

CIRCLES (Tangents to a Circle)

Worksheet 3 — Challenge Round — Answer Key & Marking Scheme

Name: _____	Roll No.: _____	Date: _____
Class: X Subject: Mathematics	Time: 90 minutes	Max. Marks: 50

Note for teachers: Marks for each step are shown in brackets. Any other correct method should be given full credit. Figures are not drawn to scale.

SECTION A — Multiple Choice Questions

1 mark each

Assertion-Reason (Q9-10): (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

1. The length of the tangent drawn from a point 6.5 cm from the centre of a circle of radius 2.5 cm is **[1]**
 (a) 4 cm (b) 6 cm (c) 7.5 cm (d) 9 cm

Ans: (b) 6 cm — $\sqrt{(6.5^2 - 2.5^2)} = \sqrt{36} = 6$ cm.

2. If a chord AB subtends 100° at the centre O and tangents at A and B meet at P, then $\angle APB =$ **[1]**
 (a) 100° (b) 90° (c) 80° (d) 50°

Ans: (c) 80° — $\angle APB = 180^\circ - 100^\circ = 80^\circ$.

3. PT is a tangent at T to a circle with centre O. If $\angle OPT = 30^\circ$ and $OP = 10$ cm, then $OT =$ **[1]**
 (a) 5 cm (b) $5\sqrt{3}$ cm (c) 10 cm (d) $10\sqrt{3}$ cm

Ans: (a) 5 cm — In right $\triangle OTP$, $\sin 30^\circ = OT/OP \Rightarrow OT = 5$ cm.

4. PQ is a chord of a circle with centre O and PT is the tangent at P. If $\angle QPT = 40^\circ$, then $\angle POQ =$ **[1]**
 (a) 100° (b) 80° (c) 40° (d) 50°

Ans: (b) 80° — Angle in the alternate segment = 40° ; angle at the centre = $2 \times 40^\circ = 80^\circ$.

5. Two concentric circles have radii 20 cm and 12 cm. A chord of the larger circle touches the smaller. Its length is **[1]**
 (a) 16 cm (b) 24 cm (c) 32 cm (d) 40 cm

Ans: (c) 32 cm — $2 \times \sqrt{(20^2 - 12^2)} = 2 \times 16 = 32$ cm.

6. ABCD circumscribes a circle touching AB, BC, CD, DA at P, Q, R, S. If $AP = 5$ cm, $BP = 4$ cm, $CR = 7$ cm and $DR = 3$ cm, the perimeter of ABCD is **[1]**
 (a) 19 cm (b) 76 cm (c) 28 cm (d) 38 cm

Ans: (d) 38 cm — Perimeter = $2(AP + BP + CR + DR) = 2 \times 19 = 38$ cm.

7. The radius of the circle inscribed in a right triangle with sides 20 cm, 21 cm and 29 cm is **[1]**
 (a) 6 cm (b) 5 cm (c) 7 cm (d) 4 cm

Ans: (a) 6 cm — $r = (20 + 21 - 29)/2 = 6$ cm.

8. Tangents PA and PB are drawn from P to a circle of radius r with centre O such that $OP = 2r$. Then $\angle APB =$ **[1]**
 (a) 30° (b) 45° (c) 90° (d) 60°

Ans: (d) 60° — $\sin \angle OPA = r/2r = \frac{1}{2} \Rightarrow \angle OPA = 30^\circ \Rightarrow \angle APB = 60^\circ$.

9. Assertion (A): The distance between two parallel tangents of a circle of radius 6 cm is 12 cm. **[1]**
 Reason (R): A tangent to a circle is parallel to the radius through the point of contact.

Ans: (c) — A is true (they are at the ends of a diameter). R is false — a tangent is perpendicular to that radius $\Rightarrow$ (c).

10. Assertion (A): A quadrilateral ABCD with $AB = 5$ cm, $BC = 7$ cm, $CD = 6$ cm and $DA = 8$ cm can circumscribe a circle. [1]
Reason (R): In a quadrilateral circumscribing a circle, the sums of opposite sides are equal.
Ans: (d) — $AB + CD = 11 \neq 15 = BC + DA$, so A is false; R is true $\Rightarrow$ (d).

