MATHEMATICS

WORKSHEET_130626

CHAPTER 02 POWER PLAY (Ganita Prakashan)

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : VIII **DURATION : 1½ hr**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If the mass of a dust particle is 7.2×10^{-11} kg, what is this mass in a number form?
(a) 0.000000000072 kg (b) 0.00000000072 kg
(c) 0.0000000072 kg (d) 0.000000072 kg
Ans. (a) 0.000000000072 kg
A negative exponent indicates we need to move the decimal point to the left. The exponent is -11, so we move the decimal point 11 places to the left from its current position in 7.2.
$7.2 \times 10^{-11} = 0.000000000072$ kg.

2. What is the value of $(20)^0 - (20)^0$?
(a) 20 (b) 0 (c) 1 (d) -20
Ans. (b) 0
Using the rule $n^0 = 1$ for any non-zero integer n, we have $(20)^0 = 1$. So, the expression becomes 1 – 1 = 0.

3. The value of $(0.5)^2 \times (0.5)^3$ is:
(a) $(0.5)^6$ (b) $(0.5)^2$ (c) $(0.5)^5$ (d) 0.5
Ans. (c) (0.5)5
Using the rule $a^m \times a^n = a^{m+n}$, we have $(0.5)^2 \times (0.5)^3 = (0.5)^{2+3} = (0.5)^5$.

4. If $2^x = 1/64$, what is the value of x?
(a) 6 (b) -6 (c) 8 (d) -8
Ans. (b) -6
We can rewrite 1/64 as a power of 2.
$64 = 2^6$. So, $1/64 = 1/2^6 = 2^{-6}$.
The equation becomes $2^x = 2^{-6}$. Equating the exponents, we get $x = -6$.

5. What is the simplified form of $\frac{18^2 \times 3^4}{6^4 \times 9}$?
(a) 9/4 (b) 3 (c) 1/3 (d) 9
Ans. (a) 9/4
$$\frac{18^2 \times 3^4}{6^4 \times 9} = \frac{(2\times 3^2)^2 \times 3^4}{(2\times 3)^4 \times 3^2} = \frac{2^2 \times 3^4 \times 3^4}{2^4 \times 3^4 \times 3^2} = \frac{3^{4+4-4-2}}{2^{4-2}} = \frac{3^2}{2^2} = \frac{9}{4}$$

6. A video game character's power level, P, is calculated as $P = 2^n$, where n is the number of missions completed. If a player completes 8 missions, what is their power level?

(a) 64 (b) 16 (c) 256 (d) 128

Ans. (c) 256

The power level is $P = 2^n$. The number of missions completed is $n = 8$. $P = 2^8 = 256$.

7. The population of a town is growing at a rate of 2×10^3 people per year. If the current population is 10^5, what will the population be in 2 years?

(a) $10^5 + 4 \times 10^3$ (b) 1.04×10^4 (c) $10^5 + 2 \times 10^3$ (d) 10^8

Ans. (a) $10^5 + 4 \times 10^3$

The population increases by 2×10^3 per year. In 2 years, the increase will be $2 \times (2 \times 10^3) = 4 \times 10^3$.

The new population is the current population plus the increase: $10^5 + 4 \times 10^3$.

This can also be written as 100,000 + 4000 = 104,000, or 1.04×10^5.

8. Which of the following is equivalent to $\left(\frac{1}{2}\right)^5$?

(a) 2^{-5} (b) 2^5 (c) $\left(\frac{1}{2}\right)^{-5}$ (d) $\frac{1}{2^{-5}}$

Ans. (a) 2^{-5}

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** The value of $\left(\frac{2}{3}\right)^{-2}$ is $\frac{9}{4}$.

Reason (R): The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Ans. (b) Both A and R are true, but R is not the correct explanation of A.

Assertion A is true. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$. Reason R is also true. However, the reason for the assertion is that $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$, which is a specific rule. The reciprocal relationship is part of this rule but not the complete explanation.

10. **Assertion (A):** The standard form of 0.0000035 is 3.5×10^{-6}.

Reason (R): A number in standard form must have a coefficient between 1 and 10.

