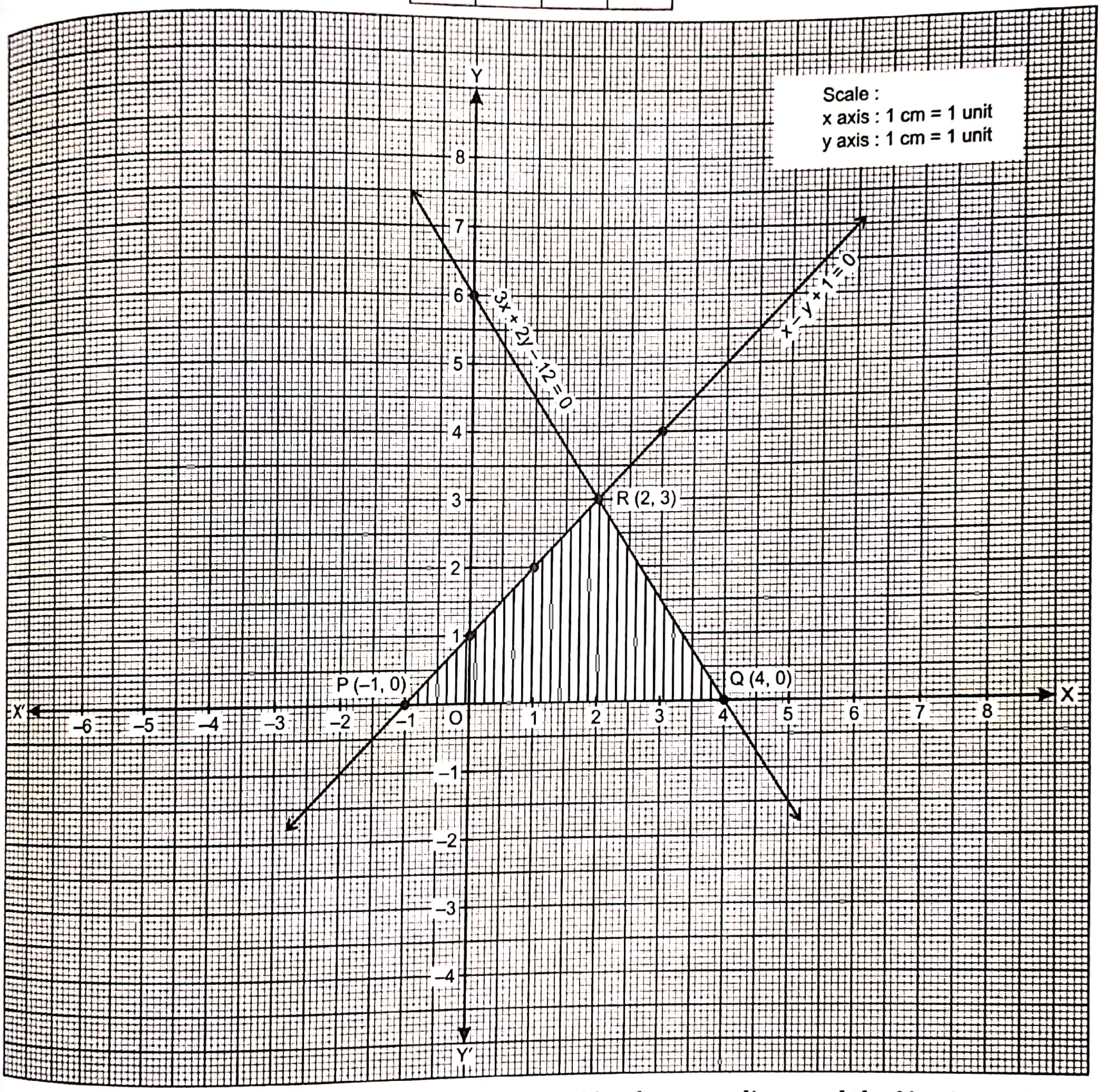
$\Rightarrow$ $y = \dfrac{12 - 3x}{2}$

If $x = 4$, $y = 0$

$x = 2$, $y = 3$

$x = 0$, $y = 6$

x	0	2	4
y	6	3	0



Hence, the coordinates of the triangle formed by these two lines and the X-axis are (– 1, 0), (2, 3) and (4, 0). **Ans.**

Sol. 11. Suppose, the total number of students is x.

Number of students who opted for visiting old age home = $\dfrac{3}{8}x$

Number of students who opted for having a nature walk = 16

Number of students who opted for tree plantation = $\sqrt{x}$.