SECTION B — Very Short Answer Questions

1 mark each

11. Fill in the blank: The tangent at any point of a circle is _____ to the radius through the point of contact. [1]
Solution: Perpendicular.
12. True or False: The tangents drawn at the ends of a diameter of a circle are parallel. [1]
Solution: True.
13. A point P is 10 cm from the centre of a circle of radius 8 cm. Find the length of the tangent from P. [1]
Solution: $\sqrt{10^2 - 8^2} = 6$ cm.
14. If the angle between two tangents PA and PB from P to a circle with centre O is 120° , find $\angle AOB$. [1]
Solution: $\angle AOB = 180^\circ - 120^\circ = 60^\circ$.
15. PA and PB are tangents from P to a circle with centre O. If $\angle OPA = 35^\circ$, find $\angle AOP$. [1]
Solution: $\angle AOP = 90^\circ - 35^\circ = 55^\circ$.
16. True or False: A circle can have infinitely many tangents. [1]
Solution: True.

SECTION C — Short Answer Questions I

2 marks each

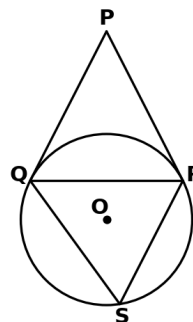
17. AB is a diameter of a circle and AT is the tangent at A. The line BT meets the circle at Q. If $\angle ABQ = 32^\circ$, find $\angle ATQ$. [2]
Solution: $AT \perp AB$ (tangent $\perp$ radius), so $\angle BAT = 90^\circ$. (1)
In $\triangle ABT$: $\angle ATB = 180^\circ - 90^\circ - 32^\circ = 58^\circ$; since Q lies on BT, $\angle ATQ = 58^\circ$. (1)
18. TP and TQ are tangents from T to a circle and $\angle PTQ = 40^\circ$. Find $\angle PQT$. [2]
Solution: $TP = TQ \Rightarrow \angle TPQ = \angle TQP$. (1)
 $\angle PQT = \frac{1}{2}(180^\circ - 40^\circ) = 70^\circ$. (1)
19. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [2]
Solution: Let AB be a diameter, with tangents at A and B. Each tangent is perpendicular to the radius through its point of contact, so both are perpendicular to AB. (1)
Alternate interior angles are each 90° , hence the tangents are parallel. (1)
20. PQ is a chord of length 12 cm of a circle of radius 10 cm. The tangents at P and Q meet at T. Find TP. [2]
Solution: Let OT meet PQ at M ($OT \perp PQ$, $PM = 6$). $OM = \sqrt{10^2 - 6^2} = 8$ cm. (1)
In right $\triangle OPT$, $PM^2 = OM \times MT \Rightarrow 36 = 8 \times MT \Rightarrow MT = 4.5$; $TP = \sqrt{6^2 + 4.5^2} = 7.5$ cm. (1)

SECTION D — Short Answer Questions II

3 marks each

21. Prove that the line segment joining the points of contact of two parallel tangents of a circle passes through the centre. [3]
Solution: Let parallel tangents touch the circle (centre O) at A and B. Through O draw line n parallel to both tangents. (1)
 $OA \perp$ tangent at A $\Rightarrow OA \perp n$; similarly $OB \perp n$. (1)
Only one perpendicular to n can pass through O, so OA and OB lie along one line. Hence AOB is a straight line, i.e. a diameter. (1)

22. In the figure, PQ and PR are tangents from P to a circle with centre O, $\angle QPR = 50^\circ$, and chord $RS \parallel PQ$. Find $\angle RQS$. [3]



Solution: $PQ = PR \Rightarrow \angle PQR = \angle PRQ = 65^\circ$. (1)
 $RS \parallel PQ \Rightarrow \angle QRS = \angle PQR = 65^\circ$ (alternate angles); and $\angle QSR = \angle PQR = 65^\circ$ (alternate segments). (1)
 In $\triangle QRS$: $\angle RQS = 180^\circ - 65^\circ - 65^\circ = 50^\circ$. (1)