Ans. (a) Both A and R are true, and R is the correct explanation of A.

The assertion is true. To get from 0.0000035 to 3.5, we move the decimal point 6 places to the right, so the exponent is −6.

The reason is also true and is the definition of standard form, which is the direct reason for the assertion.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Simplify and express the result as a power of 2: $\frac{2^5 \times 4^2}{8^2}$

Ans. $\frac{2^5 \times 4^2}{8^2} = \frac{2^5 \times (2^2)^2}{(2^3)^2} = \frac{2^5 \times 2^4}{2^6} = 2^{5+4-6} = 2^3$

12. Find the value of x for which $5^{x+1} \times 5^3 = 5^8$.

Ans. $5^{x+1} \times 5^3 = 5^8$

Using the rule $a^m \times a^n = a^{m+n}$, we have $5^{(x+1)+3} = 5^8$.

$\Rightarrow 5^{x+4} = 5^8$.

Equating the exponents: $x + 4 = 8 \Rightarrow x = 4$.

13. Simplify: $3^{-2} + 4^{-2}$

Ans. $3^{-2} + 4^{-2} = \frac{1}{3^2} + \frac{1}{4^2} = \frac{1}{9} + \frac{1}{16} = \frac{16+9}{144} = \frac{25}{144}$

14. The population of a city is 1.25×10^7. If each person consumes 10^3 litres of water per year, what is the total water consumption in a year? Express your answer in standard form.

Ans. Total water consumption = (Population) × (Consumption per person)

Total consumption = $(1.25 \times 10^7) \times 10^3 = 1.25 \times 10^{7+3} = 1.25 \times 10^{10}$ liters.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Simplify: $\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$

Ans. $\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3} = \frac{(2^2)^4 \times 3^4 \times 3^6 \times 2^2}{2^3 \times 3^3 \times 2^6 \times 3^3}$

$= \frac{2^8 \times 3^4 \times 3^6 \times 2^2}{2^3 \times 3^3 \times 2^6 \times 3^3} = \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6} = 2^{10-9} \times 3^{10-6} = 2 \times 3^4 = 2 \times 81 = 162$

16. Find the value of x for which $\left(\frac{2}{5}\right)^{x-2} = \frac{125}{8}$

Ans. $\left(\frac{2}{5}\right)^{x-2} = \frac{125}{8} \Rightarrow \left(\frac{2}{5}\right)^{x-2} = \frac{5^3}{2^3} = \left(\frac{5}{2}\right)^3$

$\Rightarrow \left(\frac{2}{5}\right)^{x-2} = \left(\frac{2}{5}\right)^{-3} \Rightarrow x - 2 = -3 \Rightarrow x = -3 + 2 = -1$

17. By what number should $\left(\frac{4}{5}\right)^{-3}$ be divided so that the quotient is 4^{-2}?

Ans. Let the number to be divided be 'x'

$\left(\frac{4}{5}\right)^{-3} \div x = 4^{-2} \Rightarrow \left(\frac{5}{4}\right)^3 \div x = \frac{1}{4^2}$

$\Rightarrow \frac{5^3}{4^3} \times 4^2 = x \Rightarrow x = \frac{125}{4}$

SECTION – D

Questions 18 carry 5 marks each.

18. Find the value of x such that $\frac{3^{2x} \times 9^{-2x}}{3^{-2}} = 27$.

Ans. $\frac{3^{2x} \times 9^{-2x}}{3^{-2}} = 27$

$\Rightarrow \frac{3^{2x} \times 3^{2}}{9^{2x}} = 27 \Rightarrow \frac{3^{2x+2}}{(3^2)^{2x}} = 27$

$\Rightarrow \frac{3^{2x+2}}{3^{4x}} = 3^3 \Rightarrow 3^{2x+2-4x} = 3^3 \Rightarrow 3^{2-2x} = 3^3$

$\Rightarrow 2-2x = 3 \Rightarrow 2x = 2-3 = -1$

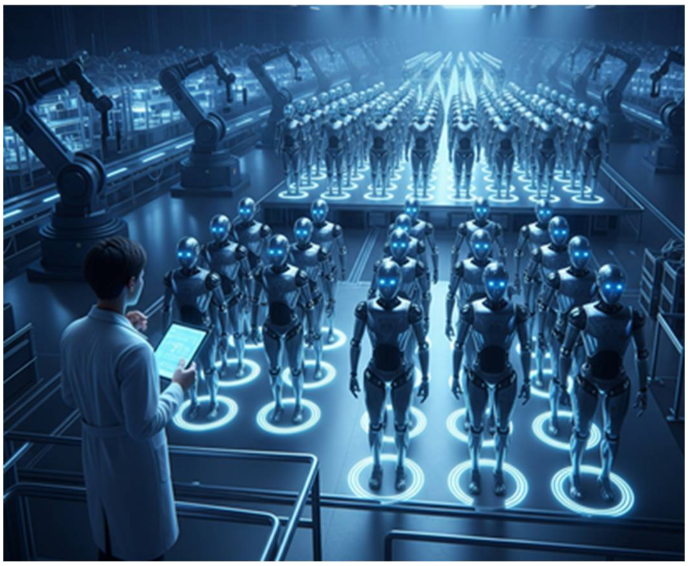
$\Rightarrow x = \frac{-1}{2}$

SECTION – E (Case Study Based Question)

Question 19 to 20 carry 4 mark

19. Case Study-1: The Robot Army

A robotics engineer is building an army of robots. The number of robots built each day is a power of 3. On the first day, 3 robots are built. On the second day, 9 robots are built, and on the third day, 27 are built. The factory has a capacity to produce 3^9 robots in a single day.



(i) Write an expression for the number of robots built on the 5th day. (1)
(ii) On which day will the number of robots built be 243? (1)
(iii) If the production rate tripled every 2 days instead of every day, starting with 3 robots, how many robots would be built on the 6th day? (1)
(iv) What is the ratio of the number of robots built on the 7th day to the number of robots built on the 3rd day? (1)

Ans. (i) The number of robots on the nth day is 3^n. On the 5th day, the number of robots will be 3^5. (1)
(ii) We need to find the day 'n' when the number of robots is 243. $3^n = 243$. Since $3^5 = 243$, the day is the 5th day. (1)
(iii) The production rate triples every 2 days. The number of tripling periods in 6 days is 6/2=3. Starting with 3 robots, the number of robots after 3 periods will be $3 \times 3^3 = 3^4 = 81$. (1)
(iv) The number of robots on the 7th day is 3^7. On the 3rd day, it is 3^3.
Ratio = $3^7 / 3^3 = 3^{7-3} = 34$. (1)

20. Case Study-2 The Architect's Blueprint:

Two treasure hunters, Ali and Zora, find a map with a riddle. The riddle leads them to a sequence of tunnels. The number of paths from the entrance of each tunnel is determined by a power of 4. From the first tunnel, there are 4 paths. From the second tunnel, there are 16 paths, and so on.

(i) How many paths are there from the 4th tunnel? (1)
(ii) If a tunnel has 1024 paths, which tunnel number is it? (1)
(iii) To find a hidden treasure, they must follow a path that has 4^3 paths from its entrance. If they find it, they gain a treasure of 4^2 gold coins. What is the total number of gold coins they could find from all such paths? (1)
(iv) The treasure is protected by a lock that requires a 6-digit code. Each digit can be any number from 0 to 9. How many possible combinations are there for the lock? Express your answer in exponential form. (1)
Ans. (i) The number of paths from the nth tunnel is 4^n. From the 4th tunnel, there are $4^4 = 256$ paths. (1)
(ii) We need to find the tunnel number 'n' when there are 1024 paths. $4^n = 1024$. We know that $4^5 = 1024$. So, it is the 5th tunnel. (1)
(iii) They find a treasure when they follow a path with 4^3 paths. This path yields 4^2 gold coins. There are 4^3 such paths.
Total gold coins = (Number of paths) × (Coins per path) Total coins
$= 4^3 \times 4^2 = 4^{3+2} = 4^5 = 1024$ gold coins. (1)
(iv) Each of the 6 digits can be any of the 10 numbers (0-9).
The total number of combinations is $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$. (1)