According to question,

$$\frac{3}{8}x = \sqrt{x} + 16$$

$\therefore$ $3x = 8\sqrt{x} + 128$

$\Rightarrow$ $3x - 8\sqrt{x} - 128 = 0$

Suppose that, $\sqrt{x} = y$

$\therefore$ $3y^2 - 8y - 128 = 0$

$\Rightarrow$ $3y^2 - 24y + 16y - 128 = 0$

$\Rightarrow$ $3y(y-8) + 16(y-8) = 0$

$\Rightarrow$ $(y-8)(3y+16) = 0$

If $y - 8 = 0$

$\Rightarrow$ $y = 8$ $[\because y > 0]$

Hence, $x = y^2 = 64$

Number of students $= 64$. Ans.

Sol. 12. Let, the speed of X and Y be x km/hr and y km/hr respectively.

Then, Time taken by X to cover 30 km $= \frac{30}{x}$ hrs.

and Time taken by Y to cover 30 km $= \frac{30}{y}$ hrs.

According to the given conditions, we have

$$\frac{30}{x} - \frac{30}{y} = 3$$

$\Rightarrow$ $\frac{10}{x} - \frac{10}{y} = 1 \Rightarrow 10y - 10x = xy$ ...(i)

If X doubles his pace, then speed of X will be $2x$ km/hr.

Time taken by X to cover 30 km $= \frac{30}{2x}$ hrs.

and time taken by Y to cover 30 km $= \frac{30}{y}$ hrs.

According to the given conditions, we have

$$\frac{30}{y} - \frac{30}{2x} = 1\frac{1}{2}$$

$\Rightarrow$ $\frac{10}{y} - \frac{10}{2x} = \frac{1}{2}$

$\Rightarrow$ $\frac{10}{y} - \frac{5}{x} = \frac{1}{2}$

$\Rightarrow$ $-\frac{10}{x} + \frac{20}{y} = 1 \Rightarrow -10y + 20x = xy$ ...(ii)

On adding equations (i) and (ii), we get

$\Rightarrow$ $10x = 2xy$

$\Rightarrow$ $y = 5$

Putting $y = 5$ in equation (i), we get

$\Rightarrow$ $10 \times 5 - 10x = 5x$

$$\Rightarrow \quad 50 = 15x$$

$$\Rightarrow \quad x = \frac{50}{15} = \frac{10}{3}$$

Hence X's speed $= \frac{10}{3} = 3\frac{1}{3}$ km/hr and Y's speed $= 5$ km/hr. **Ans.**

Sol. 13. Let the present age of the father be x years and the present age of his son be y years.

Two years ago :

$$(x-2) = 5(y-2)$$

$$\Rightarrow \quad x - 2 = 5y - 10$$

$$\Rightarrow \quad x - 5y = -8 \quad \text{...(i)}$$

Two years later :

$$(x+2) = 3(y+2) + 8$$

$$x + 2 = 3y + 6 + 8$$

$$\Rightarrow \quad x - 3y = 12 \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$x - 3y - x + 5y = 12 + 8$$

$$2y = 20$$

$$y = 10$$

Substituting $y = 10$ in equation (ii), we get

$$x - 3 \times 10 = 12$$

$$\Rightarrow \quad x - 30 = 12$$

$$\Rightarrow \quad x = 42$$

Thus, the father's age is 42 years and that of the son is 10 years. **Ans.**

Sol. 14.

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$= \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2$$

$$272 = (2^2 - 2\alpha\beta)^2 - 2\alpha^2\beta^2 \quad [\because \alpha + \beta = 2 \text{ and } \alpha^4 + \beta^4 = 272]$$

Let, $\alpha\beta = t$

Then $(4 - 2t)^2 - 2t^2 = 272$

$$\Rightarrow \quad 16 - 16t + 4t^2 - 2t^2 = 272$$

$$\Rightarrow \quad 2t^2 - 16t - 256 = 0$$

$$\Rightarrow \quad t^2 - 8t - 128 = 0$$

$$\Rightarrow \quad t^2 - 16t + 8t - 128 = 0$$

$$\Rightarrow \quad t(t - 16) + 8(t - 16) = 0$$

$$\Rightarrow \quad (t - 16)(t + 8) = 0$$

$$\Rightarrow \quad t = 16 \text{ or } -8$$

Case I : $t = 16$

$$\alpha + \beta = 2 \text{ and } \alpha\beta = 16$$

$$\Rightarrow \quad x^2 - 2x + 16 = 0$$

Case II : $t = -8$

$$\alpha + \beta = 2 \text{ and } \alpha\beta = -8$$

$$\Rightarrow \quad x^2 - 2x - 8 = 0$$

Ans.

Sol. 15. (i) Let the number of articles produced on a particular day be x.