23. A circle is inscribed in $\triangle ABC$, touching BC, CA and AB at P, Q and R. If $AB = 12$ cm, $AQ = 8$ cm and $CQ = 6$ cm, find BC and the perimeter of $\triangle ABC$. [3]

Solution: $AR = AQ = 8$ cm $\Rightarrow BR = 12 - 8 = 4$ cm = BP. (1)
 $CP = CQ = 6$ cm $\Rightarrow BC = BP + CP = 10$ cm. (1)
 $AC = AQ + CQ = 14$ cm; perimeter = $12 + 10 + 14 = 36$ cm. (1)

24. Prove that a parallelogram circumscribing a circle is a rhombus. [3]

Solution: For a circumscribed quadrilateral ABCD, $AB + CD = AD + BC$. (1)
 In a parallelogram $AB = CD$ and $AD = BC$, so $2AB = 2AD \Rightarrow AB = AD$. (1)
 Hence $AB = BC = CD = DA$: ABCD is a rhombus. (1)

SECTION E — Long Answer Questions

5 marks each

25. PA and PB are tangents from an external point P to a circle with centre O, and $\angle APB = 120^\circ$. Prove that $OP = 2AP$. [5]

Solution: Join OA, OB. $PA = PB$ and $OA = OB$, so $\triangle OAP \cong \triangle OBP$ (SSS) $\Rightarrow \angle OPA = \angle OPB = 60^\circ$. (2)
 $OA \perp AP$, so $\triangle OAP$ is right-angled at A. (1)
 $\cos 60^\circ = AP/OP \Rightarrow \frac{1}{2} = AP/OP$ (1) $\Rightarrow OP = 2AP$. (1)

26. Two circles with centres O and O' and radii 15 cm and 20 cm intersect at P and Q, such that OP and O'P are tangents to the two circles respectively. Find the length of the common chord PQ. [5]

Solution: OP is a tangent to the circle with centre O', so $O'P \perp OP$: $\angle OPO' = 90^\circ$. (1)
 $OO' = \sqrt{15^2 + 20^2} = 25$ cm. (1)
 OO' is the perpendicular bisector of PQ; let it meet PQ at M. (1)
 Area of $\triangle OPO' = \frac{1}{2} \times 15 \times 20 = 150 = \frac{1}{2} \times 25 \times PM \Rightarrow PM = 12$ cm. (1)
 $PQ = 2 \times PM = 24$ cm. (1)

SECTION F — Case Study Based Question

4 marks

- 27.** A quadrilateral plot ABCD has a circular pond inside it that touches the boundaries AB, BC, CD and DA at P, Q, R and S respectively. The boundary lengths are $AB = 14$ m, $BC = 12$ m and $CD = 10$ m. **[4]**

(i) Find the length AD. **[1]**

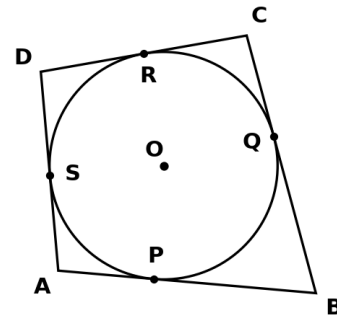
(ii) If $\angle COD = 70^\circ$, find $\angle AOB$, where O is the centre of the pond.

[1]

(iii) If $AP = 8$ m, find BP, CR and DS. **[2]**

OR

(iii) Prove that $AB + CD = AD + BC$. **[2]**

**Solution:**

(i) $AB + CD = AD + BC \Rightarrow AD = 14 + 10 - 12 = 12$ m. (1)

(ii) Opposite sides subtend supplementary angles at the centre: $\angle AOB = 180^\circ - 70^\circ = 110^\circ$. (1)

(iii) $BP = AB - AP = 6$ m; $BQ = BP = 6$ m $\Rightarrow CQ = 12 - 6 = 6$ m = CR; $DR = 10 - 6 = 4$ m = DS. (BP = 6 m, CR = 6 m, DS = 4 m) (2)

OR (iii) $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$ (tangents from a point). (1) $AB + CD = (AS + BQ) + (CQ + DS) = (AS + DS) + (BQ + CQ) = AD + BC$. (1)

— End of Worksheet —