MATHEMATICS

WORKSHEET_210726

CHAPTER 02 POWER PLAY (Ganita Prakashan)

(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : VIII **DURATION : 1½ hr**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. What is the value of $(-100)^0 / (-100)^0$?

(a) -1 (b) 0 (c) 1 (d) Undefined

Ans. (c) 1

Any non-zero number raised to the power of 0 is 1. Therefore, $(-100)^0 = 1$.

The expression becomes $1 / 1 = 1$.

2. The number of seconds in a year is approximately 3.1536×10^7. What is this number in a more common form?

(a) 3,153,600 (b) 31,536,000 (c) 315,360,000 (d) 31,53,600

Ans. (b) 31,536,000

To convert from standard form to a number, we move the decimal point to the right by the number of places indicated by the exponent. The exponent is 7, so we move the decimal point 7 places to the right. $3.1536 \times 10^7 = 31,536,000$.

3. The value of $\left(\frac{-1}{2}\right)^{-4}$ is:

(a) 16 (b) 1/16 (c) −16 (d) −1/16

Ans. (a) 16

Using the rule $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$, we get $\left(\frac{-1}{2}\right)^{-4} = \left(\frac{2}{-1}\right)^{4} = (-2)^4 = 16$

4. If $2^x \times 2^{x+1} = 32$, what is the value of x?

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b) 2

Using the rule $a^m \times a^n = a^{m+n}$, we have $2^{x+(x+1)} = 32 \Rightarrow 2^{2x+1} = 2^5$,

Equating the exponents: $2x + 1 = 5 \Rightarrow 2x = 4 \Rightarrow x = 2$.

5. What is the simplified value of $p^{-2}q^3 / p^3q^{-2}$?

(a) $(q/p)^5$ (b) $(p/q)^5$ (c) p^5q^5 (d) $p^{-5}q^{-5}$

Ans. (a) $(q/p)^5$

6. A scientist is observing a cell that divides into two new cells every 20 minutes. If the scientist starts with a single cell, how many cells will there be after 2 hours?

(a) 2^6 (b) 2^5 (c) 6^2 (d) 2^{20}

Ans. (a) 2^6
There are 60/20 = 3 divisions per hour. In 2 hours, there will be 3 × 2 = 6 divisions. The number of cells is 2^n, where 'n' is the number of divisions. So, after 6 divisions, there will be 2^6 cells.

7. A digital storage device has a capacity of 2^{40} bytes. If a single file occupies 2^{10} bytes, how many such files can be stored on the device?
(a) 2^{50} (b) 2^{30} (c) 2^4 (d) 2^{20}
Ans. (b) 2^{30}
The number of files that can be stored is the total capacity divided by the size of one file. Number of files = $2^{40} / 2^{10}$. Using the rule $a^m \div a^n = a^{m-n}$, we get $2^{40-10} = 2^{30}$.

8. Which of the following is equivalent to $10^5 \times 100^5$?
(a) 1000^5 (b) 10^{15} (c) 10^{10} (d) 100^{10}
Ans. (b) 1015
We can rewrite 100 as 10^2. $10^5 \times 100^5 = 10^5 \times (10^2)^5 = 10^5 \times 10^{10} = 10^{5+10} = 10^{15}$.

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The value of $2^3 + 2^4$ is 27.
Reason (R): For any non-zero integer 'n' and integers a and b, $n^a \times n^b = n^{a+b}$.
Ans. (d) A is false, but R is true.
Assertion A is false because there is no law of exponents for adding powers. The sum is $2^3 + 2^4 = 8 + 16 = 24$. The reason is true, but it is the law for multiplication, not addition, and does not support the assertion.

10. **Assertion (A):** The value of 10^{-3} is 0.001.
Reason (R): A number with a negative exponent is the reciprocal of the number with a positive exponent.
Ans. (a) Both A and R are true, and R is the correct explanation of A.
Assertion A is true, as $10^{-3} = 1/10^3 = 1/1000 = 0.001$. Reason R correctly defines the rule for negative exponents and explains why the assertion is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Simplify: $\left(\frac{2}{3}\right)^2 \times \left(\frac{3}{2}\right)^{-3}$

Ans. $\left(\frac{2}{3}\right)^2 \times \left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^{2+3} = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

12. If $5^x = 25$ and $3^y = 9^2$, find the value of $2x - y$.
Ans. Value of x: $5^x = 25 \Longrightarrow 5^x = 5^2 \Longrightarrow x = 2$.
Value of y: $3^y = 9^2 \Longrightarrow 3^y = (3^2)^2 \Longrightarrow 3^y = 3^4 \Longrightarrow y = 4$.
Now, Value of 2x – y = 2(2) – 4 = 4 – 4 = 0.

13. Find the reciprocal of $\left(\frac{2}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-3}$.

Ans. $\left(\frac{2}{3}\right)^{-2}+\left(\frac{1}{2}\right)^{-3}=\left(\frac{3}{2}\right)^{2}+2^{3}=\frac{3^2}{2^2}+8=\frac{9}{4}+8=\frac{9+32}{4}=\frac{41}{4}$

The reciprocal of $\frac{41}{4}$ is $\frac{4}{41}$

14. A particle has a mass of 5.2×10^{-12} kg. Another particle has a mass of 3.5×10^{-12} kg. What is the total mass of the two particles?

Ans. Total mass = $5.2 \times 10^{-12} + 3.5 \times 10^{-12}$

Since the powers of 10 are the same, we can add the coefficients.

Total mass = $(5.2 + 3.5) \times 10^{-12} = 8.7 \times 10^{-12}$ kg.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Simplify: $\frac{(2^3)^2 \times 3^{-2} \times 4^0}{12^{-1}}$

Ans. $\frac{(2^3)^2 \times 3^{-2} \times 4^0}{12^{-1}}=\frac{2^6 \times 12 \times 1}{3^2}=\frac{2^6 \times (2^2 \times 3)}{3^2}$

$=\frac{2^6 \times (2^2 \times 3)}{3^2}=\frac{2^{6+2}}{3^{2-1}}=\frac{2^8}{3^1}=\frac{256}{3}$

16. Find the value of x for which $\left(\frac{1}{5}\right)^{x-1}=125$

Ans. $\left(\frac{1}{5}\right)^{x-1}=125 \Rightarrow (5^{-1})^{x-1}=5^3$

$\Rightarrow 5^{-1(x-1)}=5^3 \Rightarrow 5^{-x+1}=5^3 \Rightarrow -x+1=3 \Rightarrow x=1-3=-2$

17. By what number should $\left(\frac{1}{3}\right)^{-2}$ be divided to obtain the result 27?

Ans. Let the number to be divided be '*x*'

$\left(\frac{1}{3}\right)^{-2} \div x=3^3 \Rightarrow 3^2 \div x=3^3$

$\Rightarrow x=\frac{3^2}{3^3}=\frac{1}{3}$

SECTION – D

Questions 18 carry 5 marks each.

18. Find the value of x and y such that $\left(\frac{3}{4}\right)^{-2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{9}{16}\right)^{3}$.

Ans. $\left(\frac{3}{4}\right)^{-2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{9}{16}\right)^{3}$

$\Rightarrow \left(\frac{4}{3}\right)^{2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{3^2}{4^2}\right)^{3} \Rightarrow \left(\frac{4}{3}\right)^{2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{3}{4}\right)^{2\times 3}$

$\Rightarrow \left(\frac{4}{3}\right)^{2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{3}{4}\right)^{6} \Rightarrow \left(\frac{4}{3}\right)^{2x} \div \left(\frac{4}{3}\right)^{x}=\left(\frac{4}{3}\right)^{-6}$

$$\Rightarrow \left(\frac{4}{3}\right)^{2x-x} = \left(\frac{4}{3}\right)^{-6} \Rightarrow \left(\frac{4}{3}\right)^{x} = \left(\frac{4}{3}\right)^{-6}$$

$\Rightarrow x = -6.$

SECTION – E (Case Study Based Question)

Question 19 to 20 carry 4 mark

19. Case Study-1: The King's Guard

In a kingdom, a king's guard is deployed in a special formation. The number of guards in each row is determined by a power of 2. In the first row, there are 2 guards. In the second row, there are 4, in the third, there are 8, and so on. The king needs to know the total number of guards for different formations.