So, cost of production of each article $= 2x + 3$

$\Rightarrow$ Total cost of production on that day $= x(2x + 3)$

$$x(2x+3) = 90$$

$\therefore$ $$2x^2+3x-90 = 0$$

$\Rightarrow$ $$2x^2+15x-12x-90 = 0$$

$\Rightarrow$ $$x(2x+15)-6(2x+15) = 0$$

$\Rightarrow$ $$(x-6)(2x+15) = 0$$

$\Rightarrow$ $$x = 6 \text{ or } x = -\frac{15}{2}$$

$\Rightarrow$

The number of articles produced cannot be negative

$\therefore$ Number of articles $= 6$

(ii) Cost of each article $= 2x+3$

$= 2 \times 6 + 3$

$= ₹\, 15$

(iii) $$x^2-2x-15 = 0$$

$\Rightarrow$ $$x^2-5x+3x-15 = 0$$

$\Rightarrow$ $$x(x-5)+3(x-5) = 0$$

$\Rightarrow$ $$(x+3)(x-5) = 0$$

$\Rightarrow$ $$x = -3 \text{ or } x = 5.$$

Sol. 16. Let, the cost of each chair be ₹x and the cost of each table be ₹y.

Thus, S.P. of chair with 10% profit $= x + \frac{10x}{100} = x + \frac{x}{10} = \frac{11x}{10}$.

and S.P. of chair with 25% profit $= x + \frac{25x}{100} = x + \frac{x}{4} = \frac{5x}{4}$

Also, S.P. of table with 10% profit $= y + \frac{10y}{100} = y + \frac{y}{10} = \frac{11y}{10}$

and S.P. of table with 25% profit $= y + \frac{25}{100}y = y + \frac{y}{4} = \frac{5y}{4}$

Now, $$\frac{5x}{4} + \frac{11y}{10} = 760$$

$\Rightarrow$ $$\frac{50x+44y}{40} = 760$$

$\Rightarrow$ $$25x+22y = 760 \times 20$$

$\Rightarrow$ $$25x+22y = 15200 \quad ...(i)$$

and $$\frac{11x}{10} + \frac{5y}{4} = 767.50.$$

$\Rightarrow$ $$\frac{44x+50y}{40} = 767.50$$

$\Rightarrow$ $$22x+25y = 767.50 \times 20$$

$\Rightarrow$ $$22x+25y = 15350 \quad ...(ii)$$

On adding equations (i) and (ii), we get

$$47x+47y = 30550$$

$\Rightarrow$ $$47(x+y) = 30550$$

$\Rightarrow$ $$x+y = 650 \quad ...(iii)$$

Subtracting equation (ii) from equation (i), we get

$$3x-3y = -150$$

$\Rightarrow$ $$x-y = -50 \quad ...(iv)$$

On adding equations (iii) and (iv), we get

$$\Rightarrow \quad 2x = 600$$

$$\Rightarrow \quad x = 300$$

Substituting $x = 300$ in equation (iii), we get

$$300 + y = 650$$

$$y = 650 - 300$$

$$y = 350$$

Thus, the cost of each chair is ₹ 300 and that of each table is ₹ 350. **Ans.**

Sol. 17. (i) (a) $2(x + 5)$ km

Chandana's car travels x km/h while Sohana's car travels 5 km/h more than Chandana's car. Thus, Sohana's car speed is $x + 5$ km/hour Distance covered in two hour is $2(x + 5)$ km.

(ii) (c) $x^2 + 5x - 500 = 0$

As per question

$$\frac{400}{x} = \frac{400}{x+5} + 4$$

$$400(x + 5) = 400x + 4x(x + 5)$$

$$2000 = 4x^2 + 20x$$

$$500 = x^2 + 5x$$

$$x^2 + 5x - 500 = 0$$

(iii) (a) 20 km/hour

We have, $x^2 + 5x - 500 = 0$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x + 25) - 20(x + 25) = 0$$

$$(x + 25)(x - 20) = 0$$

$$x = 20, x = -25$$

Since $x = -25$ is not possible, we get $x = 20$.

(iv) (d) 16 hour

Sohana's car speed = 20 + 5 = 25 km/hour

$$\text{Time taken} = \frac{400}{25} = 16 \text{ hour}$$

(v) (c) 20 hour

Chanda's car speed = 20 km/hour

$$\text{Time taken} = \frac{400}{20} = 20 \text{ hour}$$

Sol. 18. (c) Assertion (A) is true but reason (R) is false.

Explanation: Given, $f(1) + f(2) = 0$

For $f(x) = ax^2 + bx + c$, we have

$$a + b + c + 4a + 2b + c = 0$$

$$\Rightarrow \quad 5a + 3b + 2c = 0 \quad \text{...(i)}$$

Since, -1 is a root of the equation.

$f(-1) = a - b + c = 0$ ...(ii)

By eliminating c and b in turn by solving equations (1) and (2), we have

$$\frac{-b}{a} = \frac{3}{5}, \frac{c}{a} = \frac{-8}{5}$$

Since, -1 is one root, the other root is $-\frac{8}{5}$.