(i) How many guards will be in the 6th row? (1)

(ii) If a new formation requires 2^{10} guards in a single row, which row number is this? (1)

(iii) The number of guards in a different formation doubles every 2 days, starting with 4 guards. How many guards will there be after 6 days? (1)

(iv) What is the ratio of the number of guards in the 8th row to the number of guards in the 4th row? (1)

Ans. (i) The number of guards in the nth row is 2^n.

For the 6th row, it will be $2^6 = 64$ guards. (1)

(ii) We need to find 'n' such that $2^n = 2^{10}$.

Equating the exponents, n=10. So, it is the 10th row. (1)

(iii) The number of guards doubles every 2 days.

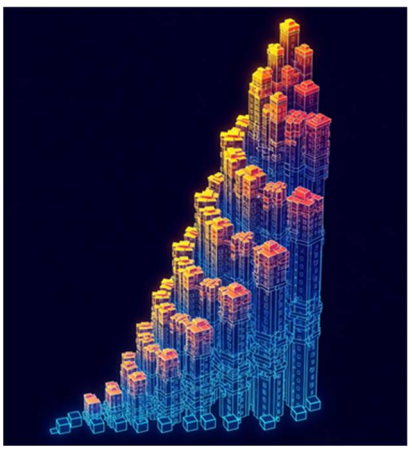
In 6 days, there are 6/2 = 3 doubling periods.

Starting with 4 guards (2^2), the number of guards will be $2^2 \times 2^3 = 2^5 = 32$ guards. (1)

(iv) The number of guards in the 8th row is 2^8. The number of guards in the 4th row is 2^4.

Ratio = $2^8 / 2^4 = 2^{8-4} = 2^4$. (1)

20. Case Study-2 The Virus Spread: The Architect's Blueprint



An architect is designing a large-scale housing project. The design is based on a pattern of blocks. The number of blocks in each layer increases exponentially. The first layer has 5 blocks. Each subsequent layer has 5 times the number of blocks as the previous layer. The total number of blocks in the final building will be 5^{10}.

(i) Write an expression for the number of blocks in the 4th layer. (1)

(ii) The total number of blocks in a building is calculated as the sum of blocks in all layers. The architect wants to build a 3-layer building. What is the total number of blocks? (1)

(iii) If another building has a design where the number of blocks triples each layer, starting with 3 blocks, how many blocks will be in the 4th layer? (1)

(iv) The architect decides to use smaller blocks, with a volume of 5^{-2} m³. How many such smaller blocks are required to construct a structure with a total volume of 5^3 m³? (1)

Ans. (i) The number of blocks in the nth layer is 5^n.

For the 4th layer, the number of blocks will be 5^4. (1)

(ii) The total number of blocks in a 3-layer building is the sum of blocks in each layer: $5^1 + 5^2 + 5^3 = 5 + 25 + 125 = 155$ blocks. (1)

(iii) The number of blocks in the nth layer is 3^n (starting with 3 blocks or 3^1 in the first layer).

The number of blocks in the 4th layer would be $3^4 = 81$ blocks. (1)

(iv) Number of smaller blocks required = Total volume / Volume of one block = $5^3 / 5^{-2}$.

Using the rule $a^m \div a^n = a^{m-n}$, we get $5^{3-(-2)} = 5^{3+2} = 5^5 = 3125$ blocks. (